Culture and Cognition

Essays in Honor of Peter Damerow

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Culture and Cognition

Essays in Honor of Peter Damerow

Jürgen Renn and Matthias Schemmel (eds.)

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Lindy Divarci, Bendix Düker, Georg Pflanz, Sylvia Szenti, Charlie Nasatir Zaharoff

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Orthostat relief with a representation of the transport of a lamassu (temple monument) placed at the entrance of Sennacherib's palace at Nineveh. British Museum.

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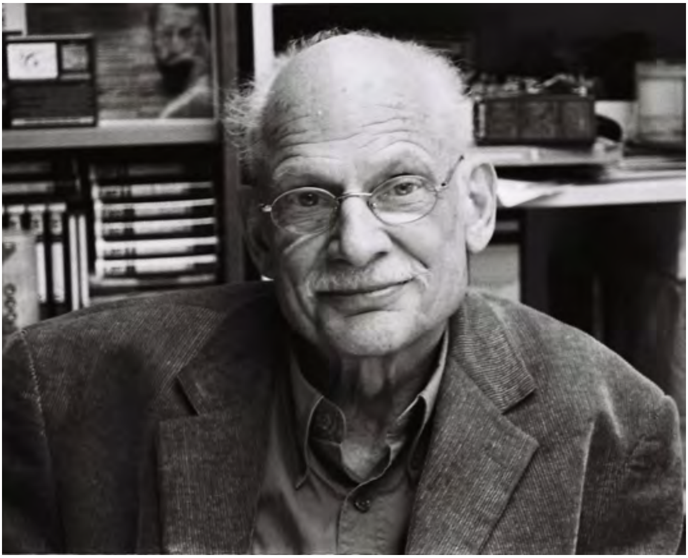

Peter Damerow (1939–2011)

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Chapter 1
Introduction
Jürgen Renn and Matthias Schemmel

Peter Damerow (1939–2011) was a visionary scholar of rare versatility. A key figure in the foundation and early development of the Max Planck Institute for the History of Science, he contributed to fields as wide-ranging as pedagogy, mathematics, philosophy, psychology, Near Eastern studies, as well as the history of knowledge and science. Through his work and his dynamic personality, he shaped the careers of many scholars worldwide. He was a paragon of the engaged scientist, having great sensitivity for political and social concerns and a perceptiveness that also shaped his scholarship. The present volume attempts to capture the vivacity of his ever-curious mind. It comprises contributions by some of his closest companions, colleagues, and friends, most of which were presented at a workshop held in Peter's honor at the Max Planck Institute for the History of Science in Berlin in December 19–20, 2013. The contributions are organized in four parts, the first three of which cover some of the areas of Peter's interests: the origins of writing and mathematics; the history of knowledge and science; and societal concerns and the role of information technologies for the humanities. The last part offers a glimpse at his life and also presents the scope of his scholarship with a bibliography of his writings.

Part 1: Early Writing and Abstraction

On the basis of extensive research by Peter Damerow and his colleagues, the first contribution by Jürgen Renn gives an overview of the origin of writing in the wider context of urbanization and early agriculture. Jens Høyrup discusses the cognitive structures inherent in arithmetical practices and their relation to an abstract concept of number. Manfred Krebernik's contribution is dedicated to an analysis of unusual early cuneiform documents bearing archaic inscriptions and reliefs. Taking up Peter's ideas about the emergence of writing from accounting practices in Mesopotamia, William Boltz ponders the possible primacy in ancient China of the sexagenary cycle signs over writing representing language. Joachim Schaper presents results of work pursued jointly with Peter on the genesis of the concept of value, comparing Marx's value theory with empirical results from Mesopotamian studies. In line with Peter's work on Late Uruk metrologies, Cale Johnson discusses bookkeeping techniques in early Mesopotamia as a link between first and higher order representations in societal activities. Mark Geller raises the question of which inferential structures are relevant to science in ancient Mesopotamia. He thus touches on an issue that was of great importance to Peter Damerow: the distinction between synthetic and analytic methods in the history of science. Florentina Badalanova Geller examines an early corpus of Slavonic para-biblical writings, paying particular attention to astronomical knowledge.

Part 2: History of Knowledge and Material Representations

Katja Bödeker elaborates on a topic that was central to Peter Damerow's oeuvre: the mutually beneficial relation between developmental psychology and the history of science. Tilman Sauer critically assesses a concrete historical instance of a relation between fundamental physics and developmental psychology, focusing on the concept of time as discussed by Albert Einstein in his work on special relativity and by the Swiss developmental psychologist Jeans Piaget. Matthias Schemmel presents the results of a research project on the long-term history of spatial concepts, which expand on insights from Peter's work, in particular, on the role of practice for the emergence of abstract concepts. Ursula Klein discusses Peter's concept of material representation (key to his analysis of the emergence of the number concept from accounting practices) from a semiotic perspective and applies it to the history of chemical notations. Wolfgang Lefèvre discusses the close interplay between practical and pure geometry in the history of knowledge, another of Peter's intellectual pursuits. Diethelm Stoller presents an early perspective construction by Piero della Francesca as an illustration of Piero's mathematical ingenuity. Oscar Abdounur discusses the emergence of a concept of real number in the context of the Renaissance scholar Erasmus of Höritz's mathematical treatment of musical intervals.

Part 3: Societal Challenges and Electronic Visions

Henning Vierck tells the history of the Comenius-Garten, a cultural institution in Berlin-Neukölln. This is a very personal story by Henning, the garden's founder, and is intimately connected with concerns about education and knowledge in society that were shared to a large extent by Peter. Wulf Schiefenhövel's contribution deals with the Eipo, an until recently non-literate society in the central highlands of West New Guinea. He explains their rapid appropriation of elements from industrialized societies, focusing on the cognitive implications of this transition. Circe Mary Silva da Silva describes a mathematics textbook for indigenous schools in Brazil which takes the cultural background of indigenous people into account, an approach long advocated by Peter. Mark Schiefsky describes the history and further development of an ingenious software tool that supports research on structured texts, a tool that was co-initiated and developed by Peter. Julia Damerow, Erick Peirson, and Manfred Laubichler present another electronic research tool for the history of science which follows Peter's visions and was subsequently adapted to the treatment of big data.

Part 4: A Glimpse of His Life

The last part of the volume begins with an obituary by Jürgen Renn that offers a brief sketch of Peter's life. The contribution by Kristina Vaillant, originally written for a volume on creative figures living in Berlin, gives a portrait of Peter's working life. The bibliography of Peter's wide-ranging publications was compiled by Anke Pietzke. Lindy Divarci curated the editorial work on this volume. We are grateful for their valuable support.

Part 1: Early Writing and Abstraction

Chapter 2
Learning from Kushim About the Origins of Writing and Farming

Jürgen Renn

Learning from Peter Damerow

Peter Damerow was a man of relentless curiosity, of great inventiveness, and of remarkable intellectual courage. His investigations into the origins of mathematics were initially inspired by his broad-ranging philosophical interests, and also by his engagement with educational science. In the early 1980s, he began to follow the traces of such great trail blazers in the exploration of ancient mathematics as Otto Neugebauer and François Thureau-Dangin, turning to the historical sources, in particular to the early documents of writing and calculating from Mesopotamia.

But at the time, the earliest documents, the so-called proto-cuneiform texts, were still generally disregarded because they seemed to provide neither important literary or historical documents, nor evidence of an advanced state of mathematics. Rather, they mostly dealt with such mundane issues as accounting and administration. In addition, they were copious, too numerous indeed, or so it seemed, to individually warrant the careful philological treatment typical of the interests and prevailing methods of philological analysis. But Peter was a resourceful man and times were changing. He was convinced that computer technology would facilitate an analysis of these proto-cuneiform texts and thus became, simultaneously, one of the pioneers of what are now called the digital humanities.

Peter was also a generous man and a master in community building. As a student of physics and of history of science in Berlin, I had the opportunity to closely follow Peter's remarkable career as one of the pioneers in the study of early writing and calculating. I participated in workshops he organized on Babylonian mathematics, which soon turned into a gathering-place of the international community of experts in Mesopotamian culture, as well as of scholars worldwide who were interested in the emergence of writing and mathematics. Many of the epistemological insights gained in this context have profoundly shaped my own work. I am deeply grateful to Peter, my mentor and close friend, with whom I collaborated on many other topics. The following text is a tribute to his extraordinary achievements in the fields of early writing and mathematics, from which I have learned so much.

Cereals, Beer, History

While history usually is distinguished from prehistory by the existence of writing, the actual introduction of writing was probably quite uneventful.[1] The Babylonian administrator Kushim could not have known that his inscriptions into soft clay would be thoroughly studied some 5,000 years later, literally inscribing him and the writing system he used to keep an account of his granular assets into the more-than-human history of humankind.

Kushim, or the scribe who was assigned to note down the calculations for him, was simply going about his duties as a SANGA: the head of an administrative unit (a term later used as a designation for a priest). Kushim was in charge of a large facility that stored the basic ingredients for the production of beer and other cereal commodities. This was no medium-size enterprise: at one point, he had to administer around 135,000 liters of barley. The tablet printed above contains calculations of the ingredients needed to produce nine different cereal products and eight different kinds of beer. The scribe working for Kushim calculates, for instance, the amount of barley groats needed to produce thirty units of a certain cereal product, the amount of barley groats and malt needed to produce five jugs of a certain beer, and the amount of barley needed for 1,800 cereal rations. In one of the calculations, the sign for "1" is written erroneously for the sign for "10," but otherwise the calculation is correct.[2]

Kushim's duties formed part of a larger administrative context of the earliest Mesopotamian cities. The administration was responsible for collecting and redistributing the agricultural products on which their wealth was mainly based. It was the agricultural surplus of this production that had enabled the development of the hierarchically stratified societies of these cities, including the division between manual and intellectual labor that gave Kushim his position in the first place. While he and his like planned the labor, and appropriated and administered its fruits, others had to do the real work. No wonder, then, that the intellectual work of Kushim, as documented by twenty-one extant clay tablets, was concerned exclusively with the economic aspects of contemporary life.

[1] This chapter also appeared in 2015 in *Grain | Vapor | Ray: Textures of the Anthropocene*, eds. K. Klingan, A. Sepahvand, C. Rosol, and B. M. Scherer, 241–259. Cambridge: MIT Press.

[2] The story of Kushim (his name is a poetic license) is based on Nissen, Damerow, and Englund (1993, 36–46). The reconstruction of the emergence of writing is based on Damerow (2012); and Damerow (1996).

The First Cities

Human history depends on ecological conditions, but it also hinges crucially on knowledge. In the fifth millennium BCE, southern Mesopotamia—the Babylonian plain lying between the rivers Euphrates and Tigris—was a landscape of swamps and marshlands suffering from regular fluvial inundations which rendered it essentially inhospitable for large-scale agricultural regimes. A climatic change that led to a reduction of precipitation during the fourth millennium BCE changed this situation. The sea level of the Persian Gulf began to drop so that the swamps and marshlands eventually became suitable for farming. Claiming that the situation had become favorable for such activities does not mean that they could actually take place without the knowledge and technology needed to turn this landscape into an agricultural resource.

Toward the end of the fourth millennium, when the early civilization in southern Mesopotamia arose, eventually giving rise to large cities and empires, such knowledge had become available. The capacity to produce food from arable land had been generated over the course of the Neolithic Revolution some 5,000 years earlier. At the turn from the fourth to the third millennium, however, a new challenge emerged as water became increasingly sparse and had to be transported to the cultivated lands. The response of the early settlers to this challenge was the construction of ever more sophisticated irrigation channels, a collaborative achievement that deeply marked the further evolution of Mesopotamian societies.[3] At this point, the relationship between humans and their environment had been reversed; rather than being determined, or at the very least constrained by environmental conditions, humans were now actively transforming and exploiting their environment.

Alien Symbols

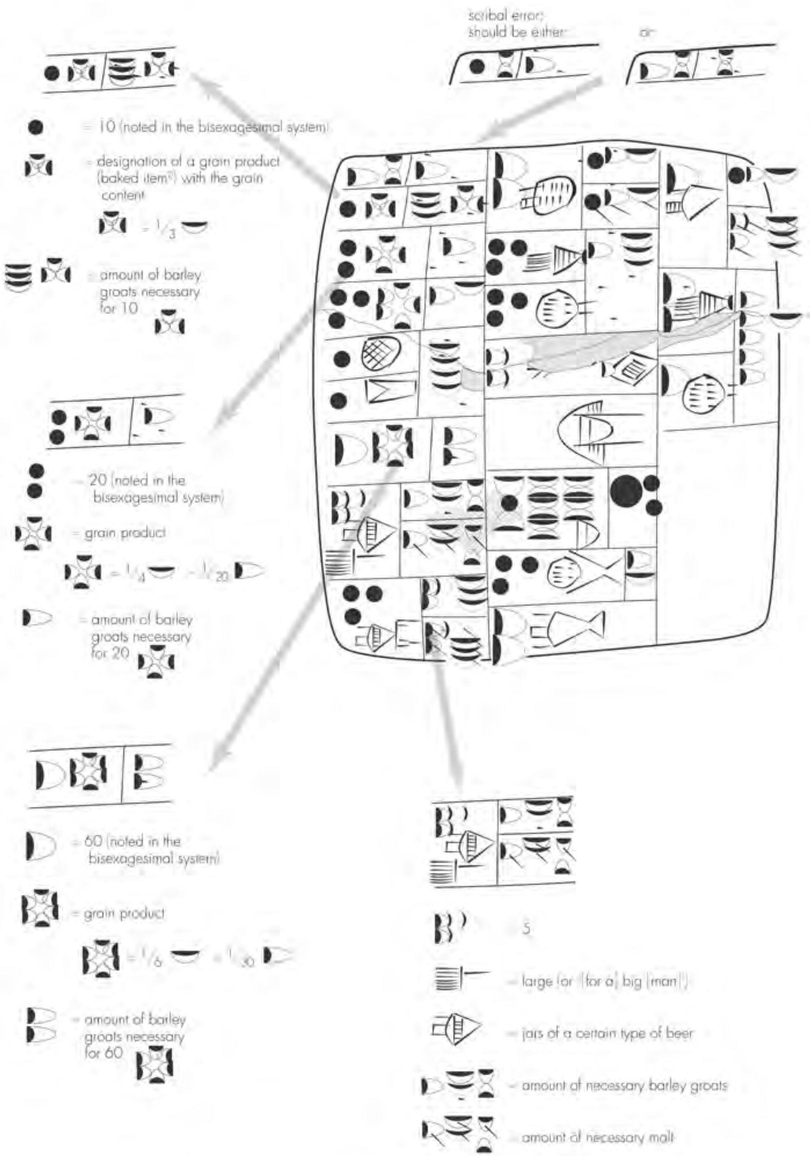
But how did Kushim come up with the glorious and very consequential idea to invent writing on what might have been a sunny day in southern Mesopotamia in 3000 BCE? Actually, he didn't. The texts that Kushim and his scribes wrote are not actually texts, since they deal with quantitative relations between different items, but have more in common with the spreadsheets used in modern administrative contexts. But they are not really spreadsheets either. The main difference here is that what Kushim used to determine the amounts of barley and malt needed for the final products were not numbers. Neither he, nor anyone else of his time, knew either writing or arithmetic. He used something else, elements of symbolic systems that we designate, for lack of a better term, as "proto-writing"

[3] This assessment is based on Nissen, Damerow, and Englund (1993, 1–3).

or "proto-arithmetic." Kushim's symbols signified neither sentences, nor calculations with abstract numbers in the sense of a manifestly context-independent arithmetic. They simply represented what he was concerned with: not language, but rather administrative acts dealing with specific products of the society of which he was a part.

On Kushim's tablet, five different numerical systems are used: one for cereal products, another for beer containers, and three different systems for the cereals themselves. In other words, the quantification of these objects was linked to their quality, while an abstract system of numbers designating quantity independent of quality and context did not yet exist. Similarly, the non-numerical signs used on these clay tablets were not elements of a writing system aiming at a rendition of human language, but just representations of certain administrative categories. We are thus confronted not with a primitive version of modern writing or mathematics, but with an evidently efficient administrative tool of the archaic Mesopotamian society, a symbolic system, and a way of thinking in its own right that is not reducible to our modern categories.

Figure 2.1: Proto-cuneiform tablet from Uruk (ca. 3 000 BCE) showing a calculation for the amounts of ingredients needed in the production of dry cereal products and beer. The transliteration is given below. See Nissen, Damerow, and Englund (1993, 42–43). Courtesy of CDLI.


scribal error:
should be either:
or
= 10 (noted in the bisexagesimal system)
= designation of a grain product (baked item?) with the grain content
= 1/3
= amount of barley groats necessary for 10
= 20 (noted in the bisexagesimal system)
= grain product
= 1/4 = 1/20
= amount of barley groats necessary for 20
= 60 (noted in the bisexagesimal system)
= grain product
= 1/6 = 1/30
= amount of barley groats necessary for 60
= 5
= large (or '(for a) big (man)')
= jars of a certain type of beer
= amount of necessary barley groats
= amount of necessary malt

This presents a typical scenario of which scholars in the humanities are very fond: a peculiar way of thinking that differs from our own modern way, situated in a remote place at a remote time, which can only be understood on its own terms by taking into account local contexts and practices. We do not even run the risk of interfering with this alien system of thinking since we can hardly ask Kushim about it. To understand it, we must simply rely on the work of archeologists, Assyriologists, and historians, in this case on the pioneering studies of Peter Damerow, Robert K. Englund, and Hans Nissen on archaic bookkeeping practices. Kushim's clay tablet, which is being considered here, is just one of many thousands documenting the Mesopotamian world. They are meanwhile openly available on the Internet thanks to the efforts of the Cuneiform Digital Library Initiative (CDLI).[4]

The Great Transitions

Although it is still favored among scholars in the humanities, there is a problem with this kind of focus on the local. As is clear from the overwhelming record of historical documents from Mesopotamia, writing in the modern sense as a more or less durable representation of human language—in this case cuneiform writing—as well as arithmetic in the sense of operations with abstract numbers *did* eventually emerge from the exotic symbol system used by Kushim. Where exactly did this system come from in the first place, and how did its transformation into writing and arithmetic take place?

These questions more generally concern the way in which human history gives rise to major transitions, such as the origins of tool use and material culture, the origin of symbolic thinking and of language, the emergence of agriculture, the development of urbanization and the invention of writing, the spread of religions, and the rise of science, or the industrial and digital revolutions. The Anthropocene is characterized by the lasting global impact of human interventions. Essentially, this impact has manifested itself since the first use of modern industrial resources or, more articulately, since the beginning of the Great Acceleration in the 1950s. The Anthropocene in fact started much earlier when humans began to transform their environment during the Neolithic Revolution (or rather during the several Neolithic Revolutions that occurred in various places around the globe) and when their knowledge became part of a long-lasting process of sedimentation in consequence of the invention of writing. If we want to understand how this global impact took place and also how we can shape it in the future to ensure human survival, we have to find out more about the relation between the long-term cu-

[4]See http://cdli.ucla.edu, accessed June 8, 2016.

mulative effect and the contingent character of human actions and knowledge, which always depends on chance constellations and local circumstances.

Exploring the Limits of Symbolic Representations

In the second half of the fourth millennium BCE, the modest accounting techniques that had been developed earlier in the context of the rural economy of Mesopotamia were extensively exploited in the administration of the emerging city-states. These city-states represented higher-order structures with centralized administrative functions within a network of smaller settlements. Among the traditional accounting techniques were small clay tokens of different shapes serving as symbolic representations of objects and used for representing and controlling their quantities; seals were used to represent certain administrative acts. The exploration of these given means, serving as external representations of administrative knowledge in the context of an expanding economy, eventually led to a transformation of the traditional symbolic culture. The potential of existing tools of symbolic representation was exploited to its limits. This led, for instance, to a proliferation of accounting practices, which originally had played only a minor role in the context of rural communities.

A critical turning point was when these two elements of traditional accounting techniques—the counters used for keeping track of the quantities of administered objects and the seal impressions documenting administrative acts—came to be represented within a single medium: a clay tablet of the kind found in Kushim's office. The seal impressions, in particular, carried information about the administrative and societal context that determined their meaning. They testified to property, legal acts, or socially correct behavior. The two elements were initially also integrated in the form of sealed hollow clay balls (*bullae*), containing certain combinations of clay counters. Sometimes the combinations of tokens inside were represented by marks on the *bullae*'s surface. In principle, sealed clay tablets served the same function but were easier to handle than the *bullae*. In any case, two initially separate accounting techniques thus became integrated into a new form of external representation whose enormous potential could and would be explored in the sequel. As we have seen, the emergence of this new form of representation was itself the result of an exploration in response to the challenge of an expanding economy.

Challenging Objects

This is a familiar feature of the development of networks of human actors embedded in environments that, at the same time, are transformed by them. They may react to external challenges, assimilating them to their existing internal structures, that is, to the mental frames of the actors and the social structures of their interactions, by expanding and differentiating them. This, in turn, creates the preconditions for their reorganization and the accommodation of their internal structures to the new situation. This internalization of external challenges by creating new regulatory regimes is, in a sense, the complementary process to the externalization of a network's internal structures in terms of the creation of a material culture and of external representations of knowledge and social institutions, in short, the complement to what biologists call "niche construction."[5]

The so-called numero-ideographic tablets were transition-stage tablets, unearthed in Kushim's Uruk as well as in ancient Iran. They contained representations of rudimentary numerical signs together with an initial set of a dozen or so non-numerical signs designating counted or measured objects. Representing an early stage of proto-writing and proto-arithmetic, they became the starting point for the exploration of new forms of information storage and processing in the archaic period of the Mesopotamian society. The tablets could hold more information than the earlier administrative techniques and this information could be more flexibly and efficiently structured. For instance, it was now easy to invent new signs to denote new semantic categories. Conversely, the existing economic and administrative activities were shaped by these new techniques of representation. The new signs and structures for information processing were standardized, giving rise to the proto-cuneiform administrative texts of the kind generated by Kushim's office. As a result of this development, the proto-cuneiform administrative texts became the external representations of a mental model of the accumulation and distribution of resources and products in the Babylonian administration. This mental model—the internal cognitive structure of the actors—was in turn generated by a reflection on the specific actions constituting this administration, including actions undertaken using these external representations.

The next step in the development of writing was shaped by a fundamental property of external representations: that the range of their possible applications is larger than the specific goals for which they had initially been introduced. The potential of the proto-cuneiform texts to represent mental constructions reached far beyond the limited field of application within Babylonian administration. In its most evolved form, reached at least by around 2600, it also included the possibility of representing spoken language and the abstract numbers of mathematics. A further characteristic feature of the development of a system of knowledge, however, is that such possibilities typically occur only as a side effect of its mainstream applications. It is also characteristic that the foundational role of these marginal applications as being constitutive of a new developmental stage is only

[5] See, for example, Odling-Smee, Erwin, and Palkovacs et al. (2013).

realized once a new perspective is introduced, often triggered by a new external context. The recontextualization of a system of knowledge thus becomes a major driving force for its reorganization.

The Introduction of a New Perspective

In the case of the emergence of writing, the further development of the proto-cuneiform writing system was at first entirely governed by its function within the Mesopotamian administration and by its growing sophistication. But precisely this sophistication also created contexts in which new applications of the system could arise and be considered from a new perspective, at some distance from the sphere of primary applications. One such context was education (Nissen 2011). Indeed, the growing complexity of the system required institutional support for its transmission from generation to generation. Schooling implies a separation of the cognitive means of administration from their immediate context of application and thus opens up a perspective in which the potential of these cognitive means could be explored independently of the constraints of their application to solve concrete administrative problems. Texts documenting this explorative quality produced in such educational contexts have indeed been preserved. What have also been preserved are so-called lexical lists documenting the concern of scribes with the standardization and semantic classification of signs. They even contain signs that seem to have been specially invented for teaching purposes.

The role of education provides a good example for the emancipation of a system of knowledge from its embedding within concrete contexts of application. But there were also other factors that may have worked similarly toward a recontextualization of the existing system of proto-writing and proto-arithmetic, thus introducing a more reflective perspective on this system that favored the discovery of the possibility to repurpose it for representing language and abstract numbers. The expansion of the system, which had to include more and more names of persons, institutions, and objects, may have triggered the search for a principle to introduce new signs and sign combinations, rather than relying on ad-hoc solutions, a search that may have led to the coding of names by phonetization. Applications of the system outside the strictly economic and administrative context, for instance to support the memorization of orally transmitted texts, must have also contributed to its recontextualization.

The Strength of Weak Ties

Further recontextualization factors contributing to recognition of the system's potential to represent language, and not just specific mental models rooted in a local administrative context, may have been the emergence of cuneiform writing within a multilingual context, the adaptation of the system to more than one language, and also the parallel development of other writing systems in the larger region. The latter provides a good illustration of what has been called “the strength of weak ties” within networks (Granovetter 1973) because the exchange even of incomplete information about writing systems or just stimulus diffusion of the knowledge of such systems may have triggered developments in marginal areas of the network that were by their very nature independent of the contexts from which they originated. Similar mechanisms affected the development of the abstract concept of number and of arithmetic in the sense that operations with these numbers were independent of the quality of the quantities with which calculations were performed.

What is Abstraction?

In the context of the genesis of the abstract number concept, however, we see even more clearly yet another mechanism at work, which we may designate as “iterative abstraction” what Jean Piaget called “reflective abstraction.” The reflection on actions with external representations, such as the clay tokens used for accounting, in the early stage generates knowledge of a higher order of abstraction than the knowledge to which these external representations originally referred. The results of such reflections may again be externally represented.

The clay tokens were simply representations of the counted objects. But representing the results of establishing a one-to-one correspondence between these counters and the counted objects in the form of numerical signs on clay tablets turned these signs into a representation of knowledge on a higher order of abstraction. On this level, the relation between the individual quantities and their sum total, for instance, can now be expressed in terms of a permanent relation between signs, whereas on the level of the original objects or the counters, it is just given by a temporal succession in which the parts are no longer distinguishable once the whole is established, for instance, after putting all fruits to be counted into one basket. Similarly, operations with these numerical signs, for instance replacing 10 signs of value “1” with one sign of the larger value “10,” no longer correspond to actions with the original objects, but to operations on a meta-level with regard to the primary actions.

Iterative abstractions such as those that eventually gave rise to the abstract mathematical concept of number ultimately depend on the material actions from which they originate, such as the concrete actions of counting material objects with the help of counters or number signs. Iterative abstractions are a constructive process in which novel cognitive structures are built by reflecting on operations with specific external representations such as language, counters, or mathematical symbols. These external representations may in turn embody previously constructed mental structures so that a potentially infinite chain of abstractions is created. It may appear as if this chain of abstractions gives rise to a predetermined hierarchy of steps leading necessarily from actions with concrete objects to ever higher-order mental operations of universal validity. This, however, is not the case. The historical development of iterative abstractions is in fact highly path-dependent, contingent as it is on a series of concrete historical experiences, as is apparent in the case of Kushim's accounting activities.

Cultural Evolution and Creativity

In this explanation of the emergence of writing and the genesis of abstract concepts, there is thus no hidden determinism at work, no teleology in the sense of progress toward a pre-established goal of the development. Everything is shaped by local contexts, by more or less accidental environmental, social, and cultural conditions, by chance constellations and, above all, by human choices. There were indeed other trajectories of the emergence of writing and other pathways along which abstract concepts were formed in other parts of the world and at other times, and they were probably shaped by different local conditions.

On a more general level, however, we still recognize in the story of Kushim the way in which human actions are not only constrained by given contexts, including the conditions they have created themselves, but also how in principle knowledge is created that offers humans, in any given historical situation, a specific freedom to choose their next steps. The invention of writing, for instance, was a step well prepared by a 1,000-year development in which all the pieces of the puzzle were assembled, yet it was a creative human invention that resulted from a deliberate reflection on these pieces. The rapidity of the last step to establish writing as a new means of representation replacing earlier techniques testifies to the autonomous, emancipatory, and intentional character of this act.

The emergence of writing, on the other hand, is clearly also a history of the gradual accumulation of knowledge, a process of sedimentation of action potentials over time, incorporated, above all, in the transmission and enrichment of material culture. At the same time, it is a history of the ever wider spread, or, as one might say, of the "globalization" of these action potentials in space and of the increasing density of links within an expanding network of interactions (Renn 2012). It is therefore more than plausible to describe this history, with its path-dependence, its cumulative and self-referential character, as an evolutionary process. But this history can only be adequately conceived as a form of cultural evolution if the autonomous character of knowledge is recognized as one of its most fundamental features (Richerson and Christiansen 2013).

Beyond the Case of Writing

The case of the invention of writing, it may be objected, is not an appropriate basis for making such far-reaching claims about cultural evolution. It specifically concerns, after all, an epistemic transformation, a development of knowledge, or even more precisely, the emergence of a new medium of representation within a larger context of societal processes such as urbanization, upon which we have hardly touched. These more fundamental processes had established the division of intellectual and manual labor in the first place, as well as the special role of administrative knowledge as knowledge about the organization of production, that is, as a particular form of *Herrschaftswissen* (social control). The result was a specific economy of knowledge with its own structures regulating the production and dissemination of knowledge, separate from the structures of society at large. In earlier periods, one may suppose that knowledge was directly incorporated into contexts of action, with language essentially being the only means of representing knowledge independently from such contexts, while symbols did not yet possess the specific function for manipulating knowledge that they acquired over the course of the emergence of writing (Damerow and Lefèvre 1998).

It seems clear, on the other hand, that knowledge (not abstract knowledge, of course, but knowledge implicit in actions) must have played a fundamental role also in the achievements of much earlier periods, as well as in those domains of any society that are not penetrated by a specialized economy of knowledge—and such domains evidently exist even today. What role did knowledge play, for instance, in the Neolithic Revolution, in the success story of agriculture, including the great irrigation projects, on which the achievements of Kushim's society were ultimately based? The great architectural projects, for instance, not only in the period immediately following the Neolithic Revolution, but also those of much later times, well into the modern period, are, in any case, largely based on knowledge that is not documented by writing or any other specialized symbolic representations. Such knowledge remains implicit in the structures of cooperative action that societies capable of realizing such ambitious projects build (Renn, Osthues, and Schlimme 2014). This does not imply, however, that such structures have no history. On the contrary, a history of cultural evolution in consideration of the fundamental role of knowledge must also include the development of such cooperative, action-implicit systems of knowledge.

The Neolithic Revolution

The evolutionary dynamics of the Neolithic Revolution conceived in these terms display remarkable similarities to the emergence of writing. It must be stressed, however, that just as there may have been many pathways to writing, there were certainly also many routes to food production in different parts of the world. Here we will concentrate on the emergence of food production in the Fertile Crescent. Developed agriculture is a comprehensive subsistence strategy involving intensive human labor. It represents an economic system by which human societies produce a large part of their food and other necessities from domesticated plants and animals. Domesticated plants such as cereals are adapted to human nutritional needs and even rely on human intervention for their reproduction. Farming based on domesticated plants and animals is as much a distinctive developmental stage, different from earlier subsistence strategies, as writing is in the sense of a representation of language as a distinctive developmental stage of what we have referred to as proto-writing.

In the history of the Neolithic Revolution, we can indeed identify an equivalent to such an earlier stage, whose expansion eventually gave rise, as with proto-writing, to a framework that channeled the development toward a full-scale agricultural economy. Long before humans began to sow harvested seeds, they practiced various forms of landscape management cultivating, for instance, wild cereals and pulses by tilling the soil.[6] Unlike fully developed agriculture, predomestication cultivation in the sense of the manipulation of wild plants and animals did not itself constitute a complete subsistence strategy, but only one component of such a strategy. It evidently existed for a very long time in human history, but played only a more or less marginal role for food production. It certainly was not motivated by the later outcomes of domestication, but constituted an activity with its own rationale and dynamics. In particular, landscape management represented, just as proto-writing, a highly local and contextualized activity. Only with the establishment of mature agriculture was it possible to reach a somewhat greater independence from local ecological conditions. The role of landscape management and cultivation in forming a scaffold for the later emergence of agriculture is another example that throws new light on the principle that the range of applications of a given means is always larger than the intentions for which they had been originally employed. This may even apply literally to some of the instruments employed in early farming.[7]

[6]See Melinda A. Zeder (2009, 32–33, n.12), Dorian Q. Fuller et al. (2011, n.13), and Asouti (2010).
[7]See the discussion in Cauvin (2000, 56–57).

Cultivation and Sedentariness

At least in the Fertile Crescent, there were several reasons why pre-domestication cultivation did not remain marginal. Ecological conditions in particular encouraged sedentariness, which favored the extension of cultivation practices bound to local environments. Given the investment of labor in cultivation practices, such local pre-domestication cultivation practices in turn stabilized sedentariness, thus creating what has been called the "labor traps" along the protracted trajectories leading to domestication (Fuller, Allaby, and Stevens 2010). This mutual reinforcement is similar to the case of writing in which the extension of symbolic practices, fostered by the administrative needs of Mesopotamian society, led to an exploration of the inherent potential of these symbolic practices, which in turn stabilized the administration constituting the institutional context of this process. Pre-domestication cultivation in the context of sedentariness thus played a similar role for the emergence of farming as proto-writing did for the invention of writing in the context of administration.

Also similar to the case of writing, globalization effects may have helped to emancipate the incipient domestication processes from the variety of local contexts in which they took place. Since cultivation was part of a network activity taking place in an extended geographical area (and not in a small core region as has been traditionally assumed), migration and exchange among different sedentary communities eventually contributed to a diversification and enrichment of cultivars at any specific location. The resulting recontextualization of cultivation also may have helped to separate wild from cultivated populations, thus contributing to a process by which human-defined plant or animal populations were transformed ultimately into biologically defined populations.

There was, in any case, initially no guarantee that pre-domestication cultivation would necessarily lead to domestication proper. Only at certain points along some trajectories may "tipping points" have been reached that then drove further development in a particular direction, whereas other trajectories may have been aborted or remained in intermediate stages (Fuller, Allaby, and Stevens 2010). Just as with the invention of writing, accidental external circumstances had thus been transformed into conditions for the internal stability and further development of a society.

Ultimately, agricultural economies were established in the Late Pre-Pottery Neolithic that included both livestock and crops. These economies in themselves constituted a successful economic model, which was capable of widespread migration and appropriation.[8] The transmission of this model must have relied on an action-implicit knowledge system represented by social interactions as well as a

[8] See Amy Bogaard (2005), cited in Fuller, Willcox, and Allaby (2011).

material culture comprising agricultural resources and technologies. By carrying seeds and animals into new regions, the "export" of this model may have had yet another recontextualization effect that contributed to its completion and recognition as an autonomous economic system, thus contrasting it with other systems or mixed economies. The expansion and transformation of settlement areas, population growth, as well as further structural changes of societies eventually turned the Neolithic Revolution, like the invention and globalization of writing ultimately based on it, into an irreversible process of global extent.

Neolithic Knowledge Systems

Even without the availability of a written record, some features of the Neolithic knowledge systems still may be recognized from a variety of sources. Sedentariness and cultivation practices favored what one may call a "horizontal organization" of human societies in space and time. This horizontal organization of societies, in the sense of cooperative actions structured by regularities in space and time, preceded the vertical stratification of societies, which was characteristic of the later period of urbanization. Larger settlements capable of food storage emerged whose economic activities were marked by seasonal changes in food supply and the corresponding labor investments. This horizontal organization may well have been what enabled the abandonment of villages at the end of the Pre-Pottery Neolithic B, when people left the foothills for the plains (Asouti 2010, 196).

Early Neolithic settlements must have been precarious institutions in the sense that their sustainability depended not only on a set of environmental factors, but also on labor investments that would not yield immediate benefits. The benefits would be evident only after some time and only if conditions remained stable. Contemporary symbolic practices may have constituted a means to cope with the awareness of this uncertainty and to strengthen social cohesion in such a way as to keep larger communities together. But ritual practices centered on mortuary and perhaps also fertility rituals could also have enabled societies to structure collective actions such as resource scheduling by normative knowledge (Warburton 2004). In any case, the overall result was the emergence of institutionalized cooperative practices that could be structured and optimized around successive steps of labor processes.

It was due to these practices, for instance, that large-scale building projects could be accomplished, far beyond the capacities of individuals or spontaneous collective activities (Kurapkat 2014). In the case of building projects, this capability to conceive and sustain "labor chains" created the conditions for such innovative developments as the invention of bricks, which enabled the separation of the preparation of building materials from the construction process itself. In the case of agriculture, labor chains involving the preparation of soils and the post-processing of crops created a framework for the emergence of domestication. The emergence of agriculture based on domesticated plants depended on the biological adaptation of plants to this new cultural regime, which constituted a niche that had not been originally created to produce this adaptation. The new regime eventually became entrenched in the biological and social conditions of this co-evolutionary process, thus transforming accidental boundary conditions into intrinsic features of the process.

For the evolution of domesticated plants and animals, human labor practices constituted an ecological niche to which they adapted. For the co-evolving human societies, on the other hand, these practices constituted not only a transformation of their environment, but also an external representation of shared knowledge about their interactions with this environment. It could therefore engender thinking processes about how to further change and optimize their living conditions according to their needs. This is why the Neolithic Revolution constitutes not only an economic transformation, or a niche construction in the biological sense, but also a stage in the evolution of knowledge. Furthermore, and coming full circle, the constructed niche of co-evolving humans and their domesticated animals and plants has also left and continues to leave observable changes in our internalized biological "knowledge system," our genome, whether this is tolerance to lactose, the ability to digest cereals and their fermented products—the beer of Kushim—or dealing with a whole new set of diseases. And, finally, this is why surviving the Anthropocene cannot simply be a matter of economic and technological adjustments, but also depends on whether or not we are capable of taking up the challenges it poses to our knowledge. This is what I believe we can learn from Kushim and his clay tablet.

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Chapter 3
What Is a Number? What Is a Concept? Who Has a Number Concept?

Jens Høyrup

Peter and I both became interested in Mesopotamian mathematics from debates about the didactics of mathematics, from Piaget, and, of course, from Marx. There were some differences, however. Peter's Marxism was very Hegelian, while mine was closer to Engels. One evening, about four decades ago, I was on a train in Copenhagen reading Engels's explanation in the middle of volume 2 of *Das Kapital*, in which he had been forced to submit arguments from numerical examples to strong editing; in the authorized English translation:

> Firmly grounded as Marx was in algebra, he did not get the knack of handling figures, particularly commercial arithmetic, although there exists a thick batch of copybooks containing numerous examples of all kinds of commercial computations which he had solved himself. Marx (1933, 289).[1]

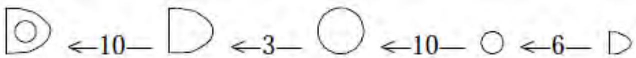
I laughed. I suspect that Peter would not have shared my appreciation and would have looked for something deeper in the numerical examples.

The same difference was revealed in our approaches to Piaget. Neither of us fell for Piaget's infatuation with group "theory" (at least I never heard Peter referring to it, and I certainly did not). But although we were both inspired by Piaget, our thinking about concepts diverged. The concept of "concepts" abounds in Piaget's work. His title *La causalité physique chez l' enfant* became *The Child's Conception of Physical Causality* in translation (other titles were changed correspondingly), and one volume in the "Jean Piaget Symposium Series" carries the title *Conceptual Development: Piaget's Legacy* (Scholnick et al. 1999). Peter maintained in one of our discussions (as I remember it) that inventors and users of protoliterate writing in Uruk in the fourth millennium BCE had no concept of number, firstly because there is no evidence that they mastered an arithmetical

[1] "So sattelfest Marx als Algebraiker war, so ungeläufig blieb ihm das Rechnen mit Zahlen, namentlich das kaufmännische, trotzdem ein dickes Konvolut Hefte existirt, worin er sämmtliche kaufmännische Rechnungsarten selbst in vielen Exempeln durchgerechnet hat." Marx (1885, 268f).

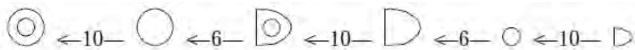
structure encompassing addition as well as multiplication (what amounts to practical multiplication may well have been seen as repeated addition),[2] and secondly because of the way their metro-numerical notations were structured, which I will discuss here (everything, of course, builds on the results obtained by Peter and Robert Englund (1987), with Jöran Friberg in the background).

First, there is the "Še-system," used for measuring quantities of grain (I leave out the "sub-unit part"):

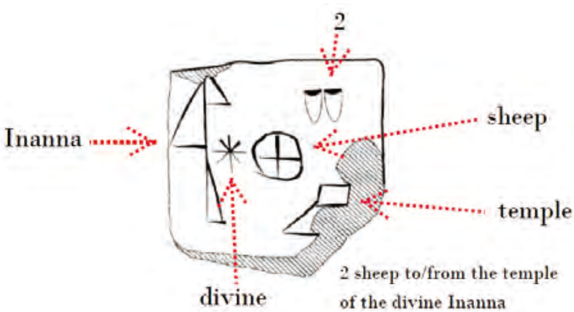


A couple of variant systems in which small markings are added to the signs were probably used for particular kinds of grain (or for the use of grain in particular processes in so far as this can be distinguished—is malt a different kind of grain or grain used in a particular process?).

Then there is "System S" ("S" for "sexagesimal"), the main number system:

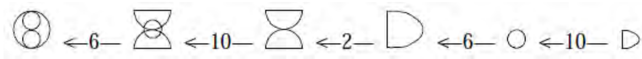


While the Še-system can be used to indicate quantity as well as quality (even though the sign še may be added as a determinative in order to avoid confusion with the same signs used in System S), System S basically designates quantity only, quality being determined separately ("2 sheep"). In this sense, System S is a system used for abstract numbers.



[2]Here, of course, group theory creeps in, but not in Piaget's metaphorical ways.

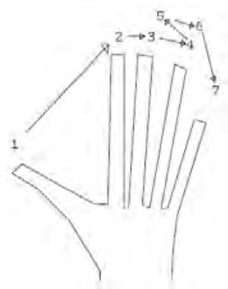
A number sequence with a more restricted use is "System B," the bisexagesimal system:



Here we see that, until level 60, it coincides with the sexagesimal system. It was apparently used for particular purposes, such as the counting of grain rations, perhaps also of milk products, and possibly, according to one text, fresh fish. A system B* derived from markings is often used without indication of what is being counted—"vermutlich weil das System B* einen so spezifischen Anwendungs bereich besaß, daß eine nähere qualitative Kennzeichnung des erfaßten Gegenstandes entfallen konnte" (Damerow and Englund 1987, 18).

In spite of this explanation, Peter tended to see the existence of systems like Še and particular counting systems like B as evidence that the protoliterate administrators possessed nothing that he would have accepted as a "number concept."

As I was also inspired by Piaget, and having made many experiments and observations of my own during the 1970s on the topic, I agreed (and agree) with Peter that speaking of a "number concept" presupposes a certain degree of structure. The intuitive ability to distinguish three items from four without counting may perhaps be seen as an "arithmetical ability," even though I would hesitate before using this characterization until we have evidence that this ability contributes to the genesis of a genuine number concept. Nor would I speak of a number concept as long as children have learned the number jingle but do not discover a problem when towards the end they "count" in circle, or as long as they have no objections to the "proof" that they have 7 fingers on one hand made by means of a backward step; both change at the time when cardinality and ordinality are merged into a single structure, and when the child knows immediately that there must be more flowers than roses in the garden without wishing to count them.



But my demands for a “number concept” do not go much further. From my experiences with teaching and explaining mathematics I have reached the conviction that concepts are dynamic structures; they grow in fullness as more and more connections are operationally integrated. That is probably also fairly Hegelian (or Hegel on his feet), and probably Peter would not have disagreed if that was what we had discussed. Possibly, our only disagreement was about where to put the lower limit for the number of integrated operations. In any case, this is the reason that I would not take the presence or possible absence of a multiplicative component (distinct from repeated addition) as a yardstick by which the presence or absence of a number concept can be decided, but only as a gauge for the richness of the concept—remembering also that even Euclid’s definition of multiplication (*Elements* VII, def. 15) refers to repeated addition.

Peter tended to regard the existence of metrological sequences where quantity and quality are merged as a proof that no concept at least of abstract number could be present. On that account I tend to follow Engels, according to whom “100.000 Dampfmaschinen [prove the principle] nicht mehr als Eine” (Engels 1962, 496). I also remember my first physics teacher explaining (I was 11 years old by then) that “density is measured in pure number”; I have no doubt that this teacher possessed a well-developed number concept himself, but he may have found it too difficult for us to understand a ratio g/cm^3.[3] So, for me “2 sheep” proves that the concept of abstract number was there,[4] even though its use was no longer compulsory for my physics teacher, as was the explication of the unit once it was decided that densities were being dealt with.

Similarly, I would see the existence of the bisexagesimal system not as proof that the Uruk-IV administrators had no unified number concept but as an early parallel to the particular brick metrologies of the late third millennium, and thus as evidence that they were skillfully adapting their mathematics to the bureaucratic standard procedures of the time.

A final disagreement of ours about number concepts concerned the implications drawn from Igor M. Diakonoff (1983, 88):

> The most curious numeral system which I have ever encountered is that of Gilyak, or Nivkhi, a language spoken on the river Amur. Here the forms of the numerals are subdivided into no less than twenty-four classes, thus the numeral ‘2’ is *mex* (for spears, oars), *mik* (for

[3] Actually, how many engineers or physicists really understand this? If they did, they would know that the apparent mystery of dimension analysis is simply a request for gauge invariance under change in unit.

[4] It had probably long been present in spoken language: the difference in structure between the Še- and the S-sequence suggests that the latter was formed when writing was introduced so as to agree with a pre-existing sequence of oral numerals.

> arrows, bullets, berries, teeth, fists), *meqr* (for islands, mountains, houses, pillows), *merax* (for eyes, hands, buckets, footprints), *min* (for boots), *met'* (for boards, planks), *mir* (for sledges) etc., etc.

Peter tended to see even this as evidence that no unified number concept was present; I, instead, would observe, as Diakonoff does in the next sentence, that "the root is m(i)- in all cases" and find nothing more than a highly elaborate parallel to the German "ein Mann/eine Frau." Perhaps we could sum up the whole thing in this way: According to Peter, we should be aware that protoliterate administrators (and so on) did not think in accordance with modern patterns; in my view, even we deviate from these ideologically prescribed patterns much more often than we usually admit. I am not generally a follower of Bruno Latour, but tend to agree that we have never been modern, or at least never as modern as we believe ourselves to be (perhaps interpreting Latour's phrase in a way that he himself would not accept).

Peter may well have argued that I have misunderstood everything he said (and I, vice versa). This is quite plausible, but this matter of disagreement was never a serious concern for us. We usually discussed our views briefly and then went on to more productive dialogue from which we could learn from each other by sharing information and through mutual critical questioning. That was much more important for both of us, but it is difficult to relate this in an interesting story. In spite of all efforts since Voltaire, war is much more conspicuous in historiography than peace; Voltaire himself had to admit as much in his historical writings.

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Chapter 4
Towards the Deciphering of the "Blau Monuments": Some New Readings and Perspectives

Manfred Krebernik

Introduction

Discussing archaic cuneiform texts with my friend Peter Damerow was one of the most exciting aspects of my visits to the Max Planck Institute for the History of Science. Peter was primarily engaged in deciphering the numeral and metrological systems attested by the earliest cuneiform texts (ca. 3300–3000 BCE, see fig. 4.1), and in reconstructing their administrative and social background. But he was also interested in the history of writing in general, and in the history of the cuneiform system in particular. Peter and I focused on different—but adjoining—periods within the early history of cuneiform. When I received the invitation to the colloquium commemorating Peter Damerow, on which this volume is based, I considered writing about the unusual early cuneiform documents which both Peter and I had studied: the so-called "Blau Monuments" or "Blau Stones." These are two stone objects of different shapes that bear archaic cuneiform inscriptions and reliefs. For their historical context, see fig. 4.1.

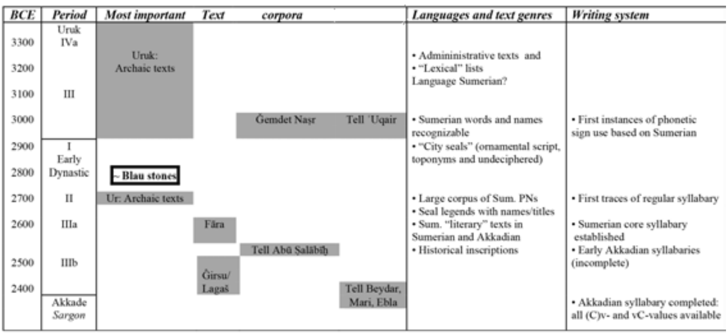

Figure 4.1: The chronological context of the "Blau Stones." MK.

The first one is called the “obelisk” (see fig. 4.2). In accordance with the orientation of its relief (and with the later direction of writing), the triangular shaped end is regarded as its top. The second one, which has a roughly semicircular shape, is called the “plaque” (see fig. 4.3). The obelisk measures 18 × 4.3 × 1.3 cm, and the plaque 15.9 × 7.2 × 1.5 cm.

Figure 4.2: The “Blau Stones”: obelisk. From Gelb, Steinkeller, and Whiting (1989–1991, plate 11).

Figure 4.3: The "Blau Stones": plaque. From Gelb, Steinkeller, and Whiting (1989–1991, plate 12).

The two pieces were allegedly bought in the vicinity of Uruk by their first, eponymous owner, Dr. A. Blau, who lived in Baghdad. In 1889, they were donated to the British Museum where they are registered as BM 86261 (obelisk) and BM 86260 (plaque), respectively. Since their publication in 1985 by W. H. Ward, they have raised much discussion and there have been many attempts to interpret them. Initially, even their genuineness was disputed (Ménant 1888, 69–88). Up to now, only several textual units—the term for "field," numerical signs, and quantified commodities—are identifiable with certainty, and a coherent interpretation of both documents together is still lacking. The two main difficulties are the correct interpretation of signs and the establishment of their correct order. The archaic sign repertoire was much larger than the later one. During the early phases of cuneiform many signs fell out of use, merged, or changed their shape. Until approximately 2500 BCE, signs were arranged freely (i.e., not reflecting linguistic serialization) within each case or line,[1] except for numeric signs, which were always placed first.

Most Recent Editions

The most recent editions and studies of the text on the Blau stones are Fenzel (1975) and Gelb, Steinkeller, and Whiting (1989–1991) (ELTS), no. 10–11 (with

[1] Early cuneiform documents are subdivided into "cases" or "boxes" rather than "lines." In the following, however, I will use the conventional term "line."

comprehensive bibliography). A new transliteration along with the copies of ELTS can be found in CDLI (ID numbers P005995, P005996). Gelb, Steinkeller, and Whiting (1989–1991) incorporated the Blau Monuments into a group of archaic documents, which they dubbed “ancient kudurrus.” The Akkadian term *kudurru*[2] originally referred to much later monuments (fourteenth–seventh centuries BCE), most of them inscribed with royal land grants; an example is given in fig. 4.4.

Figure 4.4: Middle Babylonian *kudurru* from the reign of Marduk-šāpik-zēri (1080–1068 BCE). From Hrouda (1991, 154).

[2]Cuneiform sources are rendered either in *transliteration* (graphemic level, single cuneiform signs) or in *bound transcription* (phonemic level). The two levels are distinguished in this article by the different fonts and styles exemplified here.

As a rule, the later *kudurru* texts start by providing the measurements of a field and describing its location. This seems to apply *cum grano salis* also to the "ancient *kudurrus*." The cuneiform character $\mathbf{GANA_2}$ "field" and the numerical signs referring to it are easily recognizable on top of the Blau obelisk and other "ancient *kudurrus*." Most of them are rectangular stone tablets (ELTS 1–6, no. 1 is shown in fig. 4.5) made of different material than the usual administrative tablets of clay, but some exhibit peculiar shapes such as that of a sheep or a lion-headed eagle (ELTS 8–9, no. 8 is shown in fig. 4.6).

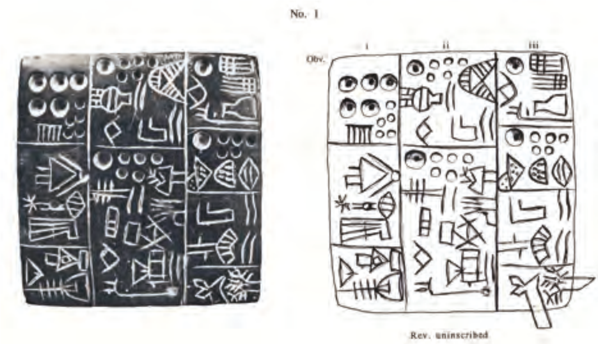

Figure 4.5: "*kudurru*" from the Uruk III period. From Gelb, Steinkeller, and Whiting (1989–1991, plate 1).

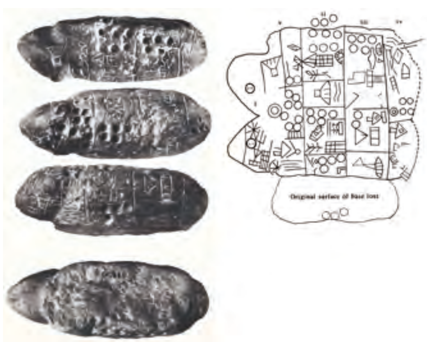

Figure 4.6: "*kudurru*" from the Uruk III period. From Gelb, Steinkeller, and Whiting (1989–1991, plate 6).

Gelb, Steinkeller, and Whiting (1989–1991), following the *communis opinio*, dated the "ancient *kudurrus*"—including the Blau monuments—to the Uruk III or Ğemdet Naṣr period (around 3000 BCE). The most recent archaeological study devoted to the Blau Monuments, Boese (2010), deals mainly with their dating. Boese quotes an article by P. Damerow and B. Englund (1989) that already expressed their doubt about the conventional dating of the monuments. Based on a more detailed paleographic analysis and adducing comparative iconographic evidence, Boese argues for a later date (Early Dynastic I). This seems to be contradicted, however, by the main figure of the relief on the plaque, the so-called "priest-king" with his characteristic "net-skirt," cap, and beard, since there are close parallels among pictorial representations, mainly on cylinder seals commonly dated to the Uruk III period and even earlier. Boese (2010) formulates the problem and its possible solution as follows: "Do these seals—and perhaps also the famous Warka-Vase with comparable pictures—equally stem, like the Blau Monuments, from the next younger phase (ED I), or did there possibly exist an unbroken tradition in theme, style and iconography, reaching from the last phase of the Protoliterate to the first stage of the Early Dynastic Period? The first alternative seems to me the more probable answer, though it cannot be proved definitely." Concerning the iconography, Boese points out that the object in the hands of the "priest-king" on the plaque is most probably "die Wiedergabe eines hohen, schlanken Gefäßes vom Typ der Warka-Vase," and that the person facing him is more likely male than female. He also addresses questions of whether the two stones really belong together, and if their inscriptions could have been added later, as suggested in Nagel, Strommenger, and Eder (2005, 11). As to the second question, he plausibly argues that the arrangement of the reliefs and the inscriptions speak in favor of their contemporaneity. As to the interrelationship between the two monuments, he repeats the obvious arguments for their belonging together: identity of material, uniformity of style, motifs, and paleography. Furthermore, he quotes an observation which I made in my review[3] of Gelb, Steinkeller, and Whiting (1989–1991) and builds upon it another possible argument for their interrelationship. While I had compared the peculiar shape of the obelisk to the early form of the cuneiform sign **KU** representing the Sumerian verb **dab$_6$** "to seize, to take," Boese proposes a corresponding interpretation for the plaque by comparing it with the cuneiform sign **BA** which represents the Sumerian verb **ba** "to allot, to assign." The two signs are illustrated below (see fig. 4.7, left) in a single archaic administrative text from the Uruk III period; a similar tablet from the same period (see fig. 4.7, right) already contains the younger, simplified form of **KU**.

[3] See Krebernik (1993–1994).

Figure 4.7: Variants of the sign **DAB$_5$** in administrative texts from the Uruk III period. From Seipel (2003, vol. IIIB, p. 37, no. 3.1.27, and p. 35, no. 3.1.25b). Sign names added by MK.

It seems noteworthy that the inscriptions on the two Blau Monuments show a clear distribution of contents: The obelisk deals with the field, whereas on the plaque quantified objects are listed, which are presumably gifts in exchange for the field. Boese does not cite an older hypothesis concerning the shape of the Blau Monuments: In 1961, M. E. L. Mallowan compared the obelisk to a craftsman’s chisel and the plaque to a pottery scraper.[4] This hypothesis deserves to be reconsidered in the light of my present contribution, in which I suggest that the Blau monuments refer to a transaction by “stone-cutters,” that is, craftsmen who make use of similar tools.

Gelb, Steinkeller, and Whiting (1989–1991) postulate a northern provenience for the “Blau Stones” on the basis of the possible toponym Urum in lines 2–3 of the obelisk, while Boese maintains that they most probably stem from Uruk because depictions of the “priest-king” are best attested there.

On the obelisk, which is inscribed only on one side, the lines run from top to bottom (= right to left according to the archaic direction of writing), and their order is unambiguous. The inscription of the plaque is more complex, but the order of columns and lines can be established on external and internal grounds with a high degree of certainty. The reliefs divide the inscription into four sections. Section 1: The text starts on the fully inscribed side (called “obverse”) with two horizontal rows of lines called “columns” according to the later direction of

[4] Mallowan (1961b, 72f; 1961a, 65f), with illustrations.

writing which implies a counter-clockwise rotation of 90%. Section 2 consists of three lines between the two standing persons (counted from top to bottom = left to right according to the later orientation). Section 3: The text continues in the back of the left standing person with two rows or columns running smoothly around the edge to the other side of the plaque. They end behind the back of a sitting person. Section 4: The signs in front of the sitting person constitute the last section of the text. Somewhat problematic is the role of the last signs in the two columns: do they constitute separate lines (with the edge functioning as line divider), or are they continuations of the last lines on the obverse? Concerning the left column, internal reasons speak clearly in favor of the first possibility: The last line on the obverse as well as the first line on the reverse contain a numerical sign like the preceding lines and the following lines (in the second column on the obverse). The end of the second column will be discussed below in connection with the structure of the text as a whole.

In my present contribution, I would like to suggest some new readings and interpretations in the hope that they may stimulate further discussion and progress. Let me show you first a synopsis of the two most recent editions of the text, Gelb, Steinkeller, and Whiting (1989–1991), and CDLI (see fig. 4.8):

<table>
<tr><th></th><th>Gelb/Steinkeller/Whiting 1991</th><th></th><th></th><th>CDLI</th></tr>
<tr><td>1</td><td>5(bùr) gán U$_8$.SAL Nin-GÍR.ḪA.RAD(ATU-850)</td><td>Obelisk</td><td>1</td><td>5(N45) , GAN2 U8 SAL NIN ZATU687 KU6~a RAD~a</td></tr>
<tr><td>2</td><td>GI$_4$.RAD</td><td></td><td>2</td><td>, GI4~a RAD~a</td></tr>
<tr><td>3</td><td>ḪA.ÚR.LAK-131</td><td></td><td>3</td><td>, KU6~a UR2 NU@g</td></tr>
<tr><td>4</td><td>ALAM.NE.PAB.KÍD?.GÍR.DU</td><td></td><td>4</td><td>, ALAN~f NE~a IB~a PAP~a ZATU687 ŠITA~a3</td></tr>
<tr><td>5</td><td>engar èš</td><td></td><td>5</td><td>, AB~a APIN~a</td></tr>
<tr><td></td><td></td><td></td><td>obv.</td><td></td></tr>
<tr><td>1</td><td>2 BA.DAR</td><td>Plaque</td><td>1.a1</td><td>2(N04)? , BA DAR~a</td></tr>
<tr><td>2</td><td>2 BA.NAM</td><td></td><td>1.a2</td><td>2(N04)? , BA NAM~d</td></tr>
<tr><td>3</td><td>2 šen</td><td></td><td>1.a3</td><td>2(N04)? , ŠEN~a</td></tr>
<tr><td>4</td><td>30 EN.ŠÀ</td><td></td><td>1.b1</td><td>3(N14) , EN~a ŠA3~a1</td></tr>
<tr><td>5</td><td>30 EN.A</td><td></td><td>1.b2</td><td>3(N14) , EN~a A</td></tr>
<tr><td>6</td><td>KA-GÍR-gal</td><td></td><td>1.b3</td><td>, KA~a GAL~a ZATU687</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>7a</td><td>2 uri</td><td></td><td>2.a</td><td>2(N04)? , URI</td></tr>
<tr><td>7b</td><td>2 gada</td><td></td><td>2.b</td><td>2(N04)? , GADA~a</td></tr>
<tr><td>7c</td><td>2 DUG+Ì+vertical-GIŠ.X</td><td></td><td>2.c</td><td>2(N04)? , ŠAKIR~b LA2 ŠE3@t</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>8</td><td>“2” (ma-na) síg</td><td></td><td>3.a1</td><td>2(N01) , ZAG~a</td></tr>
<tr><td>9</td><td>1 arád(NITA+KUR)</td><td></td><td>3.a2</td><td>1(N04)? , IR11</td></tr>
<tr><td>10</td><td>2 KUG.NA</td><td></td><td>3.a3</td><td>2(N04)? , ZATU756 NA~a</td></tr>
<tr><td>11</td><td>“1 ½” (ma-na) UŠ.BUR.TÚG</td><td></td><td>3.a4</td><td>1(N01) 1(N08) , UŠ~a TUG2~a |3(N57).GAR|</td></tr>
<tr><td>12</td><td>10 máš</td><td></td><td>3.b1</td><td>1(N14) , MAŠ2</td></tr>
<tr><td>13</td><td>2(ul) 2(bán) ninda</td><td></td><td>3.b2</td><td>2(N51)? , GAR</td></tr>
<tr><td>14</td><td>10 (dug) kaš</td><td></td><td>3.b3</td><td>1(N14) , KAŠ~d</td></tr>
<tr><td>15</td><td>ḪAŠḪUR.LÀL</td><td></td><td>3.b4</td><td>, HAŠHUR LAL3~c</td></tr>
<tr><td></td><td></td><td></td><td>rev.</td><td></td></tr>
<tr><td>16</td><td>AN.GÍR.JN-312.NUNUZ.SAG</td><td></td><td>1</td><td>, AN ZATU687 NUNUZ~a1 |NI~ax1(N57)| SAG</td></tr>
</table>

Figure 4.8: The text of the “Blau Stones” in recent transliterations. From Gelb, Steinkeller, and Whiting (1989–1991, 43); CDLI, nos. P005995, P005996.

Both editions correspond as to the order of sections and lines. The transliteration in CDLI is more abstract, more cautious, and more detailed with regard to paleographical features. Some signs—highlighted blue in the synopsis—were identified differently in the two editions.

The first discrepancy in line 1 of the obelisk shows that the interpretation of the numerical signs is in some cases doubtful. This is particularly the case with circular signs which vary in size. On clay tablets, a small circular stylus impression represents **u** “10” or 1 **bur$_3$** (a surface measure, ca. 6.5 ha), a big one **šar$_2$** “3600” or “60 **bur$_3$**.” The 5 circular holes referring to **GANA$_3$** “field” at the beginning of the obelisk inscription are interpreted as “**5(bùr)**” in ELTS, and **5(N45)** = 5 × 60 **bur$_3$** in CDLI. On the plaque, the holes in lines 4–5 = 1.b1–2 are wider than those in 12 = 3.b1 and 14 = 3.b3, but are interpreted uniformly as “10” in both editions. The numerical sign in line 13 of the plaque is read **2(ul) 2(bán)** in ELTS, but **2(N51)?** = 2 × 120 by CDLI. The latter interpretation is certainly correct, since similar forms can be found in the archaic texts from Ur and in the Fāra texts (see fig. 4.9):

Figure 4.9: Early forms of the cuneiform sign for “120.” From Gelb, Steinkeller, and Whiting (1989–1991, plate 12); Burrows (1935) (UET 2); photographs by O. Teßmer, Vorderasiatisches Museum Berlin (VAT 12624) and H. Steible, Freiburg (S 867).

In line 4 of the obelisk, the two editions differ in the reading of three signs. The sign tentatively read “**KÍD?**” in ELTS was correctly identified as **IB** already by K. Fenzel. The identification of the sign read **GIR$_2$** in Gelb, Steinkeller, and Whiting (1989–1991) and earlier editions (which I had already doubted in my review) was given up in CDLI, and the sign in question is transliterated **ZATU687**.[5]

In line 7c = 2.c of the plaque, the difficult signs following the number are transliterated **DUG+Ì+vertical-GIŠ X** in ELTS and analyzed as **ŠAKIR~b LA$_2$** (+) **ŠE$_{3\text{-tenû}}$** in CDLI. The first of the two signs seems indeed to represent a vessel, and the inscribed **NI** could specify it as an “oil” (**NI** = **ì**) vessel as suggested in ELTS. The association with later sign and term **ŠAKIR** “churn” is, however, uncertain. The following graph (**X** in ELTS) is split up into **LA$_2$** and **ŠE$_{3\text{-tenû}}$** in CDLI. This seems plausible since the two signs may be interpreted as “tied (**LA**$_2$) with a rope (**ŠE$_{3\text{-tenû}}$**),” which is a possible specification of a vessel.

The unclear sign in line 8 = 3.a, is better identified as **SIKI** (**SIG$_2$**) “wool” (ELTS) than **ZAG** “side” (CDLI) because the context requires a quantifiable commodity.

The analytical transliteration |**3(N57).GAR**| (CDLI) for **BUR** (ELTS) in line 11 = 3.a4 is highly artificial; it is by no means clear that **BUR** goes back to such a combination of signs.

Some New (and Old) Sign Identifications

Because **ZATU687** and two other signs were read differently in ELTS and CDLI, I would like to suggest interpretations of my own. Allow me to first show my transliteration and structural analysis of the text (see fig. 4.10). I have rendered the numerical signs by **n** and index numbers according to their first occurrence in the text: **n$_1$** = circular hole, **n$_2$** = horizontal, (approximately) semicircular hole etc. The different sizes of **n$_1$** are symbolized by the letters a–d: **n$_{1a}$**, **n$_{1b}$**, etc. The newly suggested readings are marked by different colors.

[5] ZATU + number symbolizes and identifies cuneiform signs (mostly of unknown reading) with reference to the *Zeichenliste der Archaischen Texte aus Uruk* (Green et al. 1990). A similar use is made of LAK + number and S + number, referring to the *Liste der archaischen Keilschriftzeichen* by A. Deimel (1922), and to the *Sign List* in Burrows (1935), respectively.

	Transliteration	Structural Units	Lexical Units
	Obelisk		
1	GANA$_2$ 5n$_{1a}$	field + size	“5 bur of field”
	U$_8$ MUNUS	person-o1 **zadim**	personal name
	ZADIM		“stone-cutter”
	NIN	institution and locations	“(of the?) queen (of?)”
	ḪA RAD		toponym (?)
2	GI$_4$ RAD		toponym (?)
3	ḪA UR$_2$ LAK131		toponym (?)
4	ALAM.PA$_4$.BIL$_4$	person-o2 **zadim**	personal name
	ZADIM		“stone-cutter (of)”
	IB	institution	temple name (?)
5	ENGAR	person-o3	“plough(-man) (of)”
	AB	institution	“temple household”
	Plaque		
1	2n$_2$ IGI/BA DARA	quantified commodities	“2 ... of DARA-bird(s)”
2	2n$_2$ IGI/BA NAM		“2 ... of NAM-bird(s)”
3	2n$_1$ ŠEN		“2 Š.-containers”
4	3n$_{1b}$ EN ŠA$_3$		“30 ...”
5	3n$_{1b}$ EN A		“30 ...”
6	KA	person-p1 **zadim-gal**	personal name
	GAL ZADIM		“chief stone-cutter”
	figure of relief		
7a	2n$_2$ URI	quantified commodities	“2 U.-containers”
7b	2n$_2$ GADA		“2 linen (cloths)”
7c	2n$_2$ ŠAKIR$^{?}_{x}$ ŠE$_3$ ***tenû*** LA$_2$		“2 Š.-vessels” + specification?
	figure of relief		
8	2n$_3$ SIKI		“2 (weight units of) wool”
9	1n$_2$ IR$_{11}$		“1 slave”
10	2n$_2$ KUG NA		“2 stones (with?) precious metal”
11	1n$_4$ UŠ.BUR.TUG$_2$		“x (weight units of?) U.-textiles”
12	1n$_{1c}$ MAŠ$_2$		“10 he-goats”
13	2n$_5$ NINDA		“240 (loaves of) bread”
14	1n$_{1d}$ KAŠ		“10 (jars of) beer”
15	ḪAŠḪUR LAL$_3$	person-p2 **zadim** (?)	“apples, honey” = personal name (?)
	figure of relief		
16	AN IL$_2$	person-p2/3 **zadim**	personal name
	ZADIM ZA		“stone-cutter of beads (?)”
	figure of relief		

Figure 4.10: The text of the “Blau Stones”: structural analysis. MK.

ZATU687 = ZADIM/MUG

Occurring twice on each stone, **ZATU687** is the most frequent sign in our text. The observation that it is a relatively rare sign in the entire corpus of archaic and Early Dynastic texts supports the suspicion that it might be the key term of the document. Already in the earliest editions, it was read as **GIR$_2$**, an identification which I doubted in my review of ELTS. I would suggest now that **ZATU687** is a precursor of the later signs **ZADIM** and **MUG**. To my knowledge, **ZADIM** and **MUG** were not yet differentiated during the Third Millennium. In later periods, a distinction seems to have been introduced only in the Assyrian ductus. **ZADIM** was clearly distinguished from **GIR$_2$** during all periods (see fig. 4.11). Nevertheless, F. Thureau-Dangin in his sign list *Recherches sur l’Origine de l’Ecriture Cunéiforme* (REC) from 1898 included the **ZADIM** of the Blau stones under **GIR$_2$**, an early error which might have influenced later studies and editions of the Blau monuments.

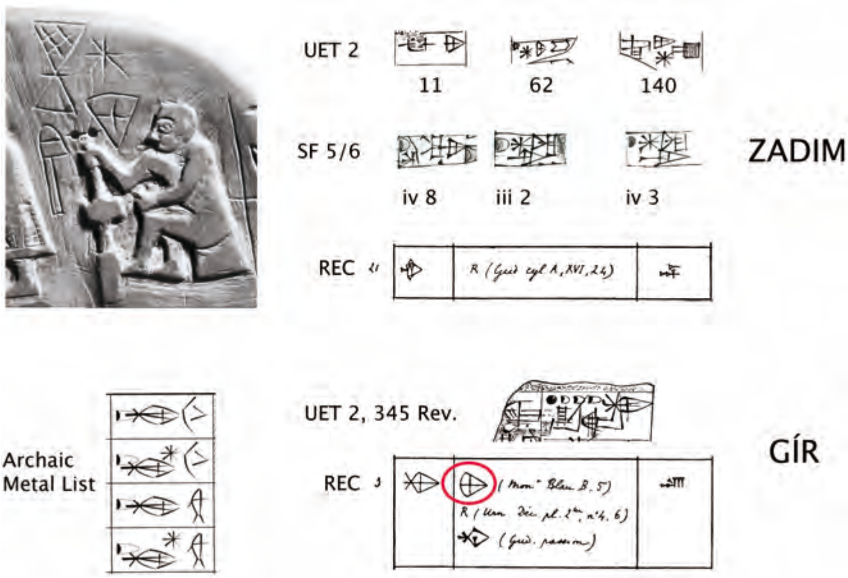

Figure 4.11: Early forms of cuneiform signs **ZADIM** and **GÍR**. From Gelb, Steinkeller, and Whiting (1989–1991, plate 12); Burrows (1935); Thureau-Dangin (1898, 1, 5) (REC); Englund, Nissen, and Damerow (1993, 32) (Archaic Metal List); MK.

The reading **mug** occurs mostly in the name of the goddess $^{\mathbf{d}}$**Nin-mug** and as a designation of wool of lesser quality (= akk. *mukku*). Since **ZATU687** and **NIN** co-occur in the first case of the obelisk, the two signs could in principle represent the divine name **Nin-mug**. ELTS assumes a divine name "**Nin-GÍR.ḪA.RAD**" which "could very well be a goddess of [the city of] **A.ḪA**." However, in view of three more occurrences of **ZATU687** in varying contexts, it seems preferable to consider "**GÍR**" = **ZATU687** here as an isolated element. The reading **zadim** designates a craftsman, namely a "stone-cutter." The term seems to be composed of **za** "stone" and $\mathbf{dim_2}$ "to make, to fashion." It can be compared with $\mathbf{ku_3\text{-}dim_2}$ "silversmith" or "goldsmith," an analogous compound of $\mathbf{ku_3}$ "precious metal" and the same verb $\mathbf{dim_2}$. If our identification is correct, and if the sign is indeed a key term for the whole document, one is tempted to connect it with the reliefs. It is impossible to cite and discuss here the many differing descriptions and interpretations of the persons, objects, and activities depicted there. My own suggestion, based on the identification of **zadim** as "stone-cutter," at the time seemed new to me, but I recently discovered that Eva Braun-Holzinger's book on *Das Herrscherbild in Mesopotamien und Elam* (Braun-Holzinger 2007) also contains a chapter on the Blau stones. She describes the scene on the reverse of the plaque as follows (p. 17): "Auf der Rückseite des 'Schabers' steht eine Figur mit der gleichen Handhaltung, im schraffierten Rock, sie ist jedoch völlig kahlrasiert; ihr zugewandt hocken zwei unbekleidete kahlköpfige Männer, die mit langen Geräten – Stößeln oder Bohrern – hantieren; hinter ihr sitzt ein dritter Handwerker. Dieser Handwerksarbeit kommt bildlich auf beiden Denkmälern eine so große Bedeutung zu, daß sie auch mit der Transaktion, die im Text festgelegt wurde, in Zusammenhang stehen könnte." In view of **zadim** as a possible key term in the text, it seems very likely that the workers are indeed using "Bohrer," that is, drills,[6] producing stone vessels or cylinders seals. Comparable representations from the Third Millennium can be found in Egypt (see fig. 4.12).

[6]For ancient drills and drilling techniques see, e.g., the articles by Gorelick and Gwinnett listed in the bibliography (with numerous illustrations and bibliography).

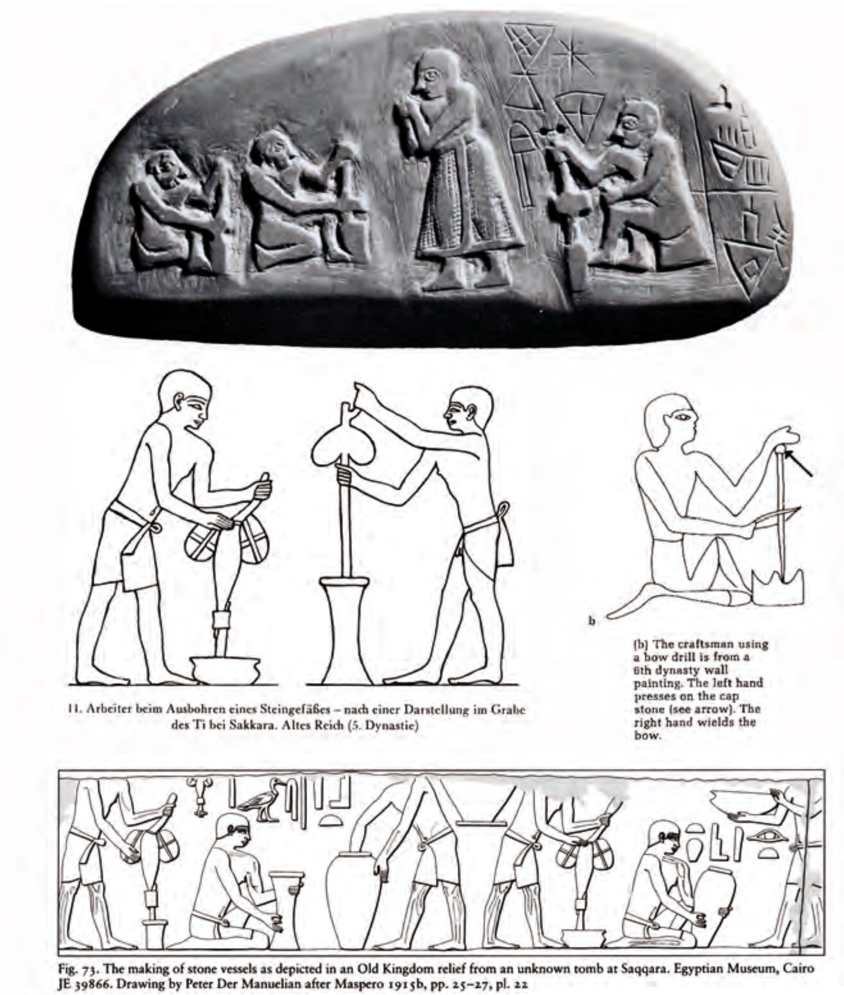

Figure 4.12: Stone-cutters using drills on the Blau plaque (?) and on monuments from Ancient Egygpt. From Gelb, Steinkeller, and Whiting (1989–1991, plate 12); Kayser (1969, 15, fig. 11); Gorelick and Gwinnett (1979, 24); O’Neill (1999, 123, fig. 73).

“DU”/“ŠITA~a3” = S377/GIŠ

The sign transliterated **DU** by Gelb, Steinkeller, and Whiting and **ŠITA~a3** by CDLI in line 4 of the obelisk is certainly identical with **S377** = sign no. 377 in E. Burrows’ list of archaic signs from Ur. As already noted by Burrows (1935, plate 30, no. 377), **S377** is attested only as part of the sign combination **S377.PA$_4$.NE**, which later became **GIŠ.GIBIL** and **GIŠ.NE**, read **bil$_3$** and **bil$_4$**, respectively. The most famous occurrence of **GIŠ.GIBIL/NE** is in early spellings of the name “Gilgamesh,” approximately pronounced (pa)bilga-mes in the time of the archaic texts. On the Blau obelisk, the alleged “**DU**” or “**ŠITA~a3**” appears in the vicinity of **NE**, **PA$_4$**, and ALAM. The combination of the four signs yields a personal name **pa$_4$-bil$_4$-alam**, which is also attested in the archaic texts from Ur (see fig. 4.13):

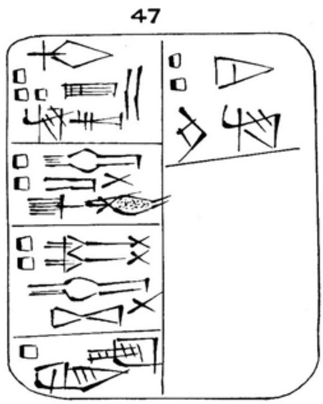

Figure 4.13: Archaic administrative text from Ur. From Burrows (1935, plate VII).

As can be observed on the same tablet (UET 2 = Burrows 1935, no. 47), the transformation of **S377** into **GIŠ** had already begun by the time of the archaic Ur archives. **S377** was already correctly identified and connected with **NE** by K. Fenzel, who saw here a personal name “**pa-bil$_4$-alam-ib-GÍR**.” It is indeed very highly probable that we deal here with a personal name. It should comprise, however, as typological parallels suggest, only the signs **pa$_4$-bil$_4$-alam** (order of signs uncertain). Thus, we find **alam-abzu**, **alam-kurta**, **lugal-alam**, and **munus-alam** in the archaic texts from Ur. The closest parallels are obviously **lugal/munus-alam**, in which **pa$_4$-bil$_4$** “older relative,” **lugal** “king,” and **munus** “women” all designate persons. The identification of **pa$_4$-bil$_4$-alam** as a personal name leaves us with two remaining signs, **IB** and **ZADIM**. If **zadim** “stonecutter” is meant here, **ib** must be a specification like “stone-cutter of the **ib**,” where **ib** could be the designation of a sanctuary well attested in Early Dynastic inscriptions.

“JN-312”/“|NI~ax1(N57)|” = IL_2

My next suggestions concern the last line of the plaque. It seems likely to me that the sign transliterated **JN-312** by Gelb, Steinkeller, and Whiting and **|NI~ax1(N57)|** by CDLI combined with the **SAG** next to it is the ancestor of the later sign $\mathbf{IL_2}$. It represented the Sumerian noun **du(b)si(g)** “basket” (> Akkadian *tupšikku*) as well as two verbs for “carry,” $\mathbf{il_2}$ and $\mathbf{gur_3}$. These notions would have been symbolized by a burden or a support together with a burden (perhaps a jar) on top of a “head” (**SAG**). A transitional sign form may be found in line 2' of a literary (?) fragment from Fāra (S 800), which seems to be older than the majority of the Fāra texts (see fig. 4.14).

Figure 4.14: Archaic form of **IL** in a fragment from Fāra. From Krebernik, Steible and Yıldız (2015, 378).

“NUNUZ” = ZA

The sign transliterated **NUNUZ** in both editions should rather be interpreted as **ZA**. According to the **ZATU** no. 423, both signs were originally identical: sign forms similar to that of the Blau plaque are registered as **NUNUZ**, but a value **ZA$_2$** is also postulated. In the archaic texts from Ur and in the Fāra texts, however, **NUNUZ** and **ZA** are clearly distinguished: **ZA** consists of circular or half-circular stylus-impressions with vertical wedges inside, whereas **NUNUZ** consists of two lozanges with vertical wedges inside (see fig. 4.15).

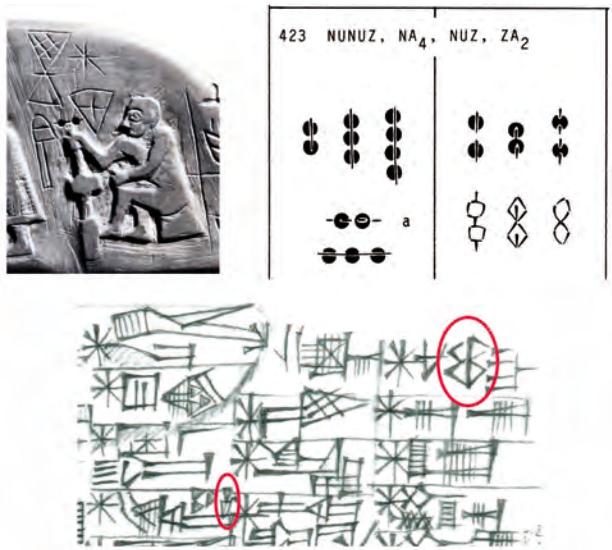

Figure 4.15: The signs **ZA** and **NUNUZ**. From Gelb, Steinkeller, and Whiting (1989–1991, plate 12); Green, Nissen, Damerow, and Englund (1990, 261); MK.

The similarity between the two signs seems to reflect the similarity of the objects originally depicted: two (and originally more) beads on a string (**ZA**) and two eggs (**NUNUZ**), respectively. If the Blau stones indeed date to the Early Dynastic period, the distinction described above should be valid, and the sign in question therefore be identified as **ZA**. It occurs next to **ZADIM** and it is possible that the two signs are to be connected. If so, **ZA** could be a phonetic indicator or a specification of **ZADIM**. Since **ZADIM** occurs earlier in the text three times without **ZA**, the first alternative is unlikely, but a meaning like “stonecutter of beads, bead maker” is conceivable.

Structure and Contents

Let us finally look back to the text as a whole and briefly discuss its structure. As already stated above, the obelisk deals with the field, and the plaque with the gifts involved in the transaction.

Obelisk: The field is present in the very beginning of the text in the shape of the sign **GANA$_2$** and a number expressing its size. The line contains 6 more signs. Their interpretation is difficult because the arrangement of signs within a line was still free in the period of our text. Thus, one has the choice among a variety of possible alignments and groupings. I have already argued against the combination **nin-mug** (name of a goddess). Both **nin** and **munus** are common elements of Early Dynastic personal names. **nin**, on the other hand, could by itself refer to a “queen,” and **zadim** most probably refers to a “stonecutter.” If both interpretations are accepted, the remaining four signs could represent a personal name, **munus-u$_8$** “the woman is (like a) ewe,” and a toponym or hydronym, **Ḫ̮A.RAD** (**RAD** “canal,” **Ḫ̮A** “fish”). Unfortunately, I was not able to find the presumed personal name or close parallels of it in other sources, but the latter assumption can be supported by structural considerations—as in later *kudurrus*, the location of the field could be specified—and by similar expressions in the following lines: **RAD.GI$_4$** and **Ḫ̮A.UR$_2$.LAK131**. Note that each has one sign in common with **Ḫ̮A.RAD**. ELTS considers **Ḫ̮A.RAD** and **Ḫ̮A.UR$_2$.LAK131** as toponyms and connects them with Urum = Tell ʿUqair (archaic spelling **Ḫ̮A.RAD.UR$_2$**) and Tub/wa (spelled **A.Ḫ̮A**). The last line can easily be related to the “field” of line 1 through the sign **APIN**, which in later texts expresses **apin** “plough,” **uru$_4$** “to till,” and **engar** “ploughman.” **AB** is most probably to be understood as “sanctuary” or perhaps better “temple (household),” a meaning associated later with its value **eš$_3$**. ELTS translates the term as “*agronomos* of the temple household” and considers it as the title of the person named in the previous line. This line contains, however, according to my analysis, a personal name, **pa$_4$-bil$_4$-alam**, together with a title, **zadim ib**. Therefore, the “*agronomos* of the temple household” must be another person, referred to only by his function (which is very often the case in Early Dynastic administrative texts). Thus, the obelisk seems to mention three persons connected with the field: (1) one **zadim** (line 1) probably associated with the **nin** and furthermore with the following toponyms, (2) a second **zadim** (line 4) associated with the **ib**(-sanctuary), and (3) a non-**zadim**, the “ploughman of the temple (household).”

Plaque: Two sections listing quantified commodities are clearly recognizable (lines 1-5, 7–14/15). As far as the individual items are concerned, I do not want to go into lexical discussions and speculations here but only comment briefly on one of them: ELTS considers the possibility that **BA.DAR** in line 1 represents a noun borrowed from Akkadian *patarru* “sharp tool, prod” and spelled **ba-da-ra** in later Sumerian texts. This is unlikely, not only because the identity of the first sign—**IGI** or **BA**—is uncertain, but also because lines 1–2 seem to contain parallel expressions, each composed of **IGI/BA** and the name of a bird: **dara** “francoline” and **NAM** = **sim** “swallow,” respectively. A similar observation can be made in the two hitherto unexplained lines 4–5, where the common element is **EN** is combined with **Š**$\mathbf{A_3}$ and **A**.

Either section is followed by the name of a **zadim** (lines 6 and 16). The second section involves, however, a problem which has already been addressed above: Is line 15 indeed a separate line containing a personal name **ḫašḫur-lal**$_3$, or do lines 14–15 constitute one single line, in which case **ḫašḫur lal**$_3$ “apple” and “honey” would specify **kaš** “beer”? Arguments for both possibilities can be adduced, but in my opinion the stronger ones speak in favor of the first possibility. Though I cannot find further evidence for the personal name **ḫašḫur-lal**$_3$, meaning something like “sweet apple,” it seems not impossible since **lal**$_3$ is a common element of Early Dynastic personal names. The function or title which one would expect can be easily supplied by referring **zadim** in line 16 to both preceding names. This interpretation can be supported by the iconography: the three craftsmen depicted on the reverse of the plaque, one of them bigger and more prominent than the two others, would neatly correspond to the **zadim gal** “chief stone-cutter” of line 6 and the two **zadim za** “stone-cutters of beads” named in lines 14–15.

If we rightly assume that the two Blau stones document the sale and purchase of a field, and if the sign **ZADIM** has been identified correctly as a key-term meaning “stone-cutter,” it follows that the sellers as well as the buyers were stone-cutters. The two contracting parties are mentioned on and probably also symbolized by the two differently shaped monuments. Even if the identification of the two parties is uncertain—most probably the sellers are the **zadim**s on the obelisk, and the buyers are the **zadim**s on the plaque—we can state that the transaction implies a guild of “stone-cutters” associated with the “queen” (**nin**) and with religious institutions (**AB**, **ib**). The property transaction obviously involved a ritual (as attested in later periods) which was headed by the dominant male figure depicted on the reliefs who is commonly identified with the En or “priest-king” of Uruk. The confirmation or rejection of the scenario suggested here as well as the further elucidation of the ritual depend, *inter alia*, on a new look at the iconographic details which I would like to recommend to my archaeological colleagues.

Postscript

After submitting my contribution, I noticed that the craftsmen depicted on the Blau plaque had previously been interpreted, on purely iconographical grounds, as stone-cutters by W. Max Müller in 1915 in an article on “Steinbohrer in Altbabylonien,” *Orientalistische Literaturzeitung*, 18, 266–268 (not mentioned in Braun-Holzinger (2007)).

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Chapter 5
The Chinese Sexagenary Cycle and the Origin of the Chinese Writing System

William G. Boltz

In early 1999 the University of Pennsylvania hosted a conference on the topic of how writing systems originate. I had the good fortune to meet Peter at that meeting for the first time and to talk with him at length about how to approach the question of the origin of writing. This was where Peter first presented his paper on the origin of writing as a problem of historical epistemology (Damerow 2006). As is well known, the central point of that paper was that we should look for evidence of pre-writing graphic notational systems, and what functions they served and what functions they did not adequately serve, as a possible source-context out of which glottographic writing arose. As is also well known, this thesis is the result of the extensive work that Peter did with Bob Englund and Hans Nissen in the 1980s on the proto-cuneiform texts (Nissen, Damerow, and Englund 1993). At about the same time Günter Dreyer published materials that he had excavated and analyzed from the U-j tomb at Abydos in upper Egypt (Dreyer 1998). These included a large number of objects with clearly discernible signs, some pictographic, some abstract (often suggesting a kind of numeracy), most recurring in the corpus, but not known from Egyptian hieroglyphic writing proper. Although these materials themselves probably do not pre-date the earliest glottographic Egyptian writing, all the same they are distinct from the recognized Egyptian writing system and seem instead to represent *grosso modo* the kind of precursor graphic notational system for Egyptian writing that the proto-cuneiform texts do for Mesopotamian writing. In this respect they are sometimes called proto-hieroglyphic texts.

The third place in the ancient world where writing arose independently of any already existing writing system is of course China, where writing first appeared about two millennia later than in Mesopotamia or Egypt. With all due respect to Peter's enduring wish to be able to explain the emergence of writing in human civilization as a one-time occurrence and thus to find a way to account for Chinese writing through some remote influence from the Ancient Near East, I have to acknowledge that there is not a shred of evidence for anything other than the independent invention of writing in China. What makes this particularly tantalizing is that there is also not a shred of archaeological evidence for any pre-writing graphic notational system or even any rudimentary notational scheme out of which Chinese writing might have arisen, comparable to the proto-cuneiform or proto-hieroglyphic materials of Mesopotamia and Egypt.

There is one curious feature of the earliest attested written Chinese texts that suggests a link with a pre-glottographic notational system. This is the set of twenty-two signs of the Chinese sexagenary cycle. (Tab. 5.1) These signs are not known in any form or context prior to their occurrence in the earliest extant texts written in Chinese, the so-called "oracle bone" inscription materials of the late Shang state, ca. 1200 BCE, so *sensu stricto* they do not constitute a pre-glottographic notational system. But they appear already as a fully functioning ordinal counting system in the earliest known Chinese texts, and their use there is distinctive enough to raise at least the question of a pre-writing existence.

The twenty-two signs are listed in table one in both their Shang period graphic guise and as modern Chinese characters. They are divided into two groups, group A of ten signs called "celestial stems" (*tiān gān* 天干), and group B of twelve signs called "terrestrial branches" (*dì zhī* 地支). Calling them "stems" and "branches" is a tradition based on mythological associations that have been secondarily imposed on the set at a time much later than their Shang inscriptional use, and has, as far as we know, nothing to do with the original meaning or function of the signs. In this later tradition the ten "stems" are conventionally correlated systematically with the five primary colors ('red' *chì* 赤, 'black' *hēi* 黑, 'yellow' *huáng* 黄, 'white' *bái* 白, 'blue-green-grey' *qīng* 青), the five traditional "natural agents" ('fire' *huǒ* 火, 'water' *shŭi* 水, 'earth' *tŭ* 土, 'metal' *jīn* 金, 'wood' *mù* 木) and with several other traditional "fives." The twelve "branches" are the signs that are associated with the animals of the so-called Chinese zodiac. There is no evidence that these signs had any of these meanings or correlative associations much earlier than the beginning of the unified Chinese empire in 221 BCE, about a thousand years after their first use as seen in the earliest written texts. For the first millennium of their orthographic life they had no meaning at all beyond their function as ordinal counters. Because they are a part of the glottographic writing system, they have pronunciations, but those pronunciations are not known as words in the Shang language apart from being the names of these twenty-two signs. To be sure, some of these signs come to be used to write free words in later stages of the language, e.g., 甲 *jiă* 'squama' and the aspectual negative 未 *wèi* 'not yet,' but these lexical identifications do not pertain to their earliest usage in the language of the Shang inscriptions.

The two groups are used together in a dual-cyclical way, pairing one graph from the "stem" group with one from the "branch" group in strict cyclical order, to wit, (01 - 01), (02 - 02), (03 - 03), …(10 - 10), (01 - 11), (02 - 12), (03 - 01), (04 - 02), (05 - 03), …(10 - 12). This gives a set of sixty distinct two-character pairings. (Tab. 5.2) The sixty pairs in this cycle can be used to keep track of anything; it is a simple counting system. In the Shang divinatory inscription texts, the earliest Chinese texts of any kind extant or known, this system is used exclusively to keep track of days, predominantly as a part of a formulaic text line specifying when a particular divinatory ceremony was performed. Transcribed in modern script the formulaic use looks like this, for example:

> 丙辰卜㱿貞 *[plastron-] cracking on the (03-05) [= 53rd] day [of the cycle], the diviner Gu ascertaining [a response to the proposition]*: … (See fig. 5.1; the text line occurs written vertically at the top left and again at the top right, marked with a red box.)

The set of ten "celestial stems" is also used in contemporaneous ceremonial Shang bronze inscriptions to refer posthumously to royal ancestors. That use is much less systematic than the pairings that we find used to track days, and its basis is not well understood, but it is all the same clearly intended to refer in an ordered way to the past generations of the royal clan (figs. 5.2, 5.3).

There are three reasons to look upon this graphic two-part cyclical counting system as possibly having something to do with a pre-glottographic notational system and perhaps with the origin of Chinese writing.

1. All graphs in the Shang writing system have both a pronunciation and a meaning. Expressed in formal feature terms this means that they are always +**P**, +**S**. Although the graphs of the "celestial stems" and "terrestrial branches" set have pronunciations, that is, formally they are +**P**, their only meaning is as ordinal "counters" in the sexagenary cycle counting system as described above. Their function in this counting system is not dependent on pronunciations. Unlike graphs that stand for words, and are therefore inherently +**P** by definition, the **P** value of these graphs is not essential to their effective use as counters. It is likely, of course, that if the set of twenty-two graphs existed as a pre-glottographic notational system, functioning as an ordinal counting device, there would have been conventional ways to verbalize them.[1] But the individual graphs may have become phoneticized as glottographic writing only when the sexagenary counting scheme became a part of some kind of oral performance, perhaps the oral divination ceremony, perhaps something else, and when this had to be recorded in written form. They would then have been adapted to the written Shang language when a written record of the event was produced.

[1] For the difference between 'verbalizing' and 'reading' in connection with writing systems see Hyman (2006).

2. Most Shang graphs, *once the word that they write is known*, can be seen in one way or another to have an iconically identifiable, or at least suggestive, origin. The graphs of the sexagenary cycle, because they have no intrinsic meaning beyond serving as units in a counting system, cannot be explained as iconic in any objective sense (though see below re the graph 丁 / ▢). Although it is possible to imagine that some of the “branch” graphs are iconic when matched with their associated animals, there are two compelling reasons that argue against this; (i) there is no evidence that the words for the graphs in question ever actually meant anything as real lexical items having to do with the various animals, and (ii) the animal associations are not attested for nearly a thousand years from the time of the first use of this set.
3. In very general terms, allowing that Peter’s premise regarding the primacy of graphic notational systems for counting or measuring purposes prior to the emergence of glottographic writing *per se* is applicable beyond the confines of the Mesopotamian ecumene, a set of graphs that is primarily designed to serve as a counting mechanism is in principle entirely feasible as a precursor to glottographic writing.

While we must acknowledge that at present there is no archaeological evidence for the existence, much less the notational use, of these graphs prior to their appearance in the earliest known written Chinese texts, there may be astronomical clues to the existence of at least one of these graphs prior to the emergence of glottographic writing in China. David Pankenier has proposed that the counting function of the twenty-two graphs was in origin explicitly and exclusively calendrical, that is, astronomical, and that the fourth of the “stems” set, the graph 丁, modern Chinese *dīng*, Old Chinese ***tteng**, iconically represents in its original graphic form (▢, see Table one, part A) the asterism known in traditional sources by the nearly homophonous name 定 *dìng*, Old Chinese ***tteng-s**, which is perceived in the sky as a slightly imperfect rectangle and seems to have been used to determine the proper alignment (正 *zhèng* < ***teng-s**) of human establishments (cities, towns, tombs, houses, ceremonial and sacrificial edifices, & c.) relative to polar meridians (Pankenier 2011) (see fig. 5.4). If the link between the “stem” name 丁 *dīng* and the traditional asterism name 定 *dìng* is more than simply fortuitous, as Pankenier argues, then the two words are cognate with each other, and both are related to the word 正 *zhèng* ‘to straighten, align, make correct,’ the shared semantic sense of all three being something like √RECT- > ‘correct, fixed, set right.’ And the rectangular graphic form ▢ for the fourth of the celestial stems 丁 *dīng*, then has an iconic origin in the *dìng* asterism. The archaeological evidence for the astronomical observations associated with using the *dìng* asterism as a guide to human activities predates the emergence of glottographic writing in China by several centuries. This allows for the possibility that the graph ▢ had a notational significance well before it became a part of the regular Shang writing system. If this speculation turns out to be correct, it would mean that writing in China, instead of originating in an accountancy context, as it seems to have in Mesopotamia and perhaps in Egypt, arose in an astronomical and calendrical context and would thus be reminiscent of early Mesoamerican writing. This, together with the fact that a dual-cyclical sexagenary counting system, albeit extending to numbers far larger than sixty, occupies a central position in early Maya calendrical texts, bespeaks a similarity between Chinese and Maya that Peter was eager to embrace for its monogenetic suggestiveness.

Figure 5.1: 丙辰殻貞 *[plastron-] cracking on the (03-05) [53rd] day [of the cycle], the diviner Gu ascertained [a response to the proposition]: ...*

A. Celestial Stems: *Tiān gān* 天干

(01		甲	*jiă*	dendro-glaucescent
(02		乙	*yĭ*	dendro-glaucicant
(03		丙	*bĭng*	flammi-rubescent
(04		丁	*dīng*	flammi-rubicant
(05		戊	*wù*	terri-flavescent
(06		己	*jĭ*	terri-flavicant
(07		庚	*gēng*	metallo-can(d)escent
(08		辛	*xīn*	metallo-can(d)icant
(09		壬	*rén*	aqui-nigrescent
(10		癸	*gŭi*	aqui-nigricant

B. Terrestrial branches *Dì zhī* 地支

01)		子	*zĭ*	murine	00	North
02)		丑	*chŏu*	bovine	02	
03)		寅	*yín*	tigridine	04	
04)		卯	*măo*	leporine	06	East
05)		辰	*chén*	dracontine	08	
06)		巳	*sì*	anguine	10	
07)		午	*wŭ*	equine	12	South
08)		未	*wèi*	ovine	14	
09)		申	*shēn*	simiine	16	
10)		酉	*yŏu*	galline	18	West
11)		戌	*xū*	canine	20	
12)		亥	*hài*	porcine	22	

Table 5.1

	1-甲	**2-**乙	**3-**丙	**4-**丁	**5-**戊	**6-**己	**7-**庚	**8-**辛	**9-**壬	**10-**癸
子**-1**	(01-01) 1		(03-01) 13		(05-01) 25		(07-01) 37		(09-01) 49	
丑**-2**		(02-02) 2		(04-02) 14		(06-02) 26		(08-02) 38		(10-02) 50
寅**-3**	(01-03) 51		(03-03) 3		(05-03) 15		(07-03) 27		(09-03) 39	
卯**-4**		(02-04) 52		(04-04) 4		(06-04) 16		(08-04) 28		(10-04) 40
辰**-5**	(01-05) 41		(03-05) 53		(05-05) 5		(07-05) 17		(09-05) 29	
巳**-6**		(02-06) 42		(04-06) 54		(06-06) 6		(08-06) 18		(10-06) 30
午**-7**	(01-07) 31		(03-07) 43		(05-07) 55		(07-07) 7		(09-07) 19	
未**-8**		(02-08) 32		(04-08) 44		(06-08) 56		(08-08) 8		(10-08) 20
申**-9**	(01-09) 21		(03-09) 33		(05-09) 45		(07-09) 57		(09-09) 9	
酉**-10**		(02-10) 22		(04-10) 34		(06-10) 46		(08-10) 58		(10-10) 10
戌**-11**	(01-11) 11		(03-11) 23		(05-11) 35		(07-11) 47		(09-11) 59	
亥**-12**		(02-12) 12		(04-12) 24		(06-12) 36		(08-12) 48		(10-12) 60

Table 5.2

Figure 5.2: From Chen (1995, 52).

Figure 5.3: From Chen (1995, 62).

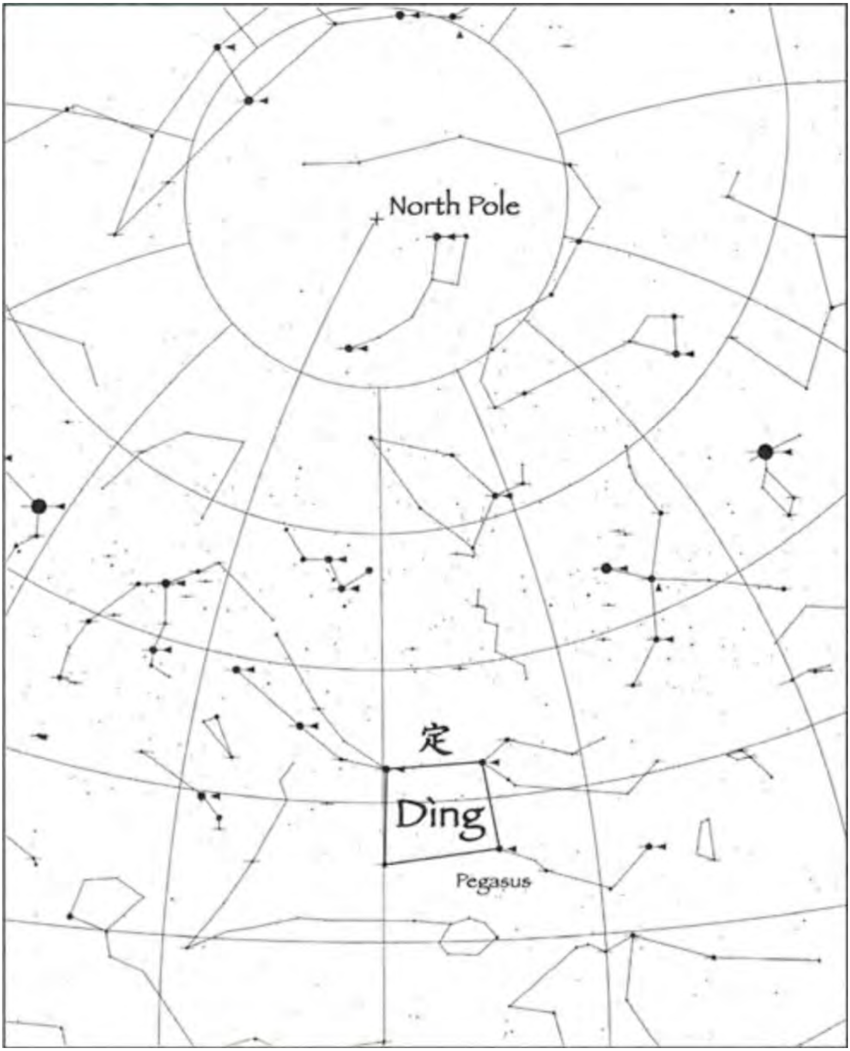

Figure 5.4: Position of the 定 *dìng* asterism as viewed from Luoyang ca. 650 BCE (from Pankenier 2011, 40).

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Chapter 6
'Real Abstraction' and the Origins of Intellectual Abstraction in Ancient Mesopotamia: Ancient Economic History as a Key to the Understanding and Evaluation of Marx's Labor Theory of Value

Joachim Schaper

The Problem

The present paper sets out to critically assess the significance of the existence of intellectual abstractions in the society of the neo-Sumerian empire of the Ur III period, and their being rooted in real abstraction, for a reappraisal of Marx's labor theory of value. The term 'real abstraction' refers to abstraction that does not spring from thought but from social being.[1] I see this paper as a *Vorarbeit* for a compendious essay which I hope to publish sooner rather than later and which will contain the fruits of work done in collaboration with Peter Damerow in 2010 and 2011 and further work by myself. What I present in this paper is therefore very much a work in progress and quite tentative.

Peter's reason for agreeing to co-author the essay in question was his view that recent insights into the early Mesopotamian economy had much to offer with regard to a reappraisal of Marx's labor theory of value. My own interest was triggered by my reading of Sohn-Rethel's work on what he called—using a term that was inspired by Marx's theory, but not used by Marx himself—'real abstraction' and its significance for the formulation of a materialist epistemology.[2] While Peter thought that the concept of 'real abstraction' was a helpful one, he criticized Sohn-Rethel for postulating that real abstraction sprang from the act of commodity exchange, while in fact, Peter thought, it was rooted in labor, that is, in the realm of production and not, as Sohn-Rethel claims, in the realm of circulation.

[1] "Das Wesen der Warenabstraktion aber ist, daß sie nicht denkerzeugt ist, ihren Ursprung nicht im Denken der Menschen hat, sondern in ihrem Tun" Sohn-Rethel (1973, 41).

[2] "Die Abstraktion kommt der Werkstatt der Begriffsbildung gleich, und wenn die Rede von der gesellschaftlichen Seinsbestimmtheit des Bewußtseins einen formgerechten Sinn besitzen soll, so muß ihr eine materialistische Auffassung von der Natur des Abstraktionsprozesses zugrundegelegt werden können. Eine Bewußtseinsbildung aus dem gesellschaftlichen Sein setzt einen Abstraktionsprozeß voraus, der Teil des gesellschaftlichen Seins ist" Sohn-Rethel (1973, 39).

Peter thus pre-empted a criticism of Sohn-Rethel which has been expressed in some of the most recent contributions to the debate on Marx's labor theory of value,[3] of which more later.

The Context

As early as 1859, in *Zur Kritik der Politischen Ökonomie* (Marx 1961, 15), Marx invoked Aristotle's treatise *De republica* as the first instance in antiquity of a discussion of the difference between use-value and exchange-value. It was so important to him that he referred to it again in a foundational passage in the first volume of *Capital* (Marx 1962, 50–51). We shall sketch the significance of some of the relevant Mesopotamian sources in order to show that they may well support Marx's reconstruction of the genesis of the concept of value. While Marx's attempt at providing a historical analysis of the development of the concept of value suffered from the lack of availability of historical sources which could have supported his reasoning, we now have documents that are much older than the texts adduced by Marx and betray the existence of value-abstraction and of the concept of exchange-value, as opposed to use-value only, in the late third millennium BCE. In fact, those documents bear witness to the existence of money—not, of course, of precious metal in *coined* form, but of precious metal (silver), in the form of coils and ingots, which gradually established itself as a "universal equivalent."[4] Those texts have received attention in the context of research into early Mesopotamian accounting techniques (Nissen, Damerow, and Englund 2004) and the earliest history of mathematics (i.e., Robson 2008). However, their significance for an evaluation of Marx's work on value-theory, and especially of the concept of commodity-abstraction[5] and its alleged role as the originator of abstract thought, has, to the best of my knowledge, never been explored.

[3] Cf. Anselm Jappe (2013) on Sohn-Rethel: "For him, the 'only' difference he has with the master resides in the fact that he wants to replace the Marxian concept of 'commodity abstraction' with that of 'exchange abstraction': for Sohn-Rethel, it is not abstract labor that confers value on products, but their exchange. But in doing so, he diverges from Marx on a very central point. For Sohn-Rethel, the exchange act is abstract because the exchangers have undertaken to renounce temporarily the use of the products. The origin of 'abstractness' is therefore the exchanger's 'abstracting' from the use they could make of the object in question, and this 'abstracting' is a 'real physical act.'"

[4] On gold as an "allgemeines Äquivalent" and the gradual process of establishing it as such, Marx (1962, 84).

[5] "For Marx, abstract labor invests products with their 'value-objectivity,' that is, confers value on them. For Sohn-Rethel, exchange accomplishes this task which is why he advocates the replacement of the Marxian concept of 'commodity abstraction' with that of 'exchange abstraction.' Unlike Marx, Sohn-Rethel does not deem labor to be the source and substance of the value form" Jappe (2013, 9).

The Genesis of the Concept of Value in Ancient Mesopotamia: Economic Practice, Social Organization, and Intellectual Abstraction in the Ur III Period

The economic texts displaying the bookkeeping techniques of the Ur III period illustrate beautifully how abstraction—including, of course, value-abstraction—develops historically and is *rooted in (a specific form of) social being*. It is helpful in this context to remind oneself of the fact that not all societies produce abstractions, and other contributions in this volume discuss some aspects of such lack of abstraction in some modern non-literate societies, for example, in Amazonia and Papua-New Guinea (see also Damerow 1996, 291–293). I cannot go into any detail here. Suffice it to say for the moment that, by contrast with the aforementioned non-literate societies, Mesopotamia produced a remarkable array of intellectual abstractions. I shall attempt (1) to demonstrate their significance and their being rooted in real abstraction, and thus in social being, and (2) to show that they therefore vindicate Marx's labor theory of value.

The Genesis of the Concept of Value in Mesopotamia

What can we know, then, about the way in which the concept of value evolved in early Mesopotamia? For the purposes of this paper, the most relevant material is found in those administrative texts which (1) keep records of economic activities, (2) establish the debits and credits of key personnel in the economy of Ur III, and (3) document value equivalents operative within the Ur III economy and commodity exchange between it and its neighbors.

Fishery was a centrally important part of the Neo-Sumerian economy, and it is fortunate that the Ur III-period records pertaining to that economic sector have been subjected to a rigorous analysis by Robert Englund (1990). Fishery was a state-regulated activity which generated a significant amount of meticulous documentation. This documentation helps contemporary scholarship not just with regard to understanding that sector of the economy in itself, but affords us insights into the actual productive forces and relations of production that were operative in the Ur III period. This in turn enables us to better understand the nexus between the "totality" of the "relations of production," that is, the "economic structure of society," and the "forms of social consciousness" of that society (Marx 1961, 8–9). And if indeed, as Marx postulates, "[t]he mode of production of material life conditions the general process of social, political and intellectual life," the material which survived in the compendious archives of cities of the Neo-Sumerian empire[6] may well enable us to arrive at an understanding of the origins of the 'real abstraction' postulated by Sohn-Rethel.

Robert Englund has drawn attention, in his study on Ur III fisheries, to the key function of the dam-gàr, probably best translated as "commercial agent" (thus Diakonoff) or "Tauschagent" ("exchange agent," thus Englund). Contrary to the view expressed by Powell, and following Diakonoff and Englund, the dam-gàr was most likely a state agent (Englund 1990, 17–18). His office was twofold: "internally," he would exchange surplus goods within the context of the Ur III economy, and "externally," it was his task to exchange surplus goods produced in his own society against surplus goods produced in neighboring societies, that is, he acted as a long-distance trader. He can be correctly described as an official who—as part of his service to a state that operated a planned economy[7]—conducted exchange operations on the periphery of that economy, exchange operations which established an ever-growing system of value equivalencies. From the point of view of the labor theory of value, the dam-gàr is fascinating, for it is—according to Marx—precisely at the periphery of pre-capitalist societies that exchange first becomes significant: it is at the periphery that commodities are first exchanged, and it is from the periphery that commodity-exchange then enters the mainstream of the economies of the participating social formations and slowly transforms them: "Der Warenaustauch beginnt, wo die Gemeinwesen enden, an den Punkten ihres Kontakts mit fremden Gemeinwesen oder Gliedern fremder Gemeinwesen. Sobald Dinge aber einmal im auswärtigen, so werden sie auch rückschlagend im innern Gemeinleben zu Waren" (Marx 1962, 102). The economy of Ur III illustrates precisely this point: Neumann has rightly pointed out that the long-distance trade carried out by the *dam-gàr* officials accelerated the acceptance of silver as the standard equivalent and ultimately had a subversive effect on the economic system of the Ur III period, in the sense that it slowly transformed the economic organization of the Neo-Sumerian empire.[8]

Long-distance trade—through quantifying goods and establishing value equivalents across an ever-growing range of goods, thus turning them into commodities[9]—thus had a significant effect on the development of the system of value-equivalencies and, concurrently, on the refinement of arithmetic,

[6] E.g., in Girsu and Umma.

[7] The Ur III state was characterized by an economic system that has correctly been described as a planned economy (Planwirtschaft) by Hans Nissen (2012, 88–89) and others.

[8] "Während das Silber in der Praxis des zentralisierten Wirtschaftslebens der Ur III-Zeit offenbar nur eine untergeordnete Rolle gespielt hat, scheint dagegen der Fernhandel dieser Zeit die Durchsetzung des Silbers als allgemeines Äquivalent beschleunigt zu haben. Dadurch förderte die Ausweitung des Ur III-Handels letzten Endes eine Entwicklung, die gegen die bestehende Organisation der Produktion gerichtet war" Neumann (1979).

[9] See Marx's general description of this process in *Zur Kritik der politischen Ökonomie*, Marx (1961, 35–36).

especially fractional arithmetic. What Marx pointed out in general terms for pre-capitalist societies is illustrated in great detail by the bookkeeping of the Ur III period: "The gradual extension of barter [Erweiterung des Tauschhandels], the growing number of exchange transactions [Austausche], and the increasing variety of commodities bartered lead, therefore, to the further development of the commodity as exchange value [Tauschwert], stimulate the formation of money and consequently have a disintegrating effect on direct barter" (Marx 1961, 36). The dam-gàr transactions and their effect on the Ur III economy also show that a concept of value equivalencies was probably first established through more or less random exchange activities on the margins of the state's planned economy. It was then adopted and refined by that economy, thus establishing a system of equivalencies which helped to integrate the diverse modes of production within the state and thus to make it more efficient both inwardly, with regard to its planning, and outwardly, with regard to its exchange relations with other states and with private individuals, *while also having the long-term subversive effect mentioned earlier*.

Bob Englund has recently marshaled new arguments (Englund 2012, 121–152) to demonstrate that, during the Ur III period, labor-time was not only integrated into the "overall system of equivalencies" (Englund 2012, 127), but that *labor* actually became a *commodity* (Englund 2012, 127)—which, to put it mildly, one would not assume to be possible in a pre-capitalist social formation.[10] The textual evidence needs further investigation, but, as Englund has demonstrated in his 1990 book, the "work day" certainly was a key feature of the system of value equivalencies in Ur III (Englund 2012, 79–90).

Just one quick remark about the Ur III economy in terms of the periodization of pre-capitalist social formations: it can be seen as a characteristic example of the second of the phases postulated by Marx, since it was "based" on a "communal system" that, as Marx puts it, "prevents the labour of an individual from becoming private labour and his product the private product of a separate individ-

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[10]With regard to texts about bala-services, Englund says: "Weder Maekawa 1988 noch Sharlach 2004 (noch, soweit feststellbar, Studevent-Hickman 2006) haben sich mit den Konsequenzen für unser Verständnis der neusumerischen Verwaltung auseinandergesetzt, die sich aus der Einbettung der bala-Dienstleistungsverpflichtungen in das allgemeingültige neusumerische Abrechnungssystem ergeben. Insbesondere scheint diese Einbettung die These einer übergreifenden Gültigkeit von der Hauptstadt Ur auferlegter Verpflichtungen in den jeweiligen Provinzarchiven zu verdeutlichen. M. E. konnten Silberlohnäquivalenzen dazu dienen, die nach Arbeiterklassen unterschiedlich gewerteten Arbeitseinsätze in leicht verwendbare Silbermengen zu konvertieren, denen letztendlich die uns noch fehlenden bala-Abrechnugen [sic] der Reichskanzlei zugrundelagen" Englund (2012, 131). This would indicate that value-equivalencies were established between, on the one hand, quantities of labor done by each of the respective types of workers and, on the other hand, specific quantities of silver, thus also establishing value equivalencies between quantities of labor done by one category of workers and quantities of labor done by another type of worker.

ual; it causes individual labour to appear rather as the direct function of a member of the social organization" (MECW 29, 275).

Marx's Labor Theory of Value Against the Background of the Ur III Texts

Historians have rightly asked the fundamental question whether Marx's theory of value can be applied to pre-capitalist social formations in the first place—"*ob also der Wert schon in gewisser Weise als regulierendes Subjekt* hinter dem Rücken der Beteiligten wirken kan[n], ohne jedoch als 'automatisches Subjekt', als Kapital entwickelt zu sein," as Rudolf Walter Müller rightly asks (Müller 1981, 109–117). Some have answered the question in the affirmative, and rightly so: as Marx pointed out, for commodities to be exchanged according to their values, the economic formation in which the exchange takes place need not have reached the stage of capitalist development.[11] Marx postulates not only the theoretical but also the historical precedence of commodity values (*Warenwerte*) over production prices (*Produktionspreise*).[12]

In the neo-Sumerian texts one can detect traces of the beginnings of commodity-production. This observation does not contradict Marx: In pre-capitalist social formations, it is not just farmers and craftsmen who can, by virtue of being owners of means of production, produce commodities, that is, goods produced directly for the purpose of exchange. Marx explicitly mentions slavery and serfdom as conditions under which goods can be produced as commodities (Marx 1964, 187), and the Ur III economy is an example of just that: several modes of production under the roof of one society, with dependent laborers of various kinds producing goods as commodities.

In the economy of Ur III we have an example of the transition from barter to commodity exchange,[13] exactly along the lines of Marx's sketch of that crucial period in pre-capitalist social formations when "direct barter" (*unmittelbarer Tauschhandel*) gives way to the "formation of money" (*Geldbildung*),[14] that is,

[11]"Der Austausch von Waren zu ihren Werten, oder annähernd zu ihren Werten, erfordert also eine viel niedrigere Stufe als der Austausch zu Produktionspreisen, wozu eine bestimmte Höhe kapitalistischer Entwicklung notwendig ist" Marx (1964, 186). Müller (1981, 109) rightly stresses this important point.

[12]"Abgesehn von der Beherrschung der Preise und der Preisbewegung durch das Wertgesetz, ist es also durchaus sachgemäß, die Werte der Waren nicht nur theoretisch, sondern historisch als das prius der Produktionspreise zu betrachten" Marx (1964, 186).

[13]"Die besonderen Gebrauchswerte, die im Tauschhandel zwischen verschiedenen Gemeinwesen Waren werden, wie Sklave, Vieh, Metalle, bilden daher meist das erste Geld innerhalb der Gemeinwesen selbst. Wir haben gesehen, wie sich der Tauschwert einer Ware in umso höherm Grade als Tauschwert darstellt, je länger die Reihe seiner Äquivalenzen oder je größer die Sphäre des Austausches für die Ware ist" Marx (1961, 36).

[14]MECW 29, 291 = MEW 13, 36.

in the case of Ur III, to the establishment of silver as the standard equivalent. In Ur III society this also led, as I have pointed out, to the flourishing of fractional arithmetic, given the need for the divisibility of value-equivalents in the practice of commodity exchange;[15] indeed, the development of fractional arithmetic received a strong impulse from the introduction of silver as the standard equivalent (Damerow 1981, 82).

This in turn ties in with the fact that Ur III society also came up with the notion of a “work-day” to measure labor, to correlate it with numerous commodities (including the money commodity, i.e., silver) (Englund 2012, passim), and thus to establish more value-equivalencies in order to increasingly facilitate the processes of economic administration. The notion of the work-day was thus an attempt at standardizing labor: a truly astonishing abstraction at such an early point in recorded history, although—and this is very important—the notion of abstract labor could not possibly have occurred to the Mesopotamian administrators, for the same reason that it could not have occurred to Aristotle: “Daß aber in der Form der Warenwerte alle Arbeiten als gleiche menschliche Arbeit und daher als gleichgeltend ausgedrückt sind, konnte Aristoteles nicht aus der Wertform selbst herauslesen, weil die griechische Gesellschaft auf der Sklavenarbeit beruhte, daher die Ungleichheit der Menschen und ihrer Arbeitskräfte zur Naturbasis hatte” (Marx 1962, 74);[16] that is, in a society based, or mainly based, on slave-labor or other kinds of dependent labor, the notion of the equality of all forms of labor simply could not arise. Nevertheless, the Ur III administrators understood that labor in some sense contributed to the establishment of value, which is demonstrated by the fact that labor-time was included in debit/credit calculations, as Englund has demonstrated.[17]

While Sohn-Rethel was right when he arrived at the result that “*abstraction precedes thought*” (Toscano 2008, 281), it is also true that Marx had already described and analyzed real abstraction, although he did not call it that (cf. MEW 23, 88). Real abstraction was generated much earlier than Sohn-Rethel thought:

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[15] Silver has all the key properties required of a commodity that is to serve as a universal equivalent: “unlimited divisibility, homogeneity of its parts and uniform quality of all units of the commodity” (MECW 29, 290 = MEW 13, 35).

[16] “M. E. konnten Silberlohnäquivalenzen dazu dienen, die nach Arbeiterklassen unterschiedlich gewerteten Arbeitseinsätze in leicht verwendbare Silbermengen zu konvertieren, denen letztendlich die uns noch fehlenden bala-Abrechnugen [sic] der Reichskanzlei zugrundelagen” Englund (2012, 131, cf. above). The point is that different categories of workers had different values assigned to their labor, according to their respective categories. The notion that one might treat labor just like, say, silver and thus see it as being characterized by “unlimited divisibility, homogeneity of its parts and uniform quality of all [its] units” (MECW 29, 290 = MEW 13, 35) simply did not occur to the Sumerian administrators, and could not have occurred to them, precisely because of the society in which they were situated.

[17] Cf. Englund, *Ur III-Fischerei*, 1990, 78–90, on TCL 5, 5670.

not in seventh-century Greece, but in late third-millennium Mesopotamia. While he was right in drawing attention to 'real abstraction,' he mistakenly traced it back to the realm of circulation instead of that of production.[18]

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[18] With regard to this point, Jappe (2013) is right; with regard to some of the details of Sohn-Rethel's argumentation, he is mistaken.

Chapter 7
Meat Distribution in Late Uruk Diacritical Feasts: Second-Order Bookkeeping Techniques and their Institutional Context in Late Fourth Millennium BCE Mesopotamia

J. Cale Johnson

Introduction

Undoubtedly one of the most important achievements in Peter Damerow's distinguished career, first at the Max Planck Institute for Human Development and Education and later at the Max Planck Institute for the History of Science (MPIWG), was the systematization of the Late Uruk metrologies in collaboration with Bob Englund and Hans Nissen in the 1980s.[1] This was part of a broader effort to locate the key moments in the earliest history of human thought and investigate those social contexts and institutional locales in which human awareness and reflection rose to new heights.[2] In memory of Peter Damerow, I would like to offer a small contribution to the further decipherment of Late Uruk metrology, namely the terminology and enumeration used in the butchering of sheep in the earliest cuneiform documents as well as the distribution of these portions of meat in so-called diacritical feasts.[3] These meat distributions were central to the long history of Mesopotamian institutions, but more importantly the textual materials that defined the offices and professions that were eligible for these distributions (and also audited the distributions themselves) represent one of the earliest examples of second-order categorization and notation in human history.

As Damerow recognized years ago, Near Eastern feasting is one of the privileged contexts, if not *the* privileged context, for understanding the social forces driving the earliest advances in the history of writing and numeracy. Thus, it is no accident that in the very same volume in which Michael Dietler first coined the concept of a diacritical feast, viz. *Food and the Status Quest* (Wiessner and

[1] See Damerow and Englund (1987); Damerow, Englund and Nissen (1988); now summarized in Englund (1998).

[2] See Damerow (2008) for an overview.

[3] As Dietler, who coined the term, puts it: "the use of differentiated cuisine and styles of consumption as a diacritical device to naturalize and reify concepts of ranked differences in social status" Dietler (1996, 98).

Schiefenhövel 1996), Damerow contributed a paper entitled "Food production and social status as documented in proto-cuneiform texts" (Damerow 1996) that sought to establish a link between the well-known professions list and the distribution of beer, a theme to which he would return in one of his last papers as well (Damerow 2012). In the years since Damerow's paper, Englund has published definitive papers on proto-cuneiform grain metrology (Englund 2001) and Susan Pollock has now laid the groundwork for a general model of Near Eastern feasting and commensal politics (Pollock 2003, 2013a, 2013b), but the decisive moment of institutional self-awareness that led to the development of second- and higher-order notations has not been a central topic of investigation in recent work. If we can reduce Mesopotamian institutional life to a slogan, and in the process advance an aphorism that summarizes the intent of this paper: beer codes solidarity, but meat codes hierarchy. In other words, if we want to get to the bottom of the articulation of Mesopotamian elites, we have to look at the distribution of meat and in particular at how meat distribution was documented in the textual record.

Deciphering the Late Uruk Butchering Texts

The decisive question, if we are to make sense of meat distributions in the Late Uruk period (ca. 3300–3000 BCE), is how the individual units or portions were defined.[4] Since only some individuals within the major institutions received meat rations (and multiple portions were closely linked to the highest levels within these hierarchies), the identification of this metric and its metrology within the Late Uruk documentation is of central importance to the question of Late Uruk diacritical feasting. Although there is significant *lexical* evidence that ŠITA$_{a1}$ represented a single portion of meat or fish (see Johnson 2014), the clearest *textual* evidence for this comes from a group of seven or eight Late Uruk tablets in which a quantity of sheep is enumerated in the first line and the remainder of the tablet then lists cuts of meat and other animal products that derive from these sheep.[5] The generic structure of these documents may be seen in a text like W 16731 (= ATU 6, pl. 85), in which six sheep yield 120 cuts of meat (ŠITA$_{a1}$), or 20 portions of meat per sheep (see table 7.1).

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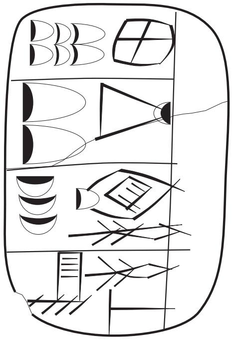
[4] This section recapitulates some material that was published in a paper entitled "Late Uruk bicameral orthographies and their Early Dynastic *Rezeptionsgeschichte*" Johnson (2014). In the interim between submission and publication, parts of this contribution have been republished in Johnson (2016).

[5] The other texts that seem to belong to this administrative genre are W 6066,b (= ATU 5, pl. 3), W 6288 (= ATU5, pl. 5), W 6573,a (= ATU 5 pl. 9), W 6573,b (= ATU 5, p. 9), and W 7343,1 (= ATU 5, pl. 30). In W 6573,a, an unspecified number of oxen are butchered alongside sheep, and the portions of meat from these two sources are carefully distinguished. See also the summary tablets W 21019,1 and W 21022,2; the latter enumerates no less than 870 pieces of ŠITA$_{b1}$ UDU as well as 1200 individual portions of dried fish (ŠITA$_{a1}$ SUHUR).



W 16731 (= ATU 6, pl. 85, copy R. K. Englund)

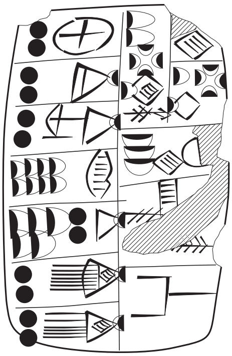
(transliteration)	(translation)
1. 6(N_1) UDU$_a$	Six sheep:
2. 2(N_{34}) ŠITA$_{a1}$	120 portions of meat,
3. 3(N_{39a}) ZATU714×HI*gunu*$_a$ MU	three bowls of soup.
4. ENDIB	The *endib*-official (is responsible).

Table 7.1: W 16731 (= ATU 6, pl. 85 [P002611], copy R. K. Englund).

Many of these texts include GEŠTU$_b$ (rather than ENDIB) in their colophon and enumerate a number of other products whose designations are related to ŠITA$_{a1}$ in one way or another, such as ŠITA$_{b1}$, BA.1(N_{57}).ŠITA$_{a1}$, and ŠA×HI*gunu*$_a$. A nice example of the fuller version of this type of document is W 6066,a (= ATU 5, pl. 3; see table 7.2). Although many of the items in this document cannot be identified, the five commodities that are designated with a sign that is related to ŠITA$_{a1}$ (viz. ŠITA$_{b1}$, BA.1(N_{57}).ŠITA$_{a1}$, ŠITA$_{a1}$, ŠA×HI*gunu*$_b$, ŠA×HI*gunu*$_a$ in lines 2–3 and 5–7 in table 7.2) amount to 400 units in the text, which yields the same ratio of 20 cuts of meat per sheep that we saw in W 16731 (see table 7.1 above). Most of the other examples of this type of document are too damaged to allow for a precise reconstruction of the numbers involved, but those parts of these other texts that do survive show roughly the same ratio.[6] There are also a number of Proto-Elamite texts that exhibit a formal similarity to these butchering texts (particularly in their use of the Proto-Elamite equivalent of GEŠTU$_b$, viz. M36, in initial position) such as MDP 17 (nos. 19, 24, 111, and

[6] The use of {e$_2$.duru$_5$^{ki}} at Ebla for a score of people seems to be unrelated (Milano 1990, cf. the discussion of /uzsula/ in Civil 1984, 162), but it would also correspond to the number of people that can be fed from a single sheep. The system used at Old Babylonian Mari for calculating cuts of meat (and the corresponding method of butchering the animal) only yields ten cuts of meat from a single sheep, Durand (1983, 16–31), *apud* Milano (1998); see also Sasson (2004, 192), particularly in reference to ARM 21, 63. The expression ZATU714×HI*gunu*$_a$ MU remains somewhat enigmatic (see Englund (2001, 12) for the most recent discussion of the term), but it is probably significant that ZATU714×HI*gunu*$_a$ MU appears in these butchering texts and then at the end of the 'grain section' of MSVO 1, 93, immediately before the descriptions of meat. This might suggest that ZATU714×HI*gunu*$_a$ MU was a 'butcher's stew' similar to the stews that form an integral part of the feasts described in the Ur III texts from Garshana, Brunke (2011a, 2011b).

136), MDP 26 (nos. 384, 385, 386), and *Revue d'Assyriologie* 50, p. 202, no. 11, but these will have to be investigated elsewhere.



W 6066,a (= ATU 5, pl. 3, copy R. K. Englund)

(transliteration)	(translation)
obv.	
i	
1. 2(N$_{14}$) UDU$_{a}$	20 sheep
2. 2(N$_{14}$) ŠITA$_{b1}$	20 …
3. 2(N$_{14}$) BA 1(N$_{57}$) ŠITA$_{a1}$	20 …
4. 8(N$_{1}$) KU$_{3a}$	eight units of dairy fat
5. 5(N$_{34}$) 2(N$_{14}$) ŠITA$_{a1}$	320 cuts of meat
6. 2(N$_{14}$) ŠA × HI*gunu*$_{b}$	20 …
7. 2(N$_{14}$) ŠA × HI*gunu*$_{a}$	20 …
ii	
1a. 2(N$_{1}$) 1(N$_{28}$) ZATU714 × HI*gunu*$_{a}$ MU	
1b1. [1(N$_{1}$)] HI*gunu*$_{a}$	
1b2. 1(N$_{1}$) 1(N$_{28}$) ZATU714	
2. 3(N$_{39a}$) HI*gunu*$_{a}$ ⌜SAG$^{?}$⌝	
3. ⌜EN$_{a}$ ŠE$_{a}$⌝ […]	
GEŠTU$_{b}$	The *geshtu*-official (is responsible)

Table 7.2: W 16731 (= ATU 6, pl. 85 [P002611], copy R. K. Englund).

Diacritical Feasts and the Surprising Continuity of Administrative Technique

Now that Late Uruk butchering practices (and the resulting metrology for portions of meat) have been clarified somewhat, we can return to the most important texts for understanding diacritical feasting in early Mesopotamia: the bookkeeping ledgers for institutional feasts that were published in MSVO 1, in particular texts such as MSVO 1, 93 (see figure 7.1). These ledgers are heavily synthetic documents, bringing together a series of individual accounts that each dealt with a particular type of highly valued product. As R. K. Englund has made clear in his review of grain metrology research (Englund 2001), MSVO 1, 93, was divided into two major subsections by a double ruling after ii 3: the initial section (i 1–ii 3) deals with grain products (including breads, pastries, and beer), ending with meat soups or stews in ii 3, while the remainder of the text records discrete objects in the sexagesimal system, including portions of meat and fish (ii 4–5), textiles (ii 8–10), and dried fruit (iii 2–3). The metadata at the end of the text (iii 5) refers to the geographical locale in which the feast took place (NI_a+RU, apparently Jemdet Nasr), gives a calendrical notation for the day on which it was held ($2N_{57}$ SU_a GIBIL), and states that these materials were all ‘consumed’ (GU_7) on that occasion.

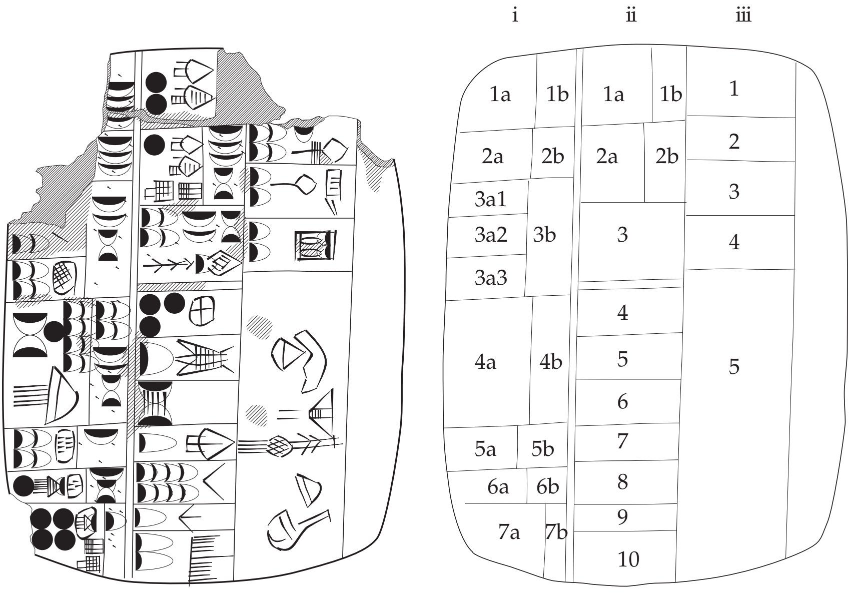


Figure 7.1: MSVO 1, 93 (obverse) [P005160].

One of the defining features of these texts (including MSVO 1, nos. 93, 107, and 108, as well as texts from other sites such as CUSAS 1, 173) is that the number of dried fish (SUHUR) is almost always twice as large as the number of sheep and goats (UDU). Thus in MSVO 1, 93 and 103, as well as CUSAS 1, 173, there are 30 sheep and goats (UDU) and 60 dried fish (SUHUR), while in MSVO 1, 108 and 109, there are 15 sheep and goats (UDU) as opposed to 30 dried fish

(SUHUR). Given the ratio of 20 portions of meat per sheep (UDU) that we described earlier, this would yield 600 pieces of mutton for the tablets that mention 30 UDU or alternatively 300 pieces for those that mention 15 UDU. Although it remains unclear whether these suspiciously round numbers are linked to a system of regular cultic offerings or to the amount required to feed the members of an institution on a particular feast day, it is surprising that we find very much the same ratio and even the same exact numbers in a couple of Ur III feast ledgers from approximately a millennium later (ca. 2100–2000 BCE). In particular, the two mortuary feasts {ki-a-nag} (CUSAS 3, 972 and 975) held for a man named *Šukabta*, who had married into the Ur III royal family, are remarkably similar to Late Uruk texts like MSVO 1, 93. In spite of the millennium that separates these two groups of texts, the types of food and the quantities involved are very similar. And in fact it seems that the same amounts of meat and fish are listed in each of these two sets of documents:

MSVO 1, 93 obv ii 4–5:	15 UDU	30 SUHUR
CUSAS 3, 972 obv i 16′–17′:	15 ki de_5-ga$^{\textit{ar-ki-núm}}$	30 ku_6 al-dar-ra

As Englund (2001, 17, n. 34) has shown, the proto-cuneiform sign SUHUR represented 'a fish that was split' (prior to being salted and dried), which is precisely what {ku_6 al-dar-ra} means in Classical Sumerian, so the equation between SUHUR and {ku_6 al-dar-ra} makes good sense. The parallel between UDU (the standard proto-cuneiform sign for sheep and goats) and {ki de_5-ga$^{\textit{ar-ki-núm}}$} is more difficult to make sense of, primarily because neither {ki de_5-ga} nor its gloss *ar-ki-núm* are attested as descriptions of meat or fish elsewhere in the cuneiform record.[7] If, however, we emend the text ever so slightly (KI and NA differ only by a single horizontal wedge), we arrive at a well-known qualification of sheep and goats ({udu}) as well as oxen ({gu_4}), namely {na de_5-ga}.[8]

We should not fall into the trap of reading purely formal or 'physiognomic' interpretations (Ginzburg 1989, 43–47) back into the historical record, so it is good to keep in mind that a well-defined feasting practice (and the bookkeeping techniques associated with it) may have been retasked or even revivified at a later date for quite different purposes than the original Late Uruk practice. In

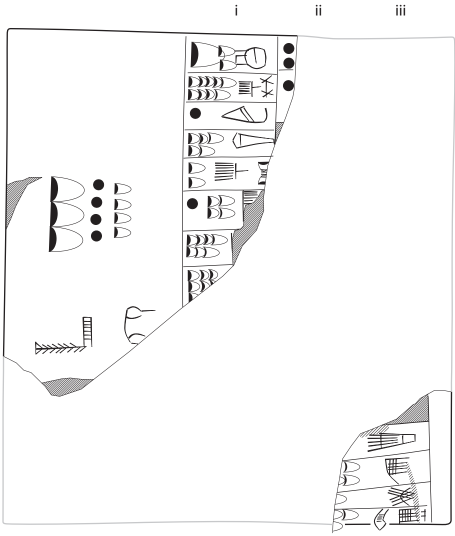
[7] See the evidence collected in Brunke (2011b, 46).

[8] The fact that all four attestations of the phrase in CUSAS 3, 972 and 975, have KI rather than NA and that none of them include the term {udu} suggests that {ki de_5-ga}, literally 'a purified or cleared place,' may have arisen as a euphemism for {udu na de_5-ga} '(ritually) purified sheep (carcass)' in the mortuary context of these texts (in the lexical tradition {udu na de_5-ga} follows immediately after {udu de_5-de_5-ga} 'a sheep that has died of natural causes'). See Klein (1980) and Sallaberger (2005) for an extended discussion of the compound verb {na—de_5(g)}.

other words, there is no reason to assume that the Late Uruk text MSVO 1, 93, necessarily documents a mortuary rite, or that these continuities in bookkeeping represent an unbroken millennium of historical continuity in the narrow sense of the term. Even if the specific goals or institutional contexts of this bookkeeping practice did change through time, however, the overarching continuities in *book-keeping* practice must have been rooted in the intersecting *chaîne opératoire* of a number of different economic and social sectors, ranging from the butchering of animals to the presentation of finished entrees at the feast itself. For example, if we can assume 300 pieces of mutton (15 sheep × 20 pieces of meat per sheep) and a roughly equivalent number of portions from the dried fish were consumed in MSVO 1, 93, these 600 pieces agree quite well with Brunke's suggestion that approximately 600 people would have enjoyed the feast described in CUSAS 3, 972 (Brunke 2011b, 46). Both the Late Uruk and the Ur III feast ledgers are oriented to very much the same goal: the documentation and presumably the auditing of the materials expended on a particular festive occasion. So it is not surprising that their contents and format are similar; the precise replication of the same numbers of animals, however, raises the possibility that some kind of macro-historical social or institutional continuity may lie hidden behind these purely textual similarities. Further evidence is needed, however.

Two Perspectives on Meat Distributions in the Late Uruk Documentation

Unlike feast ledgers such as MSVO 1, 93, which document how various economic sectors contributed to a single festive event, the Late Uruk accountants were also rather fond of second-order descriptions of meat distributions, what we might these days see as a kind of auditing. These auditing records focus exclusively on the distribution of portions of meat and seem to be rooted in the organization and hierarchy of individual offices within a concrete institution. For lack of a better name, we can refer to this unnamed entity as the UKKIN institution on the basis of a well-known lexical list that seems to have originated as its organizational plan. The best evidence for the historical reality of this institution is the Late Uruk text W 14804, a+ (see table 7.3), which recapitulates the major bureaus of the UKKIN institution on the reverse (a list that is nearly identical to the UKKIN list), but also gives the concrete number of staff members assigned to each of these bureaus (224 staff members, viz. EN.TUR, in total). This gives us a pretty clear picture of the scale of the UKKIN institution, at least at one point in its history.



W 14804, a+, reverse

(transliteration)	(position in UKKIN list)
i	
1. 1(geš$_2$) 2(diš) UKKIN	UKKIN 1
2. 9(diš) GAL TE	UKKIN 2
3. 1(u) ZATU 647	UKKIN 4
4. 5(diš) NIM	UKKIN 3
5. 2(diš) GAL KISAL$_{b1}$	UKKIN 5
6. 1(u) 2(diš) ⌜ZATU753⌝	UKKIN 8
7. 7(diš) ⌜x⌝ […]	

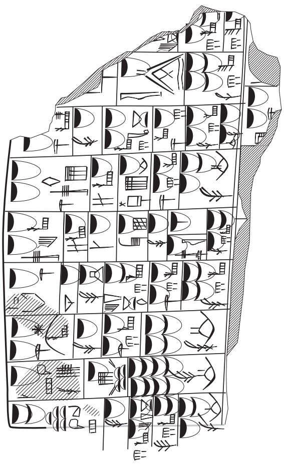
Table 7.3: W14804, a+, reverse [P002195], (columns ii and iii largely destroyed and not transliterated here).

More importantly for our purposes here, however, we know of two texts, which I have elsewhere spoken of as *subordinate staff lists*, namely MSVO 1, 112, and IM 73409,2, which break down the distribution of meat and fish by individual office-holding recipients. Recipients who occupied particular nodes within the hierarchy of the institution received a portion of meat and were marked in these lists with the term ŠITA$_{a1}$, while other recipients in other hierarchical nodes received a portion of dried fish and were marked in these lists with the term UKKIN$_a$.[9] Take, for example, the following small section from MSVO 1, 112, in which a series of office-holders that are designated as ŠITA$_{a1}$ are listed:

[9]The metadata in MSVO 1, 112, make it fairly certain that the labels ŠITA$_{a1}$ and UKKIN$_a$ correspond in a meaningful way to 'sheep and goats' (UDU) and 'dried fish' (SUḪUR), but it remains unclear whether ŠITA$_{a1}$, for example, corresponds to UDU or SUḪUR. For the sake of exposition, I assume here the former equation, but it still needs to be demonstrated.

MSVO 1, 112 ii
1′. […GAL$_a$] **ŠITA$_{a1}$**
2′. […] ⌜GEŠTUb⌝ **ŠITA$_{a1}$**
3′. ⌜1(N$_1$) NUNa+ENa⌝ **ŠITA$_{a1}$**
4′. ⌜1(N$_1$)⌝ EN AMAR **ŠITA$_{a1}$**
5′. 1(N$_1$) NAM$_2$.KAB **ŠITA$_{a1}$**
6′. 1(N$_1$) AN ŠU$_2$.EN **ŠITA$_{a1}$**

This short extract simply represents an audit, listing a group of six office-holders who received a portion of meat on a given occasion. Thanks to a second text (W 14777,c), however, we can actually see how these six individuals were distributed within the hierarchical structure of the UKKIN institution. In table 7.4, I have lined up the foregoing entries in the extract from MSVO 1, 112, with a detailed census of a particular group of bureaus and their staff members in W 14777,c (numerals and indication of damage have been largely omitted).



W 14777,c i	MSVO 1, 112 ii
1′. […]	1′. […GAL$_a$] **ŠITA$_{a1}$**
2′. […] GEŠTU$_b$?	2′. […] GEŠTU$_b$ **ŠITA$_{a1}$**
3′. NUN$_a$+EN$_a$ **BA / GI**	3′. NUN$_a$+EN$_a$⌝ **ŠITA$_{a1}$**
4′. ḪI E$_2$.NUN BA (not followed by GI)	
5′. EN ŠU NUN	
6′. EN AMAR **BA / GI**	4′. EN AMAR **ŠITA$_{a1}$**
7′. AN ŠU$_2$.EN **BA / GI**	6′. AN ŠU$_2$.EN **ŠITA$_{a1}$**
PAP NAM$_2$.KAB U$_2$ GI	
9′. LAM$_a$ SI.ME$_a$ NAM$_2$.KAB	5′. NAM$_2$.KAB **ŠITA$_{a1}$**

Table 7.4: W 14777,c [P002179] in comparison to MSVO 1, 112 [P005179], (W 14777,c column ii not transliterated here).

As the parallels between BA/ GI in W 14777,c and ŠITA$_{a1}$ in MSVO 1, 112, suggest, bureaus in which the first subcase includes BA such as NUN$_a$+EN$_a$, EN.AMAR, and AN.ŠU$_2$.EN tend to be more or less immediately followed by a subcase that includes GI (only ḪI.E$_2$.NUN in line 4′ does not have a corresponding GI subcase), and where both a BA and a GI entry occur in a given line, we find a corresponding entry for the same bureau in the secondary staff list MSVO 1, 112. Stated somewhat differently, those officials who were qualified with the ŠITA_{a1} sign in MSVO 1, 112 (presumably as recipients of a portion of meat) belong to bureaus in which the administrative terms BA and GI were used to distinguish certain staff members.

The significance of these administrative terms is far from clear, and as Englund has emphasized,[10] materials qualified with BA and GI are occasionally added together, so they cannot be equated in a facile way with materials that are 'booked out' and 'booked in' respectively. Nonetheless, comparison of these two texts (MSVO 1, 112, and W 14777,c) shows how two different genres of administrative documentation (an audit of meat distributions in MSVO 1, 112, and an organizational chart in W 14777,c) represent two different perspectives on a single fact in the life of the UKKIN institution. It may well be the case, of course, that these two texts arose in response to distinct historical events, but the importance of them for us here is that they represent two distinct second-order representations within the bureaucratic apparatus of a single type of historically contingent activity.[11]

Coda

In the same way that Friberg's typology of bricks *defines* the potential of Mesopotamian architecture or Damerow and Englund's work on beer and bread production *defines* the Mesopotamian rationing system, I would like to suggest that meat distribution *defines* elite institutional hierarchies in Mesopotamia. Moreover, in doing so, the distribution of highly valued goods in the context of diacritical feasts provides the essential link between what we might call primary economic activities (baking, brick-making, and butchering) and the second- or higher-order notational and managerial practices that sought to harness these primary activities to macro-social projects of one kind or another.

The durability of each of these distinct *first-order external representations*, as Damerow would have called them (the grain capacity system, a typology of bricks, or the way a specific butchering technique is registered in the textual record), is not an isolated fact, however. Each of these epistemological devices exists not in a vacuum, but in a coordinated system of other first-order external

[10] See the diagram of MSVO 1, 185, in Englund (1998, 63, as well as the discussion on pp. 76–77).

[11] For a general discussion of second- and higher-order administrative documents, see Cancik-Kirschbaum (2016). Much the same idea is captured by Visicato's description of "phases of compilation," see references collected in Foster (2005, 84, and n. 28) and several other discussions of early Mesopotamian accounting techniques.

representations. The fact that a set of coordinated first-order representations survive over an extended period of time already demonstrates a degree, however implicit, of second-order institutional awareness. But perhaps alone among the earliest records of human thought, the bookkeeping records from Mesopotamia in the Late Uruk period (ca. 3300–3000 BCE) already provide explicit evidence of *second-order external representations* such as the contrast between W 14777,c and MSVO 1, 112, that we looked at a moment ago.

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Chapter 8
Pappus' Theory in Mesopotamian Science

Markham J. Geller

As Woody Allen once famously said, "Confidence is what you have before you understand the problem." This was exactly my own situation when I decided to explore whether Babylonians ever employed theoretical propositions or hypotheses. Like Mr. Allen, I was confident of the results, that Babylonians and Greeks were miles apart as far as theory is concerned, and that no real connections between Greek and Babylonian theoretical sciences ever existed.

Let us begin with a relatively simple example, namely pre-Socratic theory regarding the composition of all matter deriving from four basic elements, earth, air, fire, and water, with their associated characteristics of being cold, hot, wet, and dry. No corresponding comprehensive concise statement of cosmological theory has turned up in Mesopotamia, nor is it likely to turn up; for one thing, there is no Akkadian term for "air," nor can we find any Babylonian description of the origins of matter, beyond the usual mythopoeic clichés. This is the typical pattern frequently discussed in modern secondary literature, that Babylonians were good at assembling data but were unpracticed at formulating rules. But as we shall see, this may not be entirely correct.

My view began to change after considering the work of Pappus of Alexandria, which claims to reflect the thinking of Euclid and his contemporaries. Here is what Pappus says:

> Analysis is a method of taking that which is sought as though it were admitted and passing from it through its consequences […] for in analysis we suppose that which is sought to be already done, and we inquire what it is from which this comes about, and again what is the antecedent cause of the latter, and so on until, by retracing our steps, we light upon something already known or ranking as a first principle; and such a method we call analysis, as being a reverse solution. But in synthesis, proceeding in the opposite way, we suppose to be already done that which was last reached in the analysis and arranging in their natural order as consequents what were formerly antecedents and linking them with one another, we finally

> arrive at the construction of what was sought; and this we call synthesis. (Thomas 1957, 597)

Marshall Clagett, the noted American historian of science, explained this distinction between *analysis* and *synthesis* as "one of the most important of the general methodological achievements of the early Greek mathematicians," and he explains that *analysis* "commences with the assumption of what is to be proved and then proceeds backward by successive inferences," that is, to prove the hypothesis correct. *Synthesis* works in the opposite direction, "starting with the previously accepted or proved theorem and proceeding therefrom to the proof of the new theorem." This was all explained by Clagett and many others as an example of the so-called "Greek miracle" of the fifth century BCE, during which time the Greeks far surpassed their predecessors in defining new areas of scientific thinking.[1] But could Pappus' important dictum apply to Babylonian science as well?

Mathematics

The first place to turn to is mathematics, always the best starting point for discussions of scientific method. We now know that the famous Pythagorean theorem, $a^2 + b^2 = c^2$, was known to Babylonian mathematicians a millennium before Pythagoras, but without ever formulating the rule as such (Damerow 2001). Incidentally, Babylonians always fall short of expectations by not being able to formulate simple theorems. But what we cannot judge is how Babylonian mathematicians explained their texts, since it is possible that a typical discussion within an ancient Babylonian academy may well have recognized $a^2 + b^2 = c^2$ as a rule, but never wrote it down. But why write it down? Was anyone to know that someone some 3700 years later might be interested in what was being taught in a Babylonian classroom?

However, it does seem that Babylonian mathematicians indeed worked backwards from a hypothetical proposition, à la Pappus' category of *analysis*. A typical math problem from Old Babylonian Uruk (c. 1700 BCE) computes how many man-days are required for four different teams of workers to build a ramp consisting of sections of unequal lengths. After a long and detailed calculation, the answer is 15 (as a position in the sexagesimal system, which is equal to $15 \times 60 = 900$) (Friberg 2007, 291). The only reasonable conclusion is that Babylonian mathematicians, long before Euclid, were employing the *analysis*-method of working backwards from a given answer—in this case a nice round number—in order to prove how one arrives at the hypothesis through mathemati-

[1] Clagett (1966, 33) is nuanced in his approach and his comments on Egyptian and Babylonian science are quite sober.

cal calculations. This method is standard for Babylonian mathematics and Pappus would have appreciated it.

Divination (Forecasting)

It would be useful to turn to other fields to see if Greek scientific methods have any Babylonian antecedents. We begin with celestial omens, to see if any of these patterns can be identified. Here is a typical example from the classic text *Enūma Anu Enlil*, from the sections dealing with movements of the planet Jupiter (Reiner and Pingree 2005, 40–41):

> If Jupiter (and) Venus stand against each other: the enemy will conquer the the land, as much as there is.
>
> If Jupiter (and) Venus 'eat' one another and stand against each other: end of the dynasty of the king of Amurru (i.e. the West).
>
> If Jupiter (and) Venus vie with one another and stand against each other: end of the dynasty of the king of Amurru.
>
> If Jupiter reaches Venus and they follow upon each other: the flood will come but will not irrigate the field of the commons.
>
> If Jupiter reaches Venus and passes her: flood in the land will be scarce.
>
> If Jupiter stands in the rear of Venus: there will be massacre in the land.
>
> If Jupiter enters into Venus: the prayer of the land will be in the heart of the gods.

Often these omens are repetitive (Reiner and Pingree 2005, 136–137):

> If Jupiter has a 'flare-up' (*ṣirhu*) in the evening watch toward the North: *rapādu*-disease will seize the head of Akkad.
>
> If Jupiter has a 'flare-up' (*ṣirhu*) in the middle watch toward the North: *rapādu*-disease will seize the middle of Akkad.
>
> If Jupiter has a 'flare-up' (*ṣirhu*) in the morning watch toward the North: *rapādu*-disease will seize the foundation of Akkad.

> If Jupiter has a 'flare-up' (*ṣirhu*) in the evening watch toward the South: *rapādu*-disease will seize the head of Elam. etc.

The typical pattern is that there are many more omens in the protases, in the "if-clause," than anticipated results in the apodoses clauses which follow. For instance, in a similar series of omens from the planet Venus, the results are that the king's son will be killed by his father, a city will be torn down, there will be disruption in the land, there will be a year of remission of debts or lamentation in the land, there will be revolt and famine, etc. (Reiner and Pingree 1998, 93). A similar sequence of lunar eclipse omens combined with winds from the four cardinal directions will result in the royal lands being destroyed, the land will suffer from calamity or the king will die (Rochberg-Halton 1988, 104). What we see is a finite number of such results on the right of the equation in contrast to a vast array of omens and signs on the left side of the equation.

Other examples of the imbalance between the variety of omens on the left side of the equation and the repetitive themes on the right hand column of omen results can also be found in other genres, such as in terrestrial Šumma Alû omens. Cf. the following (Freedman 1998, 296)[2]:

> If a flash of light (*birṣu*) appears in someone's house, scattering of the man's household.
>
> If a flash of light (*birṣu*) appears on the wall in someone's house, worry for the house.
>
> If a flash of light (*birṣu*) appears on the south wall in someone's house, the owner of the house will die (or) worry for the house.
>
> If a flash of light (*birṣu*) appears on the north wall in someone's house, the lady (or) owner of the house will die.
>
> If a flash of light (*birṣu*) appears in the bedroom of someone's house, a daughter of the house will die.
>
> If a flash of light (*birṣu*) appears on the exterior wall of someone's house, a house-slave will die.

[2] See also Freedman (1998, 133), in which a sequence of omens all have the identical results, i.e., a house will be deserted; in another sequence, the city will be deserted (Freedman 1998, 41). In other instances, in each omen a king of various regions will die (i.e., the king of Guti, Subartu, Akkad, Lullubu, Hanu, or Amurru); see Freedman (1998, 41). In yet another sequence in which various images of a Bailiff-demonic (*rābiṣu*) are found in someone's house (e.g., white, black, red, or yellow-green images), the household will be scattered (Freedman 1998, 277).

> If a flash of light (*birṣu*) appears on the wall of an upper room or drain pipe of someone's house, a daughter-in-law of this person's household will die.
>
> If a flash of light (*birṣu*) appears on an exterior wall[3] of someone's house, husband and wife will be divorced and the household will be scattered.
>
> If a flash of light (*birṣu*) like lightning appears in someone's house and has struck someone, that house will have a *mukīl rēši*-demon.

The same pattern occurs in many other genres as well, such as omens derived from the birth of a miscarried foetus, or so-called *Šumma Izbu* omens (Leichty 1970, 78):

> If a ewe gives birth to a lion, and it has the face of an ass—there will be severe famine in the land.
>
> If a ewe gives birth to a lion and it has the face of a dog—pestilence.
>
> If a ewe gives birth to a lion and it has the face of a pig—the lady (of the house) will die.

The predicted results of famine, pestilence, and death are all standard. Of course, from a formal point of view, all these omens represent *post hoc ergo propter hoc* fallacies (Bottéro 1982, 426). Consider the following terrestrial omens from the earlier Old Babylonian period (George 2013, 95, 20'–22'):

> If in a man's house, in the toilet, a green shoot is sprouting: [the man's household] will obtain food in future.
>
> If in a man's house snakes kiss each other: the population will diminish(?).
>
> If in a man's house mongooses kiss each other: he will achieve greatness.

It is not possible for us to know how such predictions were derived from an almost infinite range of omen topics, but this is where Pappus' observation on *analysis* vs. *synthesis* becomes relevant. In all of the above cases and many more, we can surmise that the first stage in this process was the identification of the result:

[3] Lit. desert-wall.

the death of a crown prince, regime change, an enemy at the borders, the death of a wife, pestilence, crop failure and famine or even a good crop, and so forth. The obvious way for such a list to be composed is to begin with a hypothetical or actual event: a failed military campaign, a devastating flood, or a bad harvest. The obvious next step is to ask what unusual event happened previously which might have forecast that such a thing would take place; in other words, first comes the event, then working backwards to establish the predictive omens. An earthquake occurs and then one asks if anything unusual happened lately; whatever that was, becomes the omen. Essentially, what we are suggesting is the opposite of what has been proposed for the logic of Babylonian omens: the usual assumption is phrased as, "if P, (then) Q," meaning "on the condition that (or supposing) P, Q happens, or will happen" (Rochberg 2010, 377–378), while we are supposing a somewhat different formulation, at least in the first stages of a proposition: "if Q, then P." Once the pattern has been established and the omen has been identified, then one can revert to the usual pattern of our omen texts, "if P, (then) Q," as described by Rochberg (2010, 373–397).

There is a complication, as always. Not all omens are negative (like most of the ones already cited). Omens based on examining entrails of sheep or birds have specific rules which apply, consisting of complex calculations based upon whether the omen occurs on the right or left side of the particular organ being examined, or in later Latin terminology, the *pars familiaris* or *pars hostilis* (Maul 2013, 86). Here is a rudimentary example (Koch-Westenholz 2000, 190):

> If there are two Paths [on the sheep's liver] and the right one lies above the left one: the forces of the prince will prevail over the forces of the enemy.
>
> If there are two Paths and the left one lies above the right one: the forces of the enemy will prevail over the forces of the prince.

Omens of this kind depend upon technical laws of divination—not simply a binary "yes" or "no" answer to a question—within a system as complicated as astrology, although based on different assumptions.[4] But luckily, this does not change the overall picture as defined by Pappus of Alexandria, who further explains *analysis*: "by retracing our steps, we light upon something already known or ranking as a first principle" (Thomas 1957, 597). In other words, ancient divination worked

[4]See Maul (2013, 64–109) on the complications of extispicy in first-millennium BCE omen texts. In fact, each defined area of the sheep liver is divided into upper, middle, and lower registers, each of which is subject to examination on the basis of right-left orientation of any deformities, thus applying binary opposition to numerous statistical variations. The final answer to the omen inquiry is derived from a calculation based upon numerous variables.

backwards from a general hypothesis, namely that *the answer to an oracle question can be found on the sheep's liver*, and one then works backwards step-by-step to test the proposition, examining each of the approximate 100 signs or indicators to see whether each is right or left, positive or negative. This is, in fact, like Pappus' "something already known" or "first principle," since "right" and "left" in all omen texts represent markers of positive and negative propositions, which become the basis for a statistical calculation.

Figure 8.1: Babylonian model of a liver (© Trustees of the British Museum).

Astronomy

But would *synthesis*, as defined by Pappus, have existed within this system? This would mean gathering signs or omens, working in a forward fashion to predict an event, without the benefit of hindsight. In fact, something new occurs in Babylonian science, beginning in the seventh century BCE, which does actually reflect Pappus' category of *synthesis*—the so-called Astronomical Diaries. These texts are not omens but records of celestial events, carefully recorded on a daily basis in Babylon, and the celestial data is correlated with market prices, weather, height of the river, and occasional significant events, such as Alexander the Great entering Babylon. Here is a short extract from an Astronomical Diary, from 419 BCE (Sachs and Hunger 1988, 64f):

> Night of the 29th, Mercury's [first appearance] in the west [in Aries]. […], barley, 3 sut 3 sila; at the end of the month 3 sut 4 sila; dates at the beginning of the month and at the end of the month […] … mustard, 2 pan 3 sut; cress, 3 sut; sesame, 1 sut; wool, 50 shekels … Jupiter was in front of Mercury; Mercury was in Aries; Saturn was in the end of Pisces; Venus and Mars, which had set, [were not visible]. Month XII, around the 1st, Saturn's last appearance in the end of Pisces. The 6th, a ewe gave birth, and (the young) had no jaw. (–418 BCE)

Figure 8.2: Babylonian Astronomical Diary (© Trustees of the British Museum).

Similar diaries mention that there was rain or even hail, or thunder, or that the river rose 20 fingers, and so forth. The interesting detail in the passage cited here is the reference to a lamb being born without a jaw, which had obvious omen significance, although no prediction was being made. Nonetheless, it would have been easy to look up this sign in a compendium of *Šumma Izbu* omens, as we saw earlier, to see what this omen might mean.

There is, however, a major difference between the astronomical diaries and classic divination. Astronomical diaries represent a major shift in Babylonian scholarship towards *synthesis* rather than *analysis*, in which the calculation no longer works backwards from a given hypothesis, but data is assembled in order to establish a new hypothesis. The purpose of the astronomical diaries is to record the data for astrological predictions, with the assumption that celestial movements influence terrestrial events, in particular agriculture, weather, and even politics. The way for ancient scholars to make the case for celestial influence was to gather all relevant facts on a given day, day after day, to establish the connections. This methodological innovation is supported by one other important bit of information from Babylonia.

Not long after the introduction of Astronomical Diaries, we see the beginnings of the zodiac in Babylonian astronomy and astrology, which is a major development for a variety of reasons. One characteristic of the zodiac is its simplicity: it replaced a Babylonian star and constellation list which consisted of some 300 lines; a meagre listing of 12 zodiac signs could be mathematically adapted to map the heavens in an unprecedentedly precise manner. What we lack from Babylonia, of course, is any theoretical treatise on the zodiac and how it was to be used, but the obvious simplicity of this mechanism is a statement in itself, that we are moving in the same direction as Thales and Heraclitus and others in attempting to simplify the way the data is presented and evaluated.

Medicine

Let us consider how this system would work in medicine. The Babylonian *Diagnostic Handbook* is a lengthy collection of symptoms corresponding to some parts of the Hippocratic Corpus. The *Handbook* is partially arranged from head to foot according to human anatomy, with other sections relating to epilepsy and related diseases, gynecology, and pediatrics; in all, this text consists of some 40 tablets or chapters listing several thousand different symptoms, all ending in either a prognosis or diagnosis. The point about the *Diagnostic Handbook* is that it is about disease and not patients; it does not record individual case histories. This means that numerous symptoms for a considerable number of diseases were divided up according to parts of the human body; that is, all the symptoms affecting the head, eyes, ears, nose, mouth, neck, and so forth, were listed separately, according to their associations with human anatomy, rather than listing all of the relevant symptoms for each disease (Geller 2005, 254–255):

> (Foot Disease)
>
> If his calf on the left *is short* and [throws up] dark-red blood from his mouth […].
>
> If his foot on the right hurts him, it is the Hand of Ištar; if his left foot [hurts him, it is the Hand of …].
>
> If (both) his feet hurt him, it is the Hand of Inanna; if his feet hurt him, it is the Hand of …, he will recover].
>
> If he drags his foot on the right (and) his mouth twitches, [it is a stroke (caused by) the Snatcher(-demon), it will be prolonged and he will die].
>
> If he drags his foot on the left, [he has been struck a] blow by the [Hand] of Baba, [he will die].
>
> [If] his foot on the right keeps shrinking, it is the Hand of Ištar; it is seizure by a ghost, [he will die].
>
> [If] his foot on the left keeps shrinking, it is the Hand of his God; if (both) his feet keep shrinking, [the Hand of a God; evil has seized him].

Why did they do this? One reason might be purely practical, in that a practitioner could look up which diseases might correspond to any single symptom, for example, yellow spots in the eyes or blood in the urine; we should think of the ancient *Diagnostic Handbook* as the equivalent of an index to a modern medical textbook. But one thing is clear, that it would be virtually impossible to diagnose diseases based on the several thousand symptoms scattered throughout this text, or even to recognize the diseases from this arrangement of symptoms. On the other hand, applying Pappus' methodological principle of working backwards goes a long way to explaining this puzzling text: the *Diagnostic Handbook* was assembled by working backwards from the diseases themselves. Once a disease was hypothetically recognized by a given set of symptoms, based on observation, the next step was relatively easy, to identify and organize the individual symptoms according to various parts of the human body.

If this is the case, then can we find evidence for *synthesis* in Babylonian medicine? This is not an easy question to answer, since the system of therapeutics is already well established relatively early in the second millennium BCE, and thus predates the type of distinction we have noted above between traditional astronomical divination (in *Enūma Anu Enlil*) and astronomical diaries. Nevertheless, there may be some indications of *synthesis* in medical recipes, if we consider the following therapeutic recipe for rectal disease dating roughly from the eighth BCE (Geller 2005, 145):

> If (a man's) limbs are limp and his chest and back hurt him, his arms, shins and knees hurt him, his right or left testicle aches him, and he shows blood in his urethra, that man suffers from stricture of a diseased rectum. To cure him …

This, then, could qualify as *synthesis*: the relevant data is assembled via observation in order to establish an hypothetical diagnosis, which is especially noteworthy since none of the symptoms actually refers to the anus, but to other parts of the body. This is unlikely to be a case of the backward logic of *analysis*, since each of the symptoms would be more realistically associated with other typical medical conditions within Babylonian medicine, such as kidney-disease or being lame. The question is whether there is any development in diagnostic methods as compared to earlier periods. Among the relatively few examples of medical texts stemming from the second millennium BCE, a pattern appears which differs considerably from the above-cited recipe for diagnosing and treating rectal stricture, since descriptions of symptoms from earlier periods tend to be brief and self-explanatory, such as the following:

> If a man is behexed, you dry out …
>
> If a man is ill with jaundice, you soak …
>
> If a man's tooth is attacked by a worm, you grind up … etc. (Schwemer 2010, 37)[5]

indicating relatively simple diagnoses to be treated. Occasionally, descriptions of symptoms are more elaborate, such as the following from a mid- to late-second millennium medical tablet:

> If a man has pain in his kidney, his groin constantly hurts him, and his urine is white like donkey-urine, and later on his urine shows blood, that man suffers from 'discharge' (*muṣû*-disease). (Geller 2005: 35, 23'–24')

There is no mystery here, since symptoms are all associated with the penis and the logic is transparent. The theme of witchcraft as a medical problem is also attested in mid-second millenium BCE Akkadian sources from the Hittite capital Hattuša, and these popular texts were constantly recopied in later periods,

[5] See the bibliography in Schwemer (2010, 38–39).

with somewhat more complex patterns of diagnosis than those just discussed. In one case, problems affecting the hips and toes are ascribed to the patient having walked in unclean water, while in another case his various symptoms (paralysis, fever, impotence) are explained as having been caused by witchcraft generated through a buried figurine of the patient (Schwemer 2010, 39). There is relatively little sophistication in the diagnosis, since witchcraft was standardly associated with certain physical and mental conditions in divination, magic, and medicine. There is little evidence of *synthesis* within these earlier texts and in fact the presence of *synthesis* in medical texts in general is open to question.

What we lack, in any case, is any simple statement of theory, or any rule which tells us how these symptoms are collected and sorted in order to produce the hypothetical diagnosis. In the *Diagnostic Handbook*, symptoms are often described as red, yellow, black, and white, or moist or dry, swollen or distended, similar in many respects to what is found in Greek medical writings, but without justification in Babylonia. But we can detect one step in the direction towards theory in Babylonia: simplification. One Late Babylonian tablet from the Achaemenid Babylonia, from roughly 500 BCE, was published by Irving Finkel as the *"Poor Man's" Diagnostic Handbook* (Finkel 1988, 153), in which on this single fragmentary tablet symptoms of the body are listed from head to foot; all forty chapters of the original *Diagnostic Handbook* are reduced to a text of some 25 lines. Here is a translation.

> 1 [If the patient's head(?) continually hurts him and he] constantly cries out, (it is) the Hand of Anu (var. another god).
>
> 2–7 [(If) his …] continually hurts him, (it is) the hand of [(a god)].
>
> 8 [(If) his … continually hurts him, (it is) the hand of Marduk.
>
> 9 [(If)] his [… continually hurts him], it is the hand of Anu.
>
> 10 (If) his *mouth* continually hurts him, (it is) the hand of Adad.
>
> 11 (If) his tongue and its *vessels* continually hurt him, (it is) the hand of Lisi.
>
> 12 (If) his chest continually hurts him, (it is) the hand of Ishtar.
>
> 13 (If) his shoulders continually hurts him, (it is) the hand of (gods) Shullat and Hanish.
>
> 14 (If) his right side continually hurts him, (it is) the hand of Ishtar.

15 (If) his left side continually hurts him, (it is) the hand of [(a god)].

16 (If) his insides are continually inflamed, (it is) the hand of [(a god)].

17 (If) his loins continually hurt him, (it is) the hand of [(a god)].

18 (If) his right foot continually hurts him, (it is) the hand of [(a god)].

19 (If) his left foot continually hurts him, (it is) the hand of [(a god)].

20 (If) his *brain* is struck, it is the hand of Zababa.

21 (If) his … continually hurts him, (it is) the hand of [(a god)].

22 (If) [his …] continually hurts [him, (it is) the hand of (a god)].[6]

There is another example of simplification in a unique late text from Uruk, from approximately the same period as Finkel's *Poor Man's Handbook*, in which a list of diseases within the body is associated with four internal organs. This text is more complicated than it looks, but it clearly reflects some kind of new theoretical thinking in Babylonia which has not appeared earlier, and this theory is expressed in a concise and simplified schematic form. Here is an extract (Geller 2014, 3):

ul-tu KA *kar-šú mu-ru-uṣ*	from the throat, head and
sag.du(*qaqqadi*) *u pi*-[*i*]	mouth disease
KI.MIN *pi-i šin-ni*	ditto, mouth, teeth their
⌜mur.dúr⌝.meš-*šú-nu*	'toothworms'
KI.MIN MIN ⌜*gir*$_{11}$⌝-[*gi*]-⌜*iš*⌝-*šum*	ditto, ditto, red skin lesions
KI.MIN MIN $^{\text{d}}$dìm.me	ditto, ditto, Lamaštu
KI.MIN MIN *pa-šit-t*[*u*$_{4}$] $^{\text{mí}}$*mar-tu*$_{4}$	ditto, ditto, Pašittu-daughter
KI.MIN ⌜*ma-li*⌝ *me-e*	ditto, dropsy
KI.MIN *qāt etimmi* (šu.gedim.ma)	ditto, hand of ghost
KI.MIN *maš-ka-du*	ditto, joint disease

From about this same time, we also have a unique tablet, again from Babylon,[7] containing a table of domestic and wild animals associated with zodiac signs in

[6] These attributions of symptoms to the hand of a god are likely to be technical rather than pious.

[7] Heeßel (2000, 112–130); see also the discussion in Geller (2014, 87–88). The text is also relevant to the Microzodiac, treated in the forthcoming dissertation of Marvin Schreiber.

numbered sequences, as well as material medica and dietary restrictions (not to eat leeks, pork, wheat, or drink milk) on certain days of the month, each associated with a zodiac sign. This late, unique, and exotic text, a type of zodiacal hemerology, once again attempts to simplify and record astrological and related data in a tabular form which could potentially be used to develop general rules as applications of astrology.

The Role of Motion and Concluding Remarks

There is one additional aspect of ancient Babylonian science which is worth considering. The matter is speculative, but in an article devoted to Peter Damerow, this kind of speculation is quite appropriate. It has long been noticed that traditional Babylonian divination, such as the liver omens described above, gave way to astrology and soon after the advent of the zodiac, other forms of predicting the future became increasingly obsolete. Although many reasons are given for this profound change in thinking and practice, one factor appears to have escaped notice. Most of Babylonian divination is static: a phenomenon is witnessed and recorded and an event is associated with the portent, for example, an unusual birth, an unusual node on a sheep's liver, or even the position of a star or planet in relation to fixed stars. The novelty of astrology and divination based on the zodiac, as well as astronomical diaries, is that predictions were based upon motion and movement of the stars, rather than on their static positions. The astronomical diaries, for instance, record the position of the moon at regular intervals during the month, noting its constantly changing position in relation to zodiac signs, and even remarking that the moon "passes a little to the east" (Sachs and Hunger 1988, 195, 33). The point of these observations is that the *movement* of the moon and planets was responsible for astral influences on terrestrial events, as mentioned above.

The reason why this change in perspective is significant is because the same interest in motion and movement became a basic tenet of early Greek science, especially the idea that movement generated power. We return again to Clagett (1966, 52):

> Constant attention to change and movement and speculation as to the causation involved stimulated the basic idea that when things change or move there are activating forces—no longer mythological but physical forces […]. The early emphasis on force, power, action is also reflected in the medical works contemporary with the philosophical activity. Thus Plato says that Hippocrates (b. at Cos in 460? BCE) holds that to find the fundamental nature (*physis*) of a

> thing we must examine its "power," or *dynamis*—i.e., its capacity of acting or being acted upon.

Liba Taub observes something similar in the writings of Aristotle:

> Aristotle acknowledged that the idea of the primacy of local motion had been around for a long time; he noted that many of his predecessors who had studied motion had treated local motion as the primary principle of change. (Taub 1993, 37)

What strikes us here is not only Aristotle's own attention to motion, but that earlier thinkers have been speculating about motion as well. It should not therefore surprise us that Babylonian astronomers charting the heavens also considered motion and movement of celestial bodies as worthy of careful notation. Greek philosophy naturally proceeded in a much more sophisticated way in identifying various types of motion within physics,[8] but this does not detract from the possibility that Babylonian scholars were charting celestial motion as theoretically relevant to predicting the future.

So what are the implications of this data for our view of ancient science before the Greek "miracle" suddenly appeared in Miletus and elsewhere? First of all, we still do not have evidence of Babylonian theory from academic treatises, but we can detect some movement in the right direction, that is, moving closer to the pre-Socratics. The tendency towards schematic presentations of data in tabular form is a feature of Late Babylonian scholarship, more-or-less contemporary with the beginning of Greek philosophy, and the similar tendency towards simplification of a large amount of complex data probably indicates greater interest in developing general rules. But where are these general rules from Babylonia? We lack them, but the example of the zodiac gives a pretty clear picture of how a very elaborate system of celestial divination with thousands of individual clauses could become boiled down to a bare minimum of characters—12 zodiac signs—that can be manipulated mathematically. But perhaps the most interesting implication of all is that Pappus of Alexandria describes a methodology for Greek mathematics, which applies to Babylonian science as well. It seems that Babylonian astronomical diaries, a new genre of scientific texts, represent clear examples of *synthesis*, that is, accumulating data in order to formulate a new hypothesis about celestial influences or astrology, while more traditional forms of Babylonian divination (including medical diagnostics) relied upon *analysis*, which worked backwards from a proposed hypothesis or proposition.

[8] See Taub (1993) in relation to Ptolemy.

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Chapter 9
Astronomical Knowledge in *The Slavonic Apocalypse of Enoch*: Traces of Ancient Scientific Models

Florentina Badalanova Geller

The celestial cosmography revealed in *The Slavonic Apocalypse of Enoch* (also designated among the specialists as *The Book of the Holy Secrets of Enoch*, or *2 Enoch*)[1] follows the sevenfold pattern of Creation, thus implicitly referring to the biblical scenario of *Genesis* 1–2, which is also reflected in other Judeo-Christian apocryphal writings, along with Rabbinic tradition and Byzantine hexameral literature. The symbolism of *seven* as the hallmark of esoteric wisdom in *2 Enoch* is reinforced by the fact that the visionary himself is born seven generations after Adam,[2] thus completing the first "heptad" of antediluvian ancestors and becoming its "Sabbatical" icon. Seven is also the number of heavens through which

[1] The proto-corpus of apocryphal writings attributed to the biblical patriarch Enoch was originally composed in either Hebrew or Aramaic, probably no later than the first century BCE, and after the discoveries of the Dead Sea scrolls from Qumran it became clear that some of its segments may be dated to the end of the third and beginning of the second century BCE. This ancient proto-corpus of writings was the intellectual ancestor of three main offspring: Ethiopic (*1 Enoch*), Slavonic (*2 Enoch*), and Hebrew (*3 Enoch*). As for the Slavonic corpus of *2 Enoch*, it must have originated from an earlier Greek edition of the apocryphon based on a Hebrew or Aramaic *Vorlage*; this (no longer extant) Greek protograph was fostered by Septuagint-related tradition and further influenced by Christian ideology of the Byzantine Commonwealth. The translation from Greek into Old Church Slavonic (using the Glagolitic script) was made most probably by a scholar close to the scribal circle of the Preslav Literary School, no later than the eleventh century. Various editions of a number of Church Slavonic text-witnesses were published by Popov (1880), Novaković (1884), Sokolov (1899, 1910), Ivanov (1925, 165–191), Vaillant (1952), Navtanovich (2000, 204–241), Jovanović (2003), Reinhart (2007), Mil'kov and Polianskii (2009, 459–493), Macaskill (2013), and others. For English translations of the apocryphon (with commentaries), see Morfill and Charles (1896), Andersen (1983), Pennington (1984), Badalanova Geller (2010), Macaskill (2013). See also the discussion in Bonwetsch (1896, 1922), Meshcherskii (1963, 1964), Alexander (1998), Böttrich (1991, 1995, 1996, 1997, 2012), Orlov (2004), Khristova (2008), Badalanova Geller (2012, 2014, 479–482, 2015). Several essays gathered in *New Perspectives on 2 Enoch* (2012) (under the editorship of A. Orlov and G. Boccaccini) are valuable and pertinent to the study of Enochic traditions in the Slavonic realm of the Byzantine Commonwealth.

[2] Cf. *Jude* 1:14.

Enoch ascends and learns the secrets of the Universe.[3] Seven are the planets, about the movements of which he is instructed by the Creator,[4] and the heavenly rings, where the luminaries are located, are also seven; seven are the substances from which Adam is composed,[5] and seven are the traits assigned to him. In fact, cosmogonic and anthropogonic narratives anchored by the universal formulaic number *seven* contain some unique cultural features attested exclusively in the last *lingua sacra* of Europe, Old Church Slavonic. One such case is presented by the planetary order in *2 Enoch*; its description offers one of the most enigmatic schemes of celestial topography. While revealing the secrets of the creation of heavenly bodies to the visionary, God states:

> On the first and highest ring I placed the star Kronos [= Saturn]; on the second ring, below it, I placed Aphrodite [= Venus], on the third—Ares [= Mars], on the fourth—the Sun, on the fifth—Zeus [= Jupiter], on the sixth—Hermes [= Mercury], on the seventh—the Moon. I adorned the lower ether with smaller stars [= constellations?], and I put the Sun to shine during the day, while the Moon and stars were to shine during the night; and I ordered the Sun to advance through each zodiac sign, with each of the twelve zodiac signs allocated to a [particular] month. I gave names to the zodiac signs, and the time when they enter to be born [i.e., their Heliacal risings], and the period of their rotational movement. And after that it was night and morning—the fifth day.[6]

The specific sequence of the seven luminaries in *2 Enoch* is a conceptual *hapax* which so far has not been attested in any other extant cosmographic accounts containing ancient and medieval epistemological models; it does not have a counterpart in texts reflecting exact sciences from either Babylonian or Hellenistic traditions, nor does it occur in writings concerning the astronomical knowledge from the period of Antiquity to the Middle Ages.[7] It is rather significant that in the above-quoted fragment from *2 Enoch* there is a statement that "each of the twelve zodiac signs is allocated to a [particular] month"; this is a typically Babylonian trait, since in the early development of the zodiac, which we know

[3] Yarbro Collins (2000, 37–39); Schäfer (2004, 233–274; 2009, 85). However, there are singular cases in which the number of heavens is not seven but ten; the reasons for this numerical shift will be examined elsewhere.

[4] See also Yarbro Collins (2000, 47–54).

[5] See Lincoln (1986, 11–33).

[6] See Badalanova Geller (2010, 55–57).

[7] In fact, I have consulted with Alexander Jones, Gerd Grasshoff, and John Steele, and none of them could identify any familiar pattern in the Enochic sequence of planets. I am grateful for their advice on this matter.

originated in Babylonia some time before 400 BCE, each of the twelve zodiac signs were conveniently associated with a corresponding lunar calendar month.[8] This pattern changed later in Greek astronomy, in which zodiac signs were no longer associated with a lunar month but became a feature of the solar calendar. As a result, the correspondence between a zodiac sign and a single lunar month was gradually abandoned.

A brief survey of planetary order in Slavonic texts of the Byzantine Commonwealth, which are representative of the cultural and scientific context of *2 Enoch*, reveals an intriguing epistemological template. Against this particular background, the picture becomes more transparent.

One of the earliest star/planet lists in Old Church Slavonic is found in an astronomical fragment presented in the *Symeonic Florilegium*, the oldest extant copy of which, *Sviatoslav's Miscellany*, comes from 1073.[9] In this source, the list of the seven heavenly luminaries (copied on Fol. 250 v, columns i-ii and Fol. 251 r, column i) is rendered in an astral scheme described in relation to the twelve zodiac signs. The scribe claims to be quoting one particular excerpt from the famous *Fountain of Knowledge* (or *Fountain of Wisdom*) by John of Damascus (676–749);[10] indeed, in the *Symeonic Florilegium* the section is entitled "John of Damascus on the months in the Macedonian [calendar] according to the Church tradition" [Іωа(на) Дамаскина о македоньскыихъ м҇цихъ отъ цр҇квьнаарго прѣданиꙗ]. The opening paragraph of this section reads as follows:

> Гл҇ють бо илине сѫшта дъва на десѧте животы • звѣздами на небеси противьнопе пошьстиѥ имоуштa • слъньцѫ же и лоунѣ • и инѣмъ пѧти планитомъ • и • ві҃ • животы прѣходѧща • седми тои • седми же планитъ сѫть имена се • слъньце ло(у)на • зеус • ѥрмись • ари(с) • афродит(и) • (к)ронос • планиты же нарицѫть ꙗ • имьже странъ

[8]See Brack-Bernsen (2003, 24–25) (reference courtesy F. Rochberg).

[9]This copy was made in Kiev for the Russian Prince Sviatoslav (hence its designation). As for the Slavonic protograph of the *Florilegium* itself, it was originally compiled in Bulgaria in the period between 914 and 927, during the reign of Symeon, on the basis of a Greek (Byzantine) *Vorlage*, and most certainly was commissioned by the king himself. It was designed as a compendium containing articles from various spheres of medieval knowledge: Christian theology and ethics, along with ancient science and philosophy; see Ševčenko (1981, 330–334), Lunt (1983), Dinekov (1991), Thomson (1993), Bibikov (1996). Actually, it is also in the *Symeonic Florilegium* where the first reference to the Enochic apocryphal corpus in the Slavonic realm of the Byzantine Commonwealth appears. The reference to Enoch is found at the very end of the Ms (Fol. 254), in the section devoted to the *Index of Prohibited Books*, the authorship of which is attributed to Isidore of Pelusium (d. c. 450). *Enoch* is listed at its very top, coming in second position after *The Life of Adam and Eve*; see Dinekov *et al.* (1991, 701). The Ms can be found online (http://catalog.shm.ru/api/pdf/2EC1vsn-oxUeFwTrSEVUvzwT156uM2KwKo8D9BMC8Mk.pdf). See also http://catalog.shm.ru/entity/OBJECT/178472?fund=21&index=11.

[10]The text of *De mensibus macedonicis* is contained in Book 2, Chapter 7 (*Concerning light, fire, the luminaries, sun, moon and stars*) of the *Fountain of Knowledge*.

небесе пошьⷭⷮⷡиѥ имоуть • ѥсть же по коѥмоужьдо поꙗсѫ ѥдинъ отъ з҃ • планитъ • а҃ • на прьвѣѥмъ и на вышьшнимь • кронъ • в҃ • на вътор ѣмь же дии • г҃ • на третиимь же ареи • ☿ • д҃ • на четвьрьтѣмъ • жѣ слъньце • на пѧтѣмъ же афродити • ♀ • ѕ҃ • на шестѣмь же ѥрмии • ꙁ҃ • на седмѣѥмь же и долѣшьниимъ лоуноу • ☾ да слъньце оубо по коѥмоужьдо животоу ходить мⷭць • а҃ • и на дъва на десѧте мⷭца • оба на десѧте миноуѥть животъ • і҃в • же животъ сѫть имена • и тѣхъ мⷭци • а҃ • овънь • мⷭца марта • ка҃ • приѥмлеть сльньце • в҃ • тельць • мѣсѧца априлѧ • въ • кг҃ • д҃ • ракъ • иоунѧ • кд҃ • е҃ • львь • ї оулѧ въ • ке҃ • ѕ҃ • дѣвица • аугуста въ • ке҃ • ꙁ҃ • ярьмъ • сетебра • въ • ке҃ • и҃ • скорпиос • октоврѧ • ке҃ • ѳ҃ • стрѣлець ноꙗбрѧ • въ • ке҃ • і҃ • козьльрогъ • декѧврѧ въ • ке҃ • і҃а • водолѣиць • ї ѥнъуарѧ • въ • к҃ • ві҃ • риба • феуавраара • к҃ • …[11]

The Hellenes say that there are twelve astral zodiac signs, which move in heaven in a direction opposite to that of the Sun and the Moon, along with another five planets; these seven [luminaries] pass/move through the twelve zodiac signs. These are the names of the seven planets: Sun, Moon, Zeus [= Jupiter], Hermes [= Mercury], Ares [= Mars], Aphrodite [= Venus], Kronos [= Saturn]; and these move in a direction opposite to that of heaven: that is why they are called planets [= wanderers]. On each of the seven rings there is one of these seven planets. On the first and the highest ring is the planet Kronos [= Saturn], on the second is Dii (Διός) [= Jupiter], on the third is Ares [= Mars] ☿[12], on the fourth is the Sun, on the fifth is Aphrodite [= Venus] ♀, on the sixth is Hermes [= Mercury], on the seventh and the lowest is the Moon ☾. In each of the twelve months [during the solar year] the Sun passes through one of the twelve zodiac signs. It passes through all the twelve signs. These are the names of the zodiac signs, along with the names of the months. The Sun goes into the first one, Aries, on March 21st. The second one, Taurus, [begins] on April 23rd; [then comes the third, Gemini]; the fourth, Cancer, [begins] on June 24th; the fifth, Leo, on July 25th; the sixth, Virgo, on August 25th; the seventh, Libra, on September 25th; the eighth, Scorpio, on October 25th; the ninth, Sagittarius, on November 25th; the tenth, Capricorn, on December 25th; the eleventh, Aquarius, on January 20th; the twelfth, Pisces, on February 20th …[13]

[11] For the text of the *Symeonic Florilegium*, see Dinekov *et al*. (1991, Vol. 1, 694–695).
[12] The astrological symbol ☿ designates Mercury [Hermes, Ἑρμῆς], not Mars [= Ares, Ἄρης]; obviously, the copyist made a mistake.
[13] Author's translation.

Significantly, on the page on which the above-quoted fragment appears in the *Symeonic Florilegium*, the twelve zodiac signs[14] are illuminated along the margins of the text. The order is idiosyncratic, starting from Sagittarius (стрѣльць) and going through Aquarius (водолѣиць), Aries (овънъ), Taurus (тельць), Gemini (близньцы), Leo (левъ), Libra (ѧрьмъ), Cancer (ракъ), Virgo (дѣвица), Scorpio (скорп), Pisces (рыба), and Capricorn (козълърогъ). There have been many attempts to explain the reason behind this peculiar rendition of zodiacal sequence,[15] but none of them is satisfactory. On the other hand, a more precise comparison between the astronomical section in the *Symeonic Florilegium* and the respective excerpts from the *Fountain of Knowledge* shows that the *Florilegium* comprises information from two different chapters of the *Fountain of Knowledge.* Thus in Book 2, Chapter 6 (*Concerning Heaven*), John of Damascus lists the seven luminaries in an order which differs from that offered by him in Chapter 7 (*Concerning light, fire, the luminaries, sun, moon and stars*); in Chapter 6 he states the following:[16]

> It is maintained that there are seven heavenly girdles which are situated above each other. It is said that the substance of the heavens is similar to that of a thin smoke, and on each girdle there is a planet. It is maintained that there are seven planets: the Sun, the Moon, Dii (Διός) [= Jupiter], Hermes [= Mercury], Ares [= Mars], Aphrodite [= Venus] and Kronos [= Saturn]. Aphrodite is called either the Morning Star or the Evening Star. They are called planets [= wanderers], since they move in a direction opposite to that which the heavens follow; because the heavens and other stars move from east to west, and only [the planets] move from west to east. And this can be seen [while observing] the Moon, which moves each evening a little backwards [i.e., eastwards].[17]

[14]On the duodentary animal cycle in medieval Slavonic tradition (with special emphasis on Russian texts), see Ryan (1971, 12–20). Zodiac imagery also appears frequently in the iconography of *Isaiah* 34: 4 and *The Apocalypse of John* (*The Book of Revelation*) 6: 14; see 9.1
[15]See the research on the topic by Ivan Dobrev (1979), Elisaveta Musakova (1992), Tatiana Slavova (1993, 1994), and others.
[16]For the original Church Slavonic text of John the Exarch's *Theology*, see Mil'kov and Polianskii (2008, 63).
[17]Author's translation.

Figure 9.1: "The Angel of the Lord rolls up the skies"; fresco (1476) from the Dragalevtsi Monastery of the Holy Theotokos of Vitosha (Драгалевски манастир Света Богородица Витошка), Bulgaria. The fresco represents an iconographic rendition of the eschatological accounts in *Isaiah* 34: 4 ("And all the host of heaven shall be dissolved, and the heavens shall be rolled together as a scroll: and all their host shall fall down, as the leaf falleth off from the vine, and as a falling fig from the fig tree") and *The Apocalypse of John* (*The Book of Revelation*) 6: 14 ("And the heavens departed as a scroll when it is rolled together"). Depicted on the scroll of heaven, rolled up in the hands of the angel, is the disappearing image of the Sun; next to the image of the Moon (at the bottom) is the winged *Aphrodi* [Venus], depicted in a boat (alluding to her designation as a planet, i.e., as a "floating one"). Other winged luminaries (also in boats) are depicted to the left of the angel, next to the image of the Sun. The lower figure is difficult to identify, but the one above it is labelled as *Ermis* [Mercury]. Since the inscriptions are not clear, one can only guess that on the top edge of the scroll, to the right of the angel, are depicted (counterclockwise) Aries, Taurus, Gemini, Cancer; Virgo[?]; then below, clockwise, Scorpio, Sagittarius, Capricorn; below, Aquarius[?], Pisces, and Leo. Photo: FBG.

It is relevant to note that the above-quoted passage and other abridged excerpts from the *Fountain of Knowledge* were translated into Old Church Slavonic by John the Exarch,[18] one of the most important men of letters working at the end of the ninth and the beginning of the tenth century in Preslav, then capital of Bulgaria. They were put together into one composition entitled *Theology* (also known as *Heavens*).

As for the particular planetary order, which was initially presented by John of Damascus in Chapter 6 of his *Fountain of Knowledge*, it corresponds to the ancient philosophical concept of *musica universalis* (or harmony of the Spheres) regarding proportions in the movements of the seven celestial bodies (the Sun, the Moon, and the five planets) as "seven musical tones" (octave, with $1^{st} = 8^{th}$ constituent); this theory rests on the Pythagorean idea that the Sun, Moon, and the five planets all produce their own unique tone (= "orbital resonance") based on their "orbital revolution." In fact, the concept of "orbital revolution" is presented in the concluding chapter of Plato's *Republic*, Book 10, in *The Myth of Er*, where he describes the rotation of the Spindle of Necessity. It is during the course of his afterlife journey that Er sees the cosmic mechanism of "orbital revolution," the Spindle turning on the knees of the deity Necessity, who is surrounded by her daughters, the three Fates, responsible for the present, past, and future of those born on Earth. This Spindle was stretched from the extremities of the heavens in the middle of a pillar of light. It resembled the brightness of the rainbow, its luminosity-like bonds holding the sky together:

> In the center of the light the ends of its bonds stretched from the sky: for this light was what bound the sky together, like the braces of triremes, so holding together the whole revolution. Stretching down from either end was the spindle of Necessity by means of which all the circles turn. Both its shaft and hook were made of adamant, while the whorl is a mixture of this and other sorts of material. The nature of the whorl is as follows: its shape is like the ones we use, but you have to imagine what it's like from his description of it, just as if in a large hollow whorl scooped out right through, another one of the same sort lies fitted inside it, and so on, just like boxes that fit into one another, with a third and fourth and four more. The total number of whorls is eight, each lying inside the other. Their edges seen from above are circles, forming from the back a continuous single whorl around the shaft, the latter being driven right through the center of the eighth.[19]

[18] See Thomson (1991), Trendafilov (1995, 1996, 1998a, 1998b, 1998c, 2003, 2004).

[19] See Emlyn-Jones and Preddy (2013, 470–473); see also the discussion in Badalanova Geller (2015, 387–388, fn. 19).

The cosmos is described here as a revolving vault or spindle whorl consisting of seven inner rings, which rotate concentrically within the eighth outer ring. This motif of rotating spheres resembles the description of the heavens in *2 Enoch* in similar terms, and it is possible that the celestial models from which the *Myth of Er* and *2 Enoch* were derived were comparable, or perhaps shared a common ancestry. One thing, however, is clear: the planetary order described by John of Damascus in Chapter 6 of his *Fountain of Knowledge* and its subsequent renditions in the *Symeonic Florilegium* and John the Exarch's *Theology* suggest that these traditions continued to be a vibrant part of the epistemological scope of exact sciences in the Byzantine Commonwealth at least until the tenth century.

Then again, as briefly stated above, only the first paragraph of the astronomical section in the *Symeonic Florilegium* derives from Chapter 6 of *The Fountain of Knowledge*; its second paragraph stems from Chapter 7 (*Concerning light, fire, the luminaries, sun, moon and stars*), where John of Damascus gives a totally different planetary scheme which follows the Ptolemaic system in reverse order. In the Old Church Slavonic rendition of Chapter 7, as translated by the medieval Bulgarian intellectual John the Exarch, the following is stated:[20]

> There are, it is said, seven planets amongst these luminaries; it is said that these move in a direction opposite to that of heaven: hence the name planets [= wanderers]. For, while they say that heaven moves from east to west, the planets move from west to east; but heaven bears the seven planets along with it by its swifter motion. Now these are the names of the seven planets: the Sun, the Moon, Zeus [= Jupiter], Hermes [= Mercury], Ares [= Mars], Aphrodite [= Venus], Kronos [= Saturn]. Each of these seven planets [belongs to] a separate girdle. On the first and highest is Kronos [= Saturn]; on the second—Dia (Διός) [= Jupiter]; on the third—Ares [= Mars]; on the fourth—the Sun; on the fifth—Aphrodite [= Venus]; on the sixth—Hermes [= Mercury]; on the seventh and lowest—the Moon.[21]

As we can see, the version in the *Symeonic Florilegium* (see pp. 105–107 above) represents a rather abridged redaction of the two astronomical excerpts from the *Fountain of Knowledge*; it also has some specific textual features which indicate that it was translated from a source which was not identical with the source used by John the Exarch. Still, in both accounts (the *Symeonic Florilegium* and John the Exarch's *Theology*) the pattern given is: Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon, which is the standard Ptolemaic sequence of planets but in *re-*

[20]For the original Church Slavonic text of John the Exarch's *Theology*, see Mil'kov and Polianskii (2008, 69).
[21]Author's translation.

verse order. The author of *2 Enoch*, on the other hand, puts Venus between Saturn and Mars. Still, in all three sources (*2 Enoch*, the *Symeonic Florilegium*, and Exarch's *Theology*), Kronos (Saturn) is placed on the first and highest heavenly ring. Then again, in *2 Enoch*, Kronos and other luminaries were considered to be "stars" (as in Babylonian astronomy), while in the *Symeonic Florilegium* and John the Exarch's *Theology* they are called "planets" (as in Greek astronomy).

The survey of Slavonic sources containing astronomical fragments betrays, however, a certain uneasiness on behalf of the compilers, suggesting that they considered this kind of information to be related to astrological knowledge rather than to Christian discourse. Thus, in the first Slavonic *Hexameron*, also compiled by John the Exarch in the late ninth century, the list of the planets is rendered within the framework of the zodiacal signs, and is presented in the *Homily* on the fourth day as a treaty against astrologists who define planets as benevolent and malevolent:[22]

> Since the so-called planets are in fact "floating stars," they [the astrologers] divide them into benevolent [and malevolent]. The star of *Dii* [that is, Zeus = Jupiter] and the star of Aphrodite [= Venus] they call benevolent, whereas Ares [= Mars] and the star of Kronos [= Saturn] are considered to be malevolent. As for the star of Hermes [= Mercury], it is considered to be commonly representative of either of the two groups. They further maintain that the Sun and the Moon have the power of rulers.[23]

Since John the Exarch's *Hexameron* is a compilation based on the homilies composed by one of the Cappadocian Fathers, St. Basil the Great, Bishop of Caesaerea (around 370 CE), it can also be treated as evidence for the attitude of the Church towards astronomical and astrological knowledge not only as esoteric, but also as erroneous and blasphemous. This will also explain why *2 Enoch*, densely packed with ancient scientific—astronomical, calendrical, and mathematical—information, was featured in all the Slavonic redactions of the *Indices of Prohibited Books*. As for the sources outside of these *Indices* (such as *The Interpretative Palaea*, *The Historical Palaea*, and *The Chronographical Palaea*, along with *The Chronicle of John Malalas*, *The Chronicle of George Hamartolos*, and, last but not least, *The Alexander Romance* of Pseudo-Callisthenes), the survey of astronomical schemes offered in them shows that their scribes conventionally follow either of the two universal versions of planetary lists, as outlined in John of Damascus' *Fountain of Knowledge*. Then again, the context in which the Byzantine chronicler John Malalas (c. 491–578) describes the order of planets

[22] For the original Church Slavonic text, see Barankova and Mil'kov (2001, 452).
[23] Author's translation.

(Saturn, Jupiter, Mars, Venus, Mercury, Sun, Moon) is rather symptomatic. Thus the statement attributing the authorship of the astronyms to Adam's son Seth is linked to the narrative about the mythical ancestry of kings and rulers who were named after the heavenly bodies. The *Chronicle* claims, for instance, that from the lineage of the first son of Noah came a giant called *Kronos* (named after the star) who ruled over Assyria and Persia. He was married to *Semiramin* (*Semiramis*), who also descended from Noah's son Shem. She was called *Aria* (Ares) in Syria, due to her great stature and intellect. From this union *Zeus* was born, along with his brother *Nin* and sister *Ira*, whom *Zeus* married and produced a son, *Velon*, the founder of Babylon. Another son of *Kronos* married a woman called *Astronomy*, and from this union *Aphrodite* was born.[24]

The same symptomatic link between the list of kings and list of planets (Saturn, Mars, Jupiter, Venus, Mercury) is attested in *The Chronicle of George Hamartolos* (842–867).[25] Having stated that Seth was the one who invented "Hebrew script" and possessed wisdom and knowledge about heavenly portents [= astrology], Hamartolos, following Malalas, maintains that it was also Seth who gave names to the five "wandering stars" [= planets], so that people may know them correctly. The first "wandering star" he called Kronos [= Saturn], the second—Ares [= Mars], the fourth—Aphrodite [= Venus], the fifth—Hermes [= Mercury]. Subsequently Hamartolos embarks on a description of the lineage of Nimrud as a descendant of Ham, the cursed son of Noah. According to the *Chronicle*, he founded the city of Babylon and showed his people how to hunt and do magic. He also taught Persians the "law of the stars [i.e., astronomy], astrology, and movements of the heavens, and everything generated in them." Having associated his narrative with the biblical account of the Flood, Hamartolos then hastens to describe the genealogy of the kings of Assyria, with *Kronos* being the first among them; *Semiramis*, his wife, was called *Area* (that is, Ares) by the Assyrians. Their first son was called *Dii* [Zeus = Jupiter], according to the name of the "wandering star"; the second was given the name of *Nin*, and the daughter was called *Ira*; *Nin* reigned after *Kronos*, and married his mother *Semiramis*, while *Dii* took his sister *Ira* as a wife. *Nin* ruled Syria and founded a great city in this land named after him: Nineveh. From him *Zoroaster* was born.

Despite otherwise confusing renditions of quasi-historical events, both *The Chronicle of John Malalas* and *The Chronicle of George Hamartolos* provide an important piece of information—that the ancient king lists were seen in conjunction with the lists of astronyms, with each ruler assigned a particular star. What

[24] *The Chronicle of John Malalas* was translated into Old Church Slavonic no later than the tenth century in Bulgaria (most probably in the capital Preslav by contemporaries of John the Exarch); for relevant fragments see the Appendix, Part One.

[25] *The Chronicle of George Hamartolos* was translated into Old Church Slavonic in Bulgaria, most likely in the eleventh century; for relevant fragments see the Appendix, Part Two.

then if we try to apply the same epistemological scheme to the planetary order in *2 Enoch*? Can it be suggested that the astral list in *2 Enoch* is in fact a counterpart of the list of deities?

Let us once more return to the order of heavenly bodies in *2 Enoch*: Saturn, Venus, Mars, the Sun, Jupiter, Mercury, and the Moon. Might there be a narrative behind this list concerning the gods who are identified with these luminaries? What about Babylonian god names behind these planets? Might there be some reason why Venus (Ishtar) follows Saturn (Ninurta), or Jupiter (Marduk) follows Mercury (Nabu)? Is there a relevant genealogy or mythology here? What if *2 Enoch* and this unusual order of stars/planets might be hiding something which has been missed so far: a Babylonian narrative, partially surviving in Slavonic texts from the Byzantine Commonwealth?

Finally, there may be one further clue to explain the idiosyncratic order of luminaries in *2 Enoch*, which appears to have had Venus and Jupiter exchange places: Babylonian astronomy of the Seleucid period had Venus in the second position in the order of planets,[26] and *2 Enoch* may have attempted to follow this order, which required that Venus change places with Jupiter. In this way, *2 Enoch* appears to be a compromise between the Seleucid Babylonian and Ptolemaic Greek order of planets/stars, and therefore reflects neither system precisely. Moreover, the interchange between Venus and Jupiter in the *2 Enoch* list may show their mutual links within Babylonian astrology as being both benevolent and portending favorable omens (in contrast to malevolent Saturn and Mars, and ambivalent Mercury);[27] in fact, John the Exarch's *Hexaemeron* (see p. 111 above) provides firm evidence for such a hypothesis.

Indeed, *2 Enoch* gives an idiosyncratic scheme which does not appear to follow either Babylonian or Greek astronomy (in contrast to the *Symeonic Florilegium* and John the Exarch's *Heavens*, which conform to Ptolemy's order of planets), suggesting that *2 Enoch* drew from different sources. This, in turn, indicates that in the period when the Slavonic protograph of *2 Enoch* was composed (along with the *Symeonic Florilegium* and John the Exarch's *Heavens* and *Hexaemeron*), medieval Slavonic science did not have at its disposal an established system of astronomical knowledge; scientific concepts of this period derived from various competing sources from different traditions.

[26] See Koch-Westenholz (1995, 120, footnote 2).
[27] See Rochberg-Halton (1984, 123).

Appendix

Part One: Fragments from the Slavonic translation/redaction of *The Chronicle of John Malalas*

Seth had Wisdom from God, and by God's will he invented the names of the stars and of the five planets [lit. "wanderers"], so that men may know them correctly. The first star he called *Kronos* [= Saturn], the second *Dii* [= Jupiter], the third, *Areya* [= Mars], the fourth *Aphrodit* [= Venus], and the fifth *Ermin* [= Mercury]. It is said that all together there are seven, five stars and two [great] luminaries [i.e., Sun and Moon]. He is also the one who invented Hebrew script [= "Hebrew characters"] and wrote them [down]. As for the [two] great luminaries, these are named by God Himself. The ruler of the day He called the Sun and the ruler of the night is the Moon. This is what the wise Fortunus, the Roman chronographer wrote. This work I composed in Constantinople.[28]

[…]

In that year, from the tribe of Aphraxad, a certain Moor came along. His Indian-astral name was *Gandovari*, and he was the first in India to write about astronomy. From the tribe of Ham, a Moor by the name of Hus was born. He begot Nimrud the giant, who founded Babylon. Persians call him a god, and they call the stars in heaven "planets" [lit. "wanderers"]. He was the first hunter and he gave to everyone food from his hunts. […] Syria and Persia were under the control of the tribe of Shem. From the first son of Noah, a giant was born, whose name was *Kronos*, given this name by his father Damya, after the name of the star / planet. He was very strong and showed how to rule and hold power, ruling in Assyria for many years and keeping under his control the entire land, called Persia by the Assyrians. He was a terrifying soldier, more than anyone else.

His wife was *Semiramin* [= *Semiramis*], who was called *Aria* [= Ares] in Syria, because she was great of stature and very clever. She was also from the genealogy of Shem, son of *Noah*. *Kronos* had a son whose name was *Pika*, who was called by his parents *Zeus*, since he was named after the wandering star / planet [i.e., Jupiter]. *Kronos* had yet another son whose name was *Nin* and a daughter named *Ira*. *Zeus*, who was also called *Pika*, took his sister as a lawful wife, and they all lived together. […] *Zeus* had a son from her and he was called *Velon* [abbreviation of Vavilon = Babylon?], since he was speedy. […] His grandfather *Kronos*

[28] For the original Church Slavonic text of this fragment see Istrin (1897, 6); author's translation.

left his son *Pika* in Assyria, along with his wife *Area*, also called *Semiramida* [= *Semiramis*], and with many brave men he went to the western lands which were ruled by nobody, and he conquered the lands westwards of Syria. […] He [Kronos] had a son from his wife *Filura* whose name was *Afron*, and he gave him the land of Lybia. He married the woman whose name was *Astronomy*, and by her he had a daughter whose name was *Aphrodita*. […] And by *Filura Kronos* had a son whose name was *Chiron the Philosopher*, who reigned for 20 years in Assyria.[29]

Part Two: Fragments from the Slavonic translation/ redaction of *The Chronicle of George Hamartolos*

Seth was the one who invented "Hebrew characters," and wisdom and heavenly portents [= astrology], and customs, [and] the four seasons of the year, and months and weeks, and he gave names to the stars, and to the five "wandering stars" [= planets], so that people may know them correctly. The first "wandering star" he called *Kronos* [= Saturn], the second—*Ares* [= Mars], the fourth—*Aphrodite* [= Venus], the fifth—*Hermes* [= Mercury].

After the Flood, Cainan—the son of Arphaxsad, wrote the "law of the stars," and he found the name of Seth and his children and the names of the stars written on a stone tablet, because for Seth's grandchildren it was professed from above that people will perish; and two columns were created, one of stone and one of clay. And on these he [Cainan] wrote [the knowledge which he received] from the discourse of his grandfather Seth, concerning the heavens. Thinking that if the world would perish from water, the stone column would remain, along with what is written on it. If [the world would perish] from fire, the clay column would remain, along with what is written on it. As [Flavius] Josephus says, this particular column survived the Flood and it is on Mt. Siridon [Sidon?] up until today.

Afterwards there was a giant, called a Titan, whose name was Nimrud, son of Hus the Ethiopian, from the genealogy of Ham. He founded the city of Babylon and showed how to hunt and do magic. He also taught Persians the "law of the stars" [i.e., astronomy], astrology, and movements of the heavens, and everything generated in them. […]

From the genealogy of *Semel* and *Aron*, from them are the Assyrians. And they reigned in Assyria and Persia and all the countries to the East. And after that a

[29] For the original Church Slavonic text of this fragment see Istrin (1897, 11–12); author's translation.

gigantic man, called *Kronos* [= Saturn], after the name of the "wandering star," came along. Being very strong, he subjugated and subdued many, thus being the first to constitute kingship and subdue other people. First, he was the King of Assyria for 56 years and reigned over the entire Persian lands. […] He had a wife, *Semiramis*, whom Assyrians called *Area* [= Ares?], and they had two sons and one daughter. The first son was called *Dii* [Zeus = Jupiter], according to the name of the "wandering star," and the second son he called *Nin*, and the daughter was called *Ira*, who was taken by *Pik* as a wife, just like *Zeus* took his sister as a wife. *Nin* reigned after *Kronos* for 52 years; and *Nin* married his mother *Semiramis*, since Persians had the custom of marrying their mothers and sisters. *Dii* afterwards took his sister *Ira* as a wife. *Nin* ruled mightily in Syria and founded a great city in this land […] and gave it a name after his own name, Nineveh. From him *Zoroaster* was born.[30]

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[30] For the original Church Slavonic text, see Istrin (1920, 33–35); author's translation.

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Part 2: History of Knowledge and Material Representations

Chapter 10
Können Entwicklungspsychologie und Wissenschaftsgeschichte voneinander lernen?

Katja Bödeker

Die Bildung fundamentaler Strukturen des menschlichen Denkens zu verstehen stellt eines der Hauptanliegen der historischen Epistemologie dar, so wie sie am Max-Planck-Institut für Wissenschaftsgeschichte betrieben wird. Dass Zahlen, Begriffe von Kausalität oder Räumlichkeit nicht einfach apriorische Gegebenheiten darstellen, die als zeitlose Entitäten einen platonischen Ideenhimmel bevölkern oder aber *ab initio* Universalien des menschlichen Denkens bilden, sondern dass sie vielmehr eine Geschichte haben, in die unterschiedliche Formen von Erfahrung eingehen, repräsentiert und reflektiert werden, dies hat Peter Damerow stets kämpferisch und mit Verve propagiert.

Die Entwicklung kognitiver Grundbegriffe geschieht auf mehreren Ebenen, die jeweils in unterschiedlichen Disziplinen untersucht werden. Die Ontogenese des Denkens ist Thema der Entwicklungspsychologie. Den Fokus auf die individuelle Psyche gerichtet, befasst sich der Entwicklungspsychologe mit den Veränderungen, denen das Denken von Personen über die Lebensspanne hinweg unterliegt. Die historische Entwicklung von Wissensstrukturen als kollektiver, Generationen und Populationen übergreifender Prozess ist hingegen Gegenstand der Wissenschaftsgeschichte. Da der Wissenschaftshistoriker genuin intersubjektive Prozesse der Produktion und Tradierung von Wissen mit ihren materialen und institutionellen Voraussetzungen in den Blick nimmt, kann er den methodologischen Individualismus der Psychologie nicht teilen. Das Verhältnis zwischen Ontogenese und Historiogenese, und damit auch die Beziehung von Psychologie und Geschichte war das Thema in vielen von Peter Damerows Arbeiten (Damerow 1993, 1994). Peter Damerow warf Fragen auf, die mit den Mitteln einer der beiden Disziplinen allein nicht zu beantworten waren und insbesondere in seinen Arbeiten zur Entwicklung des Zahlbegriffs schuf er, dabei kritisch an die genetische Epistemologie Jean Piagets anknüpfend, der historischen Epistemologie einen theoretischen Rahmen, welcher Psychologie und Wissenschaftsgeschichte integrierte.

In den folgenden Überlegungen möchte ich ein bescheideneres Ziel verfolgen. Ich werde am Beispiel von Arbeiten zum physikalischen Denken der Frage nachgehen, ob und was Entwicklungspsychologie und Wissenschaftsgeschichte – zwei Disziplinen mit völlig unterschiedlichen methodologischen Prämissen und Vorannahmen bezüglich ihres Gegenstandsbereiches – voneinander lernen können. Muss der Entwicklungspsychologe mit der Geschichte von Konzepten wie Kraft, Gewicht oder Bewegung vertraut sein, um die Veränderungen zu verstehen, denen diese Begriffe in der Ontogenese unterliegen? Und können umgekehrt Befunde über die Ontogenese des Gewichts- oder Bewegungsverständnisses einen Beitrag zur Untersuchung der Geschichte physikalischen Denkens leisten? Um meine Antworten vorwegzunehmen: Ich werde die erste Frage mit einem klaren Ja beantworten, bin aber unsicher, in welcher Weise Befunde aus der Entwicklungspsychologie in ihrer gegenwärtigen Form zur Wissenschaftsgeschichte beitragen können.

Querverweise von einer Disziplin auf die andere finden sich häufig. Sie gehen meist von der Beobachtung aus, dass Ideen über natürliche Prozesse, die Kinder oder Erwachsene in Befragungen äußern, mitunter verblüffende Ähnlichkeiten zu Positionen aus der Wissenschaftsgeschichte zeigen. Das bekannteste Beispiel ist die zählebige, auch angesichts der Verführungen des modernen Physikunterrichts recht widerständige Vorstellung, dass die Bewegung unbelebter Körper eine Kraft fordert, die vom Beweger auf das bewegte Objekt übertragen wird. Zur Beschreibung dieser „*motion implies a force*“-Idee, die sich bei Kindern und Erwachsenen unterschiedlichster Kulturen, ja sogar bei Physikstudenten beobachten ließ, stützten sich kognitive Psychologen auf Positionen der vorklassischen Mechanik oder auf mittelalterliche Impetustheorien.[1] Umgekehrt verweisen viele wissenschaftshistorische oder ideengeschichtliche Arbeiten über die Geschichte der Mechanik – insbesondere bei Darstellungen der aristotelischen Naturphilosophie – zur Erklärung für bestimmte inhaltliche Positionen auf Alltagserfahrungen oder auf den *common sense*.[2]

Was aber bedeutet der Verweis auf den *common sense* oder aber auf „naives physikalisches Wissen“ bzw. „Laienwissen“, wie es in der Psychologie heißt? Was ist der physikalische *common sense*? Ist dieser Begriff und das mit diesem verknüpfte methodische Vorgehen sinnvoll, Nicht-Physikern aller Altersstufen physikalische Probleme vorzulegen, um aus den Antworten später einen physikalischen *common sense* zu extrahieren? Und welchen Erkenntnisgewinn liefern dem Historiker Verweise auf den *common sense*? Was und wie kann er hier von der Entwicklungspsychologie lernen?

[1] Vgl. Clement (1983); McCloskey (1983).
[2] Vgl. Jammer (1957); Dijksterhuis (1983).

Der Vorstellung eines *common sense*, eines „Laienwissens" über die physikalische Welt, liegen unterschiedliche Intuitionen zugrunde: Zum einen setzt sie voraus, dass Menschen über Wissen zur gegenständlichen Welt verfügen, das ihren Umgang mit Objekten leitet – dies ist unstrittig, in der Psychologie und in den Kognitionswissenschaften existiert eine breite Forschungstradition zu Inhalten, zur Repräsentationsform und Entwicklung dieses Wissens. Es gibt jedoch auch eine stärkere Intuition: Die Begriffe „*Folk physics*" oder „Laienphysik" legen darüber hinaus die Vorstellung nahe, dieses Wissen über die gegenständliche Welt sei vom kanonisierten naturwissenschaftlichen Wissen gänzlich unabhängig, ja, mehr noch, stelle ein kulturfreies Sediment von Sinnes- und Handlungserfahrungen dar, das seiner Fundamentalität wegen von allen Menschen geteilt wird, also universell ist. Für den Historiker würde ein solches Wissen die natürliche Basis zum Verständnis der geschichtlichen Entwicklung liefern.

Diese stärkere Intuition wirft jedoch Probleme auf. Leitend scheint hier das Anliegen, die natürlichen Fundamente des menschlichen Denkens über die gegenständliche Welt freizulegen, zur kognitiven Basisausstattung vorzudringen, über die Menschen verfügen müssen, um sich in dieser Welt, wo sich manches niemals ändert, zurechtzufinden. In der Tat wird ein solches basales physikalisches „Kernwissen" seit mehreren Jahrzehnten in Entwicklungspsychologie und komparativer Psychologie eingehend untersucht.[3] Gewöhnlich steht diese Forschung unter universalistischen Vorzeichen: In den psychologischen Experimenten werden entweder Kleinkinder in den ersten Lebensmonaten getestet, unter der Voraussetzung, dass die aufgewiesenen Kompetenzen entweder angeboren oder aber das Ergebnis sehr basaler Lernprozesse sind, oder aber es werden kognitive Fähigkeiten von Tieren, vorrangig von Primaten, untersucht, um so die natürlichen Bedingungen physikalischen Denkens offenzulegen, die wir mit unseren ältesten Vorfahren teilen (Povinelli 2000). Die Frage, wie die hier aufgewiesenen basalen kognitiven Fähigkeiten zu begreifen sind, ob man sie zu Recht als Wissen bezeichnen darf, wird in der Entwicklungspsychologie kontrovers diskutiert (Tomasello 1999). Die physikalischen Kompetenzen, um die es hier geht, sind sehr elementar: Beispiele sind vorsprachliche Erwartungen, dass physische Objekte solide und kohäsiv sind oder sich kontinuierlich durch Raum und Zeit bewegen.

Entwicklungslogisch sind solche „Kernwissensstrukturen" jedoch weit von den Ideen entfernt, die prototypisch für physikalisches Laienwissen sind und zu Vergleichen mit wissenschaftshistorischen Modellen Anlass gegeben haben: Eine Vorstellung wie „Der Stein fliegt am Anfang geradeaus und fällt dann runter, weil sein Schwung alle ist" ist keinesfalls Element eines basalen physikalischen Kernwissens. Vorstellungen wie diese gehen aus Entwicklungsprozessen hervor, in die neben Handlungserfahrungen kulturgebundenes Wissen immer schon eingeht

[3] Vgl. Spelke (1994, 2000).

– kognitive Entwicklung ist Enkulturation von Anfang an. Die Intuition eines angeborenen bzw. ausschließlich handlungs- und wahrnehmungsbasierten Wissens über die physikalische Welt mag vielleicht in dem bei Säuglingen und Primaten aufgewiesenen Kernwissen seine Entsprechung haben. Die Vorstellungen, die Schulkinder oder wissenschaftlich wenig vorgebildeten Laien zu physikalischen Prozessen äußern, entstehen jedoch niemals kulturfrei. In der Ontogenese physikalischen Wissens wirken angeborene perzeptuelle oder kognitive Strukturen, Reflexionen von Handlungserfahrungen und die Aneignung kulturgebundenen geteilten Wissens zusammen. In den meisten Fällen kann der Wissenschaftshistoriker also nicht wissen, auf was er eigentlich verweist, wenn er die aristotelische Unterscheidung zwischen natürlicher und gewaltsamer Bewegung oder die Impetusmodelle eines Johannes Buridan mit *common sense* Konzepten über Bewegung vergleicht. Sind diese *common sense* Konzepte wirklich reine Produkte von Handlungserfahrungen? Oder sind sie von der Sprache selbst suggerierte Vorstellungen? Oder geht die ontogenetische Entwicklung solcher Vorstellungen vielmehr – über Umwege und Zwischenschritte – auf die wissenschaftshistorischen Positionen selbst zurück, deren Inhalt und Genese doch mit jenen Intuitionen gerade erklärt werden sollte? Ich glaube, dass die Entwicklungspsychologie an dieser Stelle ein Erklärungsdefizit hat und hier selbst von der Wissenschaftsgeschichte lernen kann.

Oben habe ich erwähnt, dass Psychologen ihrerseits auf die Wissenschaftsgeschichte rekurrieren, um physikalische Ideen von Kindern oder Erwachsenen zu beschreiben. Wie erklären nun Psychologen die aufgewiesenen Ähnlichkeiten zwischen historischen Modellen und intuitivem Wissen von Kindern oder Erwachsenen? Welche Bedeutung kommt der Wissenschaftsgeschichte in der Psychologie zu und wo könnten Psychologen von der Wissenschaftsgeschichte lernen?

Inhaltliche Übereinstimmungen zwischen Modellen aus der Geschichte der Physik und Vorstellungen von Kindern galten in der Entwicklungspsychologie – einer Disziplin, die in ihren Anfängen Haeckels biogenetische Grundregel als theoretischen Rahmen voraussetzte – oft als Evidenzen für eine Rekapitulation der Historiogenese in der Ontogenese: Das Kind sollte also in seiner individuellen Entwicklung Phasen durchlaufen, die denen der Wissensentwicklung in der Geschichte entsprechen (Gould 1977). Zur Erklärung dieses Parallelismus wurde vielfach vorausgesetzt, dass in beiden Prozessen dieselben genetischen Prinzipien wirken. So hat Jean Piaget, der in seinem gesamten Werk Spielarten eines solchen onto-historiogenetischen Parallelismus vertrat, universelle Mechanismen der kognitiven Entwicklung wie die Dezentrierung postuliert, welche Ontogenese und die erste Phase der Physikgeschichte bis Newton gleichermaßen bestimmen sollten. Onto- und Historiogenese des physikalischen Denkens folgen laut Piaget daher derselben Entwicklungslogik und zeigen inhaltliche Korrespondenzen (Piaget und Inhelder 1975; Piaget und García 1989). Dass Piagets Theorie historisch schwer zu halten ist, hat Peter Damerow in mehreren Arbeiten gezeigt (Damerow 1993, 1994). Für die Entwicklung des Bewegungs- und Kraftverständnisses ist dieses Modell auch entwicklungspsychologisch kaum zu plausibilisieren: Selbst in der ontogenetischen Entwicklung lassen sich individuelle Vorstellungen zur Verursachung von Bewegung in die strukturgenetische Logik Piagets kaum einfügen, und noch schwieriger ist es, Denkhaltungen ganzer Epochen als „präoperational" oder „konkret-operational" zu charakterisieren (Bödeker 2006). Vollends problematisch wird das Modell jedoch, wenn die diachrone Sichtweise einer synchronen weicht und die Rekapitulationsthese als Folie für den Kulturvergleich dienen soll: Dann droht ein altes, auf Edward Tylor zurückgehendes evolutionistisches Denkmotiv in entwicklungspsychologischem Gewande wieder aufzuerstehen: Das Denken sogenannter „traditioneller Kulturen" repräsentiert frühere Stadien der kognitiven Entwicklung, oder – sagen wir es doch gleich unverblümt – „Naturvölker" resp. „Wilde" denken wie Kinder.

War es Piagets Anliegen, die historische Entwicklung der Physik ausgehend von der Ontogenese physikalischen Denkens und der hier wirkenden Entwicklungsprinzipien zu verstehen, so entlehnen neuere entwicklungspsychologische Ansätze wie die von Susan Carey das begriffliche Werkzeug zur Beschreibung von konzeptuellem Wandel in der Ontogenese der Wissenschaftsgeschichte (Carey 1992, 2009). Überzeugungen, die Kinder über physikalische oder biologische Vorgänge hegen, stellen demzufolge intuitive Theorien dar, die mit expliziten wissenschaftlichen Theorien wesentliche Merkmale gemeinsam haben. Wissen über Gegenstände, das in der Ontogenese entsteht, so die Leitidee dieser „Theorie-Theorien", lässt sich nicht auf eine Sammlung sinnlicher Erfahrungen reduzieren, Kinder versuchen vielmehr, sich Phänomene in konsistenter Weise verständlich zu machen und stützen sich dabei auf grundlegende ontologische und kausale Konzepte, die wie wissenschaftliche Theorien Vorhersagen und Erklärungen erlauben. Auch Transformationen im Denken von Kindern werden von Susan Carey mit Verweis auf die Wissenschaftsgeschichte, in Anlehnung an Thomas Kuhns Untersuchungen zu wissenschaftlichen Revolutionen, beschrieben. Demnach unterliegen Begriffe über Materie und Gewicht, über Lebendigkeit, Bewegung und Kraft in der mittleren Kindheit so fundamentalen Veränderungen, dass die späteren Konzepte *inkommensurabel* zu ihren Vorläufern sind.

Entwicklungspsychologische Anleihen aus der Wissenschaftsgeschichte, wie sie in den „Theorie-Theorie"-Ansätzen vorgenommen werden, sind in mehrfacher Hinsicht angreifbar. Kritik erfuhren sie insbesondere von Seiten der Wissenschaftshistoriker, die ja ihrerseits in der kognitiven Psychologie nach geeigneten begrifflichen Werkzeugen suchen, um konzeptuelle Umbrüche bei wissenschaftlichen Revolutionen zu analysieren.[4] An dieser Stelle werde ich jedoch nicht diskutieren, ob Kinder zu Recht als „kleine Wissenschaftler" beschrieben werden sollten. Am Beispiel eines grundlegenden begrifflichen Wandels bei Kindern, der Veränderungen, die die Ideen von Materie und Gewicht in der mittleren Kindheit durchlaufen, möchte ich vielmehr zeigen, dass der Entwicklungspsychologe auf die Wissenschaftsgeschichte angewiesen ist, wenn er die Faktoren beschreiben möchte, die die Ontogenese physikalischen Wissens vorantreiben.

Im Vorschul- und im frühen Grundschulalter, so ein recht robuster Befund aus vielen experimentellen Studien, neigen Kinder dazu, Materialität mit physischer Realität zu identifizieren. Materiell ist alles, was man sich nicht nur einbildet, sondern was man sehen und spüren kann. Materie ist in den Augen der Kinder aber kein Kontinuum, kann daher bei wiederholter Teilung vollkommen verschwinden, das Gewicht hingegen wird mit gefühlter Schwere gleichgesetzt und ist von Dichte noch nicht unterschieden. Ab dem Ende der Grundschulzeit zeigen nordamerikanische oder europäische Schüler hingegen ein grundlegend anderes Konzept von Materie und Gewicht. Materie wird nun als Kontinuum verstanden, das jetzt additiv und extensiv begriffene Gewicht als deren wesentliches Merkmal – jedes Ding, egal wie klein es ist, muss etwas wiegen, egal ob man es merkt oder nicht.[5]

Wie kommt es zu diesem begrifflichen Wandel? Es gibt Hinweise dafür, dass Kinder und Erwachsene aus Kulturen ohne etablierte Gewichtmessung kein extensives Verständnis von Gewicht ausbilden (Bödeker 2006). Somit sind die Idee von Materie als Kontinuum und das extensive Gewichtsverständnis in der ontogenetischen Entwicklung – entgegen der Annahme Piagets – nicht einfach angelegt, stellen keine Universalien dar. Dieser konzeptuelle Wandel ist vielmehr von kulturellen Faktoren abhängig, unterliegt damit selbst einer historischen Entwicklung, die wiederum Gegenstand der Wissenschaftsgeschichte ist. Wie und wann entwickelte sich die Idee, dass materielle Objekte beliebiger Größe über ein Gewicht verfügen, egal wie klein sie sind? Wie entstand die Vorstellung von Gewicht als additiver Größe und wie haben Praktiken der Gewichtsmessung bzw. naturphilosophische Theoriebildung hierzu beigetragen? Und – die für den Entwicklungspsychologen bedeutsamste Frage – auf welchen Wegen entstanden auf der Grundlage des praktischen und theoretischen Wissens über Gewicht kulturgebundene kognitive Strukturen, die wirkmächtig genug sind, um die Bildung von Gewichtsvorstellungen in der Ontogenese zu beeinflussen? Die kognitive Entwicklung ist immer auch Aneignung kulturell repräsentierter, kognitiver Strukturen, deren Genese der Entwicklungspsychologe allein nicht aufzuklären imstande

[4] Vgl. Fine (1996); Gellatly (1997).
[5] Siehe Piaget (1975); Smith, Carey und Wiser (1985); Smith et al. (1997); Carey (2009).

ist – darauf hat Peter Damerow in seinem Verständnis historischer Epistemologie stets hingewiesen. Der Entwicklungspsychologe sollte hier nicht nur von der Wissenschaftsgeschichte lernen, in der Klärung der historischen Voraussetzung seines eigenen Gegenstandes ist er auf die Wissenschaftsgeschichte vielmehr angewiesen.

Das Wissen um die historische Entwicklung ist noch in einer weiteren Hinsicht für den Psychologen von Bedeutung: Die Untersuchung physikalischer Konzepte von Kindern findet ein Anwendungsfeld in der Physikdidaktik, die Bedingungen für gelingendes Lehren und Lernen physikalischen Schulwissens zum zentralen Gegenstand hat, also just die Aneignung kulturell repräsentierten „geteilten Wissens“ thematisiert. Dass diese Aneignung oft scheitert, davon legen die Arbeiten zu intuitivem physikalischen Wissen beredt Zeugnis ab: Selbst Physikstudenten, die – so sollte man meinen – mit Newtons Trägheitsgesetz vertraut sein sollten, verweisen bei Fragen nach den Ursachen der Wurfbewegung mitunter auf eine antreibende Kraft. Unter den Augen der Physikdidaktiker verwandeln sich die spontanen Begriffsbildungen jüngerer oder älterer Kinder dann rasch in „Misskonzepte“, die mithilfe des Schulunterrichts so rasch und effizient wie möglich durch das kanonisierte Schulwissen ersetzt werden sollen. Doch hier droht die Aneignung physikalischer Begriffe in anderer Weise zu scheitern: Schüler und Erwachsene operieren dann mit Fachbegriffen wie „Gravitation“, „Lichtwelle“, verweisen auf komplizierte Apparaturen oder Versuchsanordnungen, ohne die zugrundeliegenden Modelle wirklich verstanden und in ihrem Erfahrungsgehalt begriffen zu haben. „Unbildung im Aufputz der Bildung“ oder „verdunkelndes Wissen“ nennt das der Physikdidaktiker und Bildungsforscher Martin Wagenschein. Die wissenschaftshistorische Perspektive wirkt hier als Korrektiv. Sie zeigt, auf welch verschlungenen und komplizierten Wegen sich die Begriffsbildung in der Geschichte vollzog, welche Widerstände es dabei zu überwinden galt und wie anstößig und irritierend Konzepte einst waren, die für uns zum Schulwissen gehören und über die wir daher selbstsicher zu verfügen glauben. Somit hält die Wissenschaftsgeschichte für Pädagogen oder Entwicklungspsychologen einen reichen Fundus bereit. Sie ist ein „Verjüngungs-Elixier“, so der Physikdidaktiker Martin Wagenschein (1999), denn sie hilft uns, einen Perspektivenwechsel zu vollziehen und zu verstehen, vor welchen Herausforderungen Kinder stehen, wenn sie die Welt so verstehen sollen, wie die Erwachsenen von heute es für richtig halten.

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Chapter 11
Piaget, Einstein, and the Concept of Time

Tilman Sauer

Introduction

On various occasions, Piaget reminisced that it was Einstein who inspired him to study the genesis of temporal concepts.[1] The foreword to his 1946 study on "the child's conception of time"[2] begins like this:

> This work was prompted by a number of questions kindly suggested by Albert Einstein more than fifteen years ago, when he presided over the first international course of lectures on philosophy and psychology at Davos. (Piaget 1969, ix)

Unfortunately, we do not have independent documentation of Einstein's suggestions. But Piaget continues to specify the proposed research with the following questions:

> Is our intuitive grasp of time primitive or derived? Is it identical with our intuitive grasp of velocity? What if any bearing do these questions have on the genesis and development of the child's conception of time? (Piaget 1969, ix)

Piaget goes on to tell us that after Einstein's inspiring question, every year he devoted some investigation to these issues although initially he had no hope of finding anything out since the "time relationships constructed by young children are so largely based on what they hear from adults and not on their own experiences." It was only after his investigations on the child's conceptions of number and quantity that he found a way to disentangle the various aspects of the concept of time and to dissociate its *specific* content from the notions of space and motion.

In Peter Damerow's and Wolfgang Lefèvre's research colloquium on "problems of conceptual development in the history of the natural sciences," we studied

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[1] See, e.g., Piaget (1946, v; 1950, 45; 1957, 54).

[2] Piaget (1946). English translation in Piaget (1969).

Piaget's work for several months in the fall of 1985. We read, in particular, his investigations on the genetic conception of time. Peter criticized Piaget's "concept of reflective abstraction" because of its implication that "the material means of the actions on which cognitive activity is based are irrelevant for the development of cognitive abilities" (Damerow 1996, 9). Nevertheless, Peter tried to convince us that Piaget's analysis of the concept of time carries significance and also provides insight for a historiography of temporal concepts. Those discussions with Peter proved to be some of the formative moments in my intellectual biography. In this contribution, I want to take a look again at Piaget's analysis of the concept of time and make a few comments on the question as to how his analysis may carry over to the conceptual context of the special theory of relativity.

Piaget's Analysis of Classical Temporal Concepts

The core of Piaget's investigation is one particular experiment, which I will discuss in more detail below. It was designed against the background of Piaget's tenet of the specific characteristic of the concept of time. To begin with, Piaget pointed out that temporal judgments are actually not distinct from spatial judgments, as long as only one kind of motion is considered. In various experiments, Piaget demonstrated that correspondingly, with young children of the preoperational stage, temporal judgments of 'earlier' and 'later,' or comparisons of time spans as 'shorter' and 'longer,' are based only on spatial seriation. An object moving from left to right is *first* at point A and *then* at point B if and only if A is left of B. Similarly, the time a body needs to go from point A to B is longer than the time it needs to go from C to D if and only if the distance between A and B is larger than the distance between C and D. Children will find out that things get more complicated if non-uniform motion is involved, but structurally temporal concepts are not distinguished from spatial concepts as long as only one kind of independent motion is considered.

When two different motions have to be compared, the initial reliance on basing temporal judgments on spatial features perseveres. There is a correspondence here between the child's concept of speed and the Aristotelian concept of velocity as the finite distance traversed in a finite amount of time. Judgments of comparison between different velocities are based on various proportionalities that follow from the Aristotelian concept. One body moves faster than another one if it traverses a longer distance in the same amount of time. It is also faster if it traverses the same distance in a smaller amount of time. A composition of both proportionalities can lead to contradictions if the conditions of equal time or equal distance are violated. Thus, if two bodies start moving at the same time from point A and one body arrives at B a little later than the other body arrives at C, but B is farther away from A than C, the first body is either moving faster since it traverses a larger distance or slower since it arrives later at its terminal point.

It is only when different motions have to be judged which are largely causally independent but have to be coordinated at specific points of simultaneity that the specific concept of time needs to be invoked. In order to demonstrate that it is this coordination of different motions that constitutes the conception of time and to isolate its specific deductive capacity, Piaget devised his experiment (see fig. 11.1).

Figure 11.1: In one of his experiments reported in Piaget (1946), a colored liquid is flowing from one bottle into another one through a valve. Children are shown the process and asked to mark successive water levels on prepared sheets of paper. They are then asked to reconstruct the sequence of sketches after the individual sheets were cut in half along the valve.

Two bottle-like vessels of different shape, one pear-shaped, the other of cylindrical form, are connected in such a way that a colored liquid would flow downwards from one bottle into the other through a valve that could be opened and closed. The experimenter would then let children observe a demonstration where the entire liquid is initially in the upper vessel and then is allowed to flow down into the lower vessel in certain discrete amounts. The children would get a number of prepared papers showing the two empty vessels and were asked to draw the water levels in each vessel onto their papers at each stage of the process. Thus, at the conclusion of the demonstration, when the liquid was entirely contained in the lower vessel, the children had created a series of half a dozen or so drawings of the two bottles with different water levels in each. The drawings were then shuffled and the children were asked to put them back into order again according to a temporal sequence. In a second part of the experiment, the drawings were then cut in the middle in such a way that the two halves would show the upper or the lower vessel, respectively. Again, the drawings were shuffled, and the children were asked to reconstruct the original sequence.

Piaget's observation was that very young children were not able to reconstruct the original sequence at all, even in the first part of the experiment with the intact drawings. Older children were able to put the uncut drawings in the correct sequence but failed to reconstruct the correct sequence when the drawings had been cut apart. Typically, what would happen is that random pairs of images of the upper bottle and images of the lower bottle would be formed and children at this stage would construct a sequence based on either the lower half or the upper part but with the randomly formed pairs kept intact. Thus, the water level in the reconstructed sequence would correctly rise in the lower vessel but the upper portion would show a random or wrong sequence, or vice versa. Only at the final stage were the children able to break up pairs at will and construct two coordinated sequences of rising water level in the lower bottle and sinking water level in the upper bottle, put together in such a way that the lowest level in the lower bottle would correspond to the highest level in the upper bottle.

Several features of Piaget's experiment are worth pointing out. First, it does not matter how much time actually passes during the experiment. Since the valve is opened and closed by the experimenter at will, more or less physical time passes between subsequent stages of the experiment. The experiment thus exemplifies Piaget's conviction that time is a cognitive construct, a deductive scheme, not an intuition or form of sensibility. Second, the ability to reconstruct the correct sequence of images depends crucially on the mental ability to reverse and to anticipate the actual physical process. Whereas the actual flow of time and the causal processes are irreversible, the conception of time is dependent on the mental capacity to reverse, anticipate, and interpolate causal processes. In a process of decentration, children construct a uniform, homogeneous time that allows a coordination of different sequences of events.

Piaget captured his understanding of the concept of time as a co-seriation of different sequences of physical events in an intuitive graphical representation (see fig. 11.2).

A sequence of events, or a motion, is characterized by points O_1, A_1, etc. that follow each other in a relation of earlier and later in some causally determined way. They are coordinated with other sequences of events, or motions, O_i, A_i, etc. such that O_1, O_2, O_3, etc. are put into a relation of simultaneity. Uniform time is not bound to any one specific sequence of events or motion but rather arises from the coordinating operations as a cognitive construction that allows the co-seriation of the different sequences of events A_i, B_i, etc. and the different time spans a, a', b', etc.

$$
\begin{array}{ccccccccc}
 & a & & a' & & b' & & e' & \\
O_1 & \rightarrow & A_1 & \rightarrow & B_1 & \rightarrow & C_1 & \rightarrow & \cdots \\
\downarrow_0 & & \downarrow_0 & & \downarrow_0 & & \downarrow_0 & & \\
 & a & & a' & & b' & & c' & \\
O_2 & \rightarrow & A_2 & \rightarrow & B_2 & \rightarrow & C_2 & \rightarrow & \cdots \\
\downarrow_0 & & \downarrow_0 & & \downarrow_0 & & \downarrow_0 & & \\
 & a & & a' & & b' & & c' & \\
O_3 & \rightarrow & A_3 & \rightarrow & B_3 & \rightarrow & C_3 & \rightarrow & \cdots \\
\vdots & & \vdots & & \vdots & & \vdots & &
\end{array}
$$

Figure 11.2: Piaget's graphical illustration of his concept of co-seriation (Piaget 1969, 264).

Piaget on the Concept of Time in Relativity

Piaget's concept of time as a cognitive ability of co-seriation is a convincing analysis of the concept of "absolute time" in classical mechanics. Its genesis in the development of children illustrates Piaget's idea that cognitive development proceeds along stages of ever more far-reaching decentration, which in turn result from ever-increasing capacities of transformational operations. In his 1946 book, Piaget concluded his analysis with a perspective on the question of how his analysis would carry over to the problem of understanding time in the special theory of relativity.

It remains frustratingly unclear how Piaget would have applied his analysis to the genetic explanation of special relativity. In fact, the few remarks that he gives in his 1946 book are altogether too vague to allow us even to assess whether Piaget fully understood the difference between temporal conceptions implied by the special theory of relativity and those of classical Newtonian mechanics.

In special relativity, there is no universal time. But Piaget, when he talks about Kant, says:

> As Kant has shown so clearly, time and space are not concepts but unique 'schemes'—there is only one time and one space in the entire universe. (Piaget 1969, 33)

Are we supposed to read the assertion of the unique existence of one and only one universal time as restricted to Kant, or would Piaget approve of this assertion in general? At other places, he talks about the 'relative time' (p. 396) but also about the 'time of relativity theory.' Indeed, the final passages of the book pertain explicitly to the theory of special relativity and show the same ambivalence and ambiguity. Piaget wrote:

> As for the time of relativity theory, far from being an exception to this general rule, it involves the co-ordination of motions and their velocities even more clearly than the rest. (Piaget 1969, 278f)

So far, so good, and one would be tempted to agree with Piaget on this general level, if suspicions would not have been raised by the unqualified use of the term 'time of relativity theory.' In relativity theory, there is no such thing as 'the time,' but, of course, Piaget could have meant the 'concept of time' in relativity theory. But what follows immediately afterwards carries the same ambivalence. He goes on to say:

> Let us recall first of all that relativity theory never reverses the order of events in terms of the observer's viewpoint: if *A* precedes *B* when considered from a certain point of view, it can never follow *B* when considered from a different standpoint, but will at most be simultaneous with it. (Piaget 1969, 279)[3]

How should we interpret Piaget here? Apparently *A* and *B* would be two events. Let's coordinatize them in some frame of reference, *F* as $A = (x, y, z, t)$ and $B = (x', y', z', t')$ (see fig. 11.3). Here x, y, z, and x', y', z' denote spatial coordinates, t and t' denote the time coordinate (in the following, we will suppress the irrelevant y- and z-coordinates). To say that *A* happens before *B*, would then mean that $t < t'$ or $t' - t > 0$.

In fig. 11.3, horizontal red (dashed) lines denote events with the same t or t', respectively, that is, events on the same horizontal red (dashed) line are simultaneous in *F*. Clearly, with respect to the red (dashed) lines of simultaneity, *A* precedes *B*. Now let's look at the two events from a frame of reference Φ moving with respect to *F* with velocity v along the x-direction. In Φ, we have $A = (\xi, \tau)$ and $B = (\xi', \tau')$, and in Φ the temporal difference between the two events is $\tau' - \tau$. But if we interpret Piaget's phrase 'from a certain point of view' as 'in a certain frame of reference,' his claim is wrong. To see this, consider the Lorentz transformations that allow us to go from *F* to Φ:

$$\xi = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}; \qquad \tau = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{11.1}$$

[3] The vagueness is not an artifact of the translation; in the original French, the passage reads: "Rappelons d' abord que, en aucun cas, il n' aboutit à inverser l' ordre des phénomènes en fonction des points des vue: si A est antérieur à B, d' un certain point de vue, il ne sera jamais ultérieur à B, d' un autre point de vue, mais tout au plus simultané. " (Piaget 1946, 298).

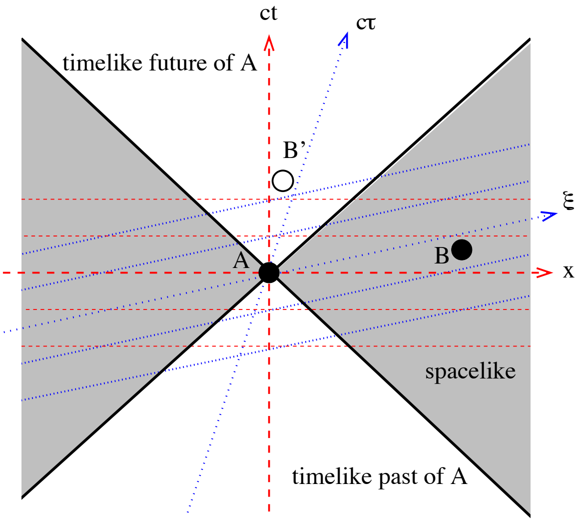

Figure 11.3: Illustration of the relativity of simultaneity of two spacelike separated events A and B in a spacetime diagram. In the red (dashed) coordinate system (x, ct), horizontal lines parallel to the x-axis represent (hyper)surfaces of simultaneous events, and in these coordinates A precedes B. But for an observer moving rapidly along the x-direction, the blue (dotted) coordinate system $(\xi, c\tau)$ is used and the tilted lines parallel to the ξ-axis represent (hyper)surfaces of simultaneous events. In the blue (dotted) frame of reference, the event B precedes A. However, for events B' in the timelike future of A, the event A precedes B' in *all* possible frames of reference.

We then get

$$\begin{aligned} \tau' - \tau &= \frac{t' - \frac{v}{c^2}x'}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}; \\ &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(t' - t - \frac{v}{c^2}(x' - x) \right). \end{aligned} \tag{11.2}$$

Clearly, we can have $\tau' - \tau < 0$ or $\tau' < \tau$, that is, a reversal in the temporal order of the two events in Φ, if

$$t' - t < \frac{v}{c^2}(x' - x). \qquad (11.3)$$

In fig. 11.3, blue (dotted) lines parallel to the ξ-axis denote events with the same τ or τ', respectively, that is, events on the same (tilted) blue (dotted) line are simultaneous in Φ. One sees that, with respect to the blue (dotted) lines of simultaneity, B precedes A.

That is to say, if the two events A and B are sufficiently far away from each other spatially in F, then an observer dashing by along their line of connection with a speed $v > c^2 \Delta t / \Delta x$ would see the two events in reverse order. And if Δx, the spatial distance of A and B as measured in F, is sufficiently large, there will be no problem in satisfying that condition. What we have shown is simply the well-known tenet that for any two events A and B whose separation is spacelike (the shaded region in fig. 11.3), that is, for which

$$(x' - x)^2 + (y' - y)^2 + (z' - z)^2 - c^2(t' - t)^2 > 0, \qquad (11.4)$$

the temporal order is undefined and depends on the state of motion of the observer.

Piaget's formulation is vague enough to allow for different, and correct, interpretations. After all, we are talking only about the final paragraph of an entire book. He could have meant two events happening along the world line of a material particle, or in other words, he could have meant that two events might be causally connectable in the sense that their separation is timelike.

In fact, it seems that we may indeed have been too critical in our reading of the above passage. Four years later, Piaget incorporated the results of his 1946 book on the genesis of the conception of time into his 1950 *Introduction à l'Épistémologie Génétique*, in its second volume dealing with *La Pensée Physique*. There, we find the same statement again in paragraph V of chapter IV, section 3, entitled "The relativistic metric."

In this paragraph, Piaget first claims quite generally that his analysis of the (classical) conception of (absolute) time as supervenient on the concept of velocity suggests the naturalness of the relativity revolution. That is because "[...] all modifications of our ideas about velocity imply a transformation of our conception of time."[4] Since in the development of physics, the velocity of light had emerged as a limiting speed that cannot be surpassed by the propagation of any causally efficacious signal, it followed with necessity that both the concept of velocity and the notion of time were subject to joint modification according to the genetic viewpoint:

[4] Piaget (1950, 44). English translations from Piaget (1950) are my own, TS.

> As soon as Michelson's and Morley's measurements had verified the special character of the speed of light and its complete isotropy, the genetic conception, which connects the idea of time with that of velocity, required a simultaneous modification of these two concepts. This revision of the physical concepts as a function of ideas about time and velocity was carried out by Einstein with well-known success. (Piaget 1950, 45)

The modification required by the axiom of the limiting value of the speed of light pertains immediately to the concept of simultaneity. If there is a maximal limit velocity, it follows that the simultaneity of *distant* events is no longer absolute but relative to the velocity of an observer, just as children will deny that two lights are switched on simultaneously if the lights are some meters apart and the children cannot see them at the same time (Piaget 1969, 110–115). It is only for spatially neighboring events that a concept of simultaneity survives, and it is here that we find the incriminating sentence again, embedded in a context which makes it clear that Piaget is thinking about "neighboring events," that is, events which are spatially close together.

> Since the simultaneity of events at *neighboring* [my emphasis, TS] places [...] is not changed, the same is true for the temporal sequence. (Piaget 1950, 47)

The sentence about the invariance of temporal sequence which we criticized above follows immediately after this qualification.

Nevertheless, Piaget offers us only analogies when it comes to the genetic explanation of relativity:

> On the other hand, this concept of relativity of duration requires an effort of coordination in order to equilibrate the viewpoints of observers moving with different velocities. This effort is only a continuation of the effort of coordination that the child has to master in order to comprehend the heterogenous durations, which it associates with different velocities, in terms of a unique common time. As paradoxical as it may seem, the relative durations and the proper times of Einstein's theory relate to absolute time as absolute time to the individual times and local times of the child's intuition. (Piaget 1950, 46)

That is to say, the coordination efforts that give rise to the child's forming a unique common time are analogous to the coordination efforts that a classically trained physicist has to master in order to overcome the limitations of the absolute time of Newtonian mechanics. But what Piaget calls paradoxical here is perhaps only a consequence, as we will see below, of the lack of a proper term that would identify the genetic analog of absolute time in a relativistic context.

In any case, Piaget puts the coordination of different velocities that underlies the genesis of the concept of absolute time in perfect parallel to the coordination of velocities that underlies the genesis of relativistic concepts:

> In both cases, time appears as a coordination of velocities, and the transition from velocities that cannot be coordinated to those that can be coordinated, which is made possible by means of a homogeneous and uniform time, is a first stage of transformation from an erroneous egocentric absoluteness into objective relations. A second stage makes possible the transition from absolute time (and the possibility of an infinite velocity) to relative time, which is connected with a precise coordination of velocities. (Piaget 1950, 46)

Einstein and the Special Theory of Relativity

As far as it goes, Piaget's interpretation of the genetic basis of special relativity provides an interesting perspective. But as it stands it is a mere program or an abstract conceptual framework. It needs to be spelled out. Here I want to make only a few comments.

I want to look at the *locus classicus* for the emergence of the special theory of relativity, Einstein's paper on the "Electrodynamics of Moving Bodies" (1905). What are the analogues of the experiences and efforts of coordination that underlie the emergence of relativistic temporal and spatial concepts? To what extent are those experiences and coordination efforts comparable to the ones that the child is confronted with and needs to master, and where are they different? Does the difference in the experiential foundation of special relativity have implications for the character of temporal concepts in the relativistic context?

The first observation we might make is that we can read the entire § 1 of "Electrodynamics of Moving Bodies" as an elaboration of the classical, Newtonian time concept along the lines of Piaget's analysis. Just in passing we may remark that the very term "coordinate system" indicates the very coordination efforts that may be interpreted in Piagetian terms. In § 1 Einstein expounds the problems of the concept of distant simultaneity and defines a global time in an inertial frame of reference by a system of clocks that are located everywhere in space and coordinated by the exchange of light signals. This system of clocks is the idealized materialization of the uniform time that allows an operational coordination of different motions, but only within the same inertial frame. As Einstein concludes: "It is essential that we have defined time by means of clocks at rest in a system at rest; because it belongs to the system at rest, we designate the time just defined as 'the time of the system at rest.'" (Einstein 1905, 894f).

It is only in the following sections that Einstein addresses the crucial question of coordinating the experiences of two observers who are in a state of uniform relative motion with each other. Very explicitly in § 2, Einstein constructs a situation where the observer dependence of spatio-temporal judgments becomes obvious. He looks at the task of determining the length of a moving rigid rod from the point of view of two observers. One observer is co-moving with the rod. In his inertial system the rod is at rest and its length can be determined by physical comparison with another rod that serves as an etalon. The other observer determines the length of the moving rod from a coordinate system at rest by looking at which points in space the end points of the rod are located at some point t of time in his coordinate system. While "the commonly used kinematics tacitly assumes that the lengths determined by the two methods are exactly identical" (Einstein 1905, 894f), Einstein finds that they are, in fact, not the same. This result raises the problem of how one can go back and forth from one frame of reference to another one that is moving with respect to the first.

This coordination task of operationally moving between different frames of reference is solved by finding coordinate transformations between the (orthogonal Cartesian) coordinates x, y, z, t of a frame of reference K, and the coordinates ξ, η, ζ, τ of a system Φ. The transformations that mediate between the two systems of coordinates are the Lorentz transformations (11.1). Their form and some of their properties were known to Einstein before, but the essential core and content of his 1905 paper is to show how the Lorentz transformations are justified and how they actually work in mediating the operational coordination between moving frames of reference.

In the "kinematical part," Einstein derives the Lorentz transformations from the two basic principles, the principle of relativity and the speed of light postulate, and he shows that they are compatible with the assumption that all moving observers measure the same vacuum speed of light. They also imply the relativity of simultaneity for observers in relative motion as well as length contraction and time dilation. He also looks at the addition of velocities and finds that the classical law of addition of velocities is modified by the Lorentz transformations to a new law, and one of its implications is that by composition of velocities one can never exceed the velocity of light.

In the "electrodynamic part," Einstein addresses the question as to how electrodynamical processes are to be transformed if viewed from different frames of references. Technically, the core result is the transformation law for the electric and magnetic fields. It is here that the capacity of transforming back and forth between moving frames of reference becomes fully operational. Einstein shows that in all frames of reference, the Maxwell equations for the electromagnetic field in a vacuum hold if the fields are properly transformed. He looks at specific problems from the point of view of different observers and explicitly performs the necessary transformational operations. One such problem is the shape of a moving electron, which is assumed to be a rigid sphere in a frame of reference in which it is at rest, and the related problem of transforming the equations of motion of a moving charge. Another such problem is the investigation of the relativistic Doppler effect by asking how a light source would appear to an observer who is rapidly moving toward it.

All this is well-known, but it illustrates the main point one might want to make from a genetic perspective. The relativistic concepts arise from a technical competence of actively changing perspectives by reversible, operational transformations between the viewpoints of moving observers. But the decentration that results from those efforts is of a special kind, which is responsible for the notorious difficulty of learning and teaching relativity theory. The change of perspective between frames of references that is relevant here is almost never—certainly not to Einstein—one that is subject to human experience. To be sure, the empirical consequences of relativity theory have amply been confirmed and even play a role in today's practical life. Nevertheless, the operational transformations that mediate between different frames of reference, to the extent that relativistic implications become important, are entirely theoretical. They are mental operations making use of conceptual and calculational means of deduction. They arise from technical operations that are connected with the manipulation of coordinate systems and coordinate transformations as well as their imagined interpretations. Humans are terrestrial observers moving slowly about the surface of the earth. The laws of physics as they would appear to an observer who is co-moving with a fast moving electron, or a canal ray or an astrophysical object, are not subject to any direct experiential concretization. Nor can we experience how a light source would look if approached with close to luminal velocities. The change of perspective and the cognitive effort of coordinating causal processes viewed from different frames of reference are mental operations mediated through specific mathematical representation.

On the other hand, the relevant transformations have to be carried out in an exact and quantitative way. The results carry empirical significance and could be confronted with observation and this is what eventually made the cognitive restructuring inevitable. For Einstein, the relevant experimental context of the early twentieth century involved the investigation of the dynamics of beta and cathode rays, that is, of fast moving electrons, as well as experiments, such as Fizeau's or Michelson's, of measuring the velocity of light, and astronomical effects like stellar aberration.

Another difficulty of transferring Piaget's analysis of pre-classical and Newtonian temporal concepts to the conceptual context of special relativity seems to be implied by the use of phrases like "time of relativity" or "relativistic time." If the classical conception of time emerges from the operational co-ordination of co-displacements or from the co-seriation of causal sequences of events, then the analog of classical time in the relativistic context should no longer be called "time." What emerges from the efforts of co-ordination of physical processes between frames of references moving relative to each other with velocities comparable to the limit speed of light is not a new concept of time. It is a new conception and conceptual framework of spatio-temporal relations that is best captured by a term that was not available to Einstein in 1905. An appropriate term only emerged a few years later with Hermann Minkowski's reinterpretation of relativistic concepts in terms of a four-dimensional "world" in which spatial and temporal relations are only projections of every individual observer, or in Minkowski's oft-quoted words:

> The views on space and time which I wish to lay before you have sprung from the soil of experimental physics. Therein lies their strength. Their tendency is radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality. (Minkowski 1909, xiv)

Minowski's "world" is nowadays called Minkowski spacetime, a four-dimensional Riemannian manifold with a Minkowski metric. In Minkowski spacetime any relative times and spaces appear as projections relative to an observer, as was illustrated graphically in fig. 11.3. It is Minkowski spacetime—the entire spatiotemporal structure, not any 'relative time' or 'time of relativity theory'—which constitutes the invariant entity emerging from the coordination efforts and allowing for the transformational operations of going back and forth at will between the possible experiences of different inertial observers.

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Chapter 12
Zur historischen Epistemologie des Raumes

Matthias Schemmel

In den letzten Jahren seines Lebens hat Peter Damerow ein Forschungsprojekt begleitet und mit eigenen Beiträgen bereichert, das mir sehr am Herzen lag: Die historische Epistemologie des Raumes, ein Projekt zur langfristigen historischen Entwicklung räumlicher Begriffe, das ich mit einer Forschergruppe im Rahmen des Projektclusters TOPOI am Max-Planck-Institut für Wissenschaftsgeschichte durchgeführt habe. Ich möchte hier zu Peters Gedenken einige Forschungsergebnisse dieses Projektes umreißen. Eine ausführliche Darstellung unserer Ergebnisse, die auch zwei bisher unveröffentlichte Beiträge von Peter enthält (Damerow 2016a, Damerow 2016b), ist kürzlich erschienen (Schemmel 2016b). Eine monographische Darstellung der historischen Epistemologie des Raumes ist Schemmel 2016a.

Das Thema der Gruppe war die langfristige Transformation räumlicher Begriffe in der Wissenschaftsgeschichte. Nun sind langfristige Wissensentwicklungen nicht allein auf der Ebene wissenschaftlichen Wissens zu verstehen. Wissenschaftliches Wissen, das ist eine Grundannahme, die sich wie ein roter Faden durch Peters Werk zieht, ist Teil einer komplexen Wissensarchitektur, in der sich verschiedene Wissensformen gegenseitig beeinflussen.[1]

In etwas vereinfachender Weise können wir drei Wissensebenen unterscheiden: elementares, instrumentelles, und theoretisches Wissen. Elementares Wissen erwirbt ein jedes Individuum erneut für sich im Prozess der Ontogenese, d. h. des Aufwachsens in seiner Umwelt. Aufgrund der Übereinstimmungen in der biologischen Konstitution und in den elementaren Eigenschaften der physikalischen Umwelt sind große Teile dieses Wissens kulturunabhängige Universalien. Dieses Wissen ist jedoch im Allgemeinen nicht begrifflich strukturiert, sondern bleibt, zum Beispiel als Handlungsschema, unbewusst.

Instrumentelles und, ganz allgemein, praktisches Wissen wird durch den Umgang mit Instrumenten und durch das Ausüben kultureller Praktiken erwor-

[1] Siehe beispielsweise Damerow und Lefèvre (1981); Damerow (1994, 1996) und Damerow, Freudenthal, et al. (2004). Insbesondere Damerow (1994) kann als ein Modelltext für das Programm einer historischen Epistemologie fundamentaler Begriffe aufgefasst werden.

ben. Das Wissen ist daher ebenso kulturabhängig wie das Vorhandensein dieser Instrumente und Praktiken. Die Weitergabe des Wissens erfolgt häufig durch gesprochene Sprache und gemeinsames Handeln.

Theoretisches Wissen hingegen setzt eine Reflexion auf vorheriges Wissen voraus. Es ist meistens im Medium der Schrift oder anderer Symbolsysteme repräsentiert und bedarf zu seiner Tradierung gesellschaftlicher Institutionen wie Schulen oder Bibliotheken.

Ein fundamentales Ergebnis unserer Arbeit besteht in der Erkenntnis, dass die diesen verschiedenen Wissensebenen zugehörigen räumlichen Wissensstrukturen sich in ihrer historischen Dynamik tatsächlich gegenseitig formen und beeinflussen.

> *Ergebnis 1*. Die Ebenen des elementaren, instrumentellen und theoretischen Wissens durchdringen sich in ihrer historischen Dynamik, indem sich die ihnen zugehörigen Wissensstrukturen gegenseitig formen und beeinflussen.

In den von uns verfolgten, konkreten Unterprojekten, die das räumliche Denken zu verschiedenen historischen Zeiten und in verschiedenen Kulturen zum Gegenstand hatten, haben wir derartige Wechselwirkungen aufzeigen können. So konnten wir etwa zeigen, wie elementares Orientierungswissen zur Erklärung von Gemeinsamkeiten in den unabhängig voneinander entstandenen, verschiedenartigen kulturellen Praktiken der räumlichen Orientierung in verschiedenen Gesellschaften herangezogen werden kann. Solches Orientierungswissen ist durch elementare Wissensstrukturen charakterisiert, wie zum Beispiel der der *kognitiven Karte* oder auch der des *mentalen Modells des Raumes im Großen*.

> *Ergebnis 2*. Die elementaren Strukturen mentaler Modelle des Raumes im Großen erklären Gemeinsamkeiten der kognitiven Strukturen, die den räumlichen Orientierungspraktiken verschiedener Gesellschaften zugrunde liegen.

Eine fundamentale Eigenschaft des mentalen Modells des Raumes im Großen ist das Vorhandensein einer Dichotomie von bewegten und unbewegten Objekten. Eine andere ist das Verständnis bestimmter Zusammenhangsverhältnisse im Raum, zum Beispiel, dass ein Ziel auf mehreren Wegen erreicht werden kann. Dieses räumliche Wissen ist nicht nur unabhängig von jeder spezifischen Kultur, die es zu untersuchen gilt, sondern geht dem Vorhandensein menschlicher Kultur überhaupt voraus. Dies zeigen Untersuchungen des räumlichen Verhaltens nichtmenschlicher Primaten. So ist es zum Beispiel bei Schimpansen nachgewiesen worden.[2]

[2] Einen umfassenden Überblick geben Tomasello und Call (1997); eine klassische Studie zu Schimpansen ist insbesondere Menzel (1973).

Unser vergleichendes Studium verschiedener kultureller Praktiken der Orientierung hat nun gezeigt, wie diese elementaren Wissensstrukturen die Grundlage für elaborierte praktische Wissenssysteme bilden. Dabei können ganz unterschiedliche Aspekte räumlichen Wissens durch kulturelle Wissensakkumulation zu einem Expertenwissen entwickelt werden, das kein außerhalb dieser Kultur stehendes Individuum jemals hätte entwickeln können. Ein Beispiel ist das für die Orientierung mikronesischer Navigatoren essentielle System absoluter Richtungen, das sich an Sternen und Sternkonstellationen ausrichtet und 32 Richtungen definiert. Mithilfe dieses *Sternkompasses* bestimmen die Navigatoren nicht nur Richtungen, sondern auch ihre eigene Position. Dazu wird die Lage einer Insel, die weder Ausgangspunkt noch Ziel der Reise ist, unter den Sternen in Betracht gezogen. Die Navigatoren wissen nämlich, in welcher Richtung diese Insel zu welchem Zeitpunkt der Reise zu sehen sein muss.[3]

Die Praxis der mikronesischen Navigatoren bietet daher auch ein interessantes Beispiel für eine Modifikation elementarer Wissensstrukturen im praktischen Wissen. Die Dichotomie von bewegten und unbewegten Objekten wird auf interessante Weise verkehrt. Es sind nun nicht mehr die Inseln, die die unbeweglichen Landmarken darstellen, zwischen denen sich das Boot bewegt. Vielmehr betrachten die Navigatoren ihr Boot als unter dem richtungsfixierenden Sternkompass ruhend, während sich die Inseln daran vorbei bewegen.

> *Ergebnis 2a*. Praktische Wissensstrukturen können die elementaren aber auch überformen, wie das Beispiel der modifizierten Dichotomie von bewegtem und unbewegtem Objekt bei den mikronesischen Navigatoren zeigt.

Eine weitere elementare Wissensstruktur, die sich in den anderen Wissensebenen widerspiegelt, ist die des mentalen Modells des permanenten Objekts. In den ersten Jahren unseres Lebens entwickeln wir ein Wissen von Objekten als Strukturen mit bestimmter Form und Größe, die sich an einem bestimmten Ort befinden oder sich bewegen, und die unterschieden sind von den Zwischenräumen, die sie trennen. Diese Wissensstrukturen spiegeln sich zunächst in der Alltagssprache wider, was bereits Wörter wie ‚groß', ‚klein', ‚dick', ‚dünn', ‚hart', ‚weich', ‚nah', ‚fern', ‚leer' und ‚voll' anzeigen.

Unter gewissen historischen Umständen, die zum Beispiel das Vorhandensein einer Kultur der Disputation und in den meisten Fällen wohl auch das einer Schriftsprache einschließen, kommt es zur theoretischen Reflexion über diese sprachliche Repräsentation elementaren Wissens. Ein prominentes Beispiel bieten die atomistischen Theorien der Antike, in denen die elementare Dichotomie

[3] Siehe Gladwin (1974) für eine umfassende Darstellung der Kultur der mikronesischen Navigatoren und Hutchins (1983) für eine Interpretation ihrer kognitiven Grundlagen.

von Objekt und leerem Zwischenraum zu einem fundamentalen Prinzip der Welterklärung absolut gesetzt wird (Damerow 2016a). Ein anderes Beispiel ist Aristoteles' Lehre vom Ort, die elementares Wissen darüber, das alle Dinge einen Ort haben, aber nicht zwei Dinge am selben Ort sein können, reflektiert. Nach dieser Doktrin ist ein leerer Ort unmöglich, da ein Ort immer der Ort von etwas sein muss.

Wir haben in unserem Projekt einen chinesischen Text, den sogenannten *Mohistischen Kanon*, aus der Zeit um 300 v. Chr. neu übersetzt, analysiert und interpretiert, um mithilfe des Vergleichs der voneinander unabhängigen westlichen und chinesischen Traditionen etwas über Notwendigkeit und Kontingenz bei der Entstehung theoretischen Wissens in der Antike herauszufinden (Boltz und Schemmel 2016).[4]

Dies ist ein Beispiel eines der knapp zweihundert Abschnitte des *Mohistischen Kanons*, die immer aus einer Definition oder einer Proposition und einer dazugehöriger Erläuterung bestehen.

> 盈，莫不有也。
>
> 盈：無盈無厚。於尺無所往而不得二。
>
> *yíng* 'ausgefüllt sein' bedeutet, nirgends nicht etwas zu haben.
>
> *yíng* 'ausgefüllt sein': Wo es kein Ausgefülltsein gibt, gibt es auch keine Ausdehnung. Auf dem Maßstab gibt es keinen Ort, zu dem er sich erstreckt, an dem nicht beides (Ausfüllung und Ausdehnung) statthätte.

Die Aussage, dass Ausdehnung nur da zu finden ist, wo auch Ausgefülltsein vorhanden ist, erinnert an die aristotelische Doktrin vom Plenum, also von der Unmöglichkeit des leeren Ortes. Tatsächlich reflektieren beide Texte, die Aristotelische *Physik* und der *Mohistische Kanon*, in weiten Teilen dieselben elementaren Strukturen räumlichen Denkens. Allerdings formuliert Aristoteles seine Ablehnung des Leeren direkt als Kritik an der atomistischen Weltsicht, der er eine eigene umfassende Weltsicht entgegenstellt. Der mohistische Text formuliert dagegen keine Kritik an einer solchen Weltsicht, geschweige denn, dass er eine solche aufstellen würde. Tatsächlich sind in der chinesischen Antike umfassende Kosmologien erst nach der Zeit der Mohisten entstanden, also nach dem Auftreten metasprachlicher Reflexionen über Raum und Wissen.

Der Vergleich von griechischer und chinesischer Antike zeigt uns also, und das ist ein weiteres Ergebnis unserer Forschung, dass das Auftreten elementarer mentaler Modelle im theoretischen räumlichen Wissen ein kulturübergreifendes

[4] Bei unserer Arbeit haben wir sehr von dem wegweisenden Werk A. C. Grahams profitiert (1978).

Phänomen ist. Die Verknüpfung solcher Reflexionen mit umfassenden Weltsystemen oder Kosmologien ist hingegen eine Besonderheit der griechischen Entwicklung. Dieser Unterschied lässt sich auf das zeitliche Verhältnis verschiedener theoretischer Traditionen innerhalb einer Kultur zurückführen.

> *Ergebnis 3*. Das Auftreten elementarer mentaler Modelle im theoretischen räumlichen Wissen ist kulturübergreifend nachweisbar.
>
> Die Verknüpfung derartiger Reflexionen mit umfassenden Weltsystemen oder Kosmologien hingegen stellt eine Besonderheit der griechischen Entwicklung dar, die von der zeitlichen Koinzidenz unterschiedlicher theoretischer Traditionen abhängt.

Die Theorien des Raumes der griechischen Antike prägten alle späteren Raumphilosophien im arabischen und im europäischen Raum bis in die Moderne. Grundsätzlich kann von zwei alternativen Modellen gesprochen werden:

- dem Raum als Gefäß aller körperlichen Dinge, wie es der atomistischen Lehre entspricht, und
- dem Raum als „Lagerungs-Qualität der Körperwelt“[5], wie es der Aristotelischen Lehre vom Ort entspricht.

Ihre Überzeugungskraft gewannen diese Modelle wohl weniger durch die Autorität einzelner Philosophen als vielmehr durch die intuitive Plausibilität der elementaren Wissensstrukturen und ihre Einbettung in umfassende Wissenssysteme.

Der theoretische Kontext mit seinen Forderungen nach Allgemeinheit und Konsistenz führte dabei zu fundamentalen Fragen über den Raum, die auf der Ebene des elementaren Wissens niemals entstanden wären, wie der Frage nach dem absolut Leeren, oder der Frage nach Endlichkeit oder Unendlichkeit des Raumes oder nach seiner Kontinuität oder Diskontinuität. Die elementaren Wissensstrukturen haben auf der Ebene elementaren Handelns ihre eindeutigen Bereiche der Anwendbarkeit. Auf der Ebene theoretischen Denkens besteht im Gegensatz dazu eine inhärente Unsicherheit darüber, auf welche Aspekte der elementaren Modelle aufgebaut werden soll. Diese Uneindeutigkeit rührt von der Abwesenheit der konkreten Handlungskontexte her, die die Bedeutung der sprachlichen Repräsentationen räumlichen Wissens in ihrem alltäglichen Gebrauch einschränken. Die Operationen reflexiven Denkens sind losgelöst von diesen ursprünglichen Kontexten und fördern Strukturen zutage, die im System der Mittel zur Wissensrepräsentation, zum Beispiel der Sprache, angelegt sind. Das Ergebnis derartiger Reflexionen ist im Allgemeinen nicht vorherbestimmt, weil der Raum

[5] So Albert Einstein in seinem Vorwort zu Max Jammers *Das Problem des Raumes* (1960, XIII), in dem er diese Unterteilung vornimmt.

der möglichen Wissensstrukturen, den die Repräsentationsmittel aufspannen, viel reichhaltiger ist, als jede konkrete Realisierung einer Theorie des Raumes.

Das heißt aber keineswegs, dass theoretisches Denken beliebig wäre. Der reflektierende Umgang mit dem sprachlich repräsentierten Wissen setzt intersubjektiv geteilte Argumentationsstandards voraus. Verschiedene Raumtheorien können daher motiviert und Argumente können für und wider sie vorgebracht und abgewogen werden, wobei Kriterien der Konsistenz, Allgemeinheit, empirischer Angemessenheit und, in entsprechenden historisch-kulturellen Kontexten, theologischer Angemessenheit angeführt werden können.

Diese Unterdeterminiertheit als allgemeine Eigenschaft theoretischen Denkens erklärt die anhaltende kontroverse Diskussion über die fundamentalen Eigenschaften des Raumes in der Spätantike, dem arabischen und dem lateinischen Mittelalter, bis in die Neuzeit. Aber kam es in der frühen Neuzeit nicht zu einer eindeutigen Entscheidung in dieser Auseinandersetzung? Newtons Begriff vom absoluten Raum lässt sich eindeutig in die Tradition atomistischer Raumauffassungen einordnen. Mit Newtons Raumbegriff scheint das Modell des Raums als „Behälter aller körperlichen Objekte" gegenüber dem Raum als „Lagerungs-Qualität der Körperwelt" zu siegen. In vielen historischen Darstellungen bis heute wird tatsächlich suggeriert, dieser „Sieg" sei einer inhärenten Überlegenheit des Gefäß-Modells gegenüber dem Lagerungsqualität-Modell geschuldet, als sei es das rationalere oder wissenschaftlichere der beiden Modelle.[6]

Unsere Analyse des Newtonschen Raumbegriffs und das Studium der weiteren Entwicklung des Raumbegriffs in der klassischen Physik haben aber deutlich gezeigt, dass es sich keineswegs um eine inhärente Überlegenheit des Modells handelte. Auch Newtons theologische Begründungen seines Raumbegriffs, die im Briefwechsel von Leibniz und Clarke eine so zentrale Rolle spielten,[7] waren immer umstritten und waren späteren Generationen von Philosophen und Physikern undienlich. Das Vorherrschen des Newtonschen Begriffs des absoluten Raumes in der klassischen Physik geht vielmehr auf den Erfolg der Newtonschen Mechanik zurück, die sich einerseits auf diesen Raumbegriff stützte und der es andererseits gelang, ein enormes Korpus empirischen Wissens erfolgreich zu integrieren: die irdische Mechanik (für Newton verkörpert in der Figur Galileis) und die himmlische Mechanik (für Newton verkörpert in der Figur Keplers). Es war also empirisches Wissen, das zunächst ganz unabhängig von der Betrachtung des Raumbegriffes über lange Zeiten astronomischer Beobachtung und mechanischen Ingenieurwesens kumulierte und zur Stützung eines bestimmten Raumbe-

[6] Ein Beispiel für eine solche Darstellung ist die Max Jammers, der den Weg zu Newtons Raumbegriff als Befreiung vom Aristotelischen Denken feiert: „Die Überwindung des Aristotelischen Raumbegriffes" (1954).

[7] Siehe zum Beispiel Leibniz and Clarke (1991).

griffes führte. Und es war das Auftreten der Trägheitskräfte bei beschleunigten Bewegungen, das Newton als einen neuen Beweis für die Existenz des absoluten Raumes anführen konnte. Der Erfolg dieses Raumkonzeptes war so durchschlagend, dass das Primat des Raumes gegenüber den ihn füllenden Körpern auch die neuzeitliche Erkenntnistheorie nachhaltig prägte, wie die Kantsche Lehre vom Raum als „reiner Anschauungsform“ gegenüber dem „empirischen Begriff“ der Materie deutlich zeigt (Kant 1997, 6).[8]

Während der Erfolg der Newtonschen Mechanik also zur Auszeichnung des Gefäß-Modells des Raumes führte, enthielt der Newtonsche Raumbegriff viele Elemente, die durch die mathematisch-physikalische Theorie der Mechanik nicht gedeckt waren. Eine solches Element war etwa die Annahme eines absoluten Standards der Ruhe. Der Zustand absoluter Ruhe ist im Newtonschen Raumbegriff prinzipiell von dem der gleichförmigen Bewegung unterschieden, obwohl dieser Unterschied in der Mechanik nicht zum Ausdruck kommt. Die Vorstellung absoluter Ruhe ist eindeutig eine Widerspiegelung der elementaren Dichotomie von bewegten und unbewegten Objekten, die auch in den Antiken Raumtheorien implizit – manchmal auch explizit – gegeben ist. Aber im Gegensatz zur elementaren Struktur, bei der es Landmarken sind, die unbeweglich sind, ist das Unbewegte Objekt bei Newton nun zum reinen Raum sublimiert.

Dies lässt sich zusammenfassen als:

> *Ergebnis 4*. Das Vorherrschen des Gefäß-Modells des Raumes gegenüber dem Lagerungsqualität-Modell im 18. und 19. Jahrhundert liegt nicht in einer inhärenten Überlegenheit dieses Modells begründet, sondern war eine Folge der langfristigen Expansion mechanischen und astronomischen Erfahrungswissens und seiner erfolgreichen theoretischen Integration.
>
> Zugleich wiesen die zeitgenössischen Raumbegriffe aber Aspekte auf, die durch dieses Erfahrungswissen nicht begründet waren.

Am Ende des neunzehnten Jahrhunderts lebte die Diskussion der Frage der Bedeutung des absoluten Raumes in der Mechanik wieder auf. In diesem Kontext wurde auch vorgeschlagen, die Newtonsche Sublimierung des ruhenden Objekts rückgängig zu machen und einen „Körper alpha“ einzuführen, der den absoluten Ruhezustand verkörpern sollte (Neumann 1870). Erst in dieser Zeit entstand das Konzept des Inertialsystems, das es möglich macht, den Raumbegriff der klassischen Physik ohne einen solchen Ruhestandard zu formulieren (Lange 1886, 133–141).

Die Entwicklung der Physik zeigt aber nicht nur, dass das Gefäß-Modell modifiziert werden musste, um eine kohärente Grundlage für den Raumbegriff der

[8] Zu Kants empirischen Begriff der Materie, siehe insbesondere Friedman (2001).

klassischen Mechanik darzustellen, sondern auch, dass die Entscheidung für dieses Modell nicht endgültig war. Die allgemeine Relativitätstheorie, die den fortgeschrittensten Raumbegriff unter den gegenwärtigen etablierten physikalischen Theorien aufweist, ist wohl eher mit dem Lagerungsqualität-Modell als mit dem Gefäß-Modell vereinbar. So gesehen ist der moderne Raumbegriff dem Aristotelischen wieder nähergerückt als es im 19. Jahrhundert der Fall war. Tatsächlich sind in der allgemeinen Relativitätstheorie die natürlichen Schwerebewegungen wieder eine Folge von Eigenschaften des Raumes, wie bei Aristoteles. Allerdings hat der fortgeschrittene Formalismus der modernen Physik die zugrundeliegenden Modelle stark modifiziert. Insbesondere der Feldbegriff, dem man eine Zwitterstellung zwischen Raum und Körper zusprechen kann, sprengt die elementare Dichotomie von Objekten und leeren Zwischenräumen und spielt in den gegenwärtigen Diskussionen über den Raumbegriff der allgemeinen Relativitätstheorie eine zentrale Rolle.

> *Ergebnis 5*. Die Autonomie des Raumbegriffs, die der erkenntnistheoretischen Sonderrolle des Raumes gegenüber der Materie zugrunde liegt, ist ein historisches Phänomen, das mit der allgemeinen Relativitätstheorie sein Ende fand.
>
> Die Rückkehr zum Lagerungsqualität-Modell des Raumes erfolgte unterdessen auf einer durch den neuen Formalismus der Feldtheorie modifizierten Grundlage.

Allerdings setzen die etablierten quantentheoretischen Behandlungen physikalischer Felder, insbesondere die Quantenelektrodynamik, nach wie vor eine (speziell-relativistische) Behälter-Raumzeit voraus. Was für einen Raumbegriff eine zukünftige Theorie hervorbringen wird, die Einsichten der allgemeinen Relativitätstheorie mit solchen der Quantentheorie verbindet, ist derzeit noch eine offene Forschungsfrage.

Damit endet diese Zusammenfassung einiger Ergebnisse aus der Forschungsarbeit zur historischen Epistemologie des Raumes, die in ihrer Gesamtheit, wie ich meine, das eingangs formulierte zentrale Forschungsergebnis stützt: Die drei Ebenen räumlichen Wissens (elementar, instrumentell und theoretisch) durchdringen sich in ihrer historischen Dynamik, indem sich die den verschiedenen Ebenen zugehörigen Wissensstrukturen gegenseitig formen und beeinflussen. Insbesondere hat sich herausgestellt, dass wissenschaftliche Raumbegriffe auf einer Vielzahl kognitiver Vorannahmen aufbauen, die ihren Ursprung in der Lösung praktischer Probleme der Orientierung und der Koordination in Raum und Zeit haben, auf die sie andersherum wieder zurückwirken. Diesen engen Zusammenhang praktischen und theoretischen Wissens hat Peter

Damerow in verschiedenen Kontexten, insbesondere in der Entwicklung des Zahlbegriffs und der Mechanik vielfach dargestellt.[9]

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[9]Zum Zahlbegriff siehe Damerow (1994), zur Mechanik siehe Renn and Damerow (2007), zur Geometrie siehe Wolfgang Lefèvres Beitrag in diesem Band.

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Chapter 13
Paper Tools

Ursula Klein

In *Rechenstein, Experiment, Sprache*—a book co-edited with Wolfgang Lefèvre—Peter Damerow published a long essay on the "Origin of Arithmetical Thinking," in which he studied the structural features of arithmetic in ancient Egypt and Babylonia (Damerow and Lefèvre 1981, 11–113). The key questions raised in this essay are the following: How can the peculiarities of arithmetic established in these two cultures be explained? Are they the result of practices involving particular "material tools" (*gegenständliche Mittel*) for calculating (Damerow and Lefèvre 1981, 14)? Typical examples of such material tools, Peter has pointed out in his essay, are the counting rods and counting boards used in ancient China.

Peter's questions relied on a broader theoretical background that was defined by the developmental psychology of Piaget and others. Our logical-mathematical structures of thinking, Piaget argued, ultimately result from the "reflection upon actions with material objects" (*Reflexion auf gegenständliche Handlungen* (Damerow and Lefèvre 1981, 13f)). If we extend this psychological argument, which refers to the *ontogenetic* development of cognitive structure, to the *historical* development of arithmetic, and more specifically to ancient Egyptian and Babylonian arithmetic, this implies that inventions of arithmetic techniques did not presuppose mathematical thinking. It was rather the other way around: mathematical concepts and thinking result from "operations with material tools" (*Operieren mit gegenständlichen Mitteln*) used for solving certain arithmetic problems (Damerow and Lefèvre 1981, 106).

A characteristic feature of arithmetic in ancient Egypt is the predominance of techniques of addition. Thus multiplication was performed by using techniques of addition. Likewise, division was carried out by means of additions of fractions. As "arithmetic begins with operative manipulations of representations of numbers," Peter has argued, these peculiar features of arithmetic relied on the Egyptian mode of representing numbers.

In ancient Egypt, arithmetic was the business of a particular social group: state officials who were concerned with the planning of production and administration in the centralized state. These men belonged to the upper social class, and they underwent a long professional education and training. Writing and script, Peter has pointed out, marked their professional life. And script was also a symbolic system they used for representing numbers. In Egyptian arithmetic, the characters of script were transformed into tools for representing numbers and calculating.

The ancient Egyptians used simplified hieroglyphic symbols for representing numbers.[1] A vertical line meant 1. Furthermore, there were distinct individual symbols for 10, 100, 1000, and so on. All other numbers were combined symbols, made up of a series of basic symbols arranged in a row. "In the most simple case," Peter has explained, "the series (*Reihung*) of one and the same symbol, namely that for one, engenders configurations that yield the number by counting [the individual symbols]" (Damerow and Lefèvre 1981, 29). He has designated this way of representing numbers a "constructive-additive" technique. We are familiar with this technique from the system of Roman numbers.

Based on this observation, Peter has argued as follows: "the *physical-geometrical* properties of the symbol are used here for constructing *material* models (*gegenständliche Modelle*) of numbers, and for designating numbers by means of the models" (Damerow and Lefèvre 1981, 29, my emphasis). And further: "Symbols of numbers that are generated in a constructive-additive way represent quantities of real objects, just like counting stones, and all operations of assembling and dividing possible with real objects can be performed in a similar way with their representations, which are *material* (*gegenständlich*) as well" (Damerow and Lefèvre 1981, 107). In this context, Peter has also spoken of "material tools of calculating" (*gegenständliche Rechenmittel*) (Damerow and Lefèvre 1981, 106).

It is certainly reasonable to argue that the Chinese counting rods are *material* tools of calculation. But it is perhaps less compelling to argue that symbols written on paper are comparable entities. Can symbols be reasonably designated "*material* tools of calculation" (*gegenständliche Rechenmittel*)? And what does Peter mean when he states that "the physical-geometrical properties of symbols are used for constructing material models of numbers"? What is the meaning of "material" in this context?

We may evade this question by saying that the German word *gegenständlich* does not necessarily mean "material." However, Peter's insistence on the analogy with the Chinese counting rods, which are common-sense material objects, bars this solution, as does the background of cognitive psychology, which refers to down-to-earth operations with material things (in "first-order representations"). It should be noted that this question is also crucial for Peter's attempt to connect

[1] It should be noted that the ancient Egyptians used two different systems of numbers. As this essay is concerned with Peter Damerow's argument, I concentrate on the hieroglyphic symbols.

Piaget's theory, which refers to the cognitive development of individuals, with studies of cognitive development in history, which refers to cultures and societies that change in history. The crucial link between the two kinds of development of cognitive structures resides in the similarity of people's reflections upon actions with real material objects and actions with symbolic devices.

A look at semiotics may be helpful in this context. In a later essay, published in 1999, Peter borrowed from semiotics as well (Damerow 1999).[2] Semioticians distinguish between the "semantics" of sign systems and their "syntax." While "semantics" refers to the meaning of signs, or the concepts they represent, "syntax" refers to the visual traces on paper (or another medium), the maneuverability of signs on the surface of paper, and the rules governing their manipulations.[3] Visibility and maneuverability are aspects of sign systems that they share with material objects, and more specifically with material tools used to achieve certain goals. We may designate these features their "material" dimension. Comparable to the materiality of ordinary tools, they define objective possibilities and constraints of work with a given type of sign system. Paper tools are material devices in the broader sense of being visible and maneuverable entities that are exterior to mental processes but help to generate mental processes.

I would like to briefly exemplify this argument by means of a different example, namely chemical formulae.[4] Chemical formulae such as H_2O for water or H_2SO_4 for sulphuric acid were introduced by the Swedish chemist Jöns Jacob Berzelius in 1813/14, and they proliferated in the chemical community in the 1830s. Their meaning was defined by a theory of chemical composition that went back to Lavoisier, Dalton, and some other chemists, including Berzelius himself. This chemical theory postulated that all chemical compounds consisted of discrete, quantitative units of elements. But it did not further define these chemical units in the light of atomism. Whereas all nineteenth-century chemists accepted Berzelian formulae, many of them were agnostic concerning the more far-going atomistic interpretation of the chemical units represented by this sign system. They restricted themselves to the postulation of discontinuous quantitative units of chemical elements, which was evinced (to some extent) by the empirical laws of stoichiometry. The semantics of Berzelian formulae presupposed the empirical

[2]It should be noted that in this later essay Peter has further elaborated his original argument. He has introduced, or further clarified, a number of additional concepts and distinctions, such as the distinction of first-order and second-order presentations, which are important for a full-fledged theory about the historical development of the concept of number. The goal of my short essay is not to recapitulate this particular theory but rather an element of it—the material dimension of operations on paper with sign systems—that has implications for the history of science and technology more broadly.

[3]It should be noted that I do not restrict syntax to cultural rules of working with symbols.

[4]For the following see Klein (2003).

laws of stoichiometry, and chemical formulae extended these laws hypothetically to all chemical compounds, including organic ones.[5] However, they neither necessitated the acceptance of the philosophical tradition of atomism nor that of the newer physical atomism and Dalton's chemical atomism.[6]

The material dimension of Berzelian formulae is even more interesting. Berzelian chemical formulae consisted of letters, taken from the initial letter (or two letters) of the Latin names of substances, and numbers. Letters are arbitrary signs, or "symbols," that did not resemble the entities they represented. A letter like S denoted a quantitative unit of sulphur in a completely arbitrary way. However, like the symbol for the Egyptian number one—the vertical line—the Berzelian letters had a certain "graphic suggestiveness" (Goodman) owing to the one-to-one correspondence between the letter and the unit of an element (Goodman 1976, 154). One letter denoted one elemental unit. The historian of art Rudolf Arnheim gave a cogent expression to what is meant here:

> In the strictest sense it is perhaps impossible for a visual thing to be nothing but a sign. Portrayal tends to slip in. The letters of the alphabet used in algebra come close to pure signs. But even they stand for discrete entities by being discrete entities: a and b portray twoness. Otherwise, however, they do not resemble the things they represent in any way [...] (Arnheim 1969, 136)

In other words, the visible, "physical-geometrical" coherence of the symbol displayed the represented unit—a defined quantity of a chemical element—in a quasi-pictorial fashion. This is perhaps what Peter meant when he claimed that the Egyptians used the "physical-geometrical properties" of the symbol [for one] for constructing "material models" of numbers. The stroke, representing the number one, was a visible, geometrical unit, and the iteration of such units visibly constructed a bundle of strokes that stood for a certain number x.

The visual letters of Berzelian formulae carried the meaning of units of chemical elements, and no more than this. The material form of the paper tool was well suited to represent a kind of chemical theory that deliberately left open many additional questions asked in the tradition of philosophical atomism. It also facilitated purely additive constructions of chemical models. In their experimental studies of chemical reactions and molecular structure, nineteenth-century chemists used Berzelian formulae to model the outcome of a reaction, and they further combined various reaction models to construct a formula representing the invisible molecular structure of a chemical compound. In so doing, they moved

[5] Stoichiometry had been established in inorganic chemistry. Experimental studies of organic compounds actually questioned the stoichiometric laws.
[6] For the distinction of early nineteenth-century physical and chemical atomism, see Rocke (1984).

the letters denoting units of chemical elements around on the two-dimensional surface of paper, and they additively combined them in new ways that fit the outcome of their experiments. Comparable to some extent to the construction of Egyptian compound numbers, the nineteenth-century chemical models were built by what Peter called a "constructive-additive" technique.

An ordinary tool must be suitable to the object involved in labor. So, too, with sign systems used as tools on paper. Chemical formulae were well suited to their application as tools for investigating chemical reactions and structure in a constructive-additive way. Chemists' constructive-additive rules could be directly translated into a mechanical model of chemical compounds and reactions, which I have called the "building-block-model" of chemical compounds. If we analyze the concrete ways in which the nineteenth-century chemists used their formulae, it is reasonable to argue that the visibility of the Berzelian symbols and the manipulation of them on paper—that is, their "materiality"—helped to generate models of chemical structure and reactions.[7]

My comparison of constructive-additive representations of numbers in ancient Egypt with the use of Berzelian chemical formulae in nineteenth-century Europe has highlighted relatively simple forms of materiality, action, and thinking, which yield representations that can be directly related to real objects ("first-order representations"). The construction of such simple representations is accompanied by abstractions from local features and contexts of actions with objects. It thus yields insights into some general features of the objects under investigation. However, such kinds of (first-order) external representations are not yet mathematical concepts or scientific theories. If you want to know more about these issues, Peter has many interesting ideas to offer as well.[8]

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[7] For examples, see Klein (2003).

[8] See Damerow (1999).

Chapter 14
Drawing Instruments
Wolfgang Lefèvre

Among Peter's writings there are quite a few that are unknown even to his friends and fans—either because they were written in German or published in edited books, which are notoriously apt hiding-places for texts. The text to which I want to draw your attention is one of those—it is written in German and, to complicate matters further, a commentary to a chapter in a reissue of an eighteenth-century book on mathematical instruments, namely of George Adams' *Geometrical and Graphical Essays* (1791). The commentary's title is "Die alltägliche Seite der Geometrie: Zum Kapitel über die Zeicheninstrumente" (*The mundane side of geometry: Regarding the chapter on drawing instruments*) (Adams 1985, 283–300).[1]

Since George Adams' chapter on drawing instruments deals with commonplace instruments such as pairs of compasses or rulers, some readers might be tempted to translate *alltägliche Seite* as the "banal" or "trivial side" of geometry. True, in some of the instruments described in this chapter—such as the pantograph, the pair of proportionable compasses, or the cyclograph—a geometry is embodied that is not that trivial. But this geometry is not Adams' topic and remains probably more often than not obscure to the addressees of the chapter—surveyors, captains, gunners, or architects. Thus, some readers may even wonder whether an employment of mathematical instruments by such practitioners should be called geometry at all. And the fact that Adams puts a list of Euclid's definitions at the head of his book,[2] although deductions of propositions or theorems cannot be found in it, will bring many a historian of science to consider this listing of Euclid's definitions as a mere rhetorical device by which the author tries to gain a higher reputation or social standing.

Now, it comes as no surprise that Peter did not belong to these historians of science. He took the obvious discrepancy between erudite geometry in the tradi-

[1] And, as if to make sure that nobody will ever find this text, this edition does not clarify which of the two editors authored the commentary to a particular chapter—Peter or the author of these lines.

[2] It is an almost complete listing of the definitions of book I of Euclid's *Elements*, albeit sometimes with slight deviations as regards the wording, and furthermore of definitions of geometrically conceived trigonometric subjects.

tion of Euclid and practical geometry of mathematical practitioners as a starting point for re-considering the relation between these two sides of geometry. And he developed this re-consideration by focusing on the functions the seemingly banal drawing instruments had for both practical and deductive geometry.

Before going a bit into his considerations, a few words about the whereabouts of this commentary might be in order. In the early 1980s, Peter and I came accidentally across a contemporary German edition of George Adams' *Geometrical and graphical essays*. Realizing that these essays by a renowned London instrument maker[3] amount to an extraordinarily rich and informative portrait of the realm of mathematical instruments and their employment before the industrial age and having a contemporary translation (Adams 1795) in our hands, we thought it worthwhile to edit a selection of it and furnish this edition with historical commentaries.

Let me now briefly sum up some of the main points of Peter's commentary to the chapter on drawing instruments. Peter started his considerations by recalling the double face of geometry—it is a deductive science and an empirical one, and these two sides cannot be completely mapped one upon another. As Albert Einstein put it in his essay *Geometry and Experience*: "As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality."[4] As an empirical science, namely as the science that explores real space, geometry is as dependent on drawing instruments as other natural sciences are upon observational and experimental instruments. Drawing instruments enable the application of geometrical knowledge in real space and reveal application limitations of geometrical ideas. Peter expected, therefore, that the importance of drawing instruments for the development of geometry is comparable to that of observational and experimental instruments for the development of natural sciences such as astronomy, physics, or chemistry.

Euclid's postulate that ruler and compass be admitted in geometry[5] is usually discussed in a philosophical mode as a means of ensuring the existence and objectivity of geometrical constructions without showing an interest in these instruments. In contrast, Peter highlighted various functions that drawing instruments have for geometry: They are means of exploration, means of demonstration, and means of representation and production (*Darstellungsmittel*). Whereas their employment for purposes of exploring and demonstrating pertains exclusively to theoretical geometry, their use as means of producing, that is, constructing geo-

[3]George Adams Jr. (1750–1795), the principal of an internationally known manufactory of mathematical and optical instruments, published several books on topics connected with these instruments.

[4]"Insofern sich die Sätze der Mathematik auf die Wirklichkeit beziehen, sind sie nicht sicher, und insofern sie sicher sind, beziehen sie sich nicht auf die Wirklichkeit" Einstein (1921, 3f); translation from Einstein (1922).

[5]Euclid *Elements* book I, postulates 1–3.

metrical figures and shapes was not restricted to theoretical geometry. As is well known, the practice of constructing forms and shapes with drawing instruments can be traced back to times long before the emergence of any theoretical geometry.

Being an expert in Babylonian mathematics, Peter knew of course that geometrical issues—first and foremost methods of determining the size of cultivable land—already got a theoretical treatment in Mesopotamian schools of scribes, namely a numerical or arithmetical treatment. However, these theories remained unconnected with the various methods of constructing shapes and patterns with ruler, compass, and other drawing instruments that were in use in the domain of ornamental arts (pottery, metal work, architecture, and so on). Against this background he realized the epoch-making significance of the fact that in ancient Greece a theoretical geometry came into being that reflected geometrical constructions. With this a completely new situation was created in which the practice of constructing geometrical figures and geometrical theorizing became related and mutually dependent on each other. In this Greek tradition of doing geometry the development of geometry became essentially a development of the relation between construction and theory.

Drawing instruments partook in this interconnected development. Becoming refined and diversified in this process, they offered new construction techniques and were at the same time indicative of the state this development had achieved at a certain point of time. The interconnection of construction and theory, once established, turned drawing instruments into embodiments of geometrical knowledge and, thus, into archaeological evidence for the historian. That's why they matter for a history of mathematics.

In this short summary of Peter's arguments, I cannot go into a very interesting and important point he made regarding Greek geometry, namely that the reflection on the action of constructing by means of ruler and compass used systematically ordinary language—more specifically: literary language—as its means of representation. This transformation of literary language into a specific means of representation and deduction capitalized on the contemporary sophists' dialectics, that is, techniques of argumentation, and particularly on their utilization of definitions, postulates, and axioms.

In the framework of Euclid's geometry, constructions play an essential, indispensable role, though also a servile, auxiliary one. They procure evidence on which the theorems base their deductions. Though absolutely subordinated to these theoretical purposes, the very fact that constructions were instrumental in a deductive theory caused offense. From idealistic philosophers like Proclos up to modern champions of pure mathematics, the constructions in Euclid's *Elements* were regarded as displeasing impurities in a deductive enterprise.

However, a complementary story could be told on the side of practical geometry. Out of the wealth of geometrical constructions employed in Greek practical geometry—just think of the refinements used in architecture—Euclid selected only a few constructions that were of use for the deductive purposes of his work. In other words, practical geometry, the geometry of practitioners, did not merge into theoretical geometry. It continued to have a life of its own although it did not remain completely untouched by or disconnected from theoretical geometry. Practitioners for their part did select and use results of theoretical geometry that were available for their purposes. But now it was the theorems' turn to play an auxiliary role.

Pursuing this productive and at the same time tense relation between practical and theoretical geometry, Peter eventually drew attention to a paradoxical situation characteristic of the early modern period. In this period, constructions lost their earlier significance for developments in the frame of learned Euclidean geometry and did so to an extent that their original role in this context was almost forgotten. At the same time, practical geometry and the art of geometrical constructing boomed in an unprecedented way exactly in this period. The traditional arsenal of geometric constructions was enormously extended and refined in the context of astronomy, cartography, surveying, and leveling, and particularly in the context of perspective and stereotomy. And so was the traditional arsenal of drawing instruments.

George Adams' *Essays* epitomize this prosperous realm of practical geometry at the close of the eighteenth century. And, by the seemingly odd listing of Euclid's definitions, they are also an indication of the bond between this realm and that of learned geometry. Summing up these considerations, Peter wrote:

> The relationship of the pure geometry that arose out of the re-appropriation of Euclid's Elements, and the practical geometry to which Adam's work must be assigned, is not that of a theory and its application. Rather, they relate in a complementary way to a shared origin in a reflective employment of drawing instruments. Pure geometry abstracts from those concrete figures whose exploration yielded geometrical insights; practical geometry abstracts from the geometrical knowledge embodied in or transmitted by the constructions of concrete figures.[6]

[6] "Die reine Geometrie wie sie aus der Wiederaneignung der Elemente des Euklid hervorgegangen ist, und die praktische Geometrie, der Adams Werk zuzurechnen ist, stehen also nicht einfach in der Beziehung einer mathematischen Theorie und ihrer Anwendung zueinander. Sie beziehen sich vielmehr komplementär auf einen gemeinsamen Ursprung im reflektierenden Umgang mit den Zeicheninstrumenten. Die reine Geometrie abstrahiert von den konkreten Figuren, an denen die geometrischen Kenntnisse gewonnen wurden, die praktische Geometrie abstrahiert von den geometrischen Kennt-

In concluding for my part, I don't hesitate to state that Peter's commentary on a chapter about drawing instruments amounts to no less than a general outline of the basic principles of a history of Euclidian geometry that captures the interplay of its different realms as the true motor of its development. To my knowledge, he never thought of writing such a history himself. And if I am right, such a desirable history has not been written up to the present day.

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nissen, auf denen die Konstruktionen konkreter Figuren beruhen oder die durch sie vermittelt werden" Adams (1985, 296f).

Chapter 15
Der Würfel auf der Spitze: Pieros inszenierter Blick
Diethelm Stoller

Ein Würfel

Ein Objekt meiner Bewunderung war immer wieder dieser Würfel (Abb. 15.1).

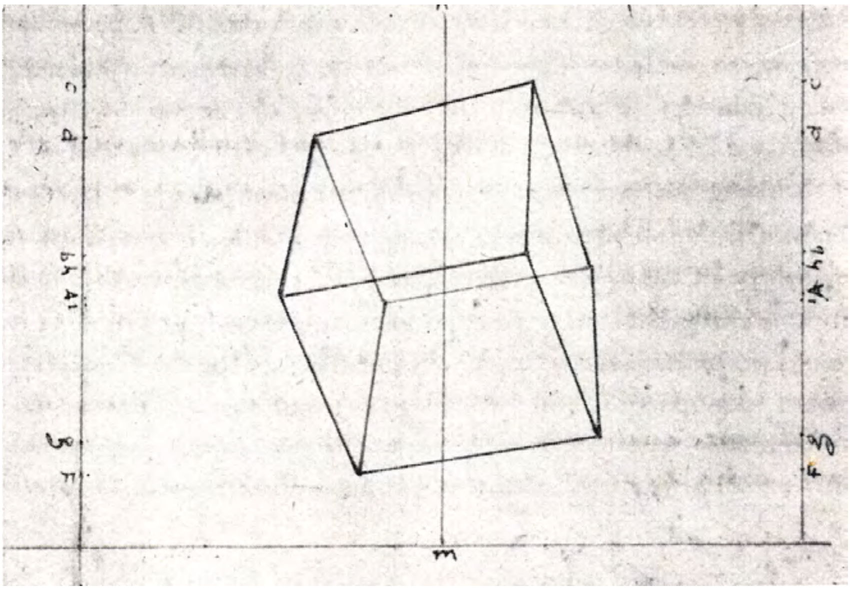

Abb. 15.1: Pieros Würfel, fig. LIV (Piero della Francesca 1942, Tav. XXIX)

Was ist denn Besonderes daran? Piero della Francesca, ein Maler, verfasst für Maler ein Lehrbuch über Zentralperspektive und zeigt hier das perspektive Bild eines Würfels (Piero della Francesca 1899, 1942, 1984).

Irgendeines Würfels? Ja, irgendeines Würfels. Der Würfel steht nicht in Frontalperspektive vor uns, nicht mit seiner Unterseite platt auf einer Standebene, nicht mit seiner Vorder- und Hinterseite parallel zur Bildebene, aber auch nicht gedreht, in Übereckperspektive. Dieser Würfel steht auf einer seiner acht Ecken, auf einer Spitze, und keine Kante hat eine ausgezeichnete Richtung, weder parallel zur Standebene noch parallel zur Bildebene. Wir haben das perspektive Bild eines Würfels in allgemeiner Lage vor uns.

In der Wirklichkeit stehen Würfel im Allgemeinen nicht auf einer Ecke. Auch keine Bauwerke. Niemals wird ein Maler einen Würfel in allgemeiner Lage zeichnen müssen, wenn er reale Welten in seinen Bildern vorzeigen will.

Und ist es wirklich ein perspektives Bild? Es gibt weder Horizont noch Hauptpunkt, keine Fluchtpunkte, keine Diagonal- oder Messpunkte.

Der Würfel auf der Spitze in Grund- und Aufriss

Stellen wir die Frage nach der Perspektive zurück. Es ist nicht die Perspektivkonstruktion, die meine Aufmerksamkeit erregt hat. Aber der fast zwanghafte Blick auf die Perspektive hat wohl verhindert, dass die zwei kleinen Schritte, die ihr vorausgehen, von der Fachwelt überhaupt bemerkt wurden. Diese beiden Schritte jedoch sind die eigentliche Sensation. Das wirklich Besondere in Pieros „De Prospectiva Pingendi“ kommt eher nebensächlich daher. Es ist die geometrisch exakte Konstruktion des Würfels in allgemeiner Lage als Voraussetzung für seine perspektive Abbildung. Es ist die Figur LII (Piero della Francesca 1942, Tav. XXVIII), die ich zum besseren Verständnis in nachkonstruierten Einzelschritten vorführen werde. Wie aus dem Nichts haben wir hier nicht nur die maßgenaue geometrische Definition eines Körpers durch seinen Grund- und Aufriss, sondern diese Risse werden souverän benutzt, um zu einer Lagedefinition des Würfels zu kommen, die man anders nicht erreichen könnte. Und noch dazu eine geometrische Lage, wie sie der Würfel niemals annehmen könnte – auf einer Spitze stehend, irgendwie, eine Inszenierung.

Folgen wir diesen ersten Konstruktionsschritten Piero della Francescas. Es gibt im Prinzip nur einen Weg, und Piero geht ihn. Vorläufer, auf denen er hätte aufbauen können, kennen wir nicht (vgl. Lefèvre 2004). Erst Monge (1989) wird diesen Weg systematisch entwickeln. Es ist der Weg über zugeordnete Risse: Ordner, also „Verbindungsgeraden“, gestatten Projektionen von einem Riss in den anderen, vom Grundriss in den Aufriss und umgekehrt, und ermöglichen so die schrittweise und verlustfreie Verwandlung des Würfels in den Würfel auf der Spitze.

Piero beginnt im Grundriss mit dem übereck gedrehten, auf der Standebene liegenden Würfel und seinem durch orthogonale Ordner in die Aufrissebene bestimmten Aufriss (Abb. 15.2).

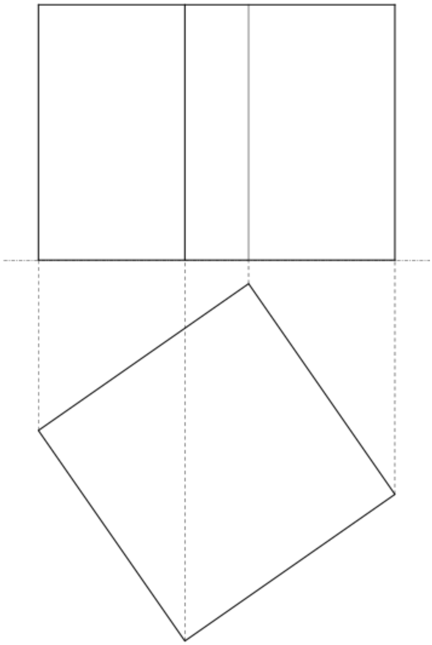

Abb. 15.2: Der Würfel in Übereck-Ansicht in Grund- und Aufriss (nach Piero, fig. LII, Piero della Francesca 1942, Tav. XXVIII)

Im nächsten Schritt dreht er den Würfel im Aufriss so, dass er auf der Ecke F zu stehen kommt. Dabei bewegen sich im Grundriss die Ecken auf Ordnern parallel zur Rissachse, der Schnittgeraden von Grund- und Aufrissebene, und durch den Schnitt dieser Ordner mit den Loten aus den Ecken des Würfels im Aufriss gewinnt er die Ecken des auf der Spitze stehenden Würfels im Grundriss (Abb. 15.3).

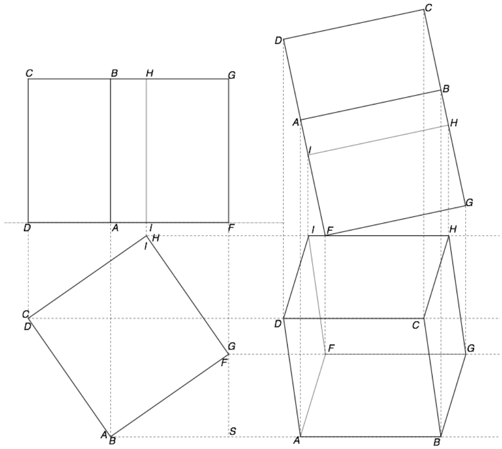

Abb. 15.3: Der im Aufriss auf eine Ecke gedrehte Würfel im Aufriss und Grundriss (nach Piero, fig. LII, Piero della Francesca 1942, Tav. XXVIII)

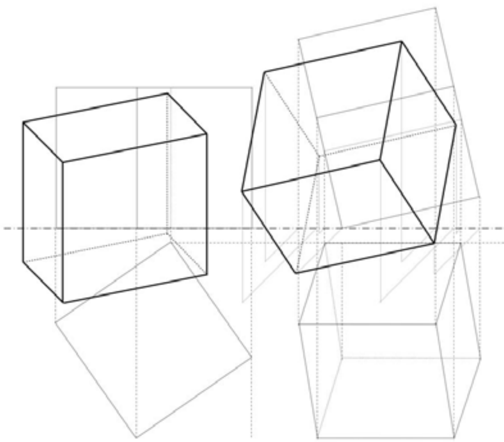

Abb. 15.4: Pieros Konstruktionsschritte in Grund- und Aufriss und in Kavaliersperspektive (Zeichnung vom Verfasser): links die Übereckperspektive des Würfels (vgl. Abb. 15.2), rechts der parallel zur Aufrissebene auf eine Ecke gedrehte Würfel (vgl. Abb. 15.3)

Damit ist der Würfel in allgemeiner Lage konstruktiv und messgenau in Grundriss und Aufriss bestimmt. In Abb. 15.4 sind beide Würfelpositionen, also der Transformationsschritt vom in Übereckposition liegenden in den auf einer Ecke stehenden Würfel von mir zusätzlich in Kavaliersperspektive veranschaulicht. So bändigt er diese flüchtige Form und erkundet, wie der Würfel Gestalt annimmt, während er ihn mit mathematischen Mitteln zeichnend auf eine Ecke stellt.

Exkurs: Die perspektive Abbildung über Grund- und Aufriss des visuellen Systems

Offenbar braucht Piero den Würfel in allgemeiner Lage, also einen besonders schwierigen Körper, als Vorwand, um eine andere Methode der perspektiven Konstruktion als notwendig erscheinen zu lassen: „[…] deshalb werde ich, da ich von schwierigern Körpern zu handeln habe, einen andern Weg und eine andere Methode bei ihren Verkürzungen einschlagen […]" (Winterberg 1899, Bd. 1, CXXV).[1]

Eine andere Methode als welche? Als die Albertis natürlich, deren Begründung und Weiterentwicklung Piero die beiden ersten seiner drei Teile umfassenden Abhandlung „De Prospectiva Pingendi" widmet. Es ist Alberti, der durch die Konstruktion des Diagonalpunkts auf dem Horizont im gleichen Abstand vom Hauptpunkt wie der des Betrachters vom Bild das Auge des Betrachters mit der Zeichnung verknüpft. Und der Versuch Pieros, Albertis Verfahren und sein eigenes, entwickelteres, aber im Prinzip äquivalentes Verfahren am Beispiel der Abbildungs eines Quadrats mathematisch über Strahlensätze zu beweisen (Abb. 15.5), wäre eine weitere kleine Untersuchung wert.

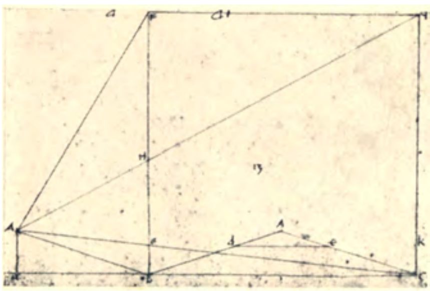

Abb. 15.5: Piero fig. XIII (Piero della Francesca 1942, Tav. IV)

[1] „[…] però avendo a tractare de corpi più deficili, pigliarò altra via et altro modo nelle loro degradationi […]" (Piero della Francesca 1984, 129).

Für die perspektive Darstellung der geschreinerten Welt in Frontal- bzw. Übereckperspektive mag Albertis oder Pieros Verfahren über Horizont, Hauptpunkt und Diagonalpunkte als Messpunkte für die Diagonalen reichen, doch nicht für die „schwierigeren Körper“.

Moritz Cantor findet eine operative Definition für das perspektive Bild, nämlich als „[…] die Kunstfertigkeit, die zu bemalende Wand als zwischen dem sehenden Auge und dem abzubildenden Gegenstande eingeschaltet zu denken und deren Durchschnittspunkte mit den Sehstrahlen nach jenem Gegenstande durch Linien zu vereinigen“ (Cantor 1880, 67). In fig. XLV (Abb. 15.6) zeigt Piero seinen anderen Weg, der wie eine konstruktive Umsetzung der anschaulichen Beschreibung Cantors daherkommt, indem er dieses visuelle System aus Auge, Sehstrahlen, Bildebene und Quadrat in Grund- und Aufriss zerlegt. Im Grundriss gewinnt er die horizontalen Koordinaten der vier benötigten Durchschnittspunkte der entsprechenden Sehstrahlen und analog im Seitenriss ihre Höhen. Mit diesen beiden Maßangaben sind die vier Bildpunkte in der Bildebene eindeutig bestimmt. Piero überträgt sie in einer eigenen Zeichnung (Abb. 15.6 unten), indem er sie auf Papierstreifen aufträgt und dann jeweils Weite und Höhe (larghezza und altezza) eines Punkts zum Schnitt bringt und die Bildpunkte zum Bild des Quadrats verbindet.

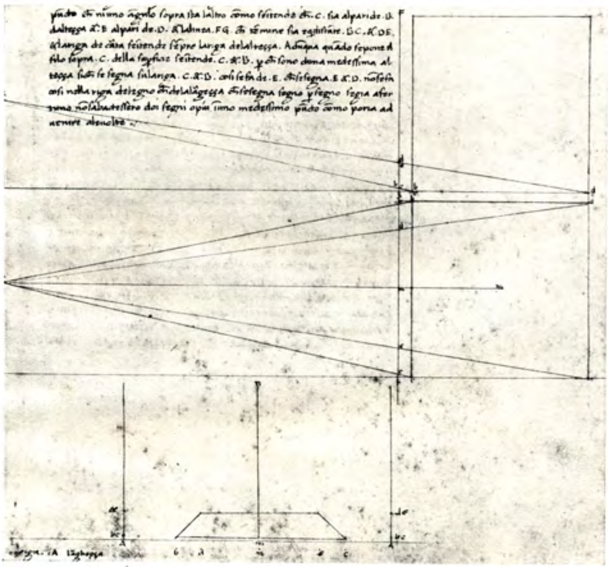

Abb. 15.6: Piero fig. XIII (Piero della Francesca 1942, Tav. IV)

Der Würfel auf der Spitze in perspektiver Abbildung

Gestützt auf diesen Exkurs können wir die perspektive Abbildung des Würfels in allgemeiner Lage mit Pieros Hilfe vollenden. Ein geeignetes visuelles System muss bestimmt und mit Grund- und Aufriss des Würfels verbunden werden. Piero wählt die Gerade KL (Abb. 15.7) als Grundriss der senkrechten Bildebene so, dass keine Kante des Würfels zu ihr parallel liegt, und setzt das Projektionszentrum O „[…] soweit als du entfernt stehen willst, um den Cubus zu sehen." Die Projektionsstrahlen von O zu den Ecken des Würfels bestimmen auf KL die horizontalen Koordinaten ihrer Bildpunkte. Bei Piero wird ein Holzstreifen an KL angelegt – „dass er gut fest liege" – und ein Faden von O nacheinander zu den Würfelecken gespannt und die Schnitte mit dem Holzstreifen auf diesem vermerkt.

Fehlt uns noch das visuelle System im Seitenriss. Das gewinnt Piero durch zugeordnete Risse (Abb. 15.8). Zur Bildebene KL zieht er eine senkrechte Gerade als Rissachse, die KL im Punkt E schneidet, so dass der Abschnitt KE zum Grundriss und LE zum Seitenriss der Bildebene wird. Den Seitenriss des Würfels findet Piero, indem er auf Parallelen zu KL durch alle Eckpunkte des Würfels im Grundriss von der Rissachse PQ aus ihre Höhen abträgt, die er dem Aufriss vom Anfang (Abb. 15.3) entnimmt, und die so gefundenen Ecken des Würfels verbindet. Das Projektionszentrum ist durch seinen Abstand zur Bildebene im Grundriss bereits bestimmt, seine Höhe wählt er nach Gefallen (Abb. 15.9). Im Seitenriss des visuellen Systems bestimmen die Projektionsstrahlen von O zu den Ecken des Würfels auf KL die Höhen der Bildpunkte über der Standebene, die er wieder auf einem Holzstreifen vermerkt. Die Fertigstellung des perspektiven Bildes des Würfels erfolgt durch Bestimmung der Bildpunkte mit Hilfe der auf den Holzstreifen eingezeichneten Koordinaten (Abb. 15.10).

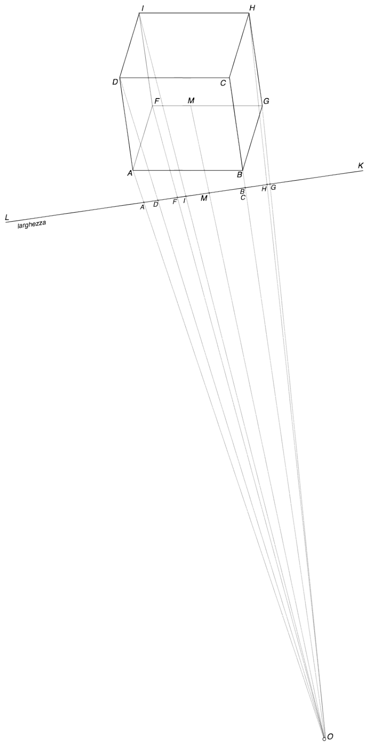

Abb. 15.7: Das visuelle System im Grundriss, mit Projektionszentrum O, Bildebene KL,Würfel in allgemeiner Lage und Projektionsstrahlen (nach Piero, fig. LII, Piero della Francesca 1942, Tav. XXVIII)

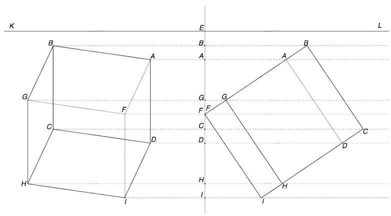

Abb. 15.8: Die Konstruktion des Würfels im Seitenriss in neuer Lage (nach Piero, fig. LIII, Piero della Francesca 1942, Tav. XXIX)

Abb. 15.9: Das visuelle System im Seitenriss (nach Piero, fig. LIII, Piero della Francesca 1942, Tav. XXIX)

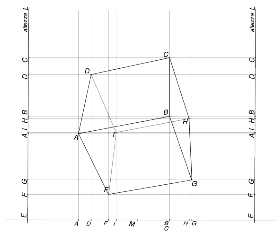

Abb. 15.10: Die Konstruktion des perspektiven Bildes (nach Piero, fig. LIV, Piero della Francesca 1942, Tav. XXIX)

Zurück zur Ausgangsfrage: Was ist Besonderes daran?

Die Konstruktion perspektiver Bilder schwieriger Körper über Grund- und Aufriss des visuellen Systems wurde möglicherweise schon von Brunelleschi bei der Herstellung seiner Bilder für sein „Perspektive-Straßentheater" vor dem Baptisterium und vor dem Palazzo Vecchio in Florenz verwendet. Mit Pieros „De Prospectiva Pingendi" von etwa 1572–75 haben wir jedenfalls zum ersten Mal den Riesenschritt schriftlich und nachvollziehbar dokumentiert, Albertis Vorstellung des Bildes als Fenster, durch das wir auf die dargestellte Welt schauen, als Schnitt mit der Sehstrahlpyramide in zugeordnetem Grund- und Aufriss geometrisch hervorgehen zu sehen.

Uns verwundert nicht so sehr Pieros Anwendung dieser Methode auf den hier behandelten Würfel, sondern eher auf wirklich schwierige Körper wie den menschlichen Kopf, den er geradezu besessen von der Idee, jeden Körper geometrisch konstruieren zu wollen, in bis zu sieben Scheiben zerlegt, von denen er fast hundert Punkte in Grund- und Aufriss bestimmt, um so das perspektive Bild dieses Kopfes zu gewinnen (Abb. 15.11, 15.12). Diese Vermessung des Kopfes wurde oft unsinnigerweise mit den Verfahren zur digitalen Darstellung des Kopfes verglichen und Piero als Prophet der Digitalisierung organischer Formen gepriesen. Piero zeigt hier, was mit konstruktiver Genauigkeit möglich ist. Ob er es in seiner Malerei etwa bei seinen majestätischen Frauen wie der Madonna del Parto in Monterchi jemals angewendet hat?

Doch Pieros Perspektivkonstruktion eines Würfels in allgemeiner Lage mit Hilfe zugeordneter Risse kommt offensichtlich so überraschend und verfrüht, dass sie lange niemand bemerkte. Vasari weiß es nicht so genau: „Er galt für einen vorzüglichen Meister in schwieriger Zeichnung regelmäßiger Körper und in der Arithmetik und Geometrie, ward aber im Alter von Blindheit und endlich durch den Tod verhindert, seine künstlerischen Anstrengungen zu nützen und seine vielen Bücher herauszugeben, welche noch in Borgo, seiner Vaterstadt, aufbewahrt werden." Und an anderer Stelle: „Piero war, wie ich schon sagte, sehr eifrig im Studium der Kunst, beschäftigte sich viel mit Perspektive und war wohl bewandert im Euklid, so daß er die wichtigsten Kreislinien regelmäßiger Körper besser als irgend sonst ein Geometer verstand, ja die meisten Aufklärungen haben wir von ihm […]" (Vasari 1988, 297). Wiener beschreibt Pieros „De Prospectiva Pingendi", deren Titel er nicht kennt, als „eine ausgedehnte Schrift mit vielen Figuren", „die aber jetzt nicht aufgefunden ist" (Wiener 1884, Bd. 1, 13). Witting zeigt sich sehr gut informiert über die „De Prospectiva Pingendi", betont die Bedeutung, die dieses Werk über Jahrhunderte als Fundamentalwerk behielt. Er redet aber nur von den ersten beiden Büchern, das dritte mit dem Verfahren der Konstruktion des perspektiven Bildes über Grund- und Aufriss des visuellen Systems erwähnt er nicht, so auch nicht die Konstruktion des Würfels auf der Spitze (Witting 1898, 152ff). Wenn es hochkommt, dann wird die orthogonale Zweitafelprojektion Pieros als „costruzione legittima“ Brunelleschi zugeschrieben, von der dann Piero auf einem geheimnisvollen Wege etwa 30–40 Jahre nach dessen Tode (1446) erfahren hätte (Salmi 1979, 201). Battisti beschränkt sich in seinem umfassenden zweibändigen Werk strikt auf die Malerei von Piero. Immerhin erwähnt er Pieros Konstruktion über Grund- und Aufriss als notwendig für den Fall, dass es etwa Schwierigkeiten bei der perspektivischen Abbildung von geometrischen Objekten komplexer Formen gibt (Battisti 1971, 95).

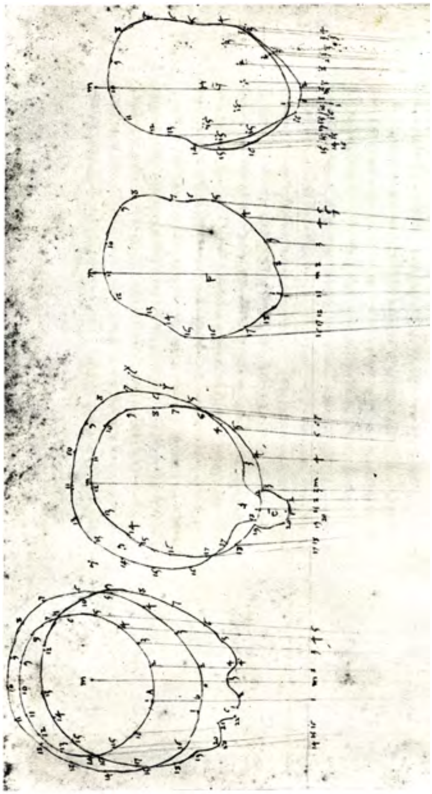

Abb. 15.11: Pieros Zerlegung eines Kopfes in Scheiben (nach Piero, fig. LXX, Piero della Francesca 1942, Tav. XLII)

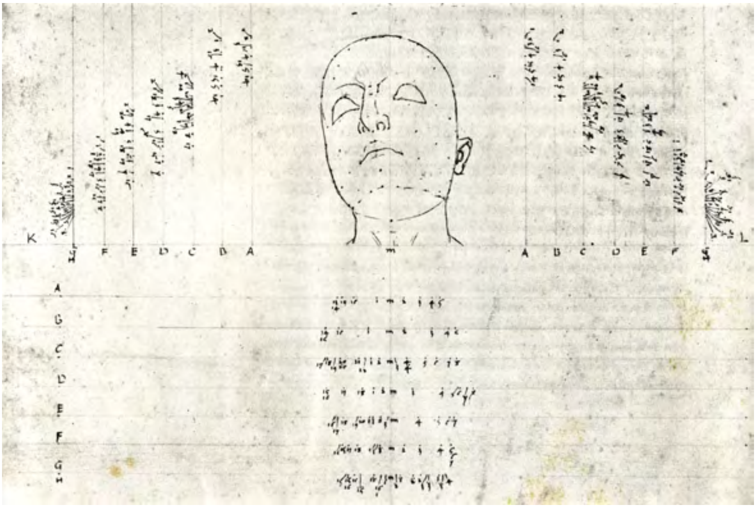

Abb. 15.12: Pieros Konstruktion eines Kopfes aus Scheiben, fig. LXXII (Piero della Francesca 1942, Tav. XLIV)

Insbesondere aber ist es auf Grund der äußerst mageren Quellenlage nicht gelungen, Pieros historisch so überraschende und singuläre meisterhafte Grundrisskonstruktion des Würfels auf der Spitze als Ergebnis einer historischen Genese oder als Reflektion einer schon vorher ausgeübten handwerklichen Technik zu begründen. In der informativen Zusammenstellung architektonischer Skizzen von Joël Sakarovitch (1998), einer umfassenden Recherche zur Genese der Darstellenden Geometrie, finden wir architektonische Grundrisse mesopotamischer und ägyptischer Provenienz, doch keine zugeordneten Risse. Von einer Wiederbelebung griechischer oder römischer Kenntnisse, wie allgemein der Renaissance als Antrieb unterstellt, kann keine Rede sein, denn architektonische Risse aus dieser Zeit sind nicht überliefert. Ansätze etwa bei Villard de Honnecourt (Hahnloser 1935) erreichen nicht einmal die begrenzten Möglichkeiten, wie wir sie am Beispiel (Abb. 15.13) aus der „Geometria Deutsch" von 1487 von Matthäus Roritzer vor uns haben. Offensichtlich wird Pieros Konstruktion des Würfels auf der Spitze über zugeordnete Risse sogar auch in einschlägigen historischen Untersuchungen einfach übersehen.[2] Verstellte etwa die (hier nicht gerechtfertigte) Bewunderung für Dürer, der in diesem Zusammenhang oft genannt wird, den Blick auf diese kleine unschätzbare Kostbarkeit bei Piero?

[2]Kleine Auswahl: Loria (1921); Field (1997, 2005); Camerota (2004); Grasselli (2008).

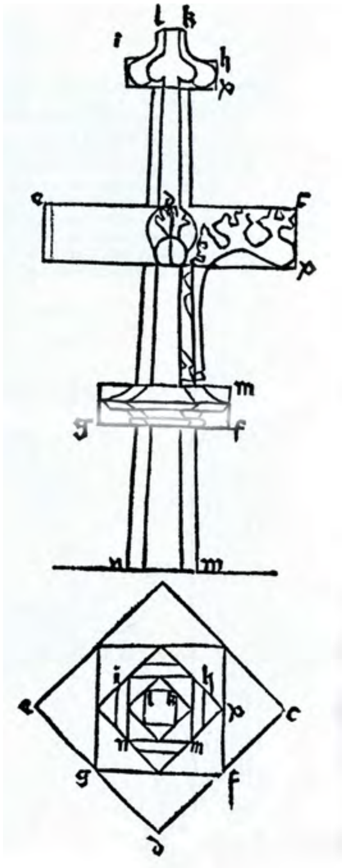

Abb. 15.13: Matthäus Roritzer „Die Geometria Deutsch“, 1487/88 (Roritzer 1999, 42)

Als ich 1986 meinen ersten Vortrag über „Piero della Francesca als Mathematiker“ hielt und Piero als Entdeckung des 20. Jh. vorstellte, mit dem Hinweis auf seine Konstruktion des perspektiven Bildes eines Würfels in allgemeiner Lage über Grund- und Aufriss des visuellen Systems und der Frage, ob er sich hier als früher Vorläufer Monges erweist (Stoller 1990, 25ff), ahnte ich nicht, dass die in dieser Frage herrschende Literaturwüste schon zwei Jahre später durch einen fulminanten Aufsatz von Kirsti Andersen urbar gemacht wurde (Andersen 1992, 2007).

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Chapter 16
The Emergence of the Idea of Real Number in the Context of Theoretical Music in the Renaissance

Oscar Abdounur

Introduction

The early modern period saw the growing use of geometry as an instrument for solving structural problems in theoretical music, a change not independent from those that occurred in the conception of ratio in the context of theoretical music. In the early sixteenth century, the Bohemian mathematician and music theorist Erasmus of Höritz emerged as a German humanist who was very articulate in musical matters. In the context of the revival of interest in Greek sources, Erasmus communicated to musical readers an important product of such a revival and was likely the first in the Renaissance to explicitly apply Euclidean geometry to solve problems in theoretical music. Although Erasmus also considered the tradition of *De institutione musica* by Boethius, he based his ideas strongly on Euclid's *The Elements*, using geometry in his own *De musica* in different ways in order to solve musical problems. It is this comprehensive geometrical work rather than the arithmetical and musical books of Boethius that served as Erasmus' starting point. However, Erasmus proposed a proportional numerical division of the whole tone interval sounding between strings with a length ratio of 8 : 9, since it was a primary arithmetical problem. This chapter aims at showing the implications of such a procedure of Erasmus for the transformation of the conception of ratio and for the emergence of the idea of real numbers in theoretical music contexts.

In order to do that, a passage will be considered here from chapter 17 of Book VI of Erasmus *De musica*, entitled *Propositio decimaseptima Toni proportionem scilicet sesquioctavam in duas proportiones equales artificialiter et geometrice dividere.*[1] It concerns the equal and proportional numerical division of the whole tone interval sounding between strings with a length ratio of 8 : 9, a problem which confused the musical theorists from antiquity up to the Renaissance, and that played an important part in the historical process leading to the emergence of

[1] Proposition Seventeen: How to Divide the Sesquioctave Ratio (that is, 8 : 9) of the Whole Tone into Two Equal Ratios, Artificially and Geometrically.

equal temperament. In this passage, Erasmus seemed to be in a position to solve such a problem.

Division of the Tone

The problem of the division of the tone arose from the Pythagorean discovery of numerical indivisibility of a superparticular or epimoric ratio, i.e., $n : n+1$, by its geometrical mean, in particular applicable to the division of the ratio $8 : 9$. Given $p < x < q$, where p and q are integers and the ratio $p : q$ is superparticular, x cannot be both an integer and at the same time fulfill the condition $p : x = x : q$; that is, it cannot be the geometric mean of p and q. Mathematically, the equal division of the tone $8 : 9$ provides ratios involving surds or incommensurable ratios underlying musical intervals. These procedures were considered impossible by Pythagoreans in theoretical music, since these intervals could be determined only by ratios of integer numbers.

Attempts to divide the tone had, however, already been done since antiquity by Aristoxenus (fourth century BCE), who conceived of the theoretical nature of music as essentially geometric, understanding pitches, musical intervals, and also distances as continuous quantities that should follow the rules of Euclidean geometry and should be capable of being divided continuously. This inevitably raises questions concerning the nature of ratio in this context. Traditionally it is considered that Aristoxenian music theory rejected the position of the Pythagoreans in the sense that musical intervals should properly be expressed only as mathematical ratios involving whole numbers, and asserting instead that the ear was the sole guide for musical phenomena (Winnington-Ingram 1995, 592). It did not mean however that Aristoxenus' theory could not be put on the same mathematical basis related to the developments in Greek mathematics of his time. Aristoxenus preferred geometry to arithmetic to solve problems involving relations between musical pitches and believed in the possibility of dividing the tone into two equal parts, conceiving of musical intervals and ratios as continuous magnitudes.

Such an idea unleashed many reactions, expressed, for instance, in the *Sectio Canonis* (Barbera 1991, 125) and much later in Boethius' *De institutione musica* (Bower and Palisca 1989, 88), which stood in a strong Pythagorean tradition in theoretical music in the Middle Ages. Following the Pythagorean tradition, many medieval musical theorists maintained the impossibility of the equal division of the tone, which would mathematically lead to incommensurable ratios underlying musical intervals. Such a position began to change in the fifteenth century and was eventually systematically overcome in the early Renaissance through scholars like Nicholas of Cusa, Erasmus of Höritz, Faber Stapulensis, Henricus Grammateus, Pedro Ciruelo, Juan Bermudo, and others, who proposed the equal division of tone mostly by means of geometry. In his *Musica*, Erasmus of Höritz made use of an abstract numerical procedure to propose a solution for the problem of the equal division of the tone, expressing, rather, as a number the geometrical mean between the terms of the ratio 8 : 9 underlying the tone.

The *De musica speculativa* from Erasmus Horicius

Erasmus' *De musica* emerged in a time when the rediscovery, translation, and publication of sources from antiquity, such as the works of Euclid, Archimedes, and Ptolemy, increased interest in and furthered the development of number theory. Gaps in the Pythagorean numerical system were quite disturbing, resulting in crisis and conceptual changes in the demarcation of the disciplines of arithmetic and geometry. So ratios involving surds, that is, incommensurable quantities, could only be discussed in the domain of continuous quantities and would demand the unification of two such disciplines as well as the conquest of a number continuum for mathematical activity.

Particularly for Erasmus, Arabic and Hindu concepts were highly influential since they promoted the development of Greek mathematics and handled entities such as negative and irrational numbers, and, with the introduction of Hindu numerals by Fibonacci, enabled computation of unprecedented complexity and the development of extremely large numbers. The latter was an important component in Erasmus' division of the whole tone ratio, as will be seen in the following.

In chapter 17 of Book VI, Erasmus refers specifically to the division of the 8 : 9 ratio, which represents the musical interval of a whole tone. In the four previous chapters of Book VI, Erasmus demonstrated incompletely the divisibility of other superparticular ratios into equal and proportional halves, like the octave (1 : 2), fourth (3 : 4), fifth (2 : 3), and minor third (5 : 6).

In chapter 17, Erasmus proposed an abstract numerical procedure to find the geometrical mean between the terms of ratio 8 : 9 underlying the tone, expressing it as a number. He did not use the geometrical construction of mean proportional to two given straight lines from Proposition 13 of Book VI of Euclid, as did, for instance, Jacques Lefèvre d'Etaples in 1496, using exclusively non-numerical Euclidian methods capable of being carried out with a straightedge and compass.

He attempted rather to reach an expression for the ratio for the supposedly equally proportional halves of the whole tone interval using very large integer numbers. He did it first using Proposition 15 of Book V of the Elements, which asserts that $a : b :: am : bm$. Following his method, the half of the 8 : 9 ratio of a tone could be obtained by the geometric mean of its expansion into the term 30958682112 : 34828517376. This ratio was derived directly from 8 : 9 by multiplying numerator and denominator by the factor 3869835264,

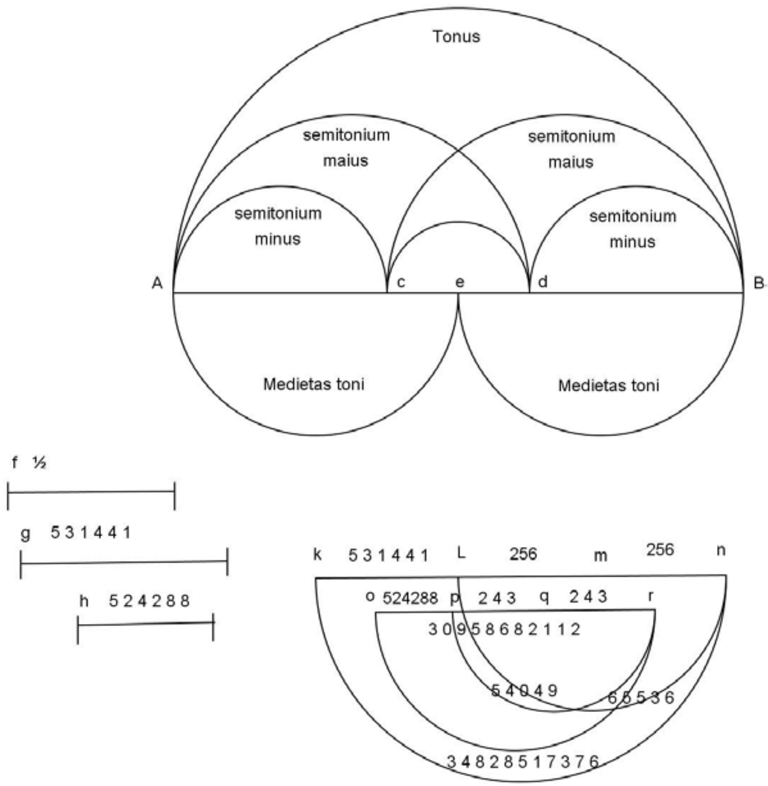

Figure 16.1: Erasmus's arithmetical division of the tone, chapter 17, Book VI, *De musica*.

a procedure guaranteed by Proposition 15 of Book V for $a = 9$, $b = 8$, and $m = 3869835264$. The proportionality between the original ratio 8 : 9 and 30958682112 : 34828517376 allows a mapping between intermediate terms of the ratio 8 : 9, including the mean. Numbers between the terms of its expansion are mapped into a large number ratio, considering that the interval determined by the expansion becomes subdivisible and that the greater the distance between the terms in the large number ratio, the greater the precision one can get for the intermediate terms of the ratio 8 : 9, represented by the large number between the two terms of the large number ratio. Since there were no decimal fractions at this time, the proportionally extended ratio is used for the purpose of extracting the square root with a high degree of precision, in this case associated with large integer numbers rather than with places after the decimal point. The larger the distance between the terms in the large number ratio, the higher the precision with which the geometrical mean is obtained. Nevertheless, Erasmus seemed not to worry about carrying out any computation in the text, and he did not present his result as an approximation of the true real number. He is the first author to propose an abstract numerical procedure for the given problem, expressing it as a number and avoiding using the construction of a geometrical line. Since it was an elementary arithmetical problem, it could be solved "*artificialiter*," that is, numerically.

Erasmus asserts that "[…] in musical demonstrations we are forced to use all kinds of ratios […] since not all shapes of consonances and also dissonances are founded in rational ratios and for that reason we must not neglect the ratios of surds" (Horitius ca. 1500, fo. 61v). Erasmus considered here incommensurable ratios or irrational numbers in musical contexts. At first sight, in order to make use of Eudoxus' theory of Book V of Euclid's Elements on which the theory of ratios of surds is based and wherein abstract quantities with continuous nature are dealt with, Erasmus established a link between continuous and discrete quantities.

It is possible to identify similar ideas concerning the relation between the numerical division of the tone proposed by Erasmus and Eudoxus' Definition V of Book 5 of the *Elements*. Whereas Erasmus confined a searched irrational number by using only integers, Eudoxus' definition corresponded, arithmetically speaking, to establishing a proportionality of ratios through the confinement of ratios with integer terms. In these analogous procedures, Erasmus and Eudoxus found precision in finding an irrational number and in establishing a proportionality between two given ratios, respectively, through ratios with big terms. Erasmus made use of *The Elements*; nevertheless, his source was the Campanus' translation, which had an arithmetical terminology not derived from the geometrical ratio theory of Book V of Euclid, but instead from a number of different sources, very likely including *Arithmetic* by Jordanus de Nemore from the thirteenth century. On the one hand, such a fact makes it implausible that Erasmus had access to Eudoxus' definition in the original sense and, on the other hand, makes the strong and curious structural analogy between both procedures very compelling.

Erasmus realized that the search for a geometrical mean to the ratio underlying the whole tone could not result in a rational number, and instead of changing the domain at this point from a discrete quantity of numbers to a continuous quantity of geometrical lines, he established a link between continuous and discrete quantities, proposing a number continuum, although not explicitly, thus creating a very dense discrete point set in the space between the original terms 8 and 9 by their expansion.

Concluding Remarks

It is conceivable that if Erasmus really thought he could divide the sesquioctave ratio in terms of a purely numerical operation, he must have possessed an at least rudimentary concept of the number continuum. Such an assumption is corroborated by a passage appearing later on in chapter 17, where he seems to refer directly to the idea of such a continuum, mentioning Boethius as a prisoner of the Pythagorean doctrine of discrete integer numbers, not accessing all ratios of numbers (Horitius ca. 1500, fo. 67v). Just before this passage, Erasmus asserts that exactly half of the whole tone interval would be provided by extracting the square root of the product of its terms 8 and 9, which would be $\sqrt{72}$ (Horitius ca. 1500, fo. 67v). He did not, however, relate this result explicitly to the computations he presented. He obtained the large number ratio, but the geometrical mean between the two terms still needed to be found. Since he presented the method of doing this by extracting the square root of 8×9, one might ask why he did not do it from the ratio $8 : 9$, or if he produced the proportionally large number ratio, how could he use this representation to the extraction mentioned above and to approach the geometrical mean. It might be assumed that he left it to the reader.

Theoretically based on many geometrical propositions and, unusually, based on the Euclidean style, *Musica* deals with ratio as a continuous quantity, announcing perhaps what would emerge as an arithmetical treatment of ratios in theoretical music contexts during the sixteenth century, approaching ratio to a real number. Interestingly, Erasmus could have easily solved the equal division of the tone by making use of the proposition of Euclid's *Elements* that provides the geometrical mean as the height of a right-angled triangle. Nevertheless, missing the concept of infinity, he preferred to use a numerical method to approach such a mean, although his procedure was not recognizable as an approximation of the true real number value of the geometrical mean. Erasmus provided a mathematical theoretical structure for a virtual pitch relation space, a continuum of rational numbers that can be seen as an important step for laying the foundations for the real number system.

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Part 3: Societal Challenges and Electronic Visions

Chapter 17
In anderen Zeiten. Die Idee des Comenius-Gartens
Henning Vierck

Der Comenius-Garten in Berlin-Neukölln ist eine wissenschaftshistorische Rekonstruktion des Natur- und Menschenbildes von Johann Amos Comenius (1592–1670). Diesen Ort zu betreten heißt, eine andere Zeit aufzusuchen. So wenigstens ist seine Idee. Und diese wiederum hat eine Entstehungsgeschichte, an der Peter Damerow wesentlich beteiligt war. Von ihm und seinen Freunden wurde über viele Jahre hinweg im Forschungskolloquium zur Begriffsentwicklung in den Naturwissenschaften, unserem Montagskreis, der Boden aufbereitet, in dem später der Comenius-Garten wachsen sollte. Die meiste und fruchtbarste Erde, die ich Montag für Montag mitnehmen durfte, lag vielfach aufgewühlt im Umfeld des Vis-viva-Streits. Ihn hatten wir uns damals, am Max-Planck-Institut für Bildungsforschung noch, immer wieder vorgenommen.

Die historische Auseinandersetzung um Lebenskraft gab mir also den Stoff, aus dem – jetzt sei es verraten – eigentlich ein Kinderbuch entstehen sollte. Alle meine Mitschriften unserer Dispute über diesen epochalen Streit in den Naturwissenschaften sind durchzogen von Notizen zu einem wissenschaftshistorischen Buch für Menschen im Alter von etwa 12 Jahren. Dieses Faktum hat bis heute keiner meiner Freunde von damals erfahren. Seine Bekanntgabe war mir unvorstellbar. Ich habe mich nicht getraut, mein Schweigen aufzugeben. Das hätte vielleicht der Ernsthaftigkeit, ja auch Wissenschaftlichkeit des Montagskreises geschadet. Das wollte ich nicht. Und selbst noch 2011, als ich mit dem Vortragstitel „Violarium, Rosarium, Viridarium … Ein Zettelkasten der Staatsbibliothek zu Berlin und die Wiederentdeckung des Comenius-Gartens" von der Kinderbuchabteilung dort ermuntert wurde, etwas von meinem literarischen Geheimnis preiszugeben, habe ich, so gut es ging, den Montagskreis rausgehalten.

Doch heute und in Zukunft, da aus meinem Kinderbuch nichts mehr werden muss, wohl aber aus dem Comenius-Garten, eine Stiftung mit wissenschaftlichem Zweck nämlich, da traue ich mich, ja fühle ich mich verpflichtet, einige Kostproben der Aufzeichnungen und Überlegungen von damals vorzutragen. Vielleicht ist dies eine Hilfe, um den Stellenwert des Kinderwissens in den gemeinsamen Projekten mit dem Max-Planck-Institut für Wissenschaftsgeschichte, in dem der

Abb. 17.1: Kinder bei der Forschung im Comenius-Garten.

Montagskreis aufgegangen ist, zu erkennen und zu benennen. Katja Bödeker und ich haben uns abgesprochen: Sie hat die methodische Reflexion unserer Arbeit übernommen[1] und ich die moralische Bewertung.

[1] Siehe ihren Beitrag in diesem Band.

Wissenschaft hat dem Leben der Menschen zu dienen. Das ist die Moral, mit der wir unsere Projekte angehen, und das ist selbstverständlich, sollte man meinen. Gewöhnungsbedürftig hingegen ist, dass wir uns deshalb in andere Zeiten zu versetzen trachten – und dies zu aller Aberwitz mit Kindern. Unser Anspruch lautet also nicht nur, dass Wissenschaft heute dem Leben verpflichtet sei, sondern vielmehr auch, dass sie ihren Gegenstand selbst immer noch und wie bei Kindern oft als belebt, als beseelt, als Quelle von Kraft erachten können darf, denn Wissenschaftsgeschichte heute muss wie Wissenschaft einst sich trauen dürfen, dem Universum und allem was darinnen ist mit einem moralischen wie epistemischen Anthropozentrismus zu begegnen. Sonst hat sie ihren Gegenstand verfehlt.

Wen wundert es also, dass Philosophie für uns wie bei Comenius die „Welt der Arbeit“ ist? Für ihn, den eigentlichen Urheber unseres Gartens, lässt sich die Frage der Erkenntnis nicht ohne die Frage nach dem Überleben stellen. „Das Erbübel“, schreibt er, „das von den Erstgeschaffenen auf uns gekommen ist, beherrscht uns so, dass wir den Baum des Lebens hintansetzen und unser verkehrtes Streben nur auf den Baum der Erkenntnis richten.“ Oder um die universelle Humanität der Wissenschaft von einst bei Gottfried Wilhelm Leibniz (1646–1716) ausfindig zu machen, darf es auch diesem Naturforscher aus Sicht unserer Projekte sehr gern so scheinen, „daß noch zur Zeit weder die Herrn Engländer noch die Cartesianer in Franckreich und Holland in den Grund der materialischen Dinge gesehen. Ich vermeine,“ so schreibt Leibniz weiter an Daniel Ernst Jablonski (1660–1741), „daß zwar alles in den cörperlichen Phaenomenis mechanisch, ursprünglich aber auch alles in der Natur zugleich metaphysisch und moral; und zwey ein ander durchdringende Reiche in allen Dingen, das Reich der Weißheit nach den Finalen, und das Reich der Krafft nach den Efficienten, indem die letzten Ursachen der mechanischen Geseze selbst von einer höhern Substanz hergehohlet werden müssen, welche auch überall gewisse Monades erschaffen hat, so keine Figur noch Theile haben, und in denen die Krafft wohnet.“[2]

In meiner Kladde ist unter dem Datum des 24. April 1982 hinsichtlich des Energieerhaltungssatzes, mit Blick also auf das Ende des Vis-viva-Streits vermerkt: „Warum hat Leibniz bei seinen physikalischen Kenntnissen solch eine Entdeckung nicht gemacht? Wieso [kam diese] erst mit Robert Mayer [1814–1878]? Wenn man sich Christian Wolff [1679–1754] ansieht, kann man es erahnen […]. Mit dem Energieerhaltungsgesetz würde man das Herzstück der Leibnizschen Physik zerstören, man müsste gegen die Lebenskraft sein.“ Stimmt diese Einschätzung, so versucht Leibniz den Baum des Lebens dem Baum der Erkenntnis gleichzustellen.

[2] Zitiert aus einem Brief (Konzept) vom 26. März 1698 an Daniel Ernst Jablonski nach einer Kladde des Autors für das Forschungskolloquium zur Begriffsentwicklung in den Naturwissenschaften mit Datum 9. Mai 1982. Vgl. Harnack (1900, 53).

Leibniz vermeint, alles in der Natur sei zugleich metaphysisch und moral. Doch ist nicht gerade diese Gleichstellung, die den Unterschied zwischen Einbildung und Wirklichkeit aufhebt, auch schon eine notwendige Voraussetzung für den Energieerhaltungssatz, für dessen die Lebenskraft zerstörende Universalität? Wie also kann der Baum des Lebens mit Gleichstellung von Metaphysik und Moral seine Existenz, seine Individualität bewahren? Das war meine Frage, die ich mir im Montagskreis stellte und die ich mithilfe eines Kinderbuchs zu beantworten suchte. Experimente aus dem Umfeld des Vis-viva-Streits sollten von Kindern, die kulturell vielleicht noch Zugang zum Denken von Leibniz hätten haben können, durchgeführt werden, um damit zu einer diskursiven Erklärung, vielleicht auch Erhaltung von Lebenskraft über Leibniz hinaus beizutragen. Der Titel des Buches sollte lauten: „Der Flaschenteufel des Herrn Kartesius. Ungewöhnliche Abenteuer zweier Ausländerkinder im Jahre Siebzehnhundertsiebenunddreißig in Rixdorf".

Es war das Jahr, in dem Nachfahren der Glaubensgeschwister von Comenius in Rixdorf, dem heutigen Berlin-Neukölln, ihre Zuflucht fanden. Sie hatten sich gut hundert Jahre in Böhmen versteckt gehalten und brachten wie heute Türken, Kosovaren, Kurden, Palästinenser, Roma, Syrer ihre Kultur mit nach Berlin. Ihr Reichtum dürfte noch aufzuspüren sein, dachte ich, zumal immer noch Nachfahren der eingewanderten Böhmen in neunter, zehnter, ja elfter Generation in Rixdorf leben. Was ich aber nicht ahnen konnte: dass es vielmehr die heutigen Migrantenkinder sein sollten, die mir mit ihrem Wissen den Zugang in andere Zeiten ermöglichten. Das erhoffte Kinderbuch wurde gar nicht von mir im Namen böhmischer Flüchtlingskinder verfasst, sondern mehr und mehr von den heutigen Migrantenkindern selbst geschrieben, nachzulesen in Forschungsheften und Werkstattberichten des Comenius-Gartens, darunter auch in einem Artikel mit dem Titel: „Die Physik des Kartesischen Teufels" (Vierck 2004).

Doch bevor ich stolz davon berichte, dass ich das Kinderbuch wirklich nicht mehr schreiben muss, habe ich, um den Wert des autonomen Kinderwissens in unseren Projekten mit dem Max-Planck-Institut für Wissenschaftsgeschichte zu betonen, noch eine Kostprobe zum verheimlichten Stand der Dinge aus meiner Kladde vorzutragen: „Einige Jahre bevor die zu lesende Geschichte tatsächlich beginnt, erhebt sich mit Schmerzen im linken Bein der müde gewordene Philosoph Lövenix, alias Gottfried Wilhelm Leibniz, und liest im Stehen noch einmal laut die zuletzt geschriebenen Sätze seines neuesten Aufsatzes: ‚Und man kann sagen, dass derjenige, der aufmerksam mehr Abbildungen von Pflanzen und Tieren, mehr Zeichnungen von Maschinen, mehr Beschreibungen und Darstellungen von Häusern oder Festungen gesehen, der mehr geistvolle Romane gelesen, der mehr wissenswerte Erzählungen gehört hat – dass derjenige, sage ich, mehr Erkenntnis als ein anderer haben wird, selbst wenn es auch kein wahres Wort in all dem gab, was man ihm schilderte oder berichtete. Denn die Übung, die er darin besitzt, augenblickliche und ausdrückbare Gedanken oder viele Begriffe im Kopf zu haben, macht ihn geeigneter, das zu begreifen, was man ihm vorlegt, und er wird sicher unterrichteter, beschlagener und fähiger sein als ein anderer, der nichts gesehen, noch gelesen, noch gehört hat – vorausgesetzt, dass er in jenen Geschichten und Darstellungen nicht etwas für wahr annimmt, was es nicht ist, und dass diese Eindrücke ihn nicht hindern, darüber hinaus das Wirkliche vom Eingebildeten oder das Tatsächliche vom Möglichen zu unterscheiden.'"[3]

Meine Kostprobe hier soll nicht verdeutlichen, dass der Versuch, ein wissenschaftsgeschichtliches Kinderbuch zu schreiben, bereits im Ansatz, bevor die zu lesende Geschichte beginnt, mit dem Erfinden, der bloßen Einbildung also, gescheitert ist. Nein, diese Kostprobe soll genau das Gegenteil belegen, dass es nämlich immer die Wirklichkeit ist, die dem Erfundenen seinen Erkenntniswert gibt. Erst in der Wirklichkeit zeigt sich, wie erfolgreich die Einbildung ist. Deshalb wollte ich so dicht wie möglich an den Vis-viva-Streit heran. Ich habe mir seinerzeit notiert: „Durch die dialektische Darstellung eines geschichtlichen Stoffes müsste auch ein Kinderbuch möglich sein, nicht als Roman (Erfundenes), sondern als Dokumentation."

Comenius, wie ich heute weiß, hätte an meiner Stelle nicht von Dokumentation, sondern von Anwendung gesprochen. Zu Einbildung und Wirklichkeit, zu Theorie und Praxis kommt bei ihm immer auch die Chresis, das gemeinschaftliche Ausprobieren. Niemand kann für sich behaupten, dass er allein im Besitz der Wahrheit sei und die Sache auszuführen wisse. Es bedarf dazu immer der Spielräume, der Schutzräume für Freiheit, in denen etwas von der eigenen Position ohne Gefahr aufgegeben werden kann – gewiss, nur zur Probe, einem Vorgang also, in dem grundsätzlich allen anderen die Einbringung ihrer Erkenntnisse ermöglicht und so die Genese des Sachverhalts verstanden wird. Chresis ist bei Comenius so etwas wie der gemeine Nutzen des Gewissens, das Reich der Normen, das auf Konsens beruhende öffentliche Verfahren, um Moral und Metaphysik, das Reich der Kraft und das Reich der Weisheit ohne Verlust ihrer Autonomie übereinstimmen zu lassen.

Stellvertretend für Jan, Anna und viele andere Kinder der Glaubensflüchtlinge von 1737 saßen im Montagskreis Wissenschaftshistoriker mit besten Kenntnissen sowohl des Reichs der Weisheit als auch des Reichs der Kraft zu Zeiten meines Flaschenteufels René. Wenn ein Kinderbuch, in dem kein wahres Wort enthalten ist, bereits zur Übung anregt, Wahrheit zu erfassen, um wie viel größer muss dann erst die Erkenntnis sein, je mehr wahre Worte in ihm enthalten sind, dachte ich. Der Vis-viva-Streit, untersucht im Kreise von Menschen, die sich einer historischen Epistemologie verschrieben hatten, dürfte mir, der ich mit

[3] Vgl. Leibniz (1904, 374f), wobei, noch einmal ausdrücklich, die Kladde des Autors zitiert wird.

einer genetischen Historiographie liebäugelte, eine enorme Hilfe sein. Viele wahre Worte, etwa über das Auftauchen meines Flaschenteufels 1648 bei Raffaello Magiotti (1597–1656) in Italien (Magiotti 1648), vier Jahre nach Erscheinen der „Prinzipien" von René Descartes (1596–1650), hätte ich für mein Kinderbuch notieren können. Doch es kam noch besser.

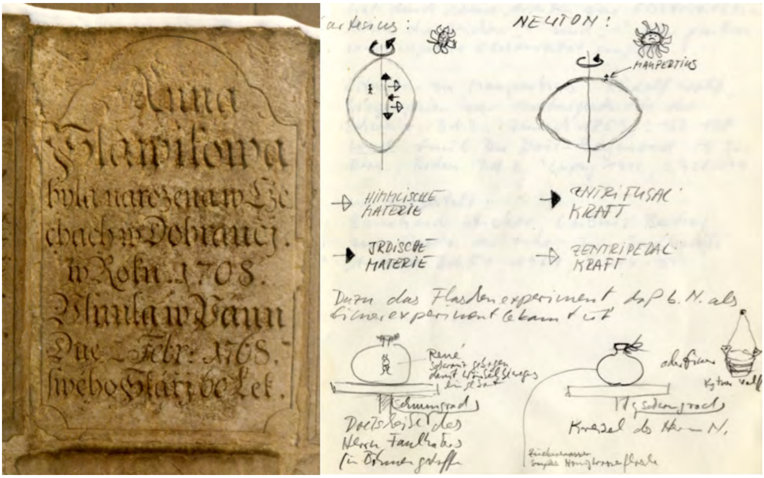

Abb. 17.2: (1) Grabstein mit Namenszug „Anna" auf dem Böhmischen Gottesacker in Berlin-Neukölln, (2) Bleistiftzeichnung zu historischen Experimenten aus einer Kladde des Autors.

Anstatt im Montagskreis etwa Experimente zur Unterscheidung von himmlischer und irdischer Materie vorzutragen, die Anna und Jan in meinem Kopf aus Anlass der Ankündigung durchführten, dass Pierre-Louis Moreau de Maupertuis (1698–1759) Nachfolger ihres verstorbenen Bischofs Daniel Ernst Jablonski an der Königlich Preußischen Sozietät der Wissenschaften werden sollte; anstatt auf mich allein gestellt im Montagskreis Experimente mit René, dem Kartesischen Flaschenteufel, vorzutragen und zu diskutieren, drängte sich mir die Idee auf, dies doch lieber gleich gemeinsam mit den erhofften Lesern von Kinderbüchern zu probieren.

Wie aber bekomme ich Anna und Jan von heute, Neuköllner Migrantenkinder also, in den Montagskreis, wenn ich mich nicht einmal traue, von meinem Kinderbuch zu reden? Vielleicht, indem ich den Montagskreis aus Berlin-Dahlem nach Berlin-Neukölln hole, in einen Philosophengarten, in dem diese Kinder von sich aus spielen. Das war die schönste, aber auch beschwerlichste Idee, um das heimlich in meinem Kopf sich befindliche Kinderbuch doch noch wissenschaftshistorische Wirklichkeit werden zu lassen. Leibniz hat mir dabei geholfen, mehr noch aber der Großvater seines Kollegen Jablonski: Johann Amos Comenius. Ich glaube, Peter Damerow und all meine anderen Freunde aus dem Montagskreis haben mir diese Hilfe nichtsahnend erhofft. Zumindest aber wird zwischen dem 19. und 26. April 1985 zum ersten Mal in einer meiner Kladden auf den Comenius-Garten eingegangen, zaghaft nur, denn dort steht allein das Wort, damit ich die Idee nicht vergesse, kurz vor meinen Notizen zu Johann Friedrich Blumenbach (1752–1840), der mit seiner Kritik an der Präformationslehre meinem Flaschenteufel eine ganz besondere Herausforderung hätte werden können. Doch dazu musste es nicht mehr kommen. Der Montagskreis beriet mich von nun an regelmäßig bei der Realisierung des Comenius-Gartens.

„Bemühen wir uns nicht“, schreibt Comenius (2001, 8), „die Absichten Wirklichkeit werden zu lassen, dann bleiben sie reine platonische Ideen, ein Begriff ohne Inhalt, eine Vorstellung ohne sachliche Gestalt.“ Doch weil wir, so füge ich in Kenntnis des Werks von Comenius hinzu, Lebewesen sind, gibt es Begriffe mit Inhalt, sowohl Ideen qua Herstellung als auch Ideen qua Geburt.

Auch wenn es so scheint, dass Ideen qua Geburt eher den Kindern und Ideen qua Herstellung eher den Wissenschaftlern zukommen, so traue ich mich doch nicht, solch eine moralische Bewertung des Stellenwerts von Kinderwissen in unseren Projekten mit dem Max-Planck-Institut für Wissenschaftsgeschichte abzugeben. Dazu bedarf es einer methodischen Reflexion unserer Arbeit, von der ich annehme, dass sie eine gar nicht so offensichtlich am Lebensalter ausgerichtete Antwort gibt. Festhalten möchte ich jedoch, dass unsere Projekte, die wir zu Themen wie dem Nichts, den Wundern, dem Himmel durchführten, Übungen sind, bei denen sich Theorie und Praxis auf eine Weise begegnen, in der sowohl Kinder als auch Wissenschaftler einen freudigen Genuss ihrer gemeinsamen Aktionen haben. Das belegen unsere Forschungshefte und Werkstattberichte. Ohne geschütztes Ausprobieren gibt es keine Literatur von oder für Kinder, auch keine wissenschaftliche.

Kinderfragen in den Wissenschaften zu stellen heißt, den Baum des Lebens immer wieder aufs Neue im Baum der Erkenntnis zu entdecken. Diese Einsicht verdanke ich unserem Forschungskolloquium zur Begriffsentwicklung in den Naturwissenschaften. Aus seinem Schutz heraus konnte der Comenius-Garten nicht nur sachliche Gestalt annehmen, sondern selbst auch ein Ort der Chresis werden, der allerdings – das ist meine Botschaft – ohne Kinderwissen unvorstellbar ist.

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Chapter 18
The Dramatic Pace of Acculturation and the Ability of So Many Eipo to Jump From Stone Age to Computer Age in One Generation … Without Having Read Aristotle

Wulf Schiefenhövel

Eipo Numerals

Peter Damerow had a deep interest in the ways members of traditional societies count, think, and see the world. He knew about our interdisciplinary research project in the isolated mountains of West New Guinea and invited me to workshop meetings on counting systems and early arithmetic. Our ways of approaching scientific questions and our personalities "clicked." We became friends and Peter was soon a guest in my little mountain house in Tyrol where we talked, our backs against the tile stove, a glass of wine in front of us, about numbers, but actually more about the world.

The Eipo (Schiefenhövel 1976, 1991, 2014a), like other Highland Papuan groups east and west of the international border between the Indonesian Province of Papua and the independent State of Papua New Guinea, have a specific system of counting and of numerals (Heeschen and Schiefenhövel 1983, 18). There are four (+ two) actual numeric terms: "*ton*" (1), "*bedinye*" (2), and "*winilye*" (3), represented by the small, the ring, and the middle finger of the left hand, "*winilyaba*" (23, middle finger of the right hand), "*bedinyaba*" (24, ring finger of the right hand) are variations of the terms for 2 and 3; "*seselekyaba*" (25) the small finger of the right hand is the fourth genuine numeral. Interestingly, these last three numerals have a different suffix ("-*yaba*") than the others ("-*digin*") represented on the left side of the body.

Returning to the ordinary way of counting: "*dumbarye*" (4) is located at the index finger of the left hand, "*fangobarye*" (5) at the thumb, "*nakobarye*" (6) the pulse, "*tekbarye*" (7) the middle of the lower arm, "*finbarye*" (8) the elbow joint, "*toubnebarye*" (9) the middle of the upper arm, the biceps, "*takobarye*" (10) the shoulder, "*koklombarye*" (11) the neck, "*obarye*" (12) the left ear, "*mekbarye*" (13) the crown of the head and the middle of the string of body-defined numerals, "*odigin*" (14) the right ear (all body points now appear in reverse order on the right side of the body), "*koklomdigin*" (15) the neck, "*takubdigin*" (16) the shoulder, "*toubnedigin*" (17) the middle of the upper arm, the biceps, "*findigin*" (18), the elbow joint, "*tekdigin*" (19) the middle of the lower arm, "*nakubdigin*" (20) the puls, "*famdigin*" (21) the thumb, "*dumdigin*" (22) the index finger, "*winilyaba*" (23) the middle finger, "*bedinyaba*" (24) the ring finger, and "*seselekyaba*" (25) the small finger.

"*Ton*," "*bedinye*," and "*winilye*" thus are real numerals for 1, 2, and 3 (they don't mean anything else), counted on the left hand. As described above, "*winilyaba*" and "*bedinyaba*" signify, in a kind of mirror reflection, 23 and 24, but really keep their semantics as numerals attached to the two specific fingers. "*Seselekyaba*" (25) represents the end of the counting row and is another real numeric term; all the others are linguistically built on the terms of the particular body part: "*dume*" is the index finger, "*fame/fango*" the thumb, "*nakob/nakub*" the region of the puls (cp. "*nakubnakub an*" – to pulsate), etc. That counting begins on the left side of the body is easy to understand; it represents, for the majority of right handers (McGrew, Marchant, and Schiefenhövel 2013), a sensomotorically intuitive way, namely pointing with the thumb of the right hand to the left hand and the other points on the left side.

When our group arrived in the southern Eipomek Valley in July 1974, everybody, including children, could count in this traditional way. The expression "*yupe bedinye*" (language/counting two, i.e., twice counted) was an approximate term for 50. In real life, however, the ability to count by using a row of numerical/body point terms was not used very often, considerably less than in our types of cultures which are very number oriented. The size of groups of people or the number of objects would rather be expressed as "*meteik*" (small) or "*weik*" (big). Only when it really mattered, like in the transaction of exchanging stone adze blades for some other valuable goods, the actual number, the sum, would be established by counting every item. Or if one talked about the number of people killed in the war between the Eipo and their hereditary enemies in the adjacent Famek Valley, then the name of the victims would be named and given a numerical point on the body. I have not seen or heard anyone, in the days before school education, doing arithmetic calculations beyond the addition of the first few numerals, that is, small numbers, even though such simple arithmetic would have been quite easy with that elegant intuitive system starting with the five fingers of the left hand and finally arriving at 25. Never have I witnessed anybody performing a subtraction based on the counting system, nor, understandably, multiplying or dividing. It was just not part of their tradition and not necessary for daily life.

I find it interesting that children in our countries learn relatively easily to say the more or less correct row of numbers (obviously a linguistic performance just like saying another string of words), but only much later learn to associate the specific numeral with a specific object, that is, really count, and, again much later, do correct simple arithmetic—in the beginning probably always connected to fingers, apples, and the like. It must be rather difficult for the brain to perform these tasks and calculations and it does not surprise me, therefore, that this cultural technique developed so late in history. It is not that the Eipo brains can't do it, far from it! They very quickly learned the basics of our Western way of arithmetic and almost everyone, including the older people of both sexes, can handle the large numbers involved in paying with Indonesian money where one euro is about 13,000 rupiah and one soon gets into astronomical numbers even calculating the price of a pig or buying a flight ticket from Eipomek to Sentani, the airport near the provincial capital of Jayapura. The human brain, it seems, does not necessarily lend itself to arithmetic if there is no good reason to do so. Peter Damerow in his scholarly work (vis-à-vis my sketchy observations) has stated that counting and counting systems occur quite late in history (2012). Human life is quite possible without them.

Early Contacts

As mentioned above it is very interesting to notice that the Eipo, in the very rapid transition from a classic non-literate Papuan society with a stone age tool kit to modernity, so quickly learned to handle numbers (the ones of Bahasa Indonesia as well as their own, which now became real arithmetic tools) and to manage the different challenges connected to paying and receiving cash money, to calculate costs, gains, and losses in case they have one of the little stores in the villages where some basic goods are sold. Every single item must be bought in the provincial capital Jayapura and its suburbs and brought by plane to Eipomek; the freight, like passenger tickets, must be paid in cash. Not only a challenge for arithmetic performance but also a complicated logistic problem (transporting the goods, storing them near the airport, getting one of the scarce flights, etc.), which many of the Eipo master extremely well.

It was not, I would argue, the very basic, often inadequate system of school education which effectuated the ability to cope with the modern world. It was much more the trickling process of innovation, which entered, in many shades and forms, life in the hitherto very isolated mountains. The gifted ones quickly and often informally learned the new techniques to count, read, write, and handle money first; from them the others learned. Just like in the case of learning Bahasa Indonesian, which spread surprisingly quickly. Many of the Eipo by far surpass my knowledge in this language.

It must have been similar in the Roman Empire, when the soldiers introduced Vulgar Latin as a lingua franca. It spread with considerable speed and so efficiently that in large parts of the conquered and administered territory (e.g., Italy, France, Spain, Portugal, Romania) hardly any of the former languages survived. Yet this process took from about 200 BCE to 100 CE (cp. Cerquiglini 1993, Roegiest 2006). The spread of such cultural techniques as building roads, growing wine, and a money-based market system happened alongside and beyond the 300 years of linguistic acculturation. People are, generally, attracted by advantageous new ways of lives, whereas their cultural institutions guard tradition. But despite its revolutionary nature and the push of the unifying Roman administrational system, linguistic acculturation in the occupied European territories took approximately 12 generations.

The Papuans, autochthonous people of New Guinea whose ancestors arrived about 50.000 years b.p. (cp. Schiefenhövel 2014a), have undergone a much more dramatic change. When the first group of our interdisciplinary project "Man, Culture and Environment in the Central Highlands of Irian Jaya, West-New Guinea," initiated by Gerd Koch (1977) of the Berlin Museum for Ethnology and funded by the Deutsche Forschungsgemeinschaft, arrived in the southern Eipomek Valley after five days of walking from Bime, the then closest airstrip, the Eipo and their neighbors had a neolithic tool kit and lived, with regard to subsistence sociopolitical structure, the stone age lives of their ancestors.

In 1959–60 the team around the French journalist and adventurer Pierre Gaisseau had walked, during an incredibly demanding expedition, the breadth of then still *Nederlands Nieuw Guinea* from the south coast to the Mamberamo River in the northern lowlands; two members even continued to the provincial capital Hollandia. After crossing the central cordillera, the team descended with great difficulty into the Eipomek Valley and thus were the first foreigners whom the Eipo ever saw. The film *Le ciel et la boue* (The sky above, the mud below) documents this quite friendly and animated encounter. From October to December 1969 Gaisseau came back, this time accompanied by a group of Indonesian military. They parachuted into what was then called X-Valley (according to the confluence of three rivers), built a camp, and stayed for several weeks with the Eipo. The team members collected (at that time unknown to fieldworkers) a first list of their language, recorded the size of settlements, production of food in the gardens, and the health condition of these pristine people (Komando Daerah Militer XVII "Cenderawasih" n.d., Hariono 2003). The team had to be rescued by helicopter after their attempt to float down one of the rapid rivers further east ended in disaster, with several members injured and all the newly made film footage of Gaisseau lost. After this, a few missionaries of the Unevangelized Fields Mission, UFM, crossed the region but did not build a mission station or airstrip in this region.

Our group consisted of the ethnologists Gerd Koch and Klaus Helfrich, the linguist Volker Heeschen, the dental anthropologist Grete Schiefenhövel, and myself, a medical doctor who had worked on anthropological and ethnomedical topics in New Guinea since 1965. When we arrived, the Eipo were still leading a neolithic life as horticulturists, gatherers (mainly a female activity), and hunters with their religion and other traditions intact. As the Eipo and their farther neighbors, of whose existence they hardly knew, had no name comprising all groups with the same language and culture, I suggested the term *Mek*, the term for "water" and "river." The people formerly called *Goliath Pygmies* (after the Dutch name Goliath given to one of the mountains south of the central chain, rather a misnomer because the Mek are of very small stature), *Kimyal* or *Ketengban* are now known under the name *Mek* in linguistics and anthropology. A dictionary of the complex Eipo language (Heeschen and Schiefenhövel 1983), a substantial number of monographs as well as ethnographic and human ethological films have been published documenting everyday behavior as well as ceremonies and rituals (cp. Human Ethology Film Archive, now at Senckenberg Research Institute and Museum, Frankfurt). The Eipo lived, between 1974 and 1980, the life which had been typical for other traditional Papuan groups of the very isolated mountainous inland of the big island New Guinea east and west of the international border—only that practically all other groups had been subject to cultural change brought about by Christian missionaries, some form of government administration, and/or, in some cases, by large-scale oil/gas and gold/copper exploitation.

The Eipo had a remarkable knowledge of flora and fauna and their taxonomic system reflected morphological similarity as well as genealogy almost in the same way as the Linnean system (Hiepko and Schiefenhövel 1987). Eipo informants were always right when they claimed, often to the disbelief of the two botany professors of the research team, that two plants were "brothers": they indeed belonged, in every single case, to the same genus or at least family. Their anatomical knowledge was astounding, too. The function of the kidneys, the ureter, the bladder, and the urethra was well known as well as the fact that when a female pig was slaughtered at the moment of being in heat one could see little yellow spots (the Graf follicles) on the ovaries. Yet they did not associate this with possible pregnancy, as they believed that an embryo was formed by the held back menstrual blood and the sperm. Their stories, legends, and myths (Heeschen 1990) as well as the lyrics of their love and mourning songs (Schiefenhövel 2014b) bear extraordinary testimony to their creative power and the quest for verbal beauty.

After the German Research Team had ended its main projects, which coincided with the destruction, by two powerful earthquakes in 1976, of the landing field for small aircraft, and the start of mission work by representatives of the UFM, the Eipo turned away from their traditional beliefs and became Christians.

This process happened around 1978–1980. They were not primarily attracted by the revolutionary theology of the New Testament. Their reason to fundamentally change their way of life was rather triggered by their insight that they had lived the basically stationary, unchanged existence of an almost completely isolated, marginalized group of stone age people in a world of high tech products, formalized education, high speed communication and transport, market economy, and globalization. In the past they had but glimpses of the outside. As all the neighboring valleys, except one, had also remained in the prehistoric mode of life, they did, for some time, not feel motivated to leave the known ground of their traditions and depart to unknown shores. To become Christians was a political rather than a religious decision. And it therefore happened in a very unified way: basically everyone in the communities of the Southern Eipomek Valley went along with the change. All received new names, many attended the Sunday service in the newly built church, listened to the stories of the Bible, and learned new songs and rituals. Some children began to go to the small elementary school erected in Eipomek. This was the beginning of a most dramatic and radical change involving all members of the society.

The Eipo, led by very intelligent men, mostly actually those who had been our informants teaching us their language and their traditional view of the world, have so far resisted the ever-present lure of cargo cult ideas (Lawrence 1964). They understand that money and other wealth won't come through some religiously fabricated miracle but only through education and hard work. When in the neighboring Famek Valley, the place of their former enemies, a "prophet" promised to cause, through the performance of a syncretistic religious ceremony, a rock to explode and spit out unimaginable amounts of Indonesian Rupiah, Philippus-Walabyan, one of the leading men from Eipomek, was able to convince them that this "technique" would not work. The Famek people then discontinued their incipient cargo cult.

Coping with the New World

In summer 2014 approximately 30 Eipo students were enrolled, in different faculties, most commonly the one for Teacher Training, at Cenderawasih (Bird of Paradise) University in Jayapura, the provincial capital. Some were studying law and other subjects at the private Muslim University in the same town and a few had been accepted at prestigious universities in the State's capital Jakarta, in Bandung, and in other cities. One young Eipo, at that time studying Administrational Science in Jakarta, had been selected to be sent to a university in the United States, but due to the sickness of a family member he stayed in Indonesia. Of those who have finished their university education, some are employed by the government and serve as administrators in their own Star-Mountains Regency, some are administrational heads or nurses of local health stations (Pusat Kesehatan Masyerakat) in the Eipomek valley or in different areas of the province, and some are primary school teachers, one of them in the six-class elementary school (*sekolah dasar*) in Eipomek. Probably more than 100 Eipo live at the coast, usually in Sentani where the airport is located, which serves domestic and international destinations. One Eipo was elected member of the local parliament; he is also the first who had a modern house built for him and his extended family in one of the suburbs of Sentani.

Almost all of those who stay permanently or periodically in this buzzing, crowded Indonesian City of more than 250.000 inhabitants cope with the challenges of everyday life—accommodation, transport, communication, finding income or family members who support them—with seeming ease. Almost everybody has a mobile phone, usually the smart phone version, and many are well trained to work with computers.

One young Eipo has created a website called "likteam" ("*lik*" is a proverbial Eipo expression meaning "I am not inclined, I don't want" and a term for the dialect in the Eipomek Valley, as it is not found in the other versions of the Mek language). He and some others have worked together with Andrew Sims, an American mission linguist, to translate the bible into the Eipo language, a work that is now finished. Many villagers use their stay at the coast to buy large quantities of basic foods (rice, noodles, cooking oil, tea, coffee, sugar, etc.), but also sophisticated items like deep-frozen chicken from Java and eggs from Sulawesi. Profit is good despite the fact that every single carton or bag must be flown in by single-engine aircraft to Eipomek. The Eipo have built, within 40 years, three airstrips: the first one with me as "engineer" in 1974–75; after its destruction by the earthquakes in 1976 the UFM mission helped them build a second one; and in recent years they have decided, on their own, to construct, with shovels and other simple tools, a third landing field at yet another spot which would allow larger airplanes to land and take off. They have succeeded. The first airstrip was 366 m long with a slope of about 5%, the latest one is about 550 m long and has a similar slope. When I asked them why they were taking on this enormous task, they said: "We need a big lapangan terbang so that big planes come and we are the center of our region." In other words: "We don't want to be Düsseldorf, but Frankfurt."

Their ambitions are unbounded and it is great to see how they try to manage with the massive cultural changes bringing not only progress but also many new problems, like dramatic population growth. Some of their answers have been ingenious. From David Cole, the first missionary who stayed with them, they learned to grow Chinese carp and other fish in ponds. He also gave them some brood to try on their own. It is now a new sign of high rank to have a number of well-built, well-functioning fishponds; some men have five or more. The number of fish is quite astounding, and some are 2 kgs or more. This is an extremely important and at the same time completely new source of protein for their growing families. Until 1980 the Eipo had never even seen fish at all; the rivers in the mountains are too swift for any fish species to make their way up to the high valleys. When one sees them now handling their fish breeding enterprises one is amazed: it is as if they had always done this.

The same is true for riding motorbikes or cars in and around Jayapura and for repairing them. Many of the young men who dare to immerse themselves in the battle of very dense and—for Europeans—crazily chaotic traffic on narrow roads don't have, as they told me, a driving license! They would need that only if they were to register a motorbike or car in their own name. It seems that police are lenient with them. I wonder whether that is also true if they are the riders of motorbike taxis (*ocek* in Bahasa Indonesia), a job that some Eipo do, that is, they take passengers, usually women with large bags of goods from the supermarket, and drive them, for small amounts of money, right to their doorstep.

In February 2010, four of our local co-workers visited Europe for one month. It was not the best time of the year (with snow in Bavaria and Tyrol) but they managed the cold just as all other challenges with remarkable calm, realism, and intrepidness. When we were inside Munich's Frauenkirche, they looked up to the ceiling far above them and asked: "Who made this, people?" and were impressed to learn that this was done 500 years ago without modern technical machinery. They were very interested to see, in the Deutsches Museum, the history of airplanes, from the first gliding devices to their present form, the technique to mine iron ore and salt with simple tools, and other primordial ways to master nature. Being below the surface of the earth, for instance in the Munich subway, caused some slight uneasiness. Other than that they felt great, especially when they got the famous, big roasted pork knuckles at Kloster Andechs. When we were looking at a large display of stone adzes in Munich's prehistory museum, one of them asked me: "Did you bring our adzes here or are these the ones of your ancestors?" That was exactly what we had hoped for: that they would understand our high-tech lives had humble origins, origins which survived in their homeland until the twenty-first century.

The New Cultural Center in Eipomek: Keeping History Alive

Knowing that there were some houses with well-functioning solar electricity by then, in 2008 I brought my laptop for the first time and started to show some of the films from the mid-1970s documenting aspects of Eipo traditional life. Soon we had to move to a school building because everyone wanted to see the images of the past, scenes the young ones had no idea of. It was, judging by the noise and the excitement, like a revolution. Obviously this was extremely important for them. A few of the leaders came to talk to me about their request. “You know, we have no history,” was their astounding statement. They used the loan word from Bahasa Indonesia *sejarah* for history: “We have no drawings, no pictures, and no writings from our old times. You have documented our customs, our culture by photograph, film, and tape recorder. Can you give this history of ours back to us, so that our young people don’t lose contact to their roots?” I was stunned. What a wise assessment and what a beautiful request. We made plans and I suggested the name “Pusat Budaya” (Cultural Center), but in time it became obvious that they wanted the English term “Center” to be part of the name because this had a special, international linguistic appeal… in Indonesia as well as in other countries, Anglicisms are en vogue.

The German Foreign Office in Berlin provided funds for the “Center Budaya Eipomek” and after some years of negotiation and finding official partners in Papua, the building and a large, nicely constructed guest house were officially opened on July 4th, exactly 40 years after the first group of German fieldworkers had arrived in the southern Eipomek Valley. The round cultural center, shaped as the traditional men’s houses, and the guesthouse are thatched with tiles of red aluminium resembling Dutch roof architecture. The walls are decorated with ethnographic items donated by the Eipo themselves and by large-scale photographs from the 1970s, depicting events and ceremonies, which the young ones have never seen. The local government of the Starmountains Regency has put up a large amount of money (at least twice the German contribution) to finalize the project.

The center is now equipped with electric wiring (power comes from a hydroelectric plant not far away) and with a good laptop, a projector, and a large screen so that the Eipo can view the documents we made. Favorites are the films of large feasts and ceremonies as well as footage showing children’s play; the former actors are now women and men in their fifties and very much enjoy watching the lively interactions they had with their peers. The Eipo know that in centralized Indonesia such a cultural center in a small remote village is something unusual. They also know that their culture and their environment are among the best researched in New Guinea and Melanesia. All monographs published in the Series “Mensch, Kultur und Umwelt im zentralen Bergland von West-Neuguinea” plus a number of other publications and many photographs are stored in a glass cupboard. The Eipo hope that this special feature of their valley will also help to attract tourists who are interested in their former and modern way of life and the beautiful alpine landscape they live in.

The Human Brain: A Flexible Organ

Summing up, it is, I believe, no exaggeration to say that the Eipo and their neighbors in the adjacent valleys have made the critical transition from the stone age to our global times surprisingly well. Many would probably hold the view that one cannot jump from prehistory to the modern world in just 40 years, that it is necessary to culturally acclimatize over a longer period of time. That one would need to know about Aristotle, the Renaissance, and at least some parts of the world's literature. This viewpoint is often taken to explain why, in developing nations, the process of sociocultural and economic development is often so painstakingly slow and sometimes does not seem to work at all. Albert Maori Kiki, one of the first generation of modern politicians, co-founder of the unions in Papua New Guinea, and a strong advocate for its independence, called his most interesting autobiography *Ten Thousand Years in a Lifetime* (1968). This is a very apt description of what has been characteristic in many parts of New Guinea. The case of the Eipo is even more dramatic than the case of Kiki's upbringing, because life in Jayapura, modern means of communication, and modern media today are even further removed from Papuan traditions than the world in which Albert Maori Kiki grew up in the 1960s.

The human brain, as evolutionary epistemologists Konrad Lorenz (1973) and Gerhard Vollmer (1975) demonstrate, is primarily an organ to ensure survival, not necessarily to find truths about the world. I hope it has become apparent from this description of the Eipo's passage from stone adze to smart phone in just one and a half generations that our brain is extremely flexible. It can handle drastic changes of environment, be they natural or cultural. That is, I think, a good message.

Figure 18.1: Prehistory lasting into the present: A man from Larye, a Mek language area, knapping, with amazing professionalism, a block of Andesit rock into a stone adze blade. Even today (2018) this technique is in practice because stone adzes are often part of bride price and other ritual payments.

Figure 18.2: Coming together after a day's work in the gardens and the rain forest. The family huts of the Eipo were very small, preserving the warmth of the fire place in the cold nights. A hafted stone adze is placed at the left wall.

Figure 18.3: An Eipo family in their modern house, provided by the government. The battery on the left is connected to a solar power system providing electricity for lighting and CD players. As these houses have no fire place (the design is borrowed from Sulawesi), the Eipo cook and eat dinner in traditional huts, where they also sleep in the night, cuddled around glowing embers.

Figure 18.4: Single engine airplanes are the only means of transport to the urban centers at the coast, 200 km direct line away. This is the third airstrip the Eipo have built in 40 years. It is 550 m long at a 5% slope and enables larger aircraft than the usual Cessna and Pilatus Porter to land: competition with other valleys.

Figure 18.5: Silas-Nyenye, son of a "Big Man," is doing "the sched": communicating, in Bahasa Indonesia, with the airlines, the pilots, and other stations via single-side-band radio. No phones work yet in the isolated mountains. The Eipo are amazingly skilful and reliable in this demanding task.

Figure 18.6: Breeding fish in specially constructed ponds has become a sign of high status; formerly, no fish lived in the fast flowing rivers of the central highlands. For the quickly growing population, carp and tilapia provide valuable protein. The owners get cash income from this new economic activity and sometimes export fish (in buckets transported by airplane) to other areas.

Figure 18.7: “Center Budaya,” the Cultural Center Eipomek, built with German and Indonesian funds. It houses a large collection of photographs, films, and publications documenting traditional life and customs before the Eipo decided to become Christians around 1980 and serves as a way to “preserve our roots,” as the Eipo say. They are proud of this institution in their valley; other museums devoted to the history and culture of Papuan peoples are restricted to the urban regions.

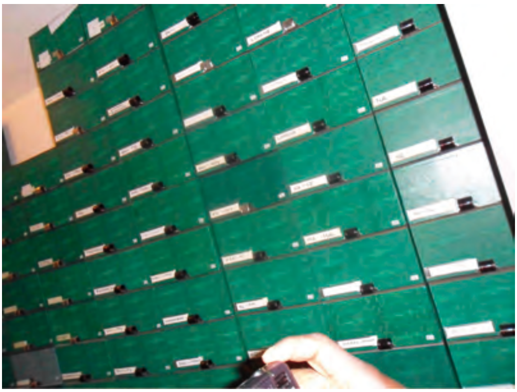

Figure 18.8: Four Eipo friends visited Europe in 2010. Upon their arrival, they got photo cameras and immediately started to document our world. They were fascinated by the filing system for reprints…

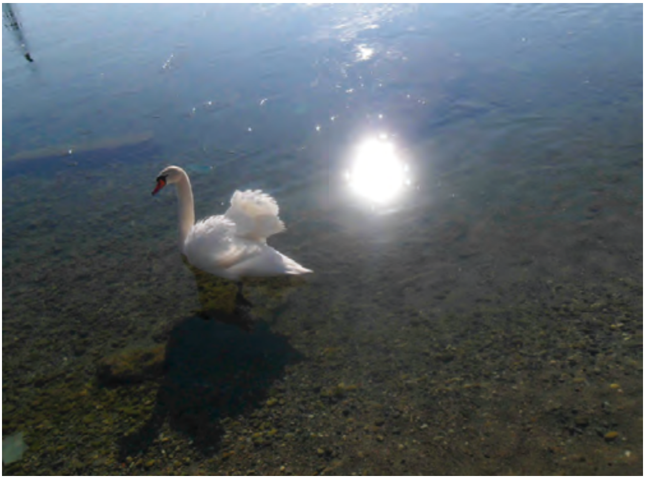

Figure 18.9: …and the swans of Lake Starnberg. More than 30 young people have successfully finished university education—their parents still lived an isolated neolithic life. A dramatic change.

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Chapter 19
A Textbook for Teaching Mathematics in Brazilian Indigenous Schools
Circe Mary Silva da Silva

Currently, the indigenous population of Brazil is about 817,000, representing 0.4 % of the country's population. Most of the indigenous people live in the north of the country. In the state of Espírito Santo 2,630 Tupinikim and 262 Guarani live on lands designated to "them" by the government, though they are constantly fighting to keep this land due to the economic interests of companies that have also settled in the region.

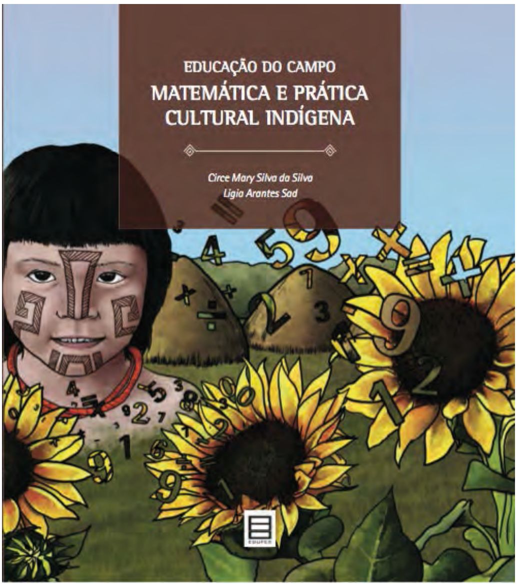

Figure 19.1: *Mathematics and Indigenous Cultural Practice.* See Silva and Sad (2019).

Between 1996 and 1999, the Tupinikim and Guarani in Espírito Santo had a new experience—the chance to get professional training to work in an intercultural and bilingual school and to be able to assume teaching activities. In assuming the actual teaching activities, the indigenous educators felt the need for continuous monitoring by teachers from whom they had received the initial training. The books used in the non-indigenous schools proved to be ill adapted to the teaching proposal. Thus, educators requested specific textbooks for teaching in indigenous schools. This chapter tells the story of the production of a mathematics textbook for indigenous schools, which was produced in cooperation with my colleague Ligia Sad (Silva and Sad 2019).

From 1984 onwards Peter Damerow advocated "mathematics for all" and drew attention to the danger of importing European standardized curricula into developing countries. In defending his viewpoint, he pointed to the issue of culture: "So I think the relations between mathematics and culture is the first and maybe the most general question which arises when mathematics for all is taken as a program" (Damerow et al. 1984, 18).

For hundreds of years indigenous Brazilians were either forgotten or received schooling by imposition that did not take their cultures into consideration. When discussing with Damerow for the first time the idea of creating a textbook of mathematics that was close to both the school and the cultural reality of the indigenous villages of Espírito Santo, his first reaction was one of great excitement. He then put his personal library at my disposal and suggested that I consult the library of the Max Planck Institute for the History of Science. As was his style, he also raised simple but rather crucial questions for the foundation of the proposal that stated: What ethnic groups are they? Where do they live? What activities do they develop for survival? What techniques do they use? What are their skills? What are their beliefs? These questions motivated a study that we developed with these two ethnic groups and the results are presented in *The Transformations of Knowledge Through Cultural Interactions in Brazil: The Case of the Tupinikim and the Guarani* (Silva and Sad 2012). For Damerow, it was essential that the activities proposed in the book were in tune with the way of life in the villages and not too far from the mathematics of non-indigenous schools. And, above all, it was important that the indigenous educators participated in the preparation of these works and tested them, because only in this way would the book reflect the aspirations of the indigenous community in Espírito Santo. It was necessary to give a voice to the community who would receive this book.

The preparation process of the book was very slow. Since the results of official assessments of student performance in indigenous schools were far below that of non-indigenous schools, courses dealing with the continuing education of indigenous educators were conducted at the same time. Moreover, the academic research of my mentees, Marcilino (2005), Magalhães (2007), and Lorenzoni (2010), helped to acquaint us with the Tupinikim and Guarani cultures as well as the realities of the school situation in the village schools.

Methodologically, we chose to prepare a text for teaching mathematics containing activities drawn from actual situations in the villages. The choice of activities involving the curriculum content was informed by the important issues related to their lives and culture.

The book proposes that the study and learning process of mathematics in the indigenous school contribute to the formation and interaction of both students and indigenous community members with the environment and society, which can strengthen the socio-political and economic organization of the Guarani and Tupinikim people in the globalized world context. According to Marcilino (2005), for these educators mathematics education should be a means for the affirmation of Tupinikim and Guarani cultures and for their self-sustaining development, thus creating conditions for students to acquire diverse knowledge. Moreover, it is expected that people who are conscious of education will have the ability to intervene in the building of a better society by promoting peace. The indigenous people understand peace as harmony with oneself, with the environment, and with nature.

Mathematics and Indigenous Cultural Practice constitutes a rather unconventional textbook because it takes the indigenous community as its starting point, and touches on themes such as agriculture, handicrafts, or body painting. It also incorporates the reports of indigenous educators and articulates the mathematical content of the lower grades in school in the form of proposed activities.

The approach of the book is to refer mathematical concepts such as the decimal system to objects and practices familiar from the daily life of the indigenous people, such as frequently used plants, or fishing activities. Thus, Chapter 1 is based on the wise indigenous men and women's experiences with healing plants. It proposes using counting activities based on the decimal system and the calculation of estimates, thereby giving voice to educators who then reported their knowledge of the *Aroeira* plant:

> In the Comboio village there is a native plant called Aroeira. It is a very important plant because besides serving several purposes it also contributes to the survival of village families over a period of four months each year. The villagers have known about the Aroeira plant for a long time, but its commercial value and usefulness were discovered only three years ago. It is important for the community because while being native, it is a source of income that does not consume community funds. We know that it is used as a substitute for black pepper, as an appetizer, in the manufacture of some glues

> as well as in the composition of a medicine to fight disease of the cornea.

Chapter 2 focuses on the topic of fishing, exploring mathematical concepts such as arithmetic operations, units of measurement, geometric objects, tables, and graphs. In Chapter 3 we introduce historical comments about the indigenous population in Brazil, the concept of place value and structure of the decimal system. For Chapter 4 we chose a topic of great importance in the lives of indigenous people—agriculture. The most common crops in the villages in Espírito Santo are manioc, beans, corn, and pineapple. We use this theme to explore the concepts of fraction and percentage. Giving voice to indigenous educators, we introduced a report showing how beans were planted: "the bean was planted by hand and without the use of chemicals, because there was enough space for the technique of 'coivara,' that is, our ancestors chose a place where they cleared the land of underwood and burned the bushes. Afterwards, they gathered in heaps branches that had not been completely burned, and set fire to them again, thus clearing the land. The ashes of burnt material served as fertilizer for the soil."

Chapter 5 presents geometric concepts related to indigenous housing construction and delves into geometric concepts such as parallelism, perpendicularism, angles, perimeter, area, and volume. "Making manioc flour in Quitungo" is the subject of Chapter 6, which provides activities for the development of mathematical skills to calculate, measure, and estimate, among others. The theme of Chapter 7 is related to beauty and art since it delves into indigenous handicrafts. In this chapter, the mathematical content addressed deals with the enlargement and reduction of figures, as well as the arithmetic operations involved in trade with these crafts. Thus, for example, an activity involving Guarani basketry and the concept of symmetry are included: "Use the basket model below and color with the same color all the vertical stripes and with another color, all horizontal stripes" (Lorenzoni 2010).

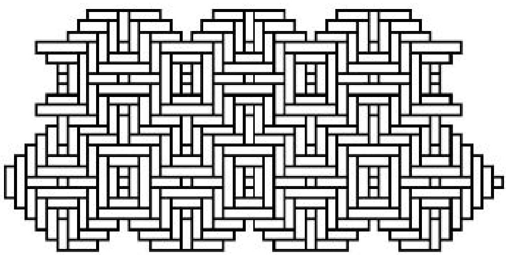

Figure 19.2: Weaving basket model. Source: Gerdes (2011).

Figure 19.3: Guarani basket. Source: Lorenzoni and Marcelino (2010).

Chapter 8 deals with an issue that involves the beauty of indigenous body painting and links it to geometry, mainly to questions of symmetry as well as curved and straight lines. The explanation of the importance of body painting is given by an educator: "The importance of the painting for us Guarani is that it is a symbol of life. There are paintings for males and females, for the newborn; there are paintings for the woman warrior, paintings of encounters, and paintings that only the leader, that is, the chief can use." An appreciation of heavenly knowledge is discussed in Chapter 9, in which we address counting, numerical operations, geometry (flat and three-dimensional geometric shapes), geometric views, tables and graphs, and flat location. The indigenous school is an important space for new generations in which school education and indigenous community education join together to meet the diverse needs of each person. For this, community involvement is paramount in the appreciation of their culture, in the strengthening of identity and collective policy, according to social goals and personal aspirations.

Mathematics and Indigenous Cultural Practice discusses mathematical concepts related to elements of cultural diversity and thus extends and complements the teaching of mathematics. This should also be thought of in the social dimension of its capacity to commit citizens to each other, regardless of their cultural or social background. We agree with Peter Damerow, who believed in the distribution of knowledge. According to him: "[…] we reject assumptions that mathematical knowledge is the prerogative of some cultural communities and not others and instead see mathematics as something potentially appropriate to all people" (Damerow et al. 1984, 25).

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Chapter 20
Beyond Archimedes: The History and Future of the Arboreal Software

Mark J. Schiefsky

This chapter describes the history and rationale of the development of Arboreal, a software application originally designed in the course of the Archimedes Project to enable the reading and analysis of structured XML documents. It also provides an update on current development of the software and sketches some directions for further work. Peter Damerow inspired and contributed to the development of this software at every stage from its inception until his death, and its existence would be unthinkable without him. I hope that this history will illuminate an important dimension of his work and convey the essence of his compelling vision for the place of information technology in humanistic scholarship. Above all, I hope that my description of the present state of this software and its future development will demonstrate that this vision is still very much alive today.

History and Rationale

The Archimedes Project was initiated by Peter Damerow and Jürgen Renn at the Max Planck Institute for the History of Science (MPIWG) in the late 1990s. It was conceived as the digital component of a major research project of Department I of the Institute on the long-term development of mechanical knowledge from the ancient world up to the early modern period. The goal of the project was to exploit the potential of emerging digital methods to study the content of mechanical knowledge and to disseminate the results of this research. Earlier work at the Perseus Project of Tufts University had demonstrated the great power of new tools for linguistic analysis, such as morphological analyzers and digitized dictionaries, for the creation of a new kind of online environment for the reading of ancient Greek and Latin texts. The possibility of extending this approach to the history of science was an exciting one, and very much called for by the large volume of textual sources involved in the study of a discipline like mechanics. More specifically, Archimedes was motivated by the need to represent the conceptual structure of mechanical thinking—relations between concepts such as "force," "weight," and "motion," such as the notion that "motion implies force" that is ubiquitous in early physical thought. To achieve a digital representation of the conceptual content of mechanical texts at this level of detail was a challenging task that went well beyond anything then existing in the Internet.

In 1999, I came to the MPIWG as a postdoc to work on the Archimedes Project, with a mandate to introduce the technology developed by the Perseus Project to the research environment of the Institute. This was a year of intellectual joys for me, not least because I had the remarkable privilege of working closely with Peter Damerow on a daily basis. We discussed everything from the origin of writing and mathematics in the third millennium BCE to markup practices for electronic texts and the history and culture of contemporary Berlin. My own scholarly perspective as a classical philologist and historian of science was immensely enriched, in ways that have continued to shape my intellectual outlook to this day. One topic that was of constant concern in our daily work during this period was the design of electronic working environments that could meet the demands of the Archimedes Project for the detailed representation of the content of mechanical knowledge. I quickly became familiar with the wide range of ingenious prototypes that Peter had created for the Macintosh using FileMaker. These were based on a simple idea: a text was split up into sentences which were then loaded into a FileMaker database, in which each sentence was an individual record. This strategy made it possible to browse through the text sentence-by-sentence, to track the terminology in which ideas were expressed in individual sentences, and to add translations or comments on sentences in a systematic manner. FileMaker's powerful indexing and viewing functions made it possible to view clusters of sentences together and to see connections between the data that were not otherwise apparent. And all of this could be achieved without any knowledge of formal programming languages. Even though the technological basis was quirky and idiosyncratic, it was clear that a powerful vision lay behind it. The use of technology was motivated by scholarly questions and tailored to scholarly aims. Peter consistently emphasized the importance of dynamic interactivity of the computing environment and human scholarly input, and the need to keep any technological solution as simple as possible. And of course the overall goal was to create resources that would be freely available online without any restrictions on access due to copyright considerations.

During the year that I spent at the MPIWG we succeeded in incorporating the results of the Perseus morphological analysis software and online dictionaries into these working environments. But despite the ingenuity of Peter's FileMaker prototypes, it was clear to all of us involved in the project that Archimedes demanded more powerful and robust tools. One of Peter's early decisions had been that the project texts would be tagged using the XML markup language, which was still relatively new at that time. This was exactly the right call, given that XML is now the de facto standard for text markup; at the time, however, there was very little user-friendly software available for working on XML texts, so the decision to use XML was something of a leap into the unknown. By the end of 1999, when I returned to Harvard as an assistant professor of Classics, we had in hand the first set of digitized texts making up the Archimedes corpus: a collection of early modern writings on mechanics in Latin and Italian by authors such as Guidobaldo del Monte, Tartaglia, and Galileo. Correcting and tagging these in a simple XML format took up the lion's share of the project's efforts in the subsequent year. In 2000 we also secured three years of funding for Archimedes from the Deutsche Forschungsgemeinschaft and the National Science Foundation; in 2001 Malcolm Hyman, a brilliant young linguist and Classicist, joined the project as a postdoc. At this point Malcom convinced me that it was time to bite the bullet and create new software from scratch that would address the basic technical challenges of making it possible to work with XML texts in the ways required by the Archimedes Project. Thus Arboreal came to be.

I will now describe the basic features of this software in the form that it reached during the Archimedes Project; for convenience we may think of this as "Arboreal 1.0."[1] Arboreal is conceived on the analogy of a traditional web browser, albeit one that works on XML rather than HTML texts. But one immediate difference is that when the user opens an XML document in Arboreal, instead of a single window we see two panes, one depicting the XML tree structure on the left and another content pane on the right. The user can navigate through the document by selecting nodes in the tree pane. As elements in the tree are selected in the tree pane they are rendered in the content pane. Arboreal allows for very complex XML structures (e.g. a text with many deletions and supplements indicated in the markup) to be rendered in whatever way is suitable for the application in question; the display of XML tags can be toggled on or off at will. Thus the XML markup is brought to life in a way that makes it accessible to the user, while the underlying format of the document is unchanged. Clicking on any word in the content pane reveals a pull-down menu that provides access to morphological and dictionary information. This information is generated on the Harvard Archimedes server (http://archimedes.fas.harvard.edu), on which we have implemented a unified frontend to various backend morphological analyzers (http://archimedes.fas.harvard.edu/donatus). Texts can contain as many languages as are supported by the software on the server, as long as these are tagged in the XML markup, and only a single server request is needed to generate the complete set of morphological analyses for all languages in a document.

[1] In terms of the actual numbering, the last version on which Malcolm Hyman worked was designated "Arboreal 5.16."

Searching can be carried out using lexical forms and regular expressions; in addition, search results can be used to navigate through the document tree. Arboreal allows for the annotation of individual words or groups of words as instances of terms, which can be manipulated and visualized in a special term editor. Terminology annotations can be saved as XML documents and subjected to further analysis. Arboreal also allows for the study of different texts in parallel to one another, making it possible to carry out systematic comparisons between texts and translations, and to create and edit translations. Finally, Arboreal enables the creation of XML content of various types. Arbitrary XSLT scripts can be applied to the text currently loaded in the main window; moreover, the terminology, morphology, and matching files that underlie the program's other functions are also in XML format.[2]

The development of this software resulted from years of intensive effort and collaboration between many individuals with different intellectual approaches and technical competences. It would have been impossible without the programming genius and intellectual vision of Malcolm Hyman, who had the original insight that such software was possible, the ability to bring it into being, and the determination to do so. The initial stage of development took place at Harvard University, in close collaboration with Peter Damerow, Jürgen Renn, and other members of Department I of the MPIWG. There were many trips between Boston and Berlin in those years. When Malcolm took up a position at the Institute in 2005, he was able to work even more closely with the colleagues there. During the period between 2005 and Malcolm's tragic death in 2009, Arboreal was used intensively in creating a translation of a Chinese text on mechanics at the MPIWG and for studying the terminology and deductive structures of Euclid's *Elements* (Schiefsky 2007).

While Arboreal was designed with the needs of the Archimedes Project in mind, it turned out to be a highly general tool. In fact, Arboreal embodies several key features that challenged—and continue to challenge—the basic way in which the Internet functions as a medium for the creation and representation of knowledge. First, Arboreal moves beyond browsing; it enables the creation of richly structured digital content in providing facilities for term annotation and the generation of new XML documents from existing texts. Second, it also moves beyond the search-dominated paradigm of the current Internet. Although Arboreal has powerful capabilities in this regard, searching is not conceived as an end in itself; rather, the program is designed to make search results the starting point for further

[2]For a longer description of the functionality of "Arboreal 1.0," see Schiefsky (2007). This description omits a number of functions provided by Arboreal that are not directly relevant to my argument here. In particular, I should note that the program is also designed to interact smoothly with image repositories and tools for image annotation, to enable the analysis of visual as well as textual content.

analysis. Third, Arboreal provides a robust platform for multilingual computing, and a model for providing the user with integrated access to diverse linguistic resources. Finally, Arboreal provides for a distinctive kind of interactivity, since the files it generates are themselves XML files that can be subjected to further analysis by the software itself. In the most general terms, Arboreal is a tool that contributes to the long-term project of making the Internet into what has been called an "epistemic web"—a domain in which unstructured, unanalyzed data is transformed into structured information and knowledge. In this vision, which descends from Peter Damerow's insights but represents the fruit of the entire course of development described in this history, the Internet becomes a vehicle for transmitting knowledge, understanding, and, we may hope, also wisdom.[3]

Since the formal conclusion of the Archimedes Project in 2004, a great deal has changed in the universe of the Internet. Social networks have exploded, enabling an exponential increase in user-generated content. Google Translate now provides a crude translation for all the world's main languages and offers the eventual possibility of a linguistically transparent Web. Natural language processing applications have greatly advanced beyond the stage of context-free morphological analysis that was still the state of the art at the time that Archimedes began. And there are vastly more digital corpora now available for research. The "big data" approach—using statistical methods to analyze and extract meaning from huge corpora—has enabled entirely new questions to be asked and new approaches to be taken to traditional questions.

Yet in many ways the ideal of the epistemic web seems more remote than ever. In the area of digital scholarship, researchers seem trapped in the "browsing" and "search" paradigms. The massive success of Google has contributed to a tendency to reduce analysis to searchability; the problem of giving the user informed assistance in *what to search for* has been neglected. Social networks offer the user the possibility to create content, but this tends to be limited to unstructured text or images. There is still an urgent need, then, for software that embodies the distinctive features of Arboreal as outlined above. In some ways the current environment renders the need for such software all the more acute. Recent challenges posed by the "big data" movement suggest that statistical approaches to large corpora may make traditional scholarly (and other kinds of) analysis irrelevant. Indeed it has recently been argued that big data and the associated modes of analysis are on the verge of eliminating the need for models and explanatory hypotheses in scholarship and in science (Anderson 2008).

But while there may be many ways of creating knowledge out of unstructured information, there is still a crucial place for human input—both in determining the questions that should be posed to automatic systems and in interpreting the

[3] For the concept of the epistemic web, see Hyman and Renn (2012).

results they produce. "Big data" approaches have their place in the sciences and—increasingly—in the humanities, but it is a mistake to think that they can answer all interesting questions. Informed input is needed not just to interpret results, but also to pose the questions. What is particularly lacking in the arguments in support of the "big data" approach is a sense of the power of interactivity—of the way in which automatically generated results only gain meaning from informed questions, and are shaped by them. Reflecting on tools like Arboreal can help us to see how technology can be used to foster these humanistic ends. Indeed I would argue that the potential of the technology itself will not be fully realized unless such interactivity is kept front and center in the development process.

To return to history. The tragic death of Malcolm Hyman in 2009 dealt a severe blow to Arboreal's development. With Peter Damerow's passing in 2011, we lost another of the program's original sources of inspiration. But I am delighted to announce that development of the code has recently resumed with the assistance of two French colleagues, Professor Said-Esteban Belmehdi of the Université de Lille and his graduate student Julien Razanajao. Working in close collaboration with me, Belmehdi and Razanajao have updated the code to work with contemporary versions of Java and succeeded in integrating some powerful new features.[4] In the remainder of this paper I shall describe these features, give some examples of their use, and outline some of the principal goals that remain for further work.

"Arboreal 2.0": Networks and Visualization

The principal innovation in this new and improved version of Arboreal is the ability to generate networks and to visualize them using the Gephi software library (http://gephi.org). This provides a standard format (.gexf) for encoding graphs in XML, as well as a powerful set of algorithms and rendering tools for viewing and analyzing graph data. The current version of Arboreal makes it possible to generate and render two kinds of graphs: graphs of the distribution of morphological variants of a given word across sections of a text, and semantic networks expressing the relations between different terms. I shall illustrate these with examples drawn from the history of mechanics and the texts of the Archimedes project corpus.

[4] It is a pleasure to acknowledge the programming skills and dedication of my two French collaborators, who worked tirelessly, successfully, and without any special institutional or financial support to decipher and extend Malcolm Hyman's brilliant but very complex Java code. Without their work none of the analysis I describe in the rest of this paper would have been possible, and the future of Arboreal would be in serious doubt.

Morphological Graphs

Morphological graphs, as defined above, have their principal use in the exploration of the relationships between the language and formal structure of texts. Consider the example of the very first ancient Greek text dedicated to theoretical mechanics, the *Problemata Mechanica* or *Mechanical Problems* attributed to Aristotle. This text contains a long introduction on the wondrous properties of the circle and circular motion, which in the author's view underlie the explanation of all mechanical movements. The introduction is followed by a set of 35 "problems" or questions that are posed using a standard formulation, then answered. Thus problem 1 asks why it is that larger balances are (allegedly) more accurate than smaller, and the author goes on to give an explanation of this fact in terms of circular motion. The text states that the balance is explained in terms of the circle, the lever in terms of the balance, and all other mechanical movements in terms of the lever. But in fact the author often appeals to circular motion directly rather than the lever. We can see this by considering the following graph of the occurrences of μοχλός ("lever") throughout the text (fig. 20.1). The large nodes represent the text itself (on the left) and the lemmatized form of the term (on the right); the lemmatized form is joined to its different morphological variants, which are themselves linked to sections (i.e. "problems") in the text. From this graph we can see at once that the term μοχλός has a fairly wide distribution across the text although the problems in which it does *not* occur also stand out clearly (these are the nodes arranged concentrically around the "Problemata Mechanica" root node).

A more complex example is provided by Guidobaldo del Monte's *Mechanicorum Liber* (1577), a key text in early modern mechanics that has a clear formal structure based on the Euclidean model. After a preface on the nature and importance of mechanics, Guidobaldo sets out a number of basic assumptions or postulates and goes on in six main sections to treat of the balance as well as the five "mechanical powers" familiar from Greek antiquity: the lever, the wheel and axle, the pulley, the wedge, and the screw. Each of the six sections is clearly divided into propositions, lemmas, and corollaries, which are tagged in the XML markup (just as the different "problems" are in the Aristotelian text mentioned above). If we consider the graph of the distribution of the term *gravitas* or "heaviness" (fig. 20.2), we find a strikingly high frequency in Proposition IV of Book I, "On the balance" (*De libra*; the thickness of the edge indicates frequency). Upon inspection of the text, we see that this is because of a large number of instances of the phrase "center of gravity" (*centrum gravitatis*) in this proposition. The frequency of this term is due to the fact that in Proposition IV, Guidobaldo is arguing against thinkers who claimed on the basis of the medieval theory of "positional heaviness" (*gravitas secundum situm*) that a balance displaced from

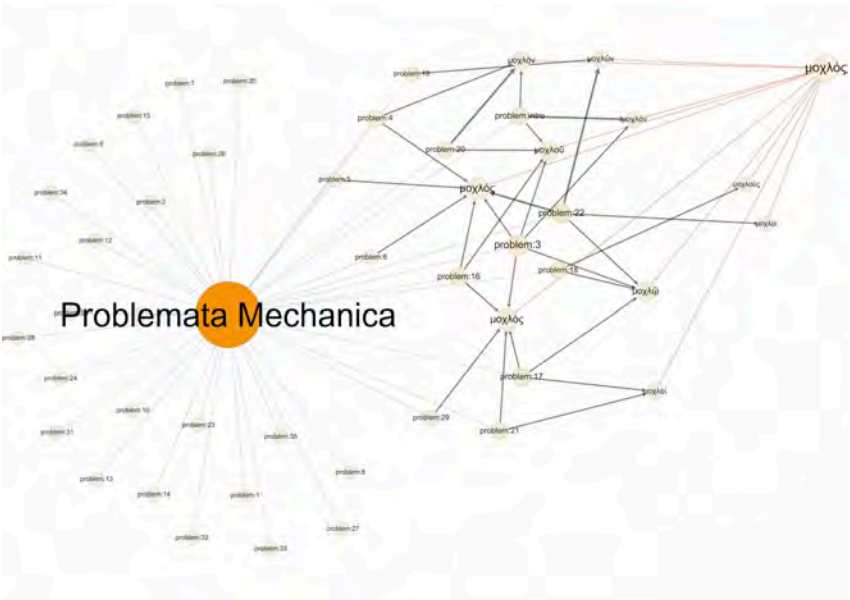

Figure 20.1: Morphological variants of μοχλός ("lever") in the *Problemata Mechanica*.

horizontal equilibrium will return to the horizontal, because the weight that is elevated has more "positional heaviness" than the one that is depressed below the original level. To this medieval theory Guidobaldo opposes the Archimedean theory of center of gravity, arguing that an object suspended from its center of gravity will not move, no matter what its orientation is to the horizontal. This "equilibrium controversy"—as Peter Damerow and Jürgen Renn have called it (Damerow and Renn 2012)—was generated by the tension between two different traditions of mechanical knowledge: one centered on the medieval concept of "positional heaviness" and another rooted in the Archimedean study of centers of gravity. The relevance of the medieval concept to Proposition IV of book I is clearly shown in the morphological graph of the distribution of *situs* (fig. 20.3). Thus, these graphs make it possible to correlate language and formal structure in a precise and visually informative way.

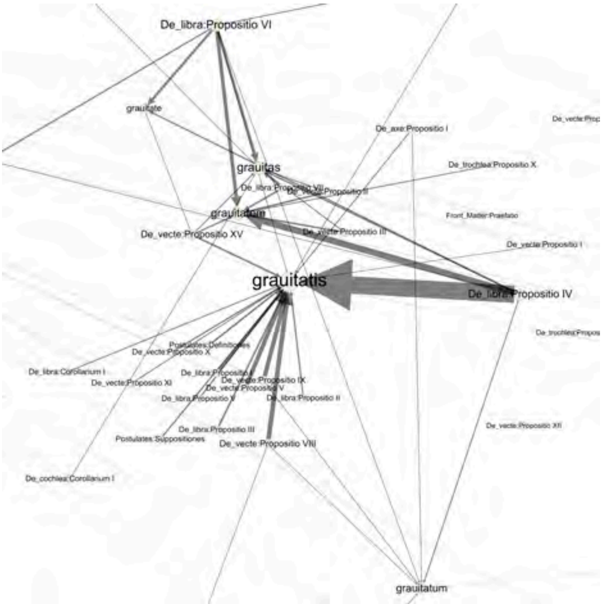

Figure 20.2: Morphological variants of *gravitas* (“heaviness”) in Guidobaldo del Monte, *Mechanicorum Liber*.

Semantic Networks

As Malcolm Hyman argued, semantic networks provide a powerful tool for studying conceptual development in the history of science (Hyman 2007). In such networks, individual terms in a text are represented as nodes in a network, and the edges linking them express the strength of their association with other terms. Very crudely speaking, terms that frequently occur in close proximity to one another have a higher association than other terms. Thus, to take an example from Hyman’s paper, “force” will have different associations in an article on theoretical physics (e.g. “potential,” “field,” “electromagnetic”) than in an article on the behavior of police towards African Americans in the American South (e.g. “violence,” “abuse”). The meaning of a term within a particular text is constituted at least in part by its relations to other terms; “force” in the sense of physical strength exerted by humans is a different concept from “force” in the context of theoretical physics, although the *term* used is the same. A semantic network is thus a kind of model of the conceptual structure of a text, in which terms serve as proxies for concepts.

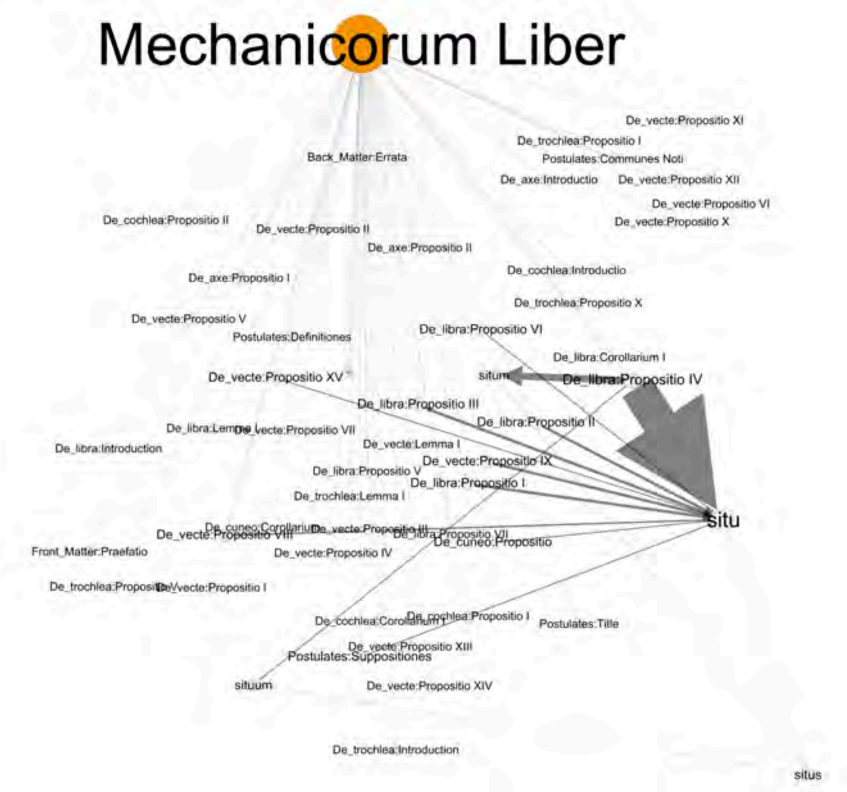

Figure 20.3: Morphological variants of *situs* ("position") in Guidobaldo del Monte, *Mechanicorum Liber*.

Such an approach is particularly useful in studying the semantic field of a concept such as *force* in texts that span a range of centuries and languages. In the early history of mechanics force is typically seen as a cause of motion; but the language of force can also be used of effects rather than causes (e.g. "a forceful blow"). There is also the distinction between physical force and a more generalized power or capacity to affect, as well as that between a force that acts instantaneously and one that persists. The Greek term δύναμις, for example, tends to refer to a general capacity or faculty to affect, rather than something that causes motion in particular; the best translation is often "power," which also expresses the tendency of a δύναμις to persist through the duration of its activity. The term ἰσχύς is glossed by Bonitz in his *Index Aristotelicus* as both “motive force” (*vis motrix*) and “bodily strength” (*robur corporis*); yet he also notes that it is typically used as a synonym for δύναμις.[5] A third Greek term, βία, has connotations of violence as well as physical strength, and is particularly important for its use to express the distinction between “natural” and “violent” motion in the Aristotelian tradition. Thus, Greek presents us with a set of terms that overlap in their meanings, though each has distinctive nuances. Similar points might be made for the Latin (*potentia*, *virtus*, and *vis*) and Italian (*forza*, *potenza/possanza*, and *virtù*) terminology on the basis of appropriate lexica. While these remarks on general usage are important, understanding the terminology of force in a particular text requires taking account of the way in which the term is actually used. It is this that the semantic network method enables us to investigate in a precise, rigorous and repeatable manner. In this perspective, the meaning of a term in a text is constituted by its place in the semantic space—a web of connections that model the text’s conceptual structure.

For the generation of semantic networks the current version of Arboreal implements the Semantic Vectors package released on GitHub (https://github.com/semanticvectors/semanticvectors/wiki). I will simply sketch the general idea here, referring to the online documentation for the details of this particular implementation. The basic idea is to represent the distribution of terms in a document via a term-document matrix. Thus, for example, the rows of the matrix may correspond to segments of text while columns correspond to particular terms; in this simple model, the value of any element of the matrix (r, c) is the number of occurrences of term c in segment r. Once such a matrix is constructed we can apply statistical methods and linear algebra techniques to derive measures for the similarity between different rows (comparing segments to segments) or columns (comparing terms to terms). A key step is dimensionality reduction, in which transformations such as singular value decomposition are used to reduce the size of the matrix; the reduced matrix is then interpreted as a representation of the document in semantic space. The net effect of this is to eliminate the “noise” caused by phenomena such as synonymy: if terms A and B are both found regularly in conjunction with the same cluster of terms C, A and B will end up very close to one another in the semantic space. Another way of looking at this is that A and B express the same *concept* within the semantic space, which is thus a model of the text’s conceptual structure. Once the semantic space has been constructed, we determine the association of terms to one another using standard metrics such as cosine similarity. These associations become the labels of the edges linking different nodes in the graph.

In analyzing a particular text, we begin by reducing morphological variants to lexical forms (thus English *is* and *was* → *be*, Latin *vires* and *vi* → *vis*) using the

[5] See Bonitz s.v. ἰσχύς, available at http://archimedes.fas.harvard.edu/pollux.

web services of the Harvard Archimedes server (http://archimedes.fas.harvard.edu) or other methods.[6] We then use Arboreal to build the semantic vectors for this reduced document and perform a pairwise comparison to determine the association that each term has with every other. (This step can take a significant amount of time for larger documents. It is also possible to compare a subset of terms T with all the terms N in the document.) The user is given the option to specify parameters used by the Semantic Vectors package, including especially the length of the segments into which the text is divided (shorter segments will result in fewer associates). Once the graph has been generated, the Gephi package provides many different algorithms for rendering the graph data in perspicuous form; these can be performed in any sequence that the user desires. Additionally, threshold values can be specified to restrict the range of associations that are rendered; we find, for example, that rendering all edges that have a score of 0.65 or above is sufficient both to eliminate noise and to reveal interesting structural features. After the graph is rendered the user can easily select subgraphs by clicking on particular nodes. Arboreal thus provides a highly interactive implementation of the Gephi algorithms that is in some ways more powerful than the Gephi application itself. Finally, I note that Arboreal can export graphs as XML files (using the .gexf format) which can then be analyzed using a range of graph-theoretic analytical tools.

Let me now turn to some examples drawn from the history of mechanics. If we apply this method to the *Problemata Mechanica*, the result is a set of groups divided into an outer ring and inner clusters (see fig. 20.4).

At the center of the largest cluster is found the term κινέω, "to move" and its close associates such as βάρος "weight," ῥᾴδιος "easy/ easily," and κίνησις "motion" (see fig. 20.5).

In this graph the larger nodes have higher *degree*, where the degree of a node is defined as the number of edges connecting it to other nodes. The rendering algorithm has driven nodes of high degree to the center. Indeed, the terms that appear as central here are the terms with the highest degree of all nodes in the graph: κινέω has degree 64, βάρος 50, ῥᾴδιος 42, and κίνησις 41, where the average degree is 3.195.[7] Now κινέω is of course a key term in the *Problemata,* which is very much concerned with the issue of moving weights (βάρος) by the use of a force (δύναμις or ἰσχύς). Indeed the author in the introduction almost immediately raises the general question why small forces can move great weights (implying that the normal or natural course of events is for a force to move a

[6]Among the most useful other tools are the Tree Tagger developed by Helmut Schmid at the Institute for Computational Linguistics of the University of Stuttgart (http://www.cis.uni-muenchen.de/~schmid/tools/TreeTagger/), which provides lemmatization and part-of-speech tagging for Latin, Italian, and other languages in the Archimedes corpus.

[7]Results according to the Gephi application.

Figure 20.4: Semantic network of the *Problemata Mechanica*.

weight that is equal to it). Moreover, the text's fundamental explanatory principle involves circular motion, and in particular the fact that the movement of a point farther from the center of a circle is quicker than one that is closer to it, assuming the two points lie along the same radius. Hence the presence of κύκλος "circle" and μέγας/μικρός "large" and "small" in this graph. Clearly the semantic analysis is capturing essential aspects of the text's conceptual content; nodes with high degree correspond to terms that are especially significant in some way. With some knowledge of the content of the text, we can explain this significance and supply meaning to the edges that goes beyond a simple numerical score.

We can study the associates of κινέω in more detail by selecting the node and rendering its associates; for this purpose it is convenient to set the lower limit of edge strength to 0 to bring out all the associations (fig. 20.6). We find among

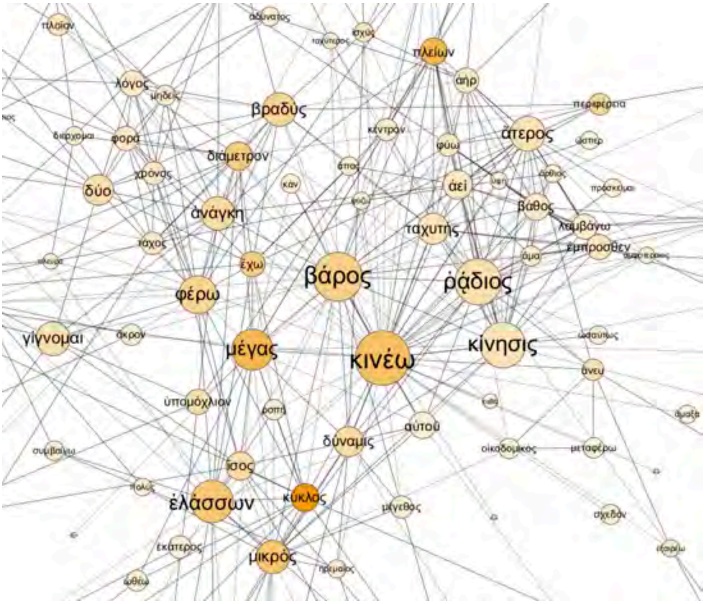

Figure 20.5: Semantic network of the *Problemata Mechanica* (central section).

the associates of κινέω not only ἰσχύς and δύναμις but also the term ῥοπή, which refers to the "inclination" or "swing" of the balance and to something (a small weight) that produces that inclination. We see that μέγας ("large") is associated with terms for force—δύναμις and ῥοπή—as well as βάρος ("weight"). For this author, mechanical movements are generated and explained by the quantitative relationships between force and weight.

The *Problemata* was a seminal text in early modern mechanics thanks to works such as the 1525 Latin translation by Niccolò Leonico Tomeo and the 1547 Latin *Paraphrasis* by Alessandro Piccolomini.[8] Examining the central clusters of the semantic networks of these texts reveals the same basic picture that we find in the Greek, with *moveo/muovere* "to move" at the center with a close linkage to *circulus/circulo* "circle" and *pondus/peso* "weight" (fig. 20.7; fig. 20.8). The graph suggests that Tomeo uses *potentia* to denote the force that causes the weight to move; in Piccolomini, however, the corresponding term is *forza*. The similarity of the place of these terms in their respective graphs suggests that the

[8] For the present analysis I have used the 1582 Italian translation of the latter work. See Drake and Rose (1971) for the basic bibliographical and biographical information pertinent to this literature.

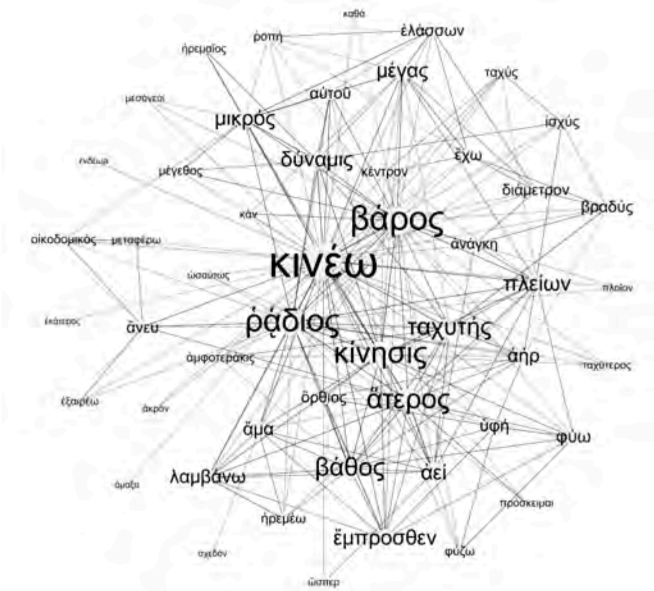

Figure 20.6: Semantic network of κινέω ("to move") in the *Problemata Mechanica*.

terminological difference is simply a matter of stylistic preference, and that the two terms express the same concept. Closer analysis of the texts is needed to confirm this hypothesis, but it is a strength of the present method that it points the reader toward such investigation.

In contrast to the concern with movement in the *Problemata* tradition, an important strand in sixteenth-century mechanics focused on problems of static equilibrium and the determination of center of gravity rather than motion. A key example is again Guidobaldo's *Mechanicorum Liber*.[9] Figure 20.9 shows the central section of the semantic network for this text.

The focus on motion has been replaced by an extremely close association between *potentia* ("power"), *pondus* ("weight"), and *sustineo* ("sustain"). Guidobaldo's text is concerned with determining the power (*potentia*) that will sustain a weight—as one weight balances another on the balance. Of course Guidobaldo also writes of moving weights; indeed he echoes the traditional formulation of the basic problem of mechanics dating back to the ancient Greco-Roman world: "to move a given weight by a given power." But his

[9] See Damerow and Renn (2010).

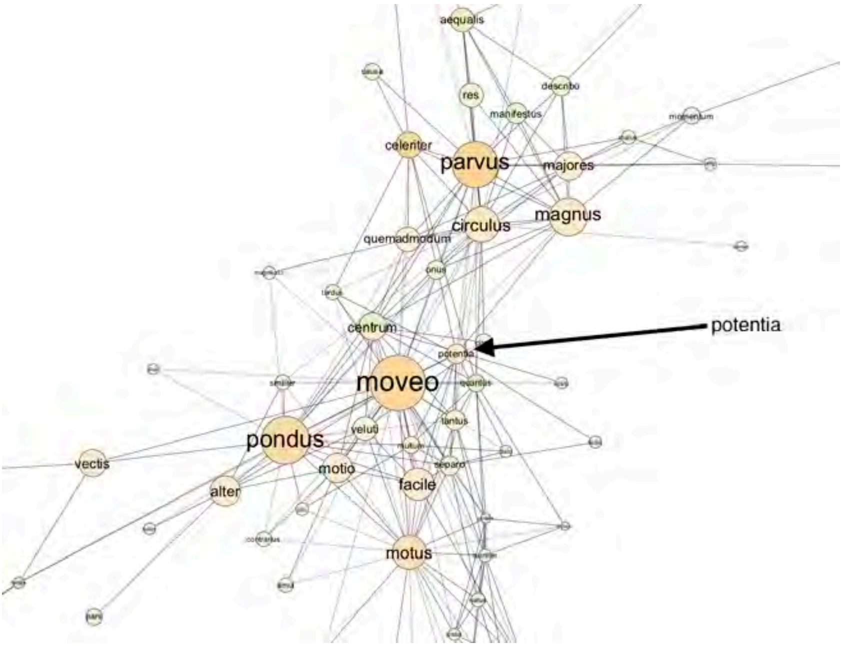

Figure 20.7: Semantic network of Tomeo's Latin translation of the *Problemata Mechanica* (central section).

approach is to determine the power that sustains the weight, on the assumption that movement will ensue if that power is increased. For him, *potentia* denotes a "force" that corresponds to "weight," and does not involve considerations of movement.

The following remark by Filippo Pigafetta from his 1581 Italian translation of Guidobaldo's text brings out the latter's concern with the "sustaining power." It follows proposition 5 in the book on the pulley:

> In this proposition it is shown reasonably that, for two pulleys and one rope, the force (*forza*) will be one-third of the weight […]. Somebody might consider this very dubious, because the pulleys and their attachments, the ropes, and so on offer resistance to the force (*forza*), and also have weight of their own, so that the [calculated] force may not be able to sustain (*sostenere*) the weight. We reply that these things may well offer resistance to the moving of the weight, but not to the sustaining of it; and it is necessary to note carefully that the au-

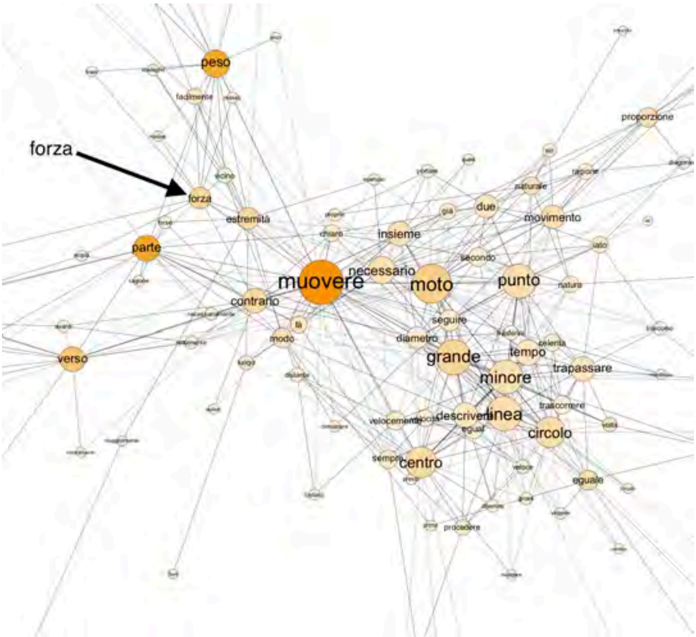

Figure 20.8: Semantic network of the 1582 Italian translation of Piccolomini's *Paraphrasis* of the *Problemata Mechanica* (central section).

> thor in these demonstrations speaks only of forces (*forze*) sustaining the weights so that they do not fall down; not about moving them.[10]

We may also note that the connection between *gravitas* and *situs* (noted above) appears in the semantic network of the term *gravis*, shown in fig. 20.10 (lower left). Here the presence of terms expressing the ideas of "ascent" and "descent" (*ascensus/ascendo*, *descensus/descendo*) also suggests Guidobaldo's engagement with the medieval theory of "positional heaviness," insofar as the texts espousing

[10] Translation Drake (in Drake and Drabkin 1969, 308). Original, full text (Pigafetta 1581, 64r): "In questa propositione si narra, che rauolgendo d'intorno à due girelle di due taglie vna corda, & quel che segue, la forza sarà vn terzo del peso, cioè se il peso sarà trecento, egli verrà sostenuto dalla possanza di cento. Direbbe alcuno ciò essere dubbioso, peroche le girelle, gli assetti suoi, le funi, & il peso della taglia di sotto fanno resistenza alla forza, & grauano sì, che ella non potrà sostenere il peso. Si risponde che queste cose ben farebbono resistenza nel mouere il peso, ma non già nel sostentarlo: & bisogna notare con diligenza che l'autore in queste dimostrationi parla sempre del sostenere solamente con le forze i pesi che non calino al basso, non del mouere."

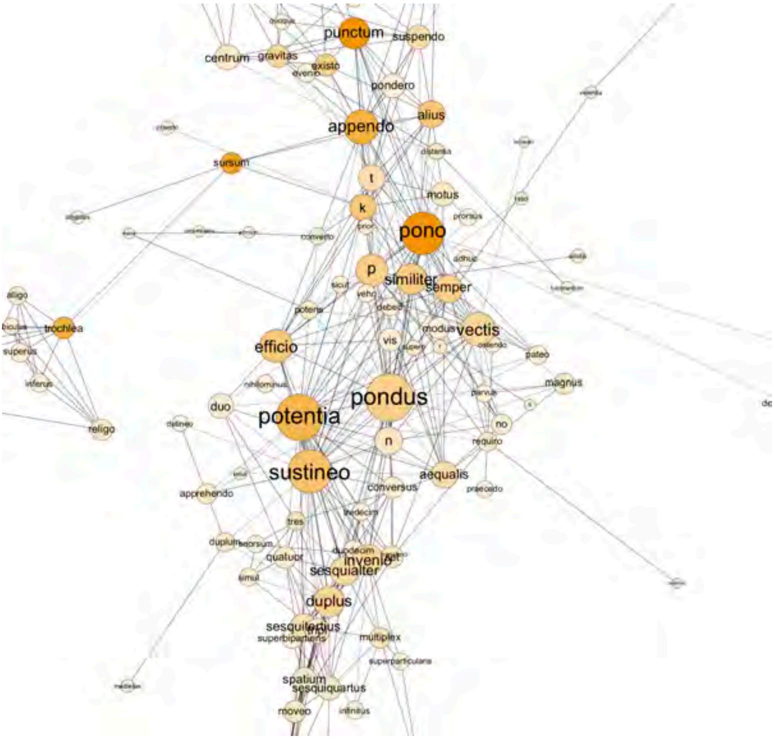

Figure 20.9: Semantic network of Guidobaldo del Monte, *Mechanicorum Liber* (central section).

this theory specified that a weight is heavier by position if its course of descent is less oblique.

I conclude this set of examples with some very brief remarks on Galileo's terminology for force. It is of course extensive and varied, and includes not only *forza*, *potenza*, and *virtù* but also other terms with a long history such as *impeto* ("impetus") and *momento* ("moment/momentum"). To illustrate the power of the semantic network approach for the analysis of his usage, we may consider two graphs generated from Galileo's *Discorsi* (1638). In fact they are components of a single graph, created by generating a semantic network for the text as a whole, then selecting the nodes for *possanza*, *potenza*, *forza*, and *virtù*. Setting a low cutoff value of 0.3, the result is two discontinuous graphs, one for the first three of these terms (fig. 20.11) and another for the last (fig. 20.12). Inspection reveals

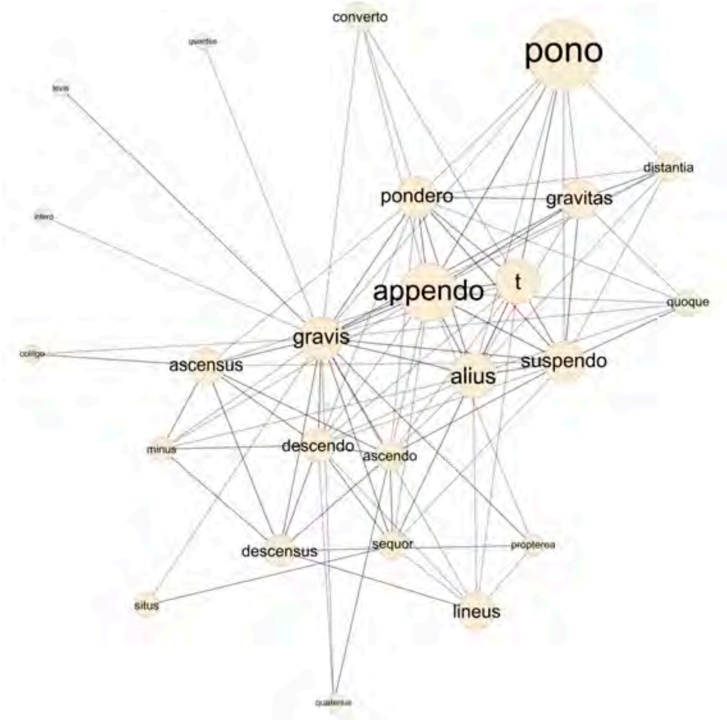

Figure 20.10: Semantic network of *gravis* (“heavy”) in Guidobaldo del Monte, *Mechanicorum Liber*.

that *virtù* is much more closely associated with the cluster of terms connected with velocity and motion, while the other terms are associated with mechanical devices such as the lever and screw (*forza*) and with problems of infinite division (*potenza*). Again these results are highly suggestive and call for further investigation by close reading of the texts.

While Arboreal’s current implementation of semantic analysis is provisional and needs substantial further work, these results serve as a proof of concept, showing that such an approach can capture some of the conceptual structure of scientific texts and contribute to the study of long-term intellectual developments. In each case we see that the meaning of the networks is apparent only within the framework of certain scholarly questions; it takes some knowledge of the texts to interpret these graphs, and conversely, they point the way to further topics of

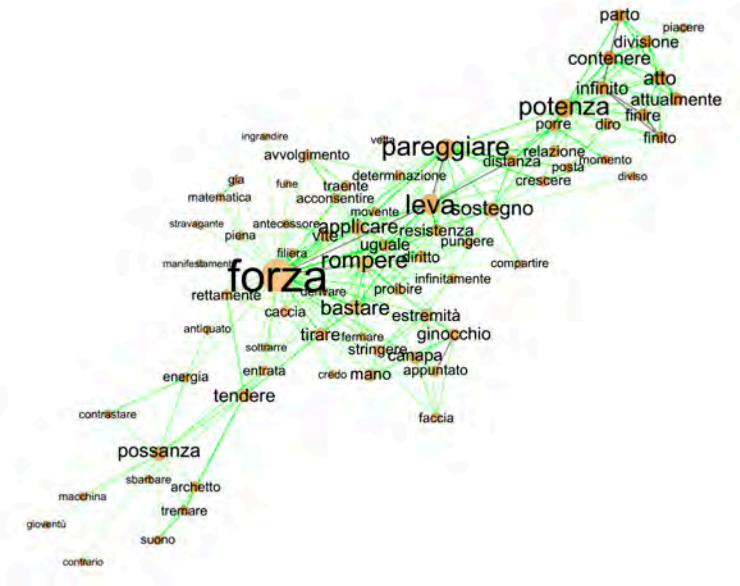

Figure 20.11: Semantic network of *potenza*, *forza*, and *possanza* in Galileo's *Discorsi*.

investigation. Considered in themselves, the semantic networks serve as a sort of "fingerprint" of the text in question, reflecting its place in the long-term development of mechanical knowledge. Moreover, this method offers the possibility of a truly multilingual approach to the history of conceptual development in science. Finally, because the method can be applied to any XML text in a language for which the necessary technology exists, it is highly general and can be used in any discipline concerned with the linguistic expression and conceptual content of textual sources. From the cuneiform archives of the third millennium BCE to the writings of Einstein, this technique has the potential to illuminate the development of human thought and to enhance our exploration of it.

Conclusions and Future Perspectives

We have much work still to do in order to complete the implementation of semantic analysis along the lines described above. One issue is that the quality of the morphological data is uneven for the various languages currently supported by our software. Insofar as the software is unable to analyze certain words in the text

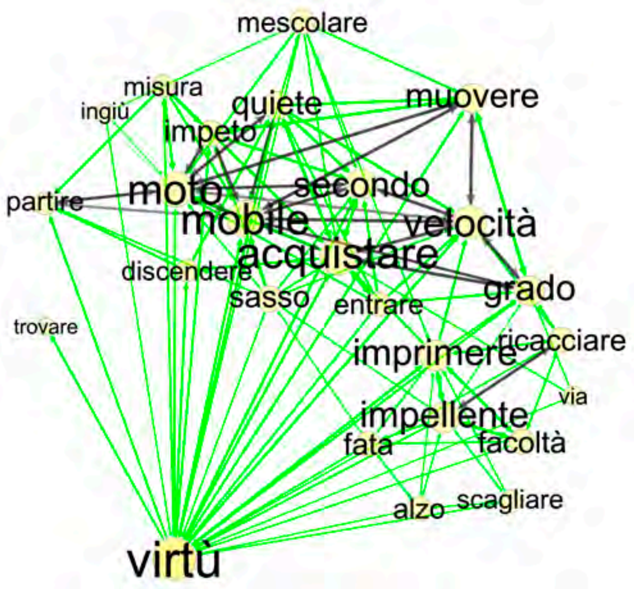

Figure 20.12: Semantic network of *virtù* in Galileo's *Discorsi*.

or refers them to multiple lemmas, the accuracy of the semantic networks that are generated is diminished. Making use of a context-sensitive part-of-speech (POS) tagger can help to avoid these problems. Yet we still have no POS tagger for ancient Greek, despite the fact that the quality of context-independent morphological analysis is very high. A related issue is that the current approach does not allow for the semantic indexing of multi-word terms such as *centrum gravitatis* or for visualizing the distribution of such terms across a text. This is so despite the fact that Arboreal provides powerful functionality for extracting and tagging such terms. We will remedy this deficiency in the near future. It would also be highly desirable to be able to navigate back into the text by clicking on nodes in a graph, which should be a straightforward function to implement.

Two more fundamental challenges remain if the full potential of Arboreal is to be realized. First, the maintenance and stability of the code base requires constant attention. Although the choice to implement Arboreal as a Java application enabled us to avoid many server-side maintenance issues, the current version consists of approximately 30,000 lines of code, and it is a significant challenge to ensure that it conforms to the latest version of Java and works on all common platforms. For the foreseeable future the latest version of the code will be available on the Harvard Archimedes server (http://archimedes.fas.harvard.edu/arboreal).

There is no doubt that the complexity of Arboreal has been an impediment to its adoption by the scholarly community. Indeed, the most successful examples of its use have been in contexts where one or more of the developers themselves were available for consultation. While the history of Arboreal's development demonstrates the importance of close collaboration, it also points to the need for better documentation and communication. A software package needs to be self-sustaining if it is to be broadly adopted. There is therefore an urgent need to simplify the software where possible and to provide better documentation of different usage scenarios. We will prioritize these tasks in the near future.

In closing, I believe that the history of Arboreal gives good reason to be optimistic about its future. Arboreal has survived many different versions of Java and many operating system updates and is still a going concern, due to the dedicated efforts of many people in various institutions. I believe that a major reason for its continued vitality lies in the way in which it embodies Peter Damerow's compelling vision of the role of information technology in the humanities. In this vision the power of computational techniques is harnessed as far as possible, but they are not treated as ends in themselves; the goal of software design is to enable researchers to engage interactively with technology and with the sources that they study; and the goals of simplicity and open access are of supreme importance. Although the challenges that remain are as as great if not greater than ever, there is every reason to believe that this vision will continue to be relevant for the foreseeable future, and that with further work we will come even closer to realizing it.

Acknowledgments

I would like to express my deep gratitude to all who have been involved in the development of Arboreal over the years; to Jürgen Renn for inviting me to Berlin in 1998 and for his steadfast support ever since; and to Matthias Schemmel for his friendship and patience as I worked to complete this paper. I am honored to be able to offer it as a small tribute to Peter's memory.

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Chapter 21
A Computational Research System for the History of Science

Julia Damerow, Erick Peirson, and Manfred D. Laubichler

Introduction

Peter Damerow focused on big data projects, digital collections, and computational tools long before digital humanities and big data became popular concepts. His pioneering efforts in these nascent fields were a logical consequence of his driving research questions: How can we understand the longue durée patterns in the history of knowledge? How can we understand the transitions from nomadic hunters and gatherers to early civilizations and what role did organized and abstract knowledge play in these transitions? What was the origin of writing and mathematics? What parts of the cognitive make-up of humans and early societies facilitated these transitions?

Clearly these are "big questions." Answering them is by necessity a collective endeavor. And, as everybody who has organized an even remotely similar research project knows, sharing information, data, interpretations, being able to work collaboratively, and being able to connect evidence from different fields is absolutely crucial. But this does not happen by itself. It was Peter's early vision to build the necessary infrastructure—from the earliest digital databases in the history of science to the most advanced open access publication platforms in our field.

In addition to sharing data and scholarly interpretations, Peter also had an active interest in and promoted the development of computational tools. For him, as a historian of early writing and mathematics, the possibilities of new algorithms that could analyze patterns of historical change or of new graph-based representations of knowledge were further steps in a process of knowledge acquisition that began deep in our evolutionary past. It is with a sense of deep gratitude and appreciation that we present a brief overview of some of the computational tools that we developed in order to analyze the complex patterns and processes within the history of knowledge. We are, in a fitting case of historical continuity, Peter's daughter Julia, a computer scientist and, as of this summer (2014), a freshly minted PhD in computational history and philosophy of science; Erick Peirson, Julia's congenial partner in the development of the computational research system introduced here, and a biologist and historian of science; and MDL, who is as proud of his academic children as Peter was of his daughter.

In Lunenfeld (2012), the authors observe that in digital humanities "[…] what we are seeing is the emergence of new conjunctions between the macro and the micro, general surface trends and deep hermeneutic inquiry, the global view from above and the local view on the ground" (Lunenfeld et al. 2012, 39). In contrast to close reading and careful studying of individual sources (the micro-scale), which are key methods in the humanities, distant reading[1] in digital humanities employs computational methods to analyze large text corpora in order to find overall patterns, trends, or connections (the macro-scale) (Lunenfeld et al. 2012). Lunenfeld et al. see "zooming in and out" between distant and close reading as a powerful tool of digital humanists. Müller calls this process "scalable reading," comparing it to the zoom function in Google Earth (Müller 2012). He states that scalable reading enables scholars to easily switch between the details of a text and its context (Müller 2012). Computers can support researchers by making vast amounts of data such as texts or images accessible through automatic extraction, analysis, and visualization of information. They can provide scholars with new tools that might help discover unknown relationships or patterns. However, they cannot replace the careful interpretation and examination of individual sources by a scholar.

This paper describes a research system called "Quadriga System" that is based on the idea of representing texts as networks of concepts that can be mathematically analyzed and visualized. These networks are created by scholars through close reading and structured annotation of texts. However, the Quadriga System follows a collaborative approach that facilitates the creation of a large-scale data repository to enable data-driven research in the history and philosophy of science. The system can therefore be placed in between the micro- and the macro-level of source analysis, on the so-called meso-level (or meso-scale). It is designed to help researchers detecting patterns and relationships of interest in their sources by transforming the materials into structured datasets on the micro-level and analyzing them on the macro-level. The Quadriga System follows a similar approach to projects in the field of bioinformatics such as GenBank that rely on different contributors from around the world to submit new entries to the database (Benson et al. 2010). The data structure underlying the Quadriga System (called *Quadruples*) enables scholars to seamlessly switch

[1] A term coined by Franco Moretti (2009). At that point Moretti used the term "distant reading" in the context of world literature and did not focus on computational methods to automatically extract information. However, the basic idea is the same: "[d]istant reading […] allows you to focus on units that are much smaller or much larger than the text" (Moretti 2009, 57).

back and forth between a single text and a whole corpus, facilitating scalable reading.

In this paper, we will first briefly describe two projects that use the Quadriga System: the Genecology Project and the EP Annotation Project. We will then detail the system's architecture and its different components. The last section will discuss how the history of science might benefit from using the Quadriga System.

The Genecology Project

The Genecology Project[2] studies genecology research in Great Britain during the twentieth century. Genecology is a branch of ecology that studies how genetic differences in plant populations relate to "geospatial variation in environmental factors (e.g. soils, altitude, climate)" (Peirson, Damerow, and Laubichler forthcoming, 3). The project analyzes how the conceptual change that occurred in genecology research was influenced by contributing researchers and their interactions and collaborations with a focus on one particular researcher: Tony Bradshaw. It also asks how ideas and theories in the field spread, and how they changed. In its first phase, the Genecology Project is therefore especially interested in identifying the main actors contributing to genecology research and who collaborated with Bradshaw, and how the patterns of collaboration among the researchers changed over time. To answer this question the project concentrates on constructing a social network from interactions, collaborations, and the institutional contexts of genecology researchers.

The Genecology Project follows a text-driven approach that is not simply based on biographical information, but also relies on acknowledgment sections of publications or other textual evidence demonstrating collaborative efforts, such as co-authorship. Texts were selected based on an initial list of papers published in 1964 that provides an overview of genecology research at that time. In the first stage, all papers from that list were digitized and annotated. In a second step, publications cited by the listed papers or other manuscripts by listed authors and co-authors were analyzed as well.

The selected texts were annotated with a set of predefined relationships using a software application called *Vogon*. Vogon allows a researcher to create a certain kind of annotation that points to the position of a word in a text and a so-called "concept" that specifies what a word refers to. Those annotations can then be put in relation to one another. For example, if a text states that a person helped the author with a certain task, the author of the text as well as the person helping

[2] See http://devo-evo.lab.asu.edu/?q=genecology-project.

him will be annotated with concepts representing the two people. The two resulting annotations will then be connected by an "engages with" relationship or any other well-defined relationship the annotator choses. Similarly, the relationship between a researcher and his affiliated institution is expressed by an "employs" relation between two annotations representing the institution and the researcher. Several annotations of this format create a network of "concepts," which in the case of the Genecology Project is a social network of persons and institutions.

Such a network can be exported from Vogon in a standard graph format such as XGMML to be visualized in a network visualization application (such as Cytoscape),[3] or if geographical information is attached to the nodes of a network, it can be plotted on a map (see fig. 21.1). A visualization as shown in figure 21.1 facilitates quick processing of the displayed data by a viewer and can reveal information that otherwise might stay hidden (Mazza 2009).

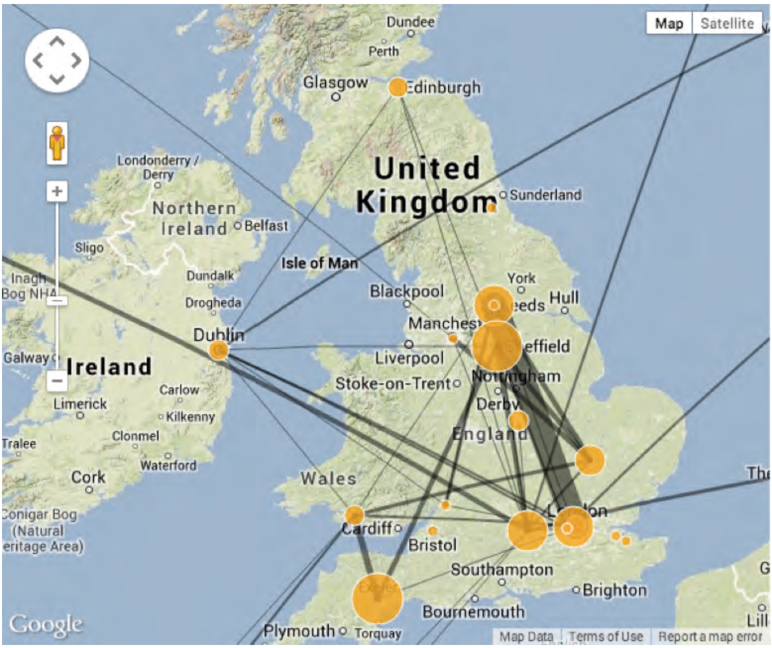

Figure 21.1: Social Network created by the Genecology Project plotted on a map.

[3]See http://www.cytoscape.org/ and Shannon (2003).

The EP Annotation Project

The goal of the EP Annotation Project is to annotate articles written for the Embryo Project with relationships that reflect how the entities described in the articles relate to each other. The Embryo Project is an online encyclopedia of embryology that aims to document embryo research in the broadest possible way (Laubichler and Maienschein 2009). Articles in the Embryo Projects "are written and marked up in such a way that they help populating the database with additional objects that have interesting and relevant relationships to the object of the entry" (Laubichler and Maienschein 2009, 11). For example, there exists an article about Hans Spemann that mentions that Spemann worked with Theodor Boveri and Wilhelm Röntgen. In the marked-up article this information is turned into annotations that represent "worked with" relationships between Spemann and Boveri and Spemann and Röntgen. However, while an entry for Boveri exists, there is no article about Röntgen. By creating relationships between Röntgen and other entities such as Spemann, information about Röntgen is stored and available for use although no article has been written yet. One motivation for creating and storing such relationships is to be able to easily answer questions such as "Who was a student of whom?" or "Who worked at a particular place? With what particular organisms?" (Laubichler and Maienschein 2009, 9).

As the EP Annotation Project is an exploratory project, it so far has been undertaken as a proof of concept project. There are about 50 articles that were annotated using Vogon; all of them describe specific persons (e.g., Hans Spemann or Viktor Hamburger) rather than institutions or organisms. For each article about 10 to 20 relationships were created, capturing information such as who was a teacher of whom, who worked with what organism, or what kind of relationship existed between a person and institution.

For the visualization of the annotations created with Vogon, annotations were transformed into graphs in which every node represents a concept of interest (for example a person or organism), and edges represent relationships between those concepts (i.e., "contributed to" or "used"). Figure 21.2 shows a network of people and organisms, techniques, or theories those people worked on. Such a network could be used to explore the articles in the Embryo Project by browsing through the concepts (represented by nodes) and their relationships to each other (edges). As Vogon allows text positions to be stored with annotations, a person using the network could jump directly to the texts that mention a specific relationship between, for example, a person and an organism. Moreover, when time information is added to the annotations, the networks resulting from the annotations could allow a user of the Embryo Project Encyclopedia to explore its content filtered by time or place, or to create timelines to visualize, for instance, who worked on a concept or theory over time.

The Quadriga System

The Quadriga System is based on the idea to represent texts as graphs. By representing unstructured texts as graphs, the information contained in a text is given a mathematical structure that can be used for computational analysis. The basic components of these graphs are so-called "Quadruples," (see fig. 21.3) also referred to as "contextualized triples." The basic idea is similar to a concept proposed in Macgregor and Ko (2003). Macgregor and Ko describe quads (a four-tuple consisting of subject, predicate, object, and context) in which a context can itself be part of a set of assertions that define the "environment" of that context.

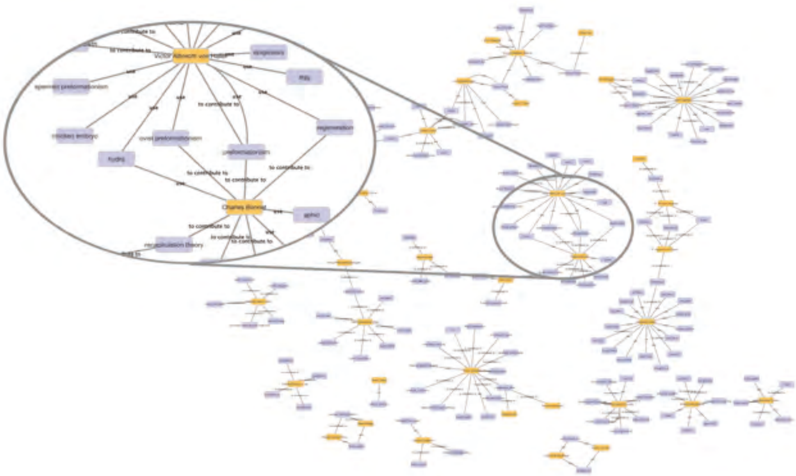

Figure 21.2: Network of people and theories, organisms, and techniques created from 35 Embryo Project articles.

Statements made in such a context are considered to be true in the environment of the context. However, Macgregor and Ko do not define a structure for environments. In the case of Quadruples in the Quadriga System, the context is well-defined. It consists of three parts: the metadata of a resource (such as publication date or author), the annotation context (such as the creator of the annotations of a text), and the creation context (such as when annotations were uploaded to a shared repository).

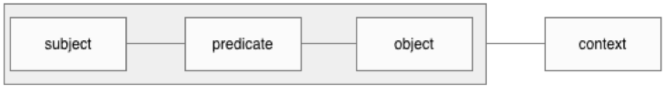

Figure 21.3: Structure of a Quadruple.

The Quadriga System has several components that support the creation of Quadruple networks and their distribution with the following workflow. A researcher annotates each text of interest with a graph that represents their interpretation of that text. Such a graph consists of relationships between concepts that the researcher created according to the relevant statements of a text. Relevant information is in this context the information that the researcher classifies as being relevant. Next, additional information such as metadata of the text is attached to the graph. The researcher then uploads his graphs to a common repository. This repository holds graphs from several researchers working on possibly different projects. Once his graphs are uploaded, the Quadriga System enables the researcher to analyze them, incorporating or excluding specific graphs created by other researchers and projects.

The Quadriga System consists of several independent components that interact with each other (see fig. 21.4). Each component has specific responsibilities. A user might directly interact with all components or with only a few depending on his role in a project. The component that users will likely interact with the most is Vogon. Vogon is a desktop application that enables users to annotate texts. This can be done with a text-based editor, in which users highlight the terms that they want to annotate, or using a graphical editor that lets users build a graph diagrammatically and then connect each node in the graph to the text.

Several annotations together form graphs. When a user has finished annotating a text, those graphs can be submitted to *Quadriga*, a network repository. Quadriga is the central component of the Quadriga System. It is a web application that provides functionality to review, annotate, store, and publish graphs consisting of Quadruples.

A basic element in the Quadriga System are texts: networks are created for texts, annotations link to positions in texts, and Quadriga manages graphs by associating them with specific texts. To use the Quadriga System to its full potential, documents that are being annotated using Vogon should be available to the whole system. This would allow, for example, visualization websites of annota-

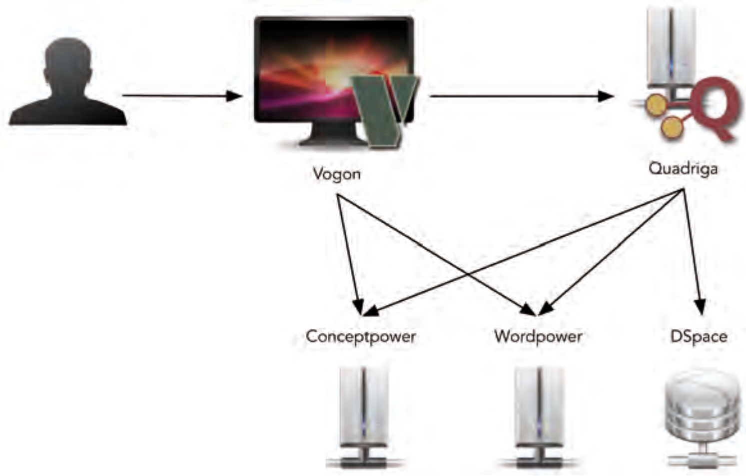

Figure 21.4: Components of the Quadriga System.

tion graphs to display the part of a text for which an annotation was created. In the Quadriga System, texts are therefore made available through a DSpace repository.[4]

The last two components in the Quadriga System are an online authority file service called *Conceptpower* and an online dictionary service called *Wordpower*. Both services are web applications. Quadriga as well as Vogon interact with these services through a web API (Application Programming Interface).[5] In contrast, human users interact with Conceptpower and Wordpower through a website using a web browser.

Conceptpower is the authority file system used in the Quadriga System. Each entry in Conceptpower represents a concept and is identified by a URI. Given such a URI, an application can retrieve a concept's properties, such as its type or the contents of the equals field. If a concept is missing in Conceptpower, a user can create a new entry in Conceptpower for the missing concept. Wordpower has many similarities with Conceptpower. As in Conceptpower, every entry in Wordpower is identified by a URI and, given the URI, other software applications can request information about a Wordpower entry. The biggest difference

[4] See http://www.dspace.org/.

[5] An API is "a way for two computer applications to talk to each other over a network (predominantly the Internet) using a common language that they both understand" (Jacobson, Woods, and Brail 2011, 5).

between the two services is that in Conceptpower each entry represents a specific meaning of a term. Even if two terms are the same, if they have different meanings there will be different entries in Conceptpower. In contrast, in Wordpower there is only one entry for a term and that entry specifies the normalized or correct spelling of a word.[6]

A typical annotation process using the Quadriga System looks like the following. A researcher starts by adding all texts he wants to annotate to Vogon. The user creates annotations for the texts and relates them to each other. During that step he queries Conceptpower and Wordpower for the terms and concepts he uses in his annotations. He also creates new entries in these two services if terms or concepts are missing. Once the researcher has finished annotating a text, he submits the annotation graphs to Quadriga for validation, publication, and visualization.

Conclusion

Quadruples, which are the underlying data structure of the Quadriga System, are contextualized triples of the form <subject - predicate - object - context>. Quadruples, in contrast to triples, store contextual information about a subject, predicate, object statement. Such contextual information contain, for instance, what text was annotated or who annotated a text. With this kind of data, it is possible to "zoom in and out" from the macro-level to the micro-level to allow scalable reading.

The Quadriga System operates on the meso-level between distance and close reading. Texts are annotated through close reading and examination of terms. However, all annotations are stored in a common repository, creating a large-scale dataset of annotation data (networks of Quadruples), which is available to other scholars. This dataset facilitates distant reading, which could assist in finding patterns and trends in the annotated corpus. However, distant reading in the context of the Quadriga System is limited by the number of annotated texts and the annotations created for those texts. Also, the annotation process itself is time-consuming and is likely to be restricted to a few texts of interest. It therefore might be practicable to use other distant reading techniques such as topic modeling on large text corpora to identify sub-corpora of interest for ingestion into the Quadriga System.

Compared to many large-scale text analysis methods such as topic modeling or co-citation analysis, the Quadriga System has the advantage of not only connecting concepts and texts but also qualifying that link. For instance, a co-citation analysis might suggest that two papers are related because they are co-cited often

[6] The normalization of a term could be singular for plural nouns, present tense for verbs, or simply the correct spelling of a word.

but it does not make any assertions about the kind of relationship between those papers. Do both papers make similar statements or does one reject the statements of the other? Similarly, topic modeling might connect two terms by placing them in the same topic. However, it does not specify what kind of relation exists between the two terms. Do texts that belong to a specific topic describe a similar relationship between these two terms, or are they using the same terms but contradict each other? The Quadriga System can answer such questions by qualifying the relationship between concepts. Two concepts are not only in relation to each other but are connected by a specific relationship. A scholar could use this property of the system by, for instance, identifying several papers connected to each other by co-citation analysis and then annotating these papers with Quadruples to determine their specific relationships. In contrast to traditional close reading methods of the identified papers, the Quadriga System would provide a researcher not only with a structured way of extracting relevant information that could then be computationally analyzed using, for instance, network analysis measures. It would also allow a researcher to publish the extracted information (the Quadruple networks) so that other scholars can examine it or use it for their own research.

The last point, publishing annotations or data to be shared among scholars, connects these tools to the vision that guided Peter Damerow throughout his distinguished career: openness and sharing of information. It also allows the history of science or any other field that uses such tools to benefit from an economy of scale that, in the fashion of big data, facilitates novel and surprising discoveries. Following again in Peter's footsteps, who devoted his whole career to collaborations, we have built this system in order to enable different researchers to share their data, collaborate on interpretations, and to expose their work beyond the narrow disciplinary boundaries of a specific discipline. It has been our own experience that all really interesting and important problems require a multi-disciplinary approach, something that hopefully just got a bit easier because of tools such as the one presented here.

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Part 4: A Glimpse of His Life

Chapter 22
Peter Damerow (1939–2011)
Jürgen Renn

With contributions from Robert K. Englund, Christine Keitel-Kreidt, Peter McLaughlin and Diethelm Stoller, among others.

The mathematician, philosopher, educational researcher, and historian of science Peter Damerow was born on December 20, 1939, in Berlin.[1] On November 20, 2011, surrounded by his family in the University Clinic Benjamin Franklin, he succumbed to cancer. He was an unusually versatile scientific personality, a realist, and at the same time a visionary who was extraordinarily generous with his talents, including the talent to forge lifelong friendships.

Peter Damerow first trained as a chemical laboratory assistant, a job he practiced for several years, including a stint in Yugoslavia. Attending night classes, he prepared for the academic qualification exams needed to begin his studies in mathematics and philosophy at the Freie Universität Berlin. There he became involved in the student movement, rising to prominence as Wolfgang Lefèvre's co-chairman at the student union (ASTA) of the Freie Universität in 1965. He was one of the student representatives on the commission that investigated the death of student activist Benno Ohnesorg. His report of this investigation was published in *Kursbuch*, Hans-Magnus Enzensberger's influential political quarterly.

His interest in philosophy focused in particular on Kant, Hegel, and Marx. His formative philosophical experiences included a Hegel colloquium which he held for years with Peter Furth, Bernhard Heidtmann, and Wolfgang Lefèvre. But even at that time, he was also interested in religious studies (as taught by Klaus Heinrich), the didactics of mathematics, and the cultural and social contexts of science.

In mathematics, he was fascinated by the systematically abstract. In 1969, Peter Damerow submitted his master's thesis in mathematics on a topic from category theory. In 1977, he was awarded his doctoral degree at Bielefeld University with a thesis titled *Die Reform der Lehrpläne für den Mathematikunterricht*

[1] This obituary was first published in German in *Archiv für Orientforschung* 52 (2013): 390–393.

der Sekundarstufe I in den Ländern der Bundesrepublik Deutschland 1963–1974 ("The reform of curricula for mathematics instruction in lower secondary education in the states of the Federal Republic of Germany 1963–1974"). His academic tutor was Karl-Peter Grotemeyer, who had founded the second institute for mathematics at the Freie Universität in 1967. This institute opened up new perspectives for university education, such as introducing discussions to the lecture hall and establishing tutorials. It was the age of what we then called the "education catastrophe." Because of his dedication to university education, Grotemeyer was often asked to apply himself above and beyond this project to improve education in mathematics and the natural sciences. He assigned to Peter Damerow and Christine Keitel-Kreidt the task of drafting an application to the Volkswagen Foundation in order to set up a central research institute for the didactics of mathematics in Berlin. The application was successful, but by the time it was approved, Grotemeyer had accepted an appointment at the new Bielefeld University. The proposed institute was founded in Bielefeld, but Peter Damerow remained in Berlin.

There were reasons for this. When a new professor was sought in 1975 for the chair of mathematics at the College of Education in Lower Saxony, Lüneburg campus, Peter Damerow was placed at the head of the list, despite the fact that he had not submitted a post-doctoral thesis to qualify for a professorship and had still to defend his doctoral thesis. Aside from his outstanding critical writings, especially on the reform of mathematics instruction, on theories of learning, on measuring performance, and on equal opportunity, it was his application lecture, titled *Didaktische Probleme der Verwendung des Rechenstabs im Schulunterricht* ("Didactic problems in the use of the slide rule in school instruction"), which drew particular attention. In taking up an apparently outdated issue, his lecture initially provoked scepticism, but the reaction turned to excitement as they came to see how Damerow had brought a fresh perspective—forward-looking, intelligent, and didactically grounded—to an apparently uninspiring topic. Although Peter Damerow was the university's first choice, in the end he was not appointed, apparently as a consequence of Lower Saxony's change from a social democratic (SPD) to a conservative (CDU) government. Nevertheless, he did give an introductory lecture in Lüneburg on the study of modern mathematics, which guided the students in cogent steps from their everyday experiences into higher mathematics and its language. Lectures followed on subject-related topics, as well as on basic historical problems. The work he performed with his colleagues in Lüneburg, especially that with Diethelm Stoller on the development of project- and student-oriented mathematics instruction, was carried out in collaboration with comprehensive school teachers in the state of Hesse as part of the KORAG (*Konkretisierung der hessischen Rahmenrichtlinien für Gesellschafts-*

lehre: "Concretization of the framework directives of the State of Hesse") and the SUGZ (*Systematische Umsetzungen gesamtschulspezifischer Zielsetzungen*: "Systematic implementations of objectives specific to comprehensive schools"). It yielded several extensive collections of instruction materials. Peter Damerow remained in close contact with his circle of friends in Lüneburg for his entire life.

In 1974, Peter Damerow became a research fellow at the Max Planck Institute for Human Development in Berlin, where he worked on the development of a mathematics curriculum. In designing instruction materials, he always remained committed to overcoming social barriers, including those limiting the propagation of mathematical and scientific knowledge. This emphasis soon directed him toward questions about the historical development of the mathematical sciences. At the Max Planck Institute for Human Development, Peter Damerow worked first under Peter M. Röder, and later in the research area dealing with development and socialization, headed by Wolfgang Edelstein, where he supervised a project on culture and cognition. He was a representative in the Humanities Section of the Max Planck Society and, for a time, even a member of the MPG Senate.

For many years, Peter Damerow and Wolfgang Lefèvre co-directed the *Begriffsentwicklung in den Naturwissenschaften* (Concept development in the natural sciences) research colloquium, a program held jointly by the Max Planck Institute for Human Development and the Freie Universität Berlin. This colloquium became, not least through Wolfgang Edelstein's initiative, one of the nuclei of the Max Planck Institute for the History of Science (founded in 1994), the proponent of a context-related, theoretically oriented historiography of science. Two of its later directors belonged to the colloquium (Jürgen Renn and Hans-Jörg Rheinberger), as did a number of the institute's staff (among them Jochen Büttner, Jörg Kantel, Hartmut Kern, Ursula Klein, Wolfgang Lefèvre, Peter McLaughlin, Staffan Müller-Wille, Jochen Schneider, and Urs Schoepflin). The book co-authored by Peter Damerow and Wolfgang Lefèvre, *Rechenstein, Experiment, Sprache: Historische Fallstudien zur Entstehung der exakten Wissenschaften*, published in 1981, blazed the trail for some of the later research projects of the institute.

In his article for the book, Peter Damerow was particularly concerned with the emergence of counting techniques in early high cultures. This soon became a central emphasis of his work, one which made him known throughout the world: the emergence of writing and counting in Mesopotamia. Starting in 1982, Peter Damerow worked closely with the archeologist Hans Nissen and the philologist Robert K. Englund on archaic texts and proto-cuneiform script. Peter Damerow was one of the pioneers of what are called today the "digital humanities." When he met Robert K. Englund in 1982, who had just begun to work as a research assistant to Hans Nissen at the time, he noticed a pile of punch cards in his office, which he brought right away to the Max Planck Institute for Human Development. His colleagues from the Near Eastern Archeology department had lost their technical support and could not find a way to make use of any of the data stored on these punch cards. Peter Damerow, in contrast, had a mathematician's confidence that there must be a way of solving this problem. He had access to the computing center of the Max Planck Institute for Human Development and the necessary programming skills in LISP to process and evaluate the data. This was the beginning of the electronic Uruk project. Even back then, Peter Damerow was deploying computer-aided methods of analysis to decode the domain-specific counting systems of early Babylonian mathematics, yielding a resounding success. This work led to his co-founding, along with Robert K. Englund, of the "Cuneiform Digital Library Initiative" (CDLI), the world's most important digital cuneiform library, which contains not only high-resolution reproductions of cuneiform tablets, but also transcriptions, catalog data, and tools for electronic publication.

Thus Peter Damerow also became one of the early advocates of the principle of "open access" for research data and publications in the humanities. Up to the very end of his life, he remained fascinated by the possibilities of new technologies for innovative research. In the end, with support from Jörg Kantel, he became one of the protagonists of applying three-dimensional scanning technologies in the institutes of the Max Planck Society dedicated to the humanities. The digitization of the famous Hilprecht collection cuneiform scripts in Jena, which he undertook with Manfred Krebernick, served as his pilot project.

In the framework of the research colloquium mentioned above, which met regularly for years on Monday evenings at the Max Planck Institute for Human Development, Peter Damerow also pursued a plethora of other history of science projects. Indeed, it is difficult to overestimate his contribution to promoting the history of science, by urging others along in heated conversation, through his critical reading of texts submitted to him for review, and through the inexhaustible energy he brought to ongoing research projects. In all of this, he was guided by the vision that from the history of science a historical, empirically based theory of the development of knowledge could be extracted, a vision discussed today in connection with "historical epistemology." His own works in this vein include exemplary studies on the development of the number concept, which were not only based on a wealth of empirical material, but also elaborated the theoretical foundations for a historical epistemology of this kind. To take one example, consider the essay "Individual Development and Cultural Evolution of Arithmetical Thinking," in: S. Strauss (ed.), *Ontogeny, Phylogeny, and Historical Development*, of 1988.

This perspective emerged not least due to his insights into the ways culture and cognition are connected, which were the focus of the research area headed by

Wolfgang Edelstein at the Max Planck Institute for Human Development. These insights involve the connections between individual learning processes, as they were studied in an extended version of Jean Piaget's genetic epistemology; the development of concepts as investigated in epistemology and logic, especially in the work of Hegel; and historical transformation processes like the ones at the core of Marx's social analysis. Peter Damerow examined these connections and their many facets to develop his own notions of a historical epistemology, which he depicts in detail in his 1995 collection of essays, *Abstraction and Representation*.

Another of Peter Damerow's central areas of interest was the history of physics. In the 1980s, the Monday colloquia in Berlin focused not only on the Scientific Revolution of early modernity, but also on the emergence of modern physics. The discussions that took place there resulted in a long-term collaboration between Peter Damerow, Gideon Freudenthal, Peter McLaughlin, and Jürgen Renn. The book they co-authored, *Exploring the Limits of Pre-Classical Mechanics*, first appeared in 1992, using concrete case studies to analyze fundamental characteristics of the development of concepts in the natural sciences. This approach later yielded the research program on the History of Mental Models of Mechanics, pursued at the Max Planck Institute for History of Science, which continues today in the work of Jochen Büttner, Matthias Schemmel, and Matteo Valleriani in collaboration with such scholars as Rivka Feldhay. In 1994, Peter Damerow obtained his postdoctoral qualification in philosophy at the University of Konstanz. There he held a number of research seminars on the development of concepts in the natural sciences–often with Peter McLaughlin and Jürgen Renn.

In the history of physics, too, Peter Damerow was active in promoting the deployment of new information technology in order to open up new perspectives for research. Together with such scholars as Jürgen Renn, Jochen Büttner, Simone Rieger, and Martin Warnke, and supported by the Florentine Institute and Museum for the History of Science, he developed the concept of an electronic representation of Galileo's manuscripts on mechanics to be made freely available on the Internet. Building on this success, further digital research libraries were developed later at the Max Planck Institute for the History of Science, in collaboration with Jochen Büttner, Jörg Kantel, Jürgen Renn, Simone Rieger, Urs Schoepflin, and Dirk Wintergrün, among others, especially the broadly designed ECHO (European Cultural Heritage Online) environment, which has been joined by developments like the Europeana and the Deutsche Digitale Bibliothek. In 2003, along with Jürgen Renn and Robert Schlögl, he was one of the spiritual fathers of the Max Planck Society's Berlin Declaration for open access to information on science and cultural heritage. In 2010, Peter Damerow developed the idea of an open access, print-on-demand publication platform, which he then realized – along with Jürgen Renn, Bernard Schutz, and Robert Schlögl, supported by Lindy Divarci, Jörg Kantel, and Matthias Schemmel, among others – as the "Max Planck Research Library for the History and Development of Knowledge," the very platform on which this book has been published.

Peter Damerow never considered the history of science to be a specialized discipline, but a research area that was part of his comprehensive interest in the development of human cognition. In this he was also a pioneer of an interdisciplinary conception of the history of science. Even his early works on the emergence of script and counting had made clear that the emergence of abstract concepts can be understood only if we take seriously the role of those material representations of thinking that are given in concrete historical cases, and the potential for actions and reflection they enable, as for instance the specific role played by cuneiform script tablets in the administration of Babylonia. This insight allowed him to contribute to completely different kinds of fields, for instance to cultural anthropology. Together with Wulf Schiefenhövel, and building on the materials Wulf collected about the life of the Eipo in Papua New Guinea, Peter Damerow investigated those culture-specific cognitive structures and their representations that allowed the Eipo to achieve astonishing mental performances in areas like house construction, setting traps, and spatial orientation. He then applied these research findings to other projects at the Max Planck Institute for the History of Science, especially to a project conceived with Jürgen Renn on an epistemic history of architecture, and to a project on the historical development of spatial thinking pursued by Matthias Schemmel.

Non-European knowledge traditions played a prominent role in Peter Damerow's thinking and actions. His close relations with Brazilian scholars go back to the mid-1980s during his work as an educational researcher, when he worked within the framework of the UNESCO, teaming up with Christine Keitel-Kreidt, Paulus Gerdes, Ubiratan d'Ambrosio, and Circe Silva da Silva Dynikoff to promote a contextualized mathematics. This, in principle, would open up to everyone the access to mathematic knowledge. These collaborations resulted in what is still a quite vibrant, regular academic exchange between the Max Planck Institute for the History of Science in Berlin and various academic institutions in Brazil, and also a concrete aid project for mathematics education in Brazil. Later, as part of his work at the institute, Peter Damerow helped establish academic relations with China and Spain. In collaboration with Zhang Baichun, Tian Miao, Jürgen Renn, and Matthias Schemmel, works on the history of mechanics in China emerged, even including a documentary film, thanks to the support of Richard Röseler. To all of these cooperative projects, Peter Damerow contributed his experience in the development of digital research environments, thus providing concrete help to overcome the "digital divide." Digitization centers were set up in La Orotava on Tenerife and in Beijing, with assistance from Urs Schoepflin and Simone Rieger, which even today continue to secure our cultural heritage, to make it available for research, and to edit it for the broader public.

Over and again, Peter Damerow's interest in a propagation of scientific knowledge also brought him to participate enthusiastically in exhibition projects. His interest in archaic cuneiform writing led in part to the Berlin Senate's 1988 bid to acquire the private Erlenmeyer collection of archaic cuneiform tablets. The Senate, using lottery funds, had teamed up with an international museum consortium to acquire the tablets when they went up for auction at Christie's in London. Since government officials seldom have experience with auctions, Peter Damerow worked out bidding tactics with a consultant experienced in such matters, and with their success, practically the entire collection ended up in public institutions—part of it is still preserved at the Pergamon Museum in Berlin. This acquisition was the foundation for Peter Damerow's first large-scale exhibition project in the late 1980s, which culminated in a widely acclaimed exhibition about the emergence of writing in Charlottenburg Palace. The accompanying catalog, co-authored with Hans Nissen and Robert K. Englund, is still used as a textbook for Assyriology, especially the English version. Later, Peter Damerow teamed up with Jochen Schneider to develop parts of the conception of the Nixdorf Computer Museum in Paderborn, which also featured an exhibit on early Babylonian calculation techniques.

In 2005, which was International Einstein Year, Peter Damerow played a key role in the major Einstein Exhibition in Berlin. His long years of cooperation with Jürgen Renn, Tilman Sauer, Giuseppe Castagnetti, Werner Heinrich, Hubert Gönner, Matthias Schemmel, Michel Janssen, John Stachel, and other Einstein scholars built the foundation for this involvement. In the late 1980s, Peter Damerow and Jürgen Renn headed the Albert Einstein working group funded by the Berlin Senate at the Max Planck Institute for Human Development, which soon became a center for international Einstein scholarship. With his eye for overarching connections, his comprehensive experience with historical issues concerning the development of concepts in the natural sciences, and not least his unerring critical inquiries, Peter Damerow made a weighty contribution to understanding the emergence of the theory of relativity, albeit one that is easily underestimated by specialized scholars. The conception and realization of this major Einstein Exhibition would have been inconceivable without his ingenuity and persistence. The idea for a virtual exhibition (not only guiding the mode of presentation but enabling the long-term storage of the exhibition's contents) can be traced back to Damerow's ideas and works, which were implemented in collaboration with Jürgen Renn, his daughter Julia Damerow, and with Malcolm Hyman, who also died too young.

In Peter Damerow we have lost a visionary teacher, a colleague, and a friend. He challenged us intellectually, radically, and without compromise—and was unconditionally loyal and helpful in all human endeavors. He left his mark on many a biography and pointed out new paths for many research institutions. For years to come, books and articles will appear that were influenced by his thought and to which he contributed decisive ideas. He had inconceivable strength and endurance, in both his work and his commitment to people. He was at once a brilliant spirit and the most cooperative person imaginable, despite or perhaps because of his unbending character. His direct manner occasionally offended, but he never refused a conciliatory conversation. Those of us who were privileged to be close to him are grateful. Peter Damerow is survived by his wife Ingrid and his two daughters, Julia and Sophie. To them we offer our condolences and support.

Chapter 23
Es begann mit den Zahlen

Kristina Vaillant

Ein ganzes Forscherleben: Als Mathematiker beschäftigte sich Peter Damerow zunächst mit Mathematikunterricht in der Grundschule, dann mit der Frage, warum in frühen Kulturen auf so unterschiedliche Weise gerechnet wurde.[1] Die Untersuchung der Zahlenkonzepte wurde für ihn zum Ausgangspunkt für die Erforschung der frühesten Schriftzeugnisse aus dem Zweistromland und später der Entwicklung der Mechanik in der frühen Neuzeit. Zu seiner Mission als Wissenschaftshistoriker gehört es, Wissen zu teilen. Diese Idee treibt er voran: konzeptionell, indem er eine Internetdatenbank zur Erforschung von Keilschrifttafeln mitbegründete und bis heute laufend weiterentwickelt; und ganz persönlich, indem er junge Forscher am Max-Planck-Institut für Wissenschaftsgeschichte an seinem Wissen und seiner Erfahrung teilhaben lässt.

Dass er einmal Wissenschaftler werden würde, "ein fanatischer Forscher", wie er sich selbst bezeichnet, war zu Beginn seines Berufslebens nicht abzusehen. Peter Damerow, 1939 in Berlin geboren, hat ursprünglich eine Lehre zum Chemielaboranten und später an der Abendschule Abitur gemacht. Seine Lehrer sagten ihm, für ihn als Lateinschüler kämen als Studienfächer nur die alten Sprachen infrage. Es kam anders: Peter Damerow studierte an der Freien Universität Berlin Mathematik und hörte nebenbei Vorlesungen in Psychologie, Philosophie und Pädagogik. "Für mich war Studieren ein Lebensziel", erzählt Peter Damerow. Er hatte Glück, denn die Studienstiftung erkannte seine Begabungen und förderte ihn. Nach dem Studienabschluss Mitte der 70er Jahre ging er an das Max-Planck-Institut für Bildungsforschung, wo er sich zunächst der Lehrerfortbildung im Fach Mathematik widmete. In seiner Promotion 1977 kritisierte er die Art und Weise, wie Grundschülern damals die Mengenlehre beigebracht wurde.

Nach der Promotion befasste sich Damerow auch mit der kognitiven Psychologie – damals ein neuer Forschungsansatz, bei dem es um die Fragen geht, wie Menschen Informationen aus der Umwelt aufnehmen, sie verarbeiten und wie sich dieses Wissen wiederum in ihrem Verhalten ausdrückt. Peter Damerow

[1] Dieser Text wurde zum ersten Mal publiziert in *Ideen, Täglich: Wissenschaft in Berlin*. Nicolai Verlag, 2010, 138–151.

interessiert sich für die verschiedenen Zahlenkonzepte, die Menschen entwickelt haben. “Die Beschäftigung damit war damals noch mehr Hobby als Beruf”, sagt er, dennoch ging er in einem wissenschaftlichen Aufsatz der Frage nach, warum Menschen der frühen Kulturen auf ganz unterschiedliche Weise rechneten. Zu seinen Untersuchungsobjekten gehörten Fragmente von beschrifteten Tontafeln aus der Zeit um 3000 v. Chr., die in Mesopotamien ausgegraben wurden – Zeugnisse der frühesten Phase der Schriftentwicklung. Doch so sehr er sich bemühte: Mit den damals gängigen Methoden der Transkription waren die Berechnungen, die in die Tontafeln geritzt waren, nicht nachzuvollziehen. “Irgendetwas konnte nicht stimmen”, erinnert sich Peter Damerow. Bisher hatten Forscher hinter solchen Ungereimtheiten Fehler der Schreiber vermutet.

Abb. 23.1: Peter Damerow an seinem Arbeitsplatz am Max-Planck-Institut für Wissenschaftsgeschichte in Berlin-Dahlem.

Für Peter Damerow war diese Erfahrung Ausgangspunkt für ein neues Forschungsvorhaben. 1983 begann er gemeinsam mit dem Archäologen Hans Jörg Nissen und dem Altorientalisten Robert K. Englund, “die archaischen Texte auf der Basis aller ausgegrabenen Tafeln von Grund auf neu zu analysieren”. Die Wissenschaftler wählten dafür einen neuen Weg: Während Forscher bislang versucht hatten, die Bedeutung der Schriftzeichen anhand einzelner Tafeln zu entschlüsseln, wollten sie über fünftausend Tontafeln, die hauptsächlich aus Grabungen in der babylonischen Stadt Uruk stammen, vergleichend untersuchen. Peter Damerow und seine Kollegen hofften, die "Schreibregeln" identifizieren zu können und so hinter das Geheimnis der Schriftzeichen zu kommen. Dies konnte nur mithilfe von Computern gelingen: "Innerhalb eines Jahres haben wir mit statistischen Methoden herausgefunden, was die Zahlzeichen bedeuteten. Das war ein Durchbruch in der Forschung", berichtet Damerow. Die Auswertung führte zu zwei wesentlichen Ergebnissen: Die frühen Schriftzeugnisse aus dem dritten Jahrtausend vor unserer Zeitrechnung sind nicht etwa die Verschriftlichung einer gesprochenen Sprache, sie dokumentieren vielmehr die Verwaltungs- und Buchhaltungsaktivitäten von Beamten. So sind auf den Tontafeln etwa Abrechnungen über die Abgabe von Saatgut, die Lieferung von Getreide oder anderer Güter festgehalten. Diesen Aufzeichnungen und Berechnungen lag kein einheitliches Zahlensystem zugrunde, sondern es wurden unterschiedliche Zahlensysteme verwendet, je nachdem, was man zählen oder abmessen wollte.

Wissenschaftler, die heute an der Entschlüsselung dieser frühen Schriftzeugnisse arbeiten, finden gänzlich andere Bedingungen vor. Sie können auf der Internetplattform "Cuneiform Digital Library Initiative" (CDLI) zehntausende Tontafeln mit Keilschrifttexten studieren. Die Bilder sind mit Transkriptionen der Inschriften, sowie mit Informationen zum Fundort, zur Datierung und zum Aufbewahrungsort versehen. Das CDLI ist ein Projekt, das Peter Damerow, seit Ende der 90er Jahre Mitarbeiter des Max-Planck-Instituts für Wissenschaftsgeschichte, zusammen mit Robert K. Englund, inzwischen Professor an der University of California in Los Angeles, initiiert hat und gemeinsam mit Wissenschaftlern und zahlreichen Museen und Archiven weiterentwickelt. Die Internetplattform soll einmal alle etwa fünfhunderttausend Keilschrifttafeln zusammenführen, die heute in Museen, Archiven und privaten Sammlungen über die ganze Welt verstreut sind.

Aber es geht Peter Damerow nicht nur um die Menge, sondern vor allem auch um die Qualität der Darstellung. In Zukunft sollen die Tontafeln nicht nur als Zeichnungen oder Fotos, sondern als dreidimensionale Objekte im Internet zu sehen sein. Bis dahin sind noch viele Probleme zu lösen: Mit welcher Software lassen sich 3-D-Bilder am besten herstellen, archivieren und darstellen? Wie geht man mit Verwertungsrechten der Eigentümer um? Um Antworten zu finden, treffen sich Peter Damerow und seine Kollegen regelmäßig mit anderen Wissenschaftlern, die ebenfalls 3-D-Technologien für ihre Forschung einsetzen.

Im November 2009 ist die Gruppe zu Gast am Lehrstuhl für Altorientalistik der Universität Jena. Die 3-D-Experten treffen sich in einem gut gesicherten Raum der Universität. Dort wird die Hilprecht-Sammlung aufbewahrt, die mit 3.300 Tontafeln die zweitgrößte Sammlung von Keilschrifttafeln in Deutschland

Abb. 23.2: Manfred Krebernik und Peter Damerow setzen 3-D-Scantechnologie ein, die ursprünglich einmal für die Automobilindustrie entwickelt wurde, um die Darstellung von Keilschrifttafeln im Internet weiter zu verbessern. Dahinter steht die umfassendere Idee der Wissenschaftler, neue Wege zu finden, wie Computertechnologie für die Erforschung historischer Dokumente genutzt werden kann.

ist. Peter Damerow will seinen Kollegen heute demonstrieren, wie sein Team die Tafeln der Sammlung scannt.

Die Hardware sieht unspektakulär aus. Sie besteht aus zwei Kameras, die auf ein Stativ montiert sind. In der Mitte, zwischen den beiden Kameras, sitzt ein Projektor, der Streifenmuster auf die Tafeln projiziert. Sarah Köhler, Studentin der Theologie und Altorientalistik, streift weiße Handschuhe über, nimmt eine Tontafel aus einem Kästchen und legt sie auf den Drehtisch, genau an die Stelle, an der die Kameras die größte Tiefenschärfe erzielen. Im Moment des Scannens gleiten die roten Lichtstreifen, erst breite, dann haarfeine, über die Tontafel. Die Messpunkte, die die Kamera aufzeichnet, liegen dicht beieinander, alle achtunddreißig Mikrometer (ein Tausendstel von einem Millimeter) einer. Das Computerprogramm berechnet aus diesen Messwerten ein Netz aus mehreren Millionen Dreiecken, das die Form der Keilschrifttafel mathematisch beschreibt, und wandelt es in ein Bild um.

Ein einziger Scan dauert nicht einmal eine Minute, aber der Vorgang muss für Vorder- und Rückseite der Tontafel je sechsmal wiederholt werden, damit sie präzise abgebildet werden kann. Dann verschwinden die Lichtstreifen und nur das Summen der Lüftung des Scansystems erfüllt den Raum. Licht fällt nur gedämpft durch die Fenster, direkte Sonneneinstrahlung würde die Scans verfälschen. Nach jeder Messung berechnet die Software in wenigen Minuten eine erste Vorschau der Aufnahmen, dafür wird aber lediglich jeder dritte Messpunkt berücksichtigt. Jetzt beginnt Sarah Köhler, am Bildschirm die einzelnen Scans zu einem Gesamtbild zusammenzusetzen.

Inzwischen hat die Software hochaufgelöste Bilder berechnet, die Sarah Köhler in einem nächsten Arbeitsschritt "reinigt". Sie schneidet überflüssige Informationen aus, beispielsweise dort, wo sich die Einzelaufnahmen überschneiden. Am Ende berechnet die Software aus den mehrere Gigabyte großen, hochaufgelösten Bilddateien das endgültige dreidimensionale Modell. Es hat jetzt einen Umfang von etwa zweihundertfünfzig Megabyte. An die zwanzig Minuten vergehen, bis die eingescannte Tontafel auf dem Bildschirm erscheint. Dieses 3-D-Modell kann jetzt so beleuchtet werden, dass alle Zeichen auf der Tafel gut zu erkennen sind. "Beim zweidimensionalen Flachscan können wir das Licht nicht beeinflussen, dann verschwinden manche Zeichen auf den Tafeln", erläutert Peter Damerow seinen Kollegen die Vorteile. "Mit dem, was wir hier tun, bewegen wir uns aber an der Grenze dessen, was heute technisch möglich ist." Es sei aber nur eine Frage der Zeit, so Damerow, denn sobald die Rechner leistungsfähiger würden, könnten die 3-D-Bilder schneller berechnet werden. "In Zukunft wird es nicht mehr notwendig sein", sagt Damerow, "dass jeder Forscher seine eigenen Ressourcen als persönliche Habe auf seinem Schreibtisch hütet. Das wird die Arbeitsweise einer ganzen Disziplin umkrempeln."

Beim nächsten Treffen der Gruppe im Frühling in Florenz werden die Teilnehmer in der Anwendung des Programms MeshLab geschult. Für die Entwickler der nichtkommerziellen Software ist es wichtig, dass sie ihr Wissen direkt an die Benutzer weitergeben können. Es würde sie zu viel Zeit kosten, das Programm in einem Handbuch zu dokumentieren. Auch für die Wissenschaftshistoriker aus Berlin zahlt sich der Workshop in Florenz aus. "Mit dem frei verfügbaren Programm MeshLab können wir die riesigen 3-D-Scandateien einzelner Keilschrifttafeln auf zwei Megabyte reduzieren", berichtet Peter Damerow seinem Kollegen Joachim Marzahn, Kustos der Keilschriftensammlung des Vorderasiatischen Museums im Pergamonmuseum auf der Berliner Museumsinsel. Allerdings, schränkt er ein, sei es noch nicht möglich, diese Dateien im Internet zur Verfügung zu stellen. Die Software ist noch nicht mit dem Internetbrowser kompatibel. Peter Damerow führt dem Keilschriftexperten im Lesesaal der Keilschriftensammlung an seinem Laptop vor, wie die neuesten 3-D-Scans aussehen: "Jetzt kannst du das

Abb. 23.3: Beim Workshop in Jena tauschen sich Wissenschaftler und Softwareentwickler über das Scannen und Darstellen dreidimensionaler Objekte aus. Jeder hat seine eigene Methode für seine eigenen Zwecke gefunden. Das Kunsthistorische Institut in Florenz beispielsweise untersucht romanische Kapitelle in Klosteranlagen mithilfe von hochauflösenden Digitalfotografien und 3-D-Modellen. Die Forscher vom Max-Planck-Institut für evolutionäre Anthropologie in Leipzig scannen Knochen und Schädel unserer urzeitlichen Vorfahren. Anders als das Team von Peter Damerow verwenden die Biologen dafür einen Computertomografen, wie er in der medizinischen Diagnostik gebräuchlich ist. Anhand der damit generierten 3-D-Modelle erweitern und präzisieren sie ihren Blick: Sie können neue Vergleiche anstellen, mögliche Entwicklungen visualisieren, Schädelfragmente zuordnen oder Schädel anhand von Fragmenten rekonstruieren. In der Fachsprache wird diese Methode als virtuelle Paläontologie bezeichnet. Zum Schluss des Workshops findet Matteo Dellepiane, Softwareentwickler und 3-D-Spezialist aus Italien, anerkennende Worte für Peter Damerow und sein Team: Aus seiner Sicht setzen sie die besten im Moment zur Verfügung stehenden Methoden ein.

Licht setzen, wie du willst. Und jedes Detail ausleuchten. Das Ergebnis ist sehr befriedigend. Wir hoffen, dass wir diese Bilder ins Netz kriegen."

Abb. 23.4: Blick durch die 3-D Brille.

Auch das Vorderasiatische Museum ist Kooperationspartner des internationalen Scanprojekts. Schon 1998 wurden alle Keilschrifttafeln des dritten vorchristlichen Jahrtausends gescannt, insgesamt etwa viertausend Tontafeln. Die Abbildungen stehen inzwischen im Internet zur Verfügung. Auch heute wird gescannt. In einem Vorraum des Lesesaals sitzen zwei Doktoranden im Dunkeln an einem Flachbildscanner, der zweidimensionale Bilder herstellt. In einem zweiten Arbeitsgang fügen sie die Ansichten der sechs Seiten der Tontafeln zu einer Gesamtdarstellung zusammen. Eigentlich sollen demnächst auch diese Tontafeln aus dem zweiten vorchristlichen Jahrtausend in der Onlinedatenbank zu finden sein, aber es geht im Moment nicht voran. Die Transkriptionen der Inschriften, berichtet Joachim Marzahn, liegen noch bei den Wissenschaftlern, die die Texte bearbeitet haben. Sie wollen sie so lange nicht zur Verfügung stellen, bis sie die Texte selbst publiziert haben. “Da gibt es noch unterschiedliche Auffassungen”, bedauert Marzahn.

Das sind die Widerstände, auf die Peter Damerow immer wieder stößt. Er selbst teilt sein Wissen täglich – auch wenn er schon seit über sechs Jahren in Rente ist. Davon profitieren vor allem seine jüngeren Forscherkollegen – so wie an diesem Tag Ende März, einem der ersten warmen Frühlingstage. Die Vögel hört

Abb. 23.5: "Jetzt gehen wir ins Allerheiligste", sagt Peter Damerow auf dem Weg ins Tontafelmagazin. Die Keilschriftensammlung des Vorderasiatischen Museums in Berlin hat über 22.000 Tontafeln katalogisiert, dreitausend Stück warten noch darauf, inventarisiert zu werden. Im Magazin schauen sich die Experten einzelne Tafeln genauer an. "Wenn du das unter dem Mikroskop betrachtest", sagt Joachim Marzahn, Kustos der Sammlung, "dann erkennst du die Korrekturen, die die Schreiber gemacht haben." Auf der Tafel ist eine Tabelle mit quadratischen Feldern zu erkennen. Die Tontafel stammt aus der Zeit um 2600 v. Chr. und wurde wahrscheinlich benutzt, um die Berechnung von Flächen zu erlernen. Sie kam 1906 ins Museum, wo man sie restauriert und nachträglich gebrannt hat, um sie haltbar zu machen.

man bis in den Besprechungsraum im Max-Planck-Institut für Wissenschaftsgeschichte zwitschern. Hier trifft sich die Forschungsgruppe von Matthias Schemmel, einem jungen Kollegen, den Peter Damerow vor einigen Jahren als Doktoranden für das Institut gewinnen konnte. Die Gruppe will sich gemeinsam mit Peter Damerow Filme anschauen, die Max-Planck-Forscher in den 70er Jahren im Hochland der Insel Neuguinea gedreht haben. Sie dokumentieren, wie Angehörige des Volks der Eipo verschiedene Arten von Fallen bauen. Die Wissenschaftshistoriker interessieren sich dafür, welches Wissen über Räume und ihre Vermessung sich in dieser Tätigkeit ausdrückt.

Abb. 23.6: Peter Damerow im Tontafelmagazin des Vorderasiatischen Museums mit Christina Tsouparopoulou und Joachim Marzahn.

Matthias Schemmel startet den Film von seinem Laptop aus. Man sieht zwei Heranwachsenden dabei zu, wie sie eine Falle bauen. Es ist nicht einfach, ihren Handgriffen zu folgen. “Also, zum ersten Mal machen die das nicht”, findet Peter Damerow. Er versteht die mechanischen Vorgänge sofort und erläutert die physikalischen Prinzipien, auf denen sie beruhen. “Die Frage ist aber, wie kommen die eigentlich dazu, ihre Fallen so zu konstruieren?”, denkt er laut. “Eines ist klar”, spricht er in die aufmerksame Ruhe, “die erfinden das nicht jedes Mal neu. Die müssen nicht darüber nachdenken, jedes Mal, wenn sie das bauen. Der Auslösemechanismus der Falle ist bei jeder Konstruktion derselbe.” Wahrscheinlich sei diese Art des Fallenbaus keine Erfindung, sondern Ergebnis eines kontinuierlichen Optimierungsprozesses. Um zu illustrieren, dass es sich nicht um abstraktes Wissen handelt, das die Fallenbauer anwenden, sondern um intuitives Handlungswissen, berichtet Peter Damerow von einer Beobachtung, die er beim Waagenbau in China gemacht hat: “Die Handwerker kannten das Hebelgesetz nicht, das war nur unsere Interpretation, aber sie konnten trotzdem Waagen bauen.” “Ja”, stimmt sein Kollege Schemmel zu und offeriert ein weiteres Beispiel: “Du kannst komplizierte Automaten bauen, auch ohne das theoretische Wissen zu haben. Galileo zum Beispiel hat das Teleskop gebaut, aber er hat es nicht verstanden."

Am Beispiel des Abstands, den die Fallenbauer in dem Film mit den Fingern ausmessen, um ein passendes Hölzchen zu finden, mit dem sie die Falle spannen, diskutiert die Forschergruppe die Frage, ob die Fallenbauer ein abstraktes Konzept von "Abstand" haben. "Dass der Begriff in ihrer Sprache nicht existiert, ist ein Indiz dafür, dass sie nicht über ein abstraktes Konzept von Abstand verfügen", davon ist Matthias Schemmel überzeugt. Peter Damerow wendet ein, dass nicht alles Wissen versprachlicht sei. Wenn man solche intuitiv vorhandenen Konzepte nachweisen wolle, müsse man das Verhalten untersuchen.

Peter Damerow beteiligt sich auch mit einer eigenen Untersuchung an dem Forschungsprojekt. "Im Moment versuche ich zu beweisen, dass die Babylonier zwar pythagoräische Zahlen, nicht aber den Satz des Pythagoras kannten. Was bedeutet das für die Vorstellung vom Raum?", erläutert er seine Forschungsfrage. Einer der Ursprünge der Geometrie sei zwar die Feldmessung, aber Feldgrößen konnten auch ohne den Satz des Pythagoras berechnet werden. Und damit kommt er zurück auf die archaischen Tontafeln: "Das Vermessen der Felder und die Berechnung der Feldflächen hat schon um 3000 vor Christi Geburt funktioniert, wenn auch nur auf eine Art und Weise, die aus Sicht der modernen Geometrie eine Näherung ist. Das kann man anhand von Keilschrifttafeln belegen."

Kurz nach seiner Pensionierung haben die Ärzte bei Peter Damerow Krebs diagnostiziert. Seitdem konnte er keine weiten Reisen mehr antreten, aber an seinem Arbeitsplatz ist er fast immer und ohne größere Unterbrechungen gewesen. "Schon wegen meiner Krankheit", sagt er. An seiner Art zu arbeiten habe sich nichts geändert, er vergesse bei der Arbeit, dass er schon siebzig Jahre alt ist. Überhaupt spielt der Altersunterschied für ihn gar keine Rolle: "Ob jung oder alt", sagt er, "die arbeiten alle fanatisch an ihren Themen." Nur die Arbeitsbedingungen seiner jüngeren Kollegen seien andere, kaum einer habe noch eine feste, unbefristete Stelle bis zur Pensionierung, so wie er. "Eigentlich", sagt Peter Damerow, "muss man heute noch verrückter sein, um Forscher zu werden."

Danksagung

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Abb. 23.7: Matthias Schemmel, Jürgen Renn und Peter Damerow am Max-Planck-Institut für Wissenschaftsgeschichte. Jürgen Renn und Peter Damerow kennen sich schon seit ihrer Studienzeit. Sie haben über Jahre gemeinsam zu Albert Einstein und dem Umbruch von der klassischen zur modernen Physik gearbeitet. Daneben verbindet sie vor allem ihr Interesse an Technik und Wissenschaft der Antike und ihrer Wiederbelebung und Weiterentwicklung in der Renaissance. Die Gründung des Max-Planck-Instituts für Wissenschaftsgeschichte und die Berufung von Jürgen Renn zum Gründungsdirektor bot ihnen die Chance, ihr gemeinsam entwickeltes Forschungsprogramm zur "historischen Epistemologie der Entwicklung des Wissens" zu verwirklichen.

Abb. 23.8: Das Gebäude des Max-Planck-Instituts für Wissenschaftsgeschichte in Berlin-Dahlem.

Chapter 24
Peter Damerow's Publications

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