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Asymptotic safety, the Higgs mass and beyond the Standard Model physics

Jan H. Kwapisz[a,b]

[a] Institute of Theoretical Physics, Department of Physics, University of Warsaw, ul. Pasteura 5, 02-093 Warsaw, Poland
[b] Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Am Mühlenberg 1 D-14476 Potsdam, Germany

Abstract

There are many hints that gravity is asymptotically safe. The inclusion of gravitational corrections can result in the ultraviolet fundamental Standard Model and constrain the Higgs mass to take exactly one value, which can be calculated. Taking into account the current top quark mass measurements this calculation gives Higgs mass ~ 131 GeV, which is in disagreement with the current experimental measurement.
This article considers the predictions of the Higgs mass in two minimal Beyond Standard Model scenarios. One is the sterile quark axion model, while the other is the $U(1)_{B-L}$ gauge symmetry model introducing a new massive Z' gauge boson. The inclusion of Z' boson gives the prediction much closer to the observed value, while inclusion of sterile quark(s) gives only a slight effect.
Also a new, gravitational solution to the strong CP problem is discussed.

Keywords: Asymptotic safety, Higgs mass, extensions of Standard Model, gravitational corrections, sterile quarks, Z' boson

1. Introduction

The couplings of the physical models change with scale, and there are two sources of this scaling. The first, classical scaling is due to canonical dimensionality of the operators. The theory, which is classically scale invariant possesses dimensionless couplings only. This is indeed the case for the Standard Model with zero bare Higgs mass, so called Conformal Standard Model [1]. The other source of scaling is caused by the quantum effects, which can spoil classical scale invariance and provide the generation of scale due to radiative corrections. In particular the Coleman-Weinberg mechanism generates masses in this pattern [2]. In quantum field theories the change ("running") of couplings with energy scale is described by renormalisation group equations

$$\mu\frac{\partial}{\partial\mu}g_i(\mu) = \beta_i(\{g_j\}). \tag{1}$$

Such a general equation can have various possible behaviours for $\mu \to +\infty$, yet only some of them makes the theory predictable up to the infinite energies. In the simplest case the couplings reach the fixed point $(\forall_i\beta_i(\{g_j\}) = 0)$ and the running stops, making the theory scale invariant on the quantum level. However, this is not only possibility, since the coupling can also be attracted to a higher dimensional structure, like a limit cycle (see for example [3] for a limit cycle behaviour in $1/r^2$ potential) or a chaotic attractor. Such theories can also be UV fundamental, yet they are not scale invariant. On the other hand, scale invariance seems to play some fundamental role in the construction of the quantum gravity theory(ies), see [4–7], so in this article we restrict to the fixed point case. The fixed point can be at zero (Gaussian fixed point), making the theory asymptotically free. Alternatively it can reach some non-zero value (non-Gaussian fixed point / residual interaction). We call such theory asymptotically safe. Steven Weinberg hypothesised that gravity possesses an interacting fixed point [8, 9]. This issue was studied in [10–12], where the calculations were done by means of ϵ expansion in the vicinity of 2 dimensions. However, in general such fixed points cannot be considered by means of ordinary perturbation theory, where one does expansion of the theory around the fixed point at zero. The study of such fixed points requires other, non-perturbative treatment.
The functional renormalisation group (FRG) is one of the tools which can be used. In the FRG approach one studies the evolution of the effective average action Γ_k, which is an quantum effective action, where all the interactions with momenta lower than k are integrated out. The Γ_k interpolates between the classical action S_Λ at the UV scale Λ and the full quantum effective action $\Gamma = \Gamma_{k=0}$. The evolution of Γ_k is given by the Wetterich equation [13–15]. Using this approach the gravitational fixed points were found for Euclidean signature, see [16, 17]. Moreover, the gravitational corrections to the matter beta functions can be calculated and they alter the UV running of the matter couplings. Despite the fact that the asymptotic safety programme for quantum gravity is far from being

Email address: Jan.Kwapisz@fuw.edu.pl (Jan H. Kwapisz)

finished, see [18, 19], yet it seems to be a very promising way to quantise gravity, not only because of its simplicity, but also due to its rich particle physics phenomenology, which can be tested. In particular two years before the discovery of the Higgs boson, its mass was calculated in [20] as $126 \pm$ few GeV. However, the authors took the top quark mass smaller than the current observed value In this article we repeat this calculation and investigate the possible sources of disparity between current experimental measurements and the theoretical predictions.

However, this is not only prediction from asymptotic safety for particle properties. Namely, the top Yukawa coupling is close to the upper bound in the basin of attraction, hence if it runs to the interacting fixed point, then it is also predictable [21]. In such a scenario the difference between the top and the bottom quark masses [22] can also be predicted. Moreover, with inclusion of new operators, the $U(1)$ coupling shows similar behaviour and then can be predictable [23]. These results are promising, however the results for the Higgs mass calculated for the top interacting fixed point scenario [21] gives $m_H \approx 132$ GeV. The authors stress that the *results arise in a truncation of the RG flow that is limited to the surmised leading-order effects of quantum gravity on matter* [21]. This might be the case, see [24–26], yet in this article we explore another possibility, namely that consideration of the Beyond Standard Model particles provides the correct predictions for the Higgs mass. The fact that most of the problems of the Standard Model can be solved at ~ 1 TeV scale [27–30] supports that view and the new physics should affect the prediction of the Higgs mass. In particular we analyse two scenarios: addition of sterile quarks and addition the Z' boson, which is related to the famous B-anomalies.

2. Calculation of the Higgs Boson mass in the Standard Model

In this paragraph we revaluate the calculations done in [20] concerning the calculation of the Higgs mass. The Higgs part of Standard Model Lagrangian is given by:

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H^\dagger D^\mu H) - \lambda \left((H^\dagger H)^2 - v^2 \right)^2, \quad (2)$$

where $v_H \approx 246.22$ GeV. On the tree level one has:

$$m_H^2 = 2\lambda v^2, \quad (3)$$

and the radiative corrections are $\mathcal{O}(1)$ GeV. The one-loop beta functions (where $\hat{\beta}_{SM} = 16\pi^2 \beta_{SM}$) in the $\overline{MS}$-scheme of the couplings are:

$$\begin{aligned} \hat{\beta}_{g_1} &= \tfrac{41}{6} g_1^3,\ \hat{\beta}_{g_2} = -\tfrac{19}{6} g_2^3,\ \hat{\beta}_{g_3} = -7 g_3^3, \\ \hat{\beta}_{y_t} &= y_t \left(\tfrac{9}{2} y_t^2 - 8 g_3^2 - \tfrac{9}{4} g_2^2 - \tfrac{17}{12} g_1^2 \right), \\ \hat{\beta}_{\lambda_1} &= 24\lambda_1^2 - 3\lambda_1 \left(3g_2^2 + g_1^2 - 4y_t^2 \right) \\ &\quad + \tfrac{9}{8} g_2^4 + \tfrac{3}{4} g_2^2 g_1^2 + \tfrac{3}{8} g_1^4 - 6 y_t^4, \end{aligned} \quad (4)$$

where g_1, g_2, g_3 are $U(1)$, $SU(2)$, $SU(3)$ Standard Model gauge couplings respectively and y_t is the top Yukawa coupling. The two-loop beta functions, we have used in our calculations, are given in [31, 32]. The gravitational corrections [20, 33–35] to the beta functions are in the leading order:

$$\beta_i^{\text{grav}}(g_i, \mu) = \frac{a_i}{8\pi} \frac{\mu^2}{M_P^2 + 2\xi_0 \mu^2} g_i, \quad (5)$$

where $M_P = 2.4 \times 10^{18}$ GeV is the low energy Planck mass, ξ_0 is related to the gravitational fixed point and depends on the matter content, see Eq. (8). For the Standard Model one has $\xi_0 \approx 0.024$ and $a_\lambda = +3$, $a_{y_t} = -0.5$, $a_{g_i} = -1$. Depending on the sign of a_i one gets repelling / attracting fixed point at zero for a given coupling. If one demands that all of the matter couplings to be asymptotically free then the ones with the repelling fixed points becomes predictable and the one with attracting fixed points have to be inside the basin of attraction, otherwise they will diverge [20]. Since $a_\lambda = +3$, then Higgs self coupling has a repelling fixed point at zero, and becomes a prediction of a theory rather than being a free parameter. On the two-loop level and for $y_t = g_1 = g_2 = g_3 = 0$ one has:

$$\beta_\lambda(\mu) = \frac{1}{16\pi^2} \left(24\lambda^2 - \frac{312}{16\pi^2} \lambda^3 \right) + \frac{a_\lambda}{8\pi} \frac{\mu^2}{M_P^2 + 2\xi_0 \mu^2} \lambda, \quad (6)$$

which has the following fixed points: $\lambda = 0$ (repeller), $\lambda \approx 21$ (attractor), $\lambda \approx -9.36$ (attractor). For this reason two basins of attraction are separated by the single trajectory going to the repelling fixed point. The numerical calculations confirm that if at any scale below Planck scale $\lambda(\mu) < 0$ then it drops to the non-perturbative fixed point. If one assumes that λ has to stay in the perturbative region, then necessarily one gets $\lambda(\mu) \geq 0$ at all scales. Furthermore in order to avoid the attractor in the positive domain one should assume that (again confirmed by numerics at two loop level):

$$\lambda = \min\{\bar{\lambda} : \forall_\mu \bar{\lambda}(\mu) \geq 0, \bar{\lambda}(M_P) \approx 0 \text{ and } \beta_{\bar{\lambda}}, (M_P) \approx 0\} \quad (7)$$

which agrees with the arguments of the authors of [20]. Then one needs stable EW vacuum in order to predict the Higgs mass in the line of [20].

However the current calculations of running of $\lambda(\mu)$ shows that $\lambda(\mu)$ drops to negative values at roughly 10^{10} GeV [36, 37], making the vacuum metastable. Also the situation is similar if one takes into account the non-minimal $H^\dagger H R$ term [38]. The current estimation of the lower stability bound is $M_H > (129.6 \pm 1.5)$ GeV [36]. On the other hand from the experimental point of view the Higgs mass is constrained as: $M_H = 125.18 \pm 0.18$ GeV [39], which corresponds to $\lambda(M_{\text{top}}) = 0.127823 \pm 0.000367$ in $\overline{MS}-$scheme for one-loop matching conditions [31, 32, 40] (and $\lambda = 0.12924 \pm 0.00037$ at the tree level), where we have taken into account the uncertainties in the measurements of the top quark $M_{\text{top}} = 173.0 \pm 0.4$ [39]. Hence the stability of EW vacuum, assumed in [20] is in contradistinction with the measured Higgs value. Yet this stability bound is close enough to the experimental value of the Higgs mass, that one can hope that a slight extensions of the SM can bring it to the correct value.

The stability argument can be confirmed by the explicit numerical calculations. Namely, to obtain the prediction for λ we do the two-loop running of the $g_1, g_2, g_3, y_t, \lambda$ with gravitational corrections and search for optimal λ for given set of g_1, g_2, g_3, y_t, such that $\lambda \geq 0$ and there are no Landau Poles (λ does not end in the non-perturbative region). Then given λ one can recover the Higgs mass via matching relations (let us note that we treat v as given from experiment). Since the result is very sensitive to $M_{\rm top}$ the result of [20] might change if one considers current observational bounds of top quark mass.

In our analysis we take one-loop-matched parameters as [36]: $g_1(M_{\rm top}) = 0.35940$, $g_2(M_{\rm top}) = 0.64754$, $g_3(M_{\rm top}) = 1.18823$, and we scan over one-loop matched y_t for various experimentally viable $M_{\rm top}$, giving $y_{top}(M_{\rm top}) = 0.94759 \pm 0.0022$, which is slightly lower than the central value obtained in [36]. As a result we get $\lambda = 0.15102 \pm 0.00158$ giving $m_H \approx 135$ GeV (at one-loop calculations) and $\lambda = 0.13866 \pm 0.00218$ (the uncertainties are due to the y_t coupling) and $m_H \approx 130.5$ GeV. Actually the two-loop result is close to the stability bound of the Higgs mass.

We have checked that if one takes the bottom quark and the taon into account, it changes the predictions for m_H less then 1 KeV, which is far below the theoretical and experimental accuracy. This can be expected since $y_b(M_{\rm top}) \approx 0.015$ [41]. Due to metastability of vacuum we see that it is necessary to introduce the beyond Standard Model operators in order for Higgs mass to be predicted in the asymptotic safety paradigm at the correct experimental value.

3. Beyond Standard Model

3.1. Gravitation constraints

First let us constrain the possible additional matter content. We have:

$$\xi_0 = \frac{1}{16\pi G_N^*} \quad \text{and} \quad G_N^* \approx -\frac{12\pi}{N_S + 2N_D - 3N_V - 46}, \tag{8}$$

where N_S, N_D, N_V are the number of scalars, fermions and vector particles respectively. We know that $G_N(M_{\rm top}) \geq 0$ and the running of G_N cannot change the sign of G_N [16]. Then we have $G_N^* \geq 0$. So the Beyond Standard Model Theories which extend the Standard Model broadly can be incompatible with the asymptotic safety paradigm. For example such theories are MSSM [42] and some of the GUTs [43, 44]. It seems [19, 45] that asymptotic safety prefers the minimal extensions of the Standard Model. However, the actual change of ξ_0 due to small modifications of the SM doesn't alter the predictions at observable level.

3.2. Models

As we have said the prediction of λ depends highly on the initial value of top Yukawa coupling. It also strongly depends on the running of y_t. So changing this running alters the prediction of λ from asymptotic safety. In this paragraph we shall discuss two extensions of the Standard Model, where β_{y_t} is slightly changed, because to predict correct value of λ it seems that only a minor effect is required. In both models we extend the Higgs sector by an additional complex scalar singlet under $SU(3)_c, SU(2)_L \times U(1)_Y$ gauge groups [46]:

$$\mathcal{L}_{\rm scalar} = (D_\mu H)^\dagger (D^\mu H) + (\partial_\mu \phi^* \partial^\mu \phi) - V(H, \phi), \tag{9}$$

$$\begin{aligned} V(H,\phi) = &-m_1^2 H^\dagger H - m_2^2 \phi^\star \phi + \lambda_1 (H^\dagger H)^2 \\ &+ \lambda_2 (\phi^\star \phi)^2 + 2\lambda_3 (H^\dagger H) \phi^\star \phi. \end{aligned} \tag{10}$$

Often one also includes right handed neutrinos coupled to ϕ [28, 47, 48], yet they won't be relevant to our discussion. The inclusion of portal interaction stabilises the vacuum [1, 28] (also with inclusion of higher order operators [49]), yet in our further analysis we shall put $\lambda_3 = 0$. So there will be no portal stabilisation effect. This is in line with the FRG analysis of such models [50], where one needs $\lambda_3 = 0$ at all scales.

3.2.1. Model I

In the Model I the global, not anomalous SM group $U(1)_{B-L}$, related to the baryon minus lepton $(B-L)$ number, is gauged and a new Gauge boson B'_μ [51–53] is introduced. Then the covariant derivatives receives an additional contribution: $D_\mu \to D_\mu + i(\tilde{g} Y + g'_1 Y_{B-L}) B'_\mu$, where Y and Y_{B-L} are hyper-charge and (B-L)-charge respectively. The B'_μ boson becomes massive due to the non-zero vacuum expectation value of ϕ, and the $\tilde{g}$ describes the mixing between Z and Z' after spontaneous symmetry breaking. Following [53] we analyse the "pure" $B-L$ model by assuming that there is no tree level mixing between Z bosons ($\tilde{g}(M_{\rm top}) = 0$), which is supported by the current data [39]. However it might be spoiled by radiative corrections. This model is also supported experimentally, since it is the most popular way of explaining so called B-anomalies [54–57], which are the observed inconsistencies of the SM with experimental data in the bottom quark decays. The beta functions at one-loop level are (for $\tilde{g} = 0$):

$$\hat{\beta}_{g'_1} = 12 g_1'^3, \quad \hat{\beta}_{y_t} = \hat{\beta}_{y_t}^{\rm SM} - \tfrac{2}{3} y_t g_1'^2 \theta(M_{Z'} - \mu). \tag{11}$$

3.2.2. Model II

The Model II is inspired by KSVZ axion [58, 59] and includes new sterile (EW singlets) quarks Q_i charged under $U(1)_{PQ}$ coupled to new scalar:

$$\begin{aligned} \mathcal{L} \; &= \; \mathcal{L}_{\rm fermions} + \mathcal{L}_{\rm Y} + \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm scalar} \\ &+ \; \sum_{i=1}^n \left(\bar{Q}_i D_\mu \gamma^\mu Q_i - y_Q \phi \bar{Q}_i Q_i + \mathbf{h.c.} \right), \end{aligned} \tag{12}$$

where we assume that Yukawa matrix $\mathbf{y}_Q$ to be diagonal and the quarks acquire masses $M_i = y_Q v_\phi/\sqrt{2}$. The "phase" of ϕ is called the axion particle and becomes massive due to instanton effects. This model was proposed to solve the Strong CP problem [60] by spontaneous symmetry breaking of $U(1)_{PQ}$ [61]. As a side comment let us note that asymptotic safety gives a possible explanation to the strong CP problem without axions. The strong CP-violation consists of two terms $\theta_{QCD} = \theta_{topological} + \arg \det M_u M_d$ and in principle $\arg \det M_u M_d$ should give much bigger contribution to the strong CP-violation. By considering the gravitational corrections the following reasoning can, at least partially, explain the smallness of the strong CP-violation effect. Namely, in the case of $\arg \det M_u M_d$ there is no running till at least 7-loops [60] Despite the fact that the gravitational corrections Eq. (5) are extremely small, yet they can overtake the dynamics even in the IR and drop $\arg \det M_u M_d$ to zero, since the matter contributions are $\mathcal{O}\left((\arg \det M_u M_d)^{17}\right)$. In order for gravitational contributions to be dominant one needs $0.01 \gtrsim \arg \det M_u M_d$, which is far beyond the experimental bounds. However, this argument requires a more detailed analysis.
Even with $\lambda_3 = 0$ the running of g_3 is affected by the inclusion of the heavy quarks:

$$16\pi^2 \frac{dg_3}{d\log\mu} = \hat{\beta}(g_3) \to \hat{\beta}(g_3) + \frac{2}{3}\sum_{i=1}^{n} \theta(M_{Q_i} - \mu) g_3^3, \quad (13)$$

which in turn alters the running of y_t. Let us note that if there are many such quarks, then even the asymptotic freedom of the QCD can be spoiled. However, in our analysis we focus on addition of one or two sterile quarks into the SM.

3.3. Calculations

At first let us note that both models agree with the condition given by Eq. (8), with $\xi_1 = 0.02$ and $\xi_2 = 0.023$ accordingly. Asymptotic safety requirement gives restrictions on the couplings of new degrees of freedom. In case of **Model I**, one gets that $g'(M_{\text{top}}) \in [0.0, 0.26]$ (with $a_{g'}, a_{\tilde{g}} = -1$). On the other hand the minimal mass for the sterile quark from **Model II** is $m_Q \sim 100$ TeV, otherwise the running becomes unstable. Furthermore if one includes one more quark, then its mass is of the order of 10^6 TeV giving a huge hierarchy, which seems to be very unnatural. Since then we shall restrict ourselves to one heavy, sterile quark. Below, on Fig. [1] we present the calculations for **Model I** for two loop beta functions for the SM β functions and one-loop g' [62–64]. We have also taken into account the one-loop $\tilde{g}$ corrections, yet they are negligible.

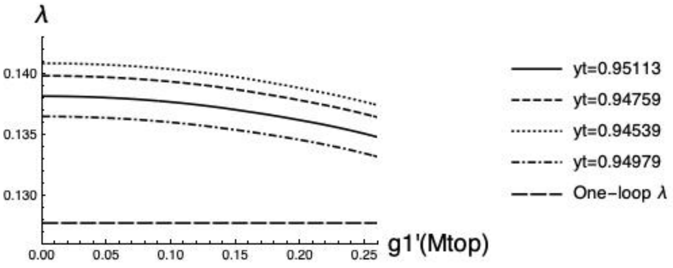

Figure 1: Optimised $\lambda(M_{\text{top}})$ for various g' and y_t

From the Fig. [1] one can observe that none of the y_t and g_1' matches the correct value of λ, yet for $y_t = 0.94539$ one gets $m_H \approx 128$ GeV with $M_{Z'} \approx 200$ GeV, which is closer to the experimental value and below the stability bound for the SM. On the other hand for $M_{Z'} \approx 3$ TeV, one gets $m_H \approx 129.5$ GeV. According to the stability argument [62, 65–67] it is possible that higher order contributions will bring Higgs mass to the desired value in **Model I** for certain space of parameters. The effect of introducing Z' boson can be even more significant if the Higgs boson is also charged under U_{B-L}, see [65]. Yet in such models Z' is highly constraint observationally with $M_{Z'} > 3$ TeV. This shows that it requires further, detailed studies. Yet if one relax the condition $\lambda_3 = 0$ then one immediately get the correct stability of vacuum bound [28, 68, 69] and hence correct Higgs mass without referring to higher loop effects, however this seems to require taking λ_3 as relevant parameter [50, 68].
In the case of **Model II** we perform the full two-loop analysis, with one quark Q and with masses in range $m_Q \in (10^5 - 10^{18})$ GeV. The results are shown below on Fig. [2].

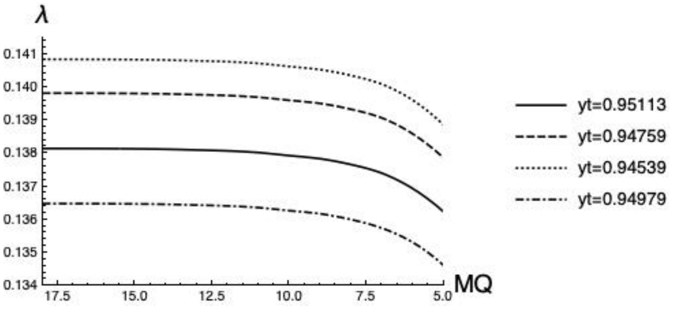

Figure 2: Optimised $\lambda(M_{\text{top}})$ for various M_Q(in logarithmic scale) and y_t

For **Model II** the new degrees of freedom influence the running of λ much less than in **Model I**. The change in predicted Higgs mass at the one-loop matching is of order 1.5 GeV downwards. There are two reasons for that. First of all addition of Q changes only running of g_3, which in turn changes the running of y_t and has only slight effect on λ. Secondly the new degrees of freedom are constrained to have mass far beyond the EW scale, while the Z' mass isn't constrained that much both theoretically and observationally. We can conclude that inclusion of additional sterile quarks cannot drop the Higgs mass to the correct value. Yet maybe **Model II** combined with **Model I** can give the correct Higgs mass.

There are a few issues which require separate discussion. First of all, to obtain the running of the considered couplings one can solve the full Wetterich equation, see for example [21, 70–72]. While Wetterich equation is exact, however it is very difficult (or even impossible) to be solved, because one has to take into account all of the operators which coincide with the symmetries. Moreover one has to choose the cutoff, which is arbitrary [19, 73]. So in order to reproduce the correct perturbative results one has to take into account many higher order operators and choose the proper cutoff. As a state of the art the current FRG calculations match the usual results at the one loop level and the leading contributions to running at two-loop level [72] for pure gauge theories. Moreover the gravitational corrections are ambiguous due to gauge dependence, for example the prediction of top Mass ranges from 130 to 171 GeV only due to this effect [21]. For the sake of phenomenology we decided to use the loop expansion and the EFT gravitational corrections [33] supplemented with the gravitational fixed point calculated with the FRG techniques [45]. Furthermore it seems that these two approaches give similar results (compare the fixed point of top Yukawa coupling in [20] and [21]).

One can also argue that a_i [20, 33, 74] are not calculated to high accuracy, making the whole calculation very sensitive to those parameters, hence not-reliable. This is indeed the case for non-Gaussian fixed point making the prediction of upper bound for top quark mass sensitive to new physics [21]. Yet in the case of Gaussian fixed point the existence of attractive / repelling fixed point at zero is much more vital than actual value of a_λ due to the stability argument.

Finally the ξ_0 depends not only on the matter content, but also on the gravity sector. For example in unimodular gravity [75] it has slightly different value, yet the effect on Higgs mass is negligible (in naive calculations one gets $\mathcal{O}(1\text{ MeV})$), yet it might be interesting to test it in the future. On the other hand there are other more fundamental modifications of gravity, like massive gravity [76] or Horava gravity [77], and their fixed point structure might be completely different. With the right theoretical and experiment accuracy one can test quantum structure of spacetime in particle colliders far below Planck scale.

4. Conclusions

In this article we have recalculated the Higgs mass in the Standard Model by taking into account the gravitational corrections and asymptotic safety requirements using the current observational bounds on M_{top}. From our calculations and due to the stability bound [36] it seems that the Higgs mass is predicted to be a higher than the experimental value. Moreover since $\lambda(\mu) \geq 0$ has to hold, then according to the stability bound one cannot hope that the higher loop effects will drop the Higgs mass to the experimental value.

We have investigated the two beyond SM models which improve the running of λ. In the **Model I** we observed that with $\lambda_3 = 0$ one gets $m_H \approx 128$ GeV as the lowest value, hence the study of higher order corrections is necessary. On the other hand we have excluded the possibility that the addition of sterile quarks gives the correct m_H.

One thing seems certain: addition of new degrees of freedom / higher order operators is required in order to predict correct value of the Higgs Boson mass.

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