Taming Uncertainty

Ralph Hertwig, Timothy J. Pleskac, Thorsten Pachur,
and The Center for Adaptive Rationality

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18 Interpreting Uncertainty: A Brief History of Not Knowing

Anastasia Kozyreva, Timothy J. Pleskac, Thorsten Pachur, and Ralph Hertwig

18.1 Introduction

> I can live with doubt and uncertainty and not knowing. I think it's much more interesting to live not knowing than to have answers which might be wrong. I have approximate answers and possible beliefs and different degrees of uncertainty about different things, but I am not absolutely sure of anything and there are many things I don't know anything about, such as whether it means anything to ask why we're here ... I don't have to know an answer. I don't feel frightened not knowing things, by being lost in a mysterious universe without any purpose, which is the way it really is as far as I can tell.
>
> —Richard Feynman, *The Pleasure of Finding Things Out*

As a physicist, Richard Feynman had trained his mind to embrace the uncertainty that is inherent in the pursuit of scientific knowledge. His words in the epigraph, however, indicate something deeper—a view in which uncertainty touches on the existential state of not knowing that permeates all areas of human life and understanding. This book examines some of the uncertainties that people, both as individuals and in groups, encounter in a range of experiences. The specific focus is on how boundedly rational decision makers find their way despite limited knowledge and environmental complexity. In this final chapter, we turn to the philosophical origins of the concept of uncertainty and explore how its interpretation transformed with time, with each conceptualization of uncertainty being shaped by a different view of rationality.

Our historical survey focuses on a few milestones in the understanding of uncertainty. The first can be found in the philosophical skepticism of ancient Greek and Roman thought. Its main contribution to the topic was

the recognition that human knowledge has limits—human existence is fundamentally intertwined with uncertainty and ignorance—and that the essence of wisdom and rationality consists in finding solutions to this human condition. The second milestone was reached with the emergence of probability and decision theory in the 17th century. These new concepts radically changed the understanding of rationality in Western thought, led to a differentiation between degrees and sources of uncertainty, and offered new tools to deal with the unknown. The next series of crucial developments took place in the 20th century, when a number of thinkers proposed formal normative theories of choice and introduced different interpretations of probability. Each interpretation gave rise to a unique conception of uncertainty in decision making, thereby exposing the limitations of approaches to the unknown based on probability theory. Highlighting these limits opened up space for what is perhaps the most recent milestone in the conceptual journey of uncertainty: the development of the approaches of bounded and ecological rationality. These developments put the focus on the role of heuristic mental tools in conditions of limited knowledge and on the mapping of those tools onto environmental structures. The aim is to describe how real people—as opposed to the idealized "rational men" or optimizing agents implicitly assumed in probability theory—make decisions under uncertainty. In so doing, the approaches of bounded and ecological rationality suggest a new view on uncertainty—a systemic one, in which the uncertain world and the cognitive tools of the human mind are intertwined.

18.2 Uncertainty and Philosophical Skepticism

> For in things uncertain there is nothing probable, but in things where there is probability the wise man will not be at a loss either what to do or what to answer. (Cicero, 45 BCE/1933, p. 609)

The idea that uncertainty is inherent to human life and knowledge has a long history. Roman stoic philosopher Seneca wrote in 49 CE that "all things that are still to come lie in uncertainty" (Seneca, 49 CE/1990, p. 313) and emphasized how dreadful it is to constantly worry about the unknown future (Seneca, ca. 65 CE/2000, p. 121). At least since the time of Socrates, awareness of one's ignorance has been seen as a sign—or even the essence—of

wisdom; and the pursuit of truth and knowledge has been balanced against the recognition of people's epistemic limitations. Socrates himself embodied this balance, coupling a love of wisdom and self-reflective deliberation ("unexamined life is not worth living"; cited in Plato, 1966, 38a) with the acknowledgment of his own ignorance ("what I do not know I do not think I know"; cited in Plato, 1966, 21d).

Skeptical philosophers of the Academic and Pyrrhonist schools became the most devoted scholars of uncertainty in ancient Greek and Roman thought (ca. third century BCE–second century CE). Skepticism as a philosophical view or movement refers to the tradition of thought concerned with the possibility of knowledge and rational beliefs, and ancient skepticism was generally meant to describe "a way of life devoted to inquiry" (Vogt, 2010). Skeptical philosophers were consumed by the idea that neither the human senses nor reason could provide knowledge that is definitely true or false, and widely discussed how to navigate this human condition. Pyrrhonists valued the tranquility of the mind above all and believed it wise to suspend all judgment (*epochê*) and act undogmatically (adapting by default to society's customs; see Annas & Barnes, 2000). They held that uncertainty cannot be remedied by any rational means and should thus simply be accepted as an inevitable condition. Skeptics of the Academic school, by contrast, believed in a life rooted in reason and considered it rational to follow plausible opinions and adopt views that are likely to be true. The ideas of the Academic school on uncertainty are perhaps best captured by the Roman politician and scholar Cicero.[1] Following the Greek skeptic Carneades, he claimed that even if all things uncertain or hidden could not be fully comprehended, a wise person would be foolish not to rely on plausible opinions when it comes to practical matters and decisions (Cicero, 45 BCE/1933, p. 595). In fact, it was Cicero who introduced the terms "uncertain" and "probable" into Latin—and by extension European—philosophical vocabulary. He used *incertus* to translate the Greek term *ádēlos*, meaning not manifest or not evident (p. 535); and *probabilis* for the Greek *pithanós*, meaning "persuasive," "plausible," or "similar to the truth" (*veri simile*; p. 508).

1. In *Academica* (45 BCE/1933), his work on philosophical skepticism, Cicero discussed uncertainty and probability based mainly on the teachings of Carneades (214–129/8 BCE), an Academic skeptic who himself left no writings.

The significance of the skeptical approach was not limited to ancient Greek and Roman thought. After a long hiatus during the Middle Ages, skeptical philosophy experienced a powerful comeback in the late Renaissance and in early modern philosophy (16th and 17th centuries). Thanks in no small part to translations of Sextus Empiricus's *Outlines of Pyrrhonism* and the influential work of Michel de Montaigne in the 16th century, the specter of skepticism returned to haunt European philosophy and quickly became "the avant-garde view of the new intellectual era dawning in early seventeenth-century France" (Popkin, 2003, p. 79). As Popkin noted, the skeptical challenges in the philosophy of the time were so prominent that they encompassed three broad domains of thought: religion, humanism, and science (pp. 54–55). While the Reformation and Counter-Reformation triggered a theological crisis of faith, the crisis of humanistic knowledge was precipitated by the discovery of the New World and its novel cultural universe. The most consequential challenge for the understanding of rationality and uncertainty, however, was the crisis of scientific knowledge that stemmed from the growing realization that establishing a science in the Aristotelian sense of certain truths was impossible. This atmosphere of total skepticism in the 17th century was particularly important because it gave rise to two disparate variants of rationality, each with its own consequences for overcoming this crisis of thought.

One variant of rationality was Cartesian philosophy, with its method of proceeding from universal doubt to establish the absolute certainty of the thinking self (*ego cogito*). Pushed to the extreme, this method allowed Descartes to formulate one unshakable truth—the existence of the thinking being—that served as a first principle for his metaphysical system (Descartes, 1641/1996). From this perspective, the uncertainty of empirical knowledge was still contrasted with the ideal of scientific certainty and demonstrative reason. An alternative solution to the challenges posed by skepticism came from a source slightly more mundane than metaphysics—namely, practical matters such as gambling, legal disputes, and annuities. Contrary to Descartes, Pascal and his colleagues at Port Royal not only accepted uncertainty as an inevitable dimension of human knowledge, but also made use of it in the then-new mathematical calculus of probability (Arnauld & Nicole, 1662/1850; Pascal, 1670/2000). The invention of the calculus of probability marked the emergence of a probabilistic approach to

the unknown. To this day, it remains the most influential contribution to the conceptual comprehension of uncertainty.

18.3 Uncertainty and Classical Probability

> Now when we work for tomorrow, and the uncertain, we are acting reasonably; for we ought to work for the uncertain according to the demonstrated rules of chance. (Pascal, 1670/2000, p. 758)

The skeptical philosophers realized that human cognition exists in an epistemic space between truth and ignorance and that reasonable people should seek methods to overcome the practical and moral difficulties associated with this condition. What separates approaches to uncertainty before and after the 17th century is, above all, a difference in these methods. While Cicero's *sapiens* ("wise man") had to navigate uncertainty according to what is likely and plausible, he had no formal rules for doing so at his disposal. The Enlightenment's own human species, the "reasonable man," could rely on a new mathematical tool: classical probability. From its very beginnings it aimed to assist people in finding better—or even rational—decisions and was generally regarded as a "reasonable calculus" for "reasonable men" (Daston, 1988/1995, p. 56). As Laplace, a key figure in the conceptualization of classical probability, famously put it: probability is simply "common sense reduced to calculus" (Laplace, 1814/1902, p. 196).

The development of this new type of pragmatic rationality was due to two closely connected conceptual events: the emergence of mathematical probability and the invention of decision theory. The first is usually associated with the correspondence between Blaise Pascal and Pierre de Fermat in 1654 that gave rise to the concept of mathematical expectation and the probability calculus (Edwards, 1982; Todhunter, 1865/2014). They were concerned with puzzles such as the "problem of points": two players stake 32 coins each on a three-point game of chance. The game is interrupted at a moment when one player has two points and the other player one point. How should the stakes be divided between the two players? (Pascal 1654/1998, p. 147). The two French mathematicians proposed solutions for this and other problems by determining the chances of uncertain events based on mathematical expectation (Pascal) and rules of combinatorics (de Fermat).

The second crucial event, the invention of decision theory, took place several years later, when Pascal (1670/2000) introduced the famous wager about God's existence in the passage *"Infini-rien"* of his *Pensées*. This step marked one in a series of many applications of mathematical probability to decision making. The question of whether it is rational to believe in God was introduced in the form of a bet: "A game is being played at the extremity of this infinite distance where heads or tails will turn up. What will you wager?" (Pascal, 1670/2000, p. 677). Unlike a typical betting game, the stakes in this "game of life" are infinitely high: if one acts as a believer and God exists, one wins eternal salvation; however, if one wagers against God and God exists, the loss brings eternal damnation. By contrast, if one chooses to live a pious life and God does not exist, one forgoes the finite but still valuable reward of having lived a life of worldly pleasures. What should a person do: act as a believer or as an atheist? Following Pascal's argument, one should reason according to the rules of probability and choose a pious life over a life of worldly pleasures because its expectation—derived on the crucial assumption of a nonzero chance of God's existence times infinite reward—is infinitely greater than that of a "cheerful party-going atheist" life strategy and its finite gratification (Hacking, 2001, pp. 121–122). The value of this argument lies not in any indisputable line of reasoning, but rather in its illustration of a new way of approaching decision problems: using the logic of probability.

A similar probabilistic argument can be found in *Port-Royal Logic*, whose last chapter explicitly examines the numerical measurement of probability. It considers a game where 10 people stake a crown (a French coin) and only one person can win the prize ("there are nine degrees of probability of losing a crown, and only one of gaining the nine"; Arnauld & Nicole, 1662/1850, p. 360). Arnauld and Nicole claimed that this logic for understanding the chances of winning can be applied to other matters, thus making "us more reasonable in our hopes and fears" (p. 361). For instance, the fear of death by thunderstorm, so the argument goes, should be proportionate not only to the magnitude of the possible damage but also to the chance of it occurring. Just as it would be unreasonable to stake too much on a gamble with a high potential payoff but merely a slight chance of winning, it would also be unreasonable to take extreme precautions against highly unlikely events such as death by thunderstorm.

This analogy reveals one of the most important aspects of the classical notion of probability: it combines degrees of subjective beliefs and rules

of chance. The insight that the two can be combined lies at the heart of Hacking's view that the modern notion of probability is essentially dual, as it is related to both "the degree of belief warranted by evidence" (*epistemic* probability) and the stable relative frequencies produced by some chance devices, such as coins or dice (*aleatory* probability; Hacking, 1975/2006, p. 1). Hacking argued that the emergence of probability became possible only when Pascal explicitly linked epistemic and aleatory probabilities and applied them to reasoning about decision making.[2]

Before we move on to modern approaches to probability, let us highlight two key aspects of classical probability: the distinction between epistemic and aleatory uncertainty, and the measurement of uncertainty on a scale from complete knowledge to total ignorance. These aspects still underlie the two main ways of classifying uncertainty in the modern day: one is based on the *source of uncertainty* and the other on the *degree of uncertainty* in one's knowledge (figure 18.1). The sources of uncertainty are characterized as aleatory (objective) or epistemic (subjective), a distinction that stems from the duality of probability discussed above[3] and that has become deeply ingrained in treatments of uncertainty across domains (Budescu & Wallsten, 1987; Elvers, Jandrig, & Tannert, 2007; Fox & Ülkümen, 2011; Kahneman & Tversky, 1982; Thunnissen, 2003; W. E. Walker et al., 2003). In general, epistemic uncertainty refers to incompleteness of knowledge, whereas aleatory uncertainty stems from the statistical properties of the environment that exist independent of a person's knowledge.

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2. Leibniz followed the same logic in applying the calculus of probability to legal reasoning, as did Jakob Bernoulli when he ordered different degrees of assurance as "ranging from total disbelief or doubt to greatest certainty" (Gigerenzer et al., 1989, p. 7).

3. This duality of epistemic and aleatory aspects of probability did not amount to different interpretations of probability in the classical period, which only became the case in the 20th century. The subjective and objective sides of probability initially enjoyed a rather unproblematic coexistence; the end of the classical interpretation was closely related to the emergence of the *clear-cut* differentiation around 1837–1843 (Daston, 1988/1995). This aspect of the classical approach to probability is in line with the fact that early theoreticians of probability—from Pascal to Laplace—generally committed to the deterministic view of the universe and therefore regarded uncertainty as belonging to people's epistemic limitations or ignorance. This view was famously expressed in Laplace's (1814/1902) *Philosophical Essay on Probabilities*, in which he claimed that a sufficiently vast intelligence (Laplace's demon) could calculate the entire future of the universe and eliminate all uncertainty inherent to the current state of human knowledge.

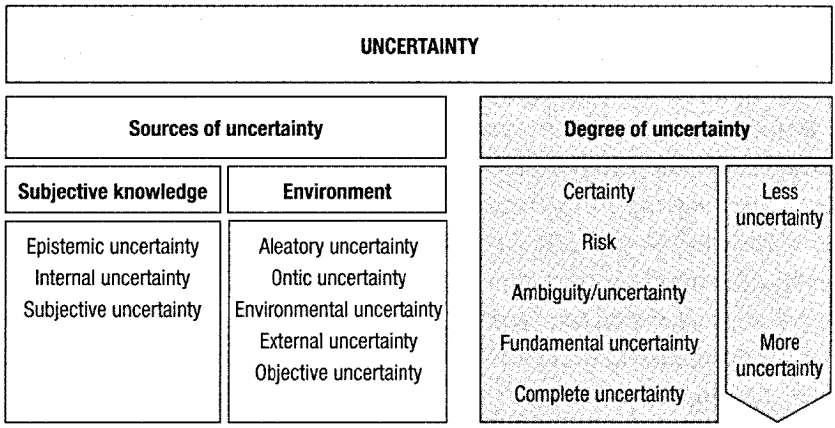

Figure 18.1
Variants of uncertainty. Uncertainty has frequently been classified in two fundamentally different ways, with one focusing on the person or the world as the source of uncertainty and the other focusing on degrees of uncertainty. The concepts listed represent an incomplete collection of terminology that has been used in the literature.

The aleatory–epistemic distinction has been shown to be psychologically meaningful as well. For instance, Fox and Ülkümen (2011) have argued that judgments under uncertainty comprise attributions to both epistemic and aleatory sources of uncertainty, which reflect different coping strategies and have distinct markers in natural language (p. 24). In this view, aleatory or environmental uncertainty is attributed to stochastic outcomes of external events and is expressed in likelihood statements (e.g., "I'd say there is a 90% chance," or "I think there is a high probability"); epistemic uncertainty, by contrast, is attributed to inadequate or missing information on the part of the subject and is expressed in confidence statements (e.g., "I am 80% sure," or "I am reasonably certain"; see also Ülkümen, Fox, & Malle, 2016). The distinction between epistemic and environmental uncertainty is addressed in various ways throughout this book (e.g., chapters 3, 7, 8, 12, and 16), showing that uncertainty can ensue either from an individual's epistemic constraints or from the properties of the world that an individual experiences. Sources of uncertainty become even more varied when one takes into account situations involving other people. For instance, strategic uncertainty associated with unknown actions and preferences of interacting

agents can be an additional source of uncertainty in the social environment (see chapters 5, 12). Social uncertainty can also constitute the main source of uncertainty in collective decision making (see chapters 13, 14).

The second type of classification refers to the degree of uncertainty and its quantitative and qualitative measurement. Mapping uncertainty according to degrees of subjective knowledge predates quantitative probability theory, but it was the probabilistic approach that first made defining and measuring degrees of uncertainty possible. Modern decision sciences have arguably adopted more or less the same logic of measuring epistemic uncertainty as was outlined in the 17th century—namely, distinguishing between different uncertainty situations as a function of the amount and nature of information available to the individual. Although degrees of knowledge in human judgment can range from total ignorance to complete certainty, it has become customary to classify such degrees in terms of *certainty*, *risk*, and *uncertainty* (Luce & Raiffa, 1957), with the last often referred to as *ambiguity* in economics (Ellsberg, 1961; Trautmann & van de Kuilen, 2015). The scale of uncertainty further includes fundamental and severe uncertainty (see, e.g., Bradley & Steele, 2015; Mousavi & Gigerenzer, 2014), as well as the extreme state of complete uncertainty and total ignorance.

Distinguishing between degrees of uncertainty has become equally relevant in behavioral decision theory. Ward Edwards's seminal paper "The Theory of Decision Making" (1954) introduced the distinction between risk and uncertainty to behavioral research. At the time, decision scientists were primarily focused on studies of risky choice, as exemplified in experiments using fully described monetary gambles—a line of research that culminated in the development of prospect theory (Kahneman & Tversky, 1979; see chapter 8). Expanding decision research to areas beyond risky choice led to the discovery of distinct patterns of behavior in decisions under risk (description) and uncertainty (experience), known as the description–experience gap (see Hertwig & Erev, 2009). As shown in chapters 7 and 8, research on decisions from experience demonstrated the empirical value of this distinction by revealing systematic behavioral differences between description-based and experience-based choice (see also chapter 10).

The integration of these classical distinctions between different sources and degrees of uncertainty into research in cognitive science shows that they are not merely philosophically meaningful—they also represent relevant

psychological categories that describe how people distinguish between different sources or degrees of uncertainty. However, in order to understand the conceptual origins of uncertainty in decision research, it is important to look into the developments that probabilistic interpretations of uncertainty underwent in the 20th century. These developments highlighted the limits of quantitative approaches to uncertainty based on probability theory.

18.4 Uncertainty and Modern Interpretations of Probability

The value of classical probability consisted in its applicability to practical problems, ranging from gambling, legal disputes, and matters of insurance to questions of moral conduct and religious commitment. In the 20th century, the scope of probability theory and statistical thinking extended to include the natural and social sciences, as well as parts of law, medicine, and industry (Gigerenzer et al., 1989, p. 271). At the same time, several crucial developments occurred in both basic mathematical theory and its philosophical interpretation. One turning point was Kolmogorov's (1933) axiomatization of probability theory, which elevated the theory to the status of a legitimate branch of mathematics. The other development was more philosophical in nature: as distinct interpretations of probability unfolded, the question arose of what probability was supposed to measure. The three most prominent philosophical interpretations of probability advanced in the 20th century were objective, subjective, and logical (Gillies, 2000). Importantly, these delineations did not concern the mathematical calculus of probability, but only the interpretation of its extramathematical and philosophical properties. Whereas objectivists maintained that probability represents the physical frequencies (von Mises, 1928/1961) or propensities (Popper, 1959) of external events, subjectivists held that if reflects degrees of personal confidence (De Finetti, 1931/1989, 1937; Ramsey, 1926). According to the logical interpretation, probability is neither purely objective nor subjective, but rather relational, because it expresses degrees of certainty given the available evidence (Carnap, 1950; Keynes, 1921/1973b).

These three interpretations of probability played a major role in the theories of uncertainty developed in the 20th century, arguably accounting for three different ways of explaining relations between uncertainty and probability. Knight's objective interpretation limited probabilistic calculations

to risk (i.e., measurable uncertainty) and liberated true uncertainty from calculations altogether (Knight, 1921/2002). Keynes, as one of the main proponents of a logical interpretation, offered an account of unmeasurable uncertainty that reflected cases with missing evidence or with "probability-relation," as he called it (Keynes, 1921/1973b; see section 18.4.2). Finally, the subjective interpretation of probability in Savage's work placed uncertainty in the realm of measurable probability, thus including personal beliefs as something that could be assigned a probability (Savage, 1954). The resulting concepts of true and unmeasurable uncertainty proposed by Knight and Keynes, as well as the incorporation of the Knightian distinction between risk and uncertainty in modern decision theory, remain among the century's most impactful legacies for understanding uncertainty in decision making.

18.4.1 Knightian Uncertainty

In his book *Risk, Uncertainty, and Profit* (1921/2002), economist Frank Knight introduced the distinction between risk and uncertainty. Whereas risk applies to situations of measurable probability, uncertainty applies to situations where such measurement is not possible, causing people to rely on "estimates," a concept to which we will return shortly. The distinction between risk and uncertainty reflects Knight's objective interpretation of probability, which he considered to be a property of the real external world. Knight further distinguished three "probability situations," each representing a distinct type of judgment about uncertain events: a priori probability, statistical probability, and estimates (Knight, 1921/2002, pp. 224–225). In *a priori probability*, the chances of random events occurring can be computed based on general mathematical principles. Examples can be found primarily in games of chance (e.g., the throw of a die), but are rare in real life. *Statistical probability* refers to the chances of events occurring that can be determined empirically by applying statistical methods to experiential data. Examples include an insurance company determining whether a building will burn, or a champagne producer quantifying the risk of bottles bursting. Finally, *estimates* describe situations of unmeasurable uncertainty, which concern unique or unprecedented events and situations. These singular events require practical judgments or decisions to be made in the absence of a "*valid basis of any kind* for classifying instances" (p. 225), and are therefore not amenable

to objective probabilistic or statistical calculations. In Knight's view, "most of conduct" (p. 226) as well as typical business decisions (e.g., a substantial investment to increase a manufacturer's capacity) rest on an estimate of the probable outcome of the considered course of action. Here, uncertainty is a matter of degree—not quantitative in a mathematical or statistical sense, but subjective, as a degree of confidence—which is, in Knight's view, itself of practical significance. As Knight supported an objective interpretation of probability, he lamented that the term "probability" was applied to estimates ("the usage is so well established that there is no hope of getting away from it," p. 224) and proposed that these estimates should at least be treated as a distinct type of judgment.

18.4.2 Keynesian Uncertainty

Like Knight, John Maynard Keynes also placed uncertainty at the heart of his economic theory. However, he proposed a different interpretation of probability, viewing it in terms of a logical relation between a hypothesis and the supporting evidence (Keynes, 1921/1973b). This relational approach imposes certain restrictions on what can be expressed by probability. More specifically, Keynes discussed cases when a probability relation is unknown or numerically immeasurable, as, for instance, when relevant evidence is missing or cannot be meaningfully used to support a proposition (Keynes, 1921/1973b; see also Lawson, 1985). In this view, fundamentally uncertain situations—such as those referring to the distant future—might lie beyond the numerical probability relation and allow for only very weak confidence. Consistent with this interpretation of probability, in *The General Theory of Employment, Interest, and Money* (1936/1973a; see also 1937), Keynes characterized uncertainty as strongly linked to the future and long-term expectations.[4] From this perspective, uncertainty is more or less equivalent to unpredictability, which carries the implication that statistical analyses and the theory of probability are useless for calculating the chances of some future events, because there is no data or evidence that could be used to do

4. Not to be confused with the notion of mathematical expectation, Keynesian expectation has a psychological meaning that designates subjective beliefs and predictions about the future. For example, long-term expectations might concern predictions about future returns on investments, while short-term expectations might designate a producer's estimates of profit from a finished product.

so. Even though there are situations in which it may be possible to make informed and principled forecasts, they should be carefully distinguished from situations that are fundamentally unpredictable. Here, Keynes's examples are similar to Knight's: he also mentions games of chance and insurance cases, in which probability calculations are admissible or even necessary. For other situations, however—such as the prospect of a war, the long-term prices of assets and commodities, or the future of inventions and major technological changes—"there is no scientific basis on which to form any calculable probability whatever. We simply do not know" (Keynes, 1937, p. 214). In sum, Knight and Keynes entertained distinct conceptions of unmeasurable uncertainty, which largely ensued from their different theoretical views of probability. It was, however, a third major interpretation of probability—subjective, or Bayesian—that proved to have the most impact on how modern decision theory conceptualizes uncertainty and, by extension, on the psychology of decision making.

18.4.3 Uncertainty in Bayesian Decision Theory

A key characteristic of modern decision theory is that it has axiomatized principles of rational decision making under uncertainty (Peterson, 2009, p. 13). In his essay "Truth and Probability," Ramsey (1926/1990) proposed axioms for the consistent treatment of subjective probabilities in rational choice under uncertainty. In their groundbreaking book *Theory of Games and Economic Behavior,* von Neumann and Morgenstern (1944/2007) put decision rules and Bernoulli's expected utility theory on axiomatic grounds and established the maximization of expected utility as the dominant rational choice strategy. Although the latter approach was not tied to any particular interpretation of probability, the further development of decision theory took a distinctively subjectivist turn.[5] Notably, in his book *The Foundations of Statistics,* Savage (1954) presented another influential

5. Another important adjustment of the expected utility framework for decision making in Savage's work concerned the normative interpretation of utility theory. Savage himself underlined that his approach followed from the von Neumann–Morgenstern theory of utility, but that "one idea held by me that I think von Neumann and Morgenstern do not explicitly support, and that so far as I know they might not wish to have attributed to them, is the normative interpretation of the theory" (Savage, 1954, p. 97).

axiomatic approach to decision making and the maximization of expected utility; he used the subjective interpretation of probability to extend expected utility to situations with unknown objective probabilities. Savage's subjective expected utility framework incorporates two concepts: a personal utility function capturing the subjective value of the possible outcomes and a personal probability distribution over future states of the world capturing subjective beliefs about the likelihood of those states to be realized. Savage argued that a rational decision maker should comply with the axioms of internal coherence and choose the option with the highest expected utility, taking into account both subjective probability and the desirability of the option relative to all other options.

Luce and Raiffa (1957) made what may be the clearest contribution of Bayesian decision theory to the conceptual shaping of uncertainty when they distinguished between certainty, risk, and uncertainty. In *Games and Decisions*, the authors proposed that in situations involving certainty, decision makers know that their actions invariably lead to specific outcomes. In decision making under risk, "each action leads to one of a set of possible specific outcomes, each outcome occurring with a known probability. The probabilities are assumed to be known to the decision maker" (Luce & Raiffa, 1957, p. 13). In contrast, the realm of decision making under uncertainty encompasses situations in which "either action or both has as its consequence a set of possible specific outcomes, but where the probabilities of these outcomes are completely unknown or are not even meaningful" (Luce & Raiffa, 1957, p. 13).

The main application of expected utility theory is in decision making under risk (i.e., decisions involving outcomes with known probabilities). Nevertheless, in the form of subjective expected utility theory, it is similarly applicable to decisions under uncertainty (i.e., decisions involving outcomes with unknown probabilities). Moreover, assuming a subjective interpretation of probability, situations of uncertainty and ambiguity can be reduced to risk by assigning probabilities to possible outcomes based on the individual's subjective degrees of belief. This point is important: in the framework of Bayesian decision theory, uncertainty is probabilistically unmeasurable only in the sense that it cannot be assigned objectively known probabilities. Yet, as long as one can form subjective distributions of probability that are consistent and add up to 1, uncertainty can be treated in a way similar to

risk. According to Ellsberg's (1961) critical portrayal of this subjectivist spirit: "for a 'rational' man—*all* uncertainty can be reduced to *risks*" (p. 645).

In this regard, the Bayesian decision-theoretical interpretation of uncertainty differs considerably from Knight's and Keynes's notions of unmeasurable uncertainty. Whereas Knight's understanding of uncertainty was motivated by an objective approach to probability, and Keynesian uncertainty hinged on Keynes's logical interpretation, Luce and Raiffa (1957) used Savage's subjective expected utility theory as their theoretical framework. This framework largely motivated their succinct "certainty–risk–uncertainty" classification based on clear criteria of actions, outcomes, and the associated probabilities of these outcomes. The resulting classification continues to underlie most contemporary views on uncertainty in decision making.

The limitations of this framework did not escape the attention of decision theorists. Ellsberg (1961) showed that people violate Savage's axioms when presented with gambles that involve risk (i.e., known probabilities of the outcomes) and ambiguity (i.e., unknown probabilities of the outcomes). In his 1955 paper "A Behavioral Model of Rational Choice," Simon attacked the normative approach to rationality provided by subjective expected utility theory, highlighting that the rules and choice criteria it imposes on the human mind are unrealistic (Simon, 1955). Savage himself was well aware that he was presenting "a certain abstract theory of the behavior of a highly idealized person faced with uncertainty" (Savage, 1954, p. 5). Notably, his distinction between small and large worlds shows that Savage supported the application of his axiomatic approach and, by extension, Bayesian decision theory, mainly to the limited realm of small-world choice problems. Under small-world situations, all possible actions and their consequences can be anticipated, enumerated, and ordered according to subjective preferences. In Savage's words, you have enough information about your decision situation and can "look before you leap" (1954, p. 16). There is no possibility of surprises, pleasant or unpleasant. In large worlds, however, you must "cross th[e] bridge when you come to it" (Savage, 1954, p. 16)—that is, you must make a decision without being able to fully anticipate all decision variables in the present and in the future (partial or complete ignorance). In these worlds, "knee-jerk consistency is then no virtue" (Binmore, 2009, p. 117). Binmore emphasized that Savage restricted the sensible application of his theory to small worlds and in fact devoted the latter half of *The Foundations of*

Statistics to understanding decision making under complete ignorance—by invoking *heuristic principles*.

Notwithstanding these issues, the probabilistic approach to uncertainty remains to this day the default in rational and behavioral theories of choice. The Bayesian decision theorist Lindley (2014) went so far as to claim that "if you have a situation in which uncertainty plays a role, then probability is the tool you have to use" (Lindley, 2014, p. 376). It is undoubtedly a valuable tool that can substantially facilitate how people deal with uncertainty, and as a research methodology it is indispensable. However, when viewed from a historical perspective, the relation between uncertainty and probability turns out to be less harmonious than one might think. The realization that the probabilistic framework is hardly a "one size fits all" approach, and that it cannot account for unmeasurable uncertainty opened up the possibility that other tools are at work in taming the unknown. This brings us to what we consider to be the most recent milestone in the conceptual history of uncertainty: the adaptive and ecological approach to human rationality taken in this book.

18.5 Uncertainty in Bounded and Ecological Rationality

> Bounded rationality [is] the key to understanding how actual people make decisions without utilities and probabilities. (Gigerenzer & Selten, 2001, p. i)

The probabilistic revolution of the 17th century was a revolution in both methods and theoretical views of rationality. It turned uncertainty into a measurable notion and provided consistent rules for rational behavior under conditions of incomplete knowledge. As probability theory developed, diverse interpretations of probability revealed the limits of quantitative approaches to the unknown and further refined the conceptual meaning of uncertainty. Partly as a response to these developments and partly due to progress in cognitive science in the second half of the 20th century, approaches to rationality and uncertainty experienced another revolution. This revolution was inspired by both advances in behavioral research on decision making (e.g., Edwards, 1954) and theoretical developments in normative theories of choice. Simon's notion of bounded rationality stood at the intersection of these advances. His goal was to formulate a psychologically realistic theory of rational choice that would offer insight into how real people make decisions

and how they can achieve their goals under the unmeasurable uncertainty and constraints of the real world (Simon, 1955, 1956).

Simon's main concern with the dominant normative models of choice (e.g., the subjective expected utility theory built upon the Bayesian interpretation of probability and Bernoulli's expected utility theory) was that their norms and postulates were inappropriate as descriptive accounts of behavior in real-world environments (see chapter 2). He called these theories "Olympian models" to reflect their idealistic assumption that decision makers are omniscient, with unlimited cognitive resources (Simon, 1983). He argued that the more successful tools of human rationality would need to differ considerably from the tools offered by the existing normative approaches. When faced with a choice where the computational costs of finding an optimal solution are too high or where such a solution is not available, the decision maker may look for an approximate and "good enough" solution instead of an optimal one (Simon, 1982, p. 295). Simon's ideas of satisficing and bounded rationality set in motion psychological research on boundedly rational heuristics—simple, adaptive tools—that, based on a modest amount of effort and computation, can render good performance possible (Gigerenzer, Hertwig, & Pachur, 2011; Gigerenzer, Todd, & the ABC Research Group, 1999; Hertwig, Hoffrage, & the ABC Research Group, 2013; Todd, Gigerenzer, & the ABC Research Group, 2012).

We should note there is also a somewhat different and influential view of heuristics in psychology. In their heuristics-and-biases research program, Kahneman and Tversky mapped the impact of cognitive limitations on people's judgments and decisions and documented a large catalog of systematic deviations from norms of rationality, drawn from probability theory, statistics, and axioms of rational choice (Kahneman, Slovic, & Tversky, 1982; Tversky & Kahneman, 1974). From this perspective, bounded rationality was interpreted in terms of behavior that diverges from such norms and, by extension, optimality. In this spirit, Kahneman (2003a) concluded: "Our research attempted to obtain a map of bounded rationality, by exploring the systematic biases that separate the beliefs that people have and the choices they make from the optimal beliefs and choices assumed in rational-agent models" (p. 1449).

By contrast, our view is that heuristics and the other tools of the adaptive toolbox (see chapter 1) offer a different way to deal with the type of uncertainty that people—as well as perhaps other species—commonly encounter

in their daily lives. Far from optimizing tools provided by the calculus of probability and axioms of Bayesian decision theory, heuristics are "approximate methods" adapted to particular environments (Simon, 1990, p. 6). This last characteristic is of particular note: the efficiency and the success of decision strategies ultimately depend on how well they fit specific choice conditions. In this view, the essence of rational behavior consists in how an organism can adapt in order to achieve its goals under the constraints posed by both the environment and its own cognitive limitations. Here, rationality is defined in terms of correspondence rather than the content-free norms of coherence (Hammond, 2000).

In the concept of ecological rationality, the importance of the environment is emphasized even further. Rather than considering the organism and the environment as two independent but related systems (Brunswik, 1957/2001), this perspective views the organism and the environment as part of one shared system. The environment is not divorced from the agent; instead, it represents the "subjective ecology of the organism that emerges through the interaction of its mind, body, and sensory organs with its physical environment" (Todd & Gigerenzer, 2012, p. 18).

Viewing the organism and the environment as interdependent components of one system changes the conception of uncertainty. The contribution of the ecological approach consists in replacing the dualistic view invoked by distinguishing two major sources of uncertainty with a synthesis of aleatory and epistemic uncertainty. Uncertainty is thus no longer to be blamed solely on the actor or the environment but instead emerges as a property of the mind–environment system (Todd & Gigerenzer, 2012, p. 18). This suggests a *systemic* view in which uncertainty comprises both environmental unpredictability and uncertainties that stem from the mind's boundaries (e.g., limits in available knowledge and cognitive capabilities). As a consequence of the interdependence of mind and environment, domain-specific rather than domain-general methods are required to make decisions in the face of uncertainty (see also Arkes, Gigerenzer, & Hertwig, 2016). Different environments represent different statistical structures and affordances, which can arise naturally or through design from a variety of environmental considerations—including physical, biological, social, and cultural factors. Ecological rationality means that specific decision-making strategies fit to particular environmental structures but not to others. It suggests that there cannot be a single universal and domain-general tool—whether Bayesian

decision theory, expected utility theory, or game theory—for making decisions. Rather, the concept of ecological rationality is linked to that of the adaptive toolbox (see chapter 1). The toolbox has often been thought to include a wide range of heuristics for games against nature and social games (Gigerenzer et al., 2011; Hertwig et al., 2013). This book proposes a more encompassing conception of the adaptive toolbox. Heuristics—as important as they are—are not the only tools at a person's disposal. They are complemented by tools such as learning through experience, social intelligence, aggregation of information, and others not contained within these pages (e.g., deliberate ignorance; see Hertwig & Engel, 2016). This rich repertoire helps people, both as individuals and as groups, to navigate different regions of uncertainty across the mind's lifelong development.

Taming Uncertainty focuses on several aspects of this systemic view of uncertainty. The first is that the tools people use to handle uncertainty cannot be understood separately from the structure of their environment. Part II, "The Heuristic Mind," zooms in on this very topic, analyzing simple cognitive strategies that embody strong but successful assumptions about the environment. A second aspect, the focus of Part III, "The Exploring Mind," is that in order to understand how the mind copes with uncertainty one must understand how the mind and the environment intersect—in particular, how the mind searches for and learns from information in the world. Part IV, "The Social Mind," highlights a third aspect—namely, that uncertainty looms large in environments shared with others—and examines the tools people enlist to deal with this source of unpredictability and how they turn the collective mind itself into a tool. Finally, taking a systemic view of uncertainty highlighted a new issue: the mind–environment interface is not stationary because its components are subject to change. Recognizing this, Part V, "The Unfinished Mind," turns to the ontogenetic and phylogenetic changes of the human mind and ever-changing environmental demands.

18.6 Optimism in the Face of Uncertainty

In this concluding chapter, we have presented a brief conceptual history of uncertainty as seen through the lenses of different approaches to human rationality: starting with skeptical philosophy, discussing in detail the time-honored probabilistic approach, and concluding with an ecological

perspective. This look back across the larger conceptual map of uncertainty allows us to see that different theories of rationality offer different *interpretations* of uncertainty and different *tools* for dealing with it. In particular, the ecological perspective on uncertainty espoused in this book suggests that the human mind has developed various mental tools to adapt to its environment and navigate the numerous ways in which the unknown takes shape.

After looking at the past and the present, it is natural to turn to the future. But predicting the future is difficult—remember Günter Schabowski and the fall of the Berlin Wall (see chapter 1). There are many factors that seem to accentuate this difficulty today. The simultaneous developments of digitalization, automation, and globalization, compounded with challenges such as social inequality, climate change, exploding health care costs, migration, aging societies, terrorism, and the rise of authoritarian nationalism and demagogic populism—to name just a few—are sources of a soaring sense of uncertainty that evokes strong emotional reactions. The world, however, has always been uncertain; living with uncertainty has been and will continue to be a part of the human condition. Modern human societies are also seeing cumulative progress on many fronts—including better medicine, sanitation, rising IQ and life expectancy, and less violence and poverty than ever before. Pessimism has its place: it encourages individuals and collectives to be cautious and can even foster progress by preparing the ground for necessary change. But human progress needs more than pessimism and caution. The authors of this book remain optimistic. Throughout the history of thought, humankind has developed an ever-better understanding of how to cope with uncertainty. What this book aims to contribute is a deeper insight into how real people act, decide, and grapple with uncertainty. This means replacing omniscience with a limited search for information; substituting complex calculations by simple, tractable, robust solutions; and moving away from optimizing and toward satisficing and exploiting the mind's environment. A better grasp of the rationality of mere mortals will help people—both individually and collectively—to conceive new and humane ways of confronting the inescapable uncertainties of the future.