Taming Uncertainty

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The MIT Press
Cambridge, Massachusetts
London, England

8 The Weight of Uncertain Events

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8.1 The St. Petersburg Paradox

Probably no decision problem has shaped the world of ideas as much as the St. Petersburg game. Originally conceived by Swiss mathematician Nicolas Bernoulli in the early 18th century, it was one of five problems he submitted to Pierre Rémond de Montmort, a French mathematician and man of letters who corresponded with eminent scholars such as Leibniz. Nicolas Bernoulli's cousin Daniel Bernoulli described the St. Petersburg game as follows:

> Peter tosses a coin and continues to do so until it should land "heads" when it comes to the ground. He agrees to give Paul one ducat if he gets "heads" on the very first throw, two ducats if he gets it on the second, four if on the third, eight if on the fourth, and so on, so that with each additional throw the number of ducats he pays is doubled. Suppose we seek to determine the value of Paul's expectation. (Bernoulli, 1738/1954, p. 31)

The puzzling aspect of this seemingly straightforward problem resides in the gap between the "rationally" determined value of the game (i.e., Paul's "expectation") and people's valuations of it (i.e., the price at which they would sell the chance to play the game). According to expected value theory,[1] which was the accepted method of calculating the expectation at the time, the St. Petersburg game gives Paul the opportunity to win an infinite amount of money. Theoretically, he should therefore demand a very high price for it. In reality, "any fairly reasonable man would sell his chance, with great

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1. As explained by Daniel Bernoulli, "[e]xpected values are computed by multiplying each possible gain by the number of ways in which it can occur, and then dividing the sum of these products by the total number of possible cases" (Bernoulli, 1738/1954, p. 23).

pleasure, for twenty ducats" (Bernoulli, 1738/1954, p. 31). This gap between theory and reality constitutes the St. Petersburg paradox.

Why is the expected value of the St. Petersburg game infinitely large? Expected value theory evaluates a risky option in terms of the sum of all the possible payoffs, each multiplied by the probability of its occurrence. The probability of Peter throwing heads the first time is .5; in this case, Paul would receive 1 ducat, thus contributing $1 \times .5$ to his expectation. The probability of Peter throwing tails on his first attempt and heads on his second one is .25, thus contributing $2 \times .25$ to Paul's expectation. More generally, the overall expectation of the game (in which, in theory, the coin could be tossed an infinite number of times) is

$$\sum_{n=1}^{N}\left(\frac{1}{2}^{n} 2^{n-1}\right) = N\frac{1}{2}. \quad (1)$$

From this it follows that, given an infinite number of throws, Paul's expectation is infinite (Menger, 1934). As mentioned above, however, typical valuations of the game are rather modest, often in the range of twice the amount of money one would win if the game were to end with the first throw (e.g., Hayden & Platt, 2009). How is it possible to explain the St. Petersburg paradox—and to reconcile theoretical expectations with people's actual behavior?

Nicolas and Daniel Bernoulli came from one of the most illustrious families in the history of mathematics and science; it produced half a dozen outstanding mathematicians in just a couple of generations. Daniel Bernoulli was intrigued by the St. Petersburg paradox and offered an explanation that was to become the most influential theory of individual decision making under risk—today known as *expected utility theory* (he referred to "moral expectation"; Bernoulli, 1738/1954, p. 24). Specifically, he retained the core of expected value theory—the multiplication and maximization components—but suggested replacing objective monetary amounts with subjective utilities. He argued that the pleasure, or utility, of money does not increase linearly with the monetary amount (as assumed in expected value theory); instead, the increases in utility diminish, with the result that the expected utility of the St. Petersburg game becomes finite (and can become rather small). A century later, the notion of diminishing sensitivity implied by the marginally decreasing utility entered psychophysics in the form of the Weber–Fechner function, and economics in the concept of diminishing returns.

But Daniel Bernoulli's proposal was not the only possible solution to the St. Petersburg paradox raised at the time (for overviews, see Hayden & Platt, 2009; Jorland, 1987; Menger, 1934). Another began with the observation that for all practical purposes, the very rare possibility of the coin landing heads up only after very many tosses, which would lead to a large gain, can be disregarded. Originally suggested by Nicolas Bernoulli himself (see van der Waerden, 1975), this idea was further developed by French naturalist Buffon (1777). In one of the first scientific simulations (see S. M. Stigler, 1991), Buffon had a child perform a series of 2,048 sets of coin tosses; each time the set continued until the coin landed heads up. On the basis of the resulting statistics, he concluded that the value of the game was about 5 ducats, irrespective of its theoretically infinite expected value. Taking this value as an anchor, one may argue that small probabilities in the St. Petersburg game—specifically, probabilities smaller than 3% (i.e., throwing at least five tails in a row)—can, will, or even should receive less weight than their objective magnitude. In the extreme, they may be completely disregarded (there is some debate about the exact threshold below which a probability is disregarded; see Dutka, 1988).

Building on Buffon's account and emphasizing the role of probabilities, Menger (1934) proposed a subjective function across the entire probability range and argued that "chances are undervalued both where the probabilities are very small (that is, close to 0) and where the probabilities are very high (that is, close to 1). Only chances with medium probabilities are valued in a way which begins to correspond to mathematical expectation" (p. 269). Although this explanation of the St. Petersburg paradox focusing on the probability dimension received less attention than Daniel Bernoulli's account, which focused on the monetary dimension, it planted the idea that people do not perceive or treat probabilities linearly when making risky decisions. In the guise of probability weighting, it later became one of the cornerstones of modern descriptive models of decision making under risk (e.g., Birnbaum & Chavez, 1997; Edwards, 1962b; Kahneman & Tversky, 1979; Lopes & Oden, 1999; Luce, 2000; Prelec, 1998; Savage, 1954; Tversky & Fox, 1995; Tversky & Kahneman, 1992).

Probability weighting also came to be used as a framework for conceptualizing and measuring how people respond to uncertainty. For instance, it has been used to characterize decisions from experience, an important type of decision under uncertainty, and to explain how they differ from decisions from description (see chapter 7). In decisions from experience, there is always at least residual uncertainty about the actual probabilities of the events experienced. Several analyses have suggested that one key difference between decisions from experience and decisions from description consists in their different probability weighting patterns (e.g., Glöckner, Hilbig, Henninger, & Fiedler, 2016; Hertwig, Barron, Weber, & Erev, 2004; Kellen, Pachur, & Hertwig, 2016; Regenwetter & Robinson, 2017)—that is, in how much weight people give to the possible outcomes of an option as a function of their probability of occurrence.

Our goal in this chapter is to illustrate that the concept of probability weighting—although rooted in Daniel Bernoulli's utility framework, which Herbert Simon (1955, 1983) criticized for making unrealistic assumptions about the decision maker (see chapter 2)—can help characterize and measure how an adaptive and boundedly rational individual responds to uncertainty. We start by sketching the historical roots of probability weighting (see section 8.2) and reviewing studies that have compared patterns of probability weighting in experience-based and description-based choice (see section 8.3). Some analyses have concluded that people making decisions from experience choose as if they underweight rare events; others have found that rare events are overweighted, even more strongly than in decisions from description. We clarify these divergent conclusions and explain how, although seemingly contradictory, they can in fact coexist. Second, we discuss how a pattern of probability weighting that overweights small probabilities and underweights large probabilities can constitute an adaptive response to uncertainty (see section 8.4). Third, we highlight how heuristics that can be used to tackle uncertainty result in distinct shapes of the probability weighting function (see section 8.5). In other words, we identify which "footprints" specific choice heuristics leave when a probability weighting function is estimated for the choices that they generate.

8.2 A Brief History of Probability Weighting

Buffon's (1777) simulation was probably the first investigation into the role of (small) probabilities in decisions under risk. Probabilities again became an explicit target of interest in the 20th century. Preston and Baratta (1948) pioneered attempts to measure how people treat probabilities when making decisions under risk. Their participants were presented with lotteries offering the chance of winning a number of points with some probability (e.g., 250 points with a probability of 5%) and asked to make a bid for each lottery. By comparing these bids with the lotteries' expected values—determined by multiplying the number of points by the probability of winning—Preston and Baratta sought to establish the "psychological probability" (p. 189) of outcomes. Not surprisingly, participants indicated higher bids for lotteries offering a particular payoff with a higher probability than for lotteries offering the same payoff with a lower probability. However, participants did not seem to weight the possible payoffs according to their probabilities, as expected value theory would predict. Instead, as shown in figure 8.1a, "the mean winning bid exceeds the mathematical expectation for small values of the probability and is less than the mathematical expectation for large values of the probability" (p. 186). In other words, low probabilities seemed to be overweighted in people's bids relative to their objective probability, and high probabilities seemed underweighted. This distortion of objective probabilities in the decision weights that people attach to events seemed to reflect their preferences to engage in risk when making a decision.

Edwards (1955) investigated choices between lotteries and likewise concluded that probabilities, like monetary outcomes, are not taken at face value but are subjectively represented in a distorted fashion (see also Edwards, 1962b; Tversky, 1967). He even went so far as to conclude, more than 200 years after Daniel Bernoulli (1738/1954), that subjective representations of probabilities are much more important than utilities (Edwards, 1955, p. 214).

Several theories have since introduced mathematical functions to describe the transformation of objective probabilities into subjective decision weights (e.g., Birnbaum & Chavez, 1997; Lopes & Oden, 1999). Kahneman and Tversky (1979) coined the term *probability weighting function* for these formal descriptions.[2] Arguably the most prominent theory postulating a probability weighting function is cumulative prospect theory

2. In some cases, the weighting function applies directly to the objective probability of an outcome; in others, it applies to the rank-dependent, cumulative probability distribution (e.g., Quiggin, 1982). For example, when deriving decision weights, π, from cumulative probabilities for a lottery offering €20, €30, or €40 with probabilities of 55%, 30%, and 15%, respectively, the decision weight for the outcome €30 would follow from the (transformed) probability of obtaining €30 or more, which is $w(.30+.15)=w(.45)$, minus the (transformed) probability of obtaining exactly €30, which is $w(.30)$. The function $w(p)$ formalizes the transformation of the probability (see box 8.1 for details).

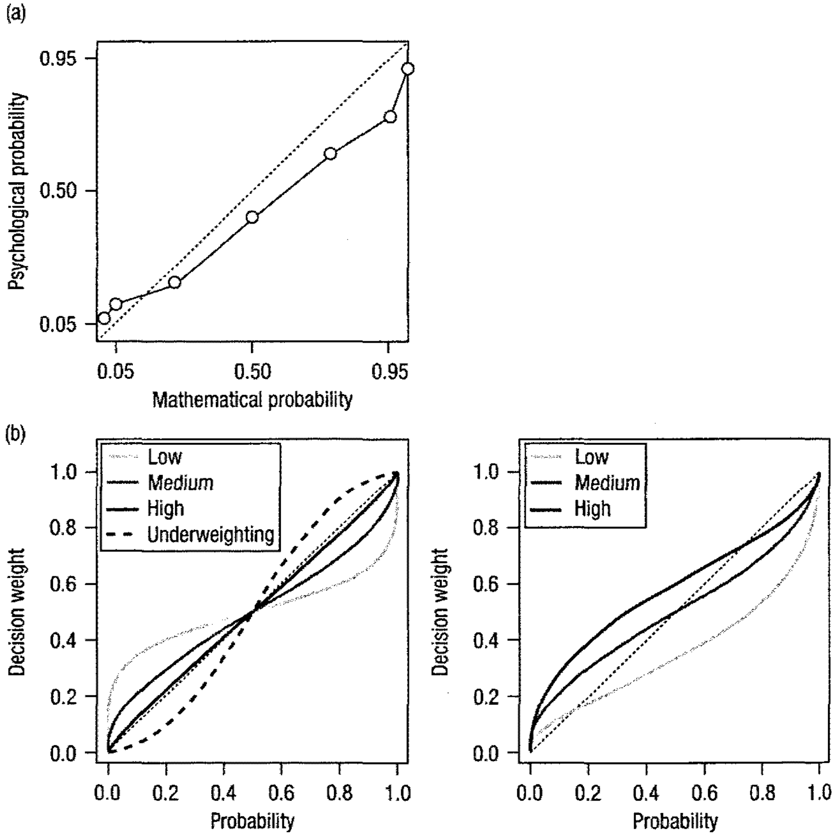

Figure 8.1

(a) Relationship between the objective probability of winning and the weight people seem to attach to the possibility of winning, as derived from bids made in Preston and Baratta (1948). (b) Cumulative prospect theory's weighting function. The left panel shows functions with varying values of the curvature parameter γ (with $\gamma < 1$ yielding overweighting, reflecting low, medium, and high probability sensitivity, and $\gamma > 1$ yielding underweighting). The right panel shows functions with varying values of the elevation parameter δ (with low, medium, and high levels of optimism and pessimism in the gain and loss domain, respectively; see box 8.1 for details).

(Tversky & Kahneman, 1992). Figure 8.1b illustrates cumulative prospect theory's probability weighting function. The function is parameterized, allowing it to assume different types of curvature. Consistent with earlier empirical observations (e.g., Edwards, 1955; Preston & Baratta, 1948; Wu & Gonzalez, 1996; see figure 8.1a), the typically assumed curvature is inverse S-shaped, indicating that small probabilities are overweighted relative to their objective counterparts, and that intermediate and large probabilities are underweighted. The parameterization also allows for different degrees of over- and underweighting (see figure 8.1b). An inverse S-shaped curvature of the weighting function can account for several notable violations of expected value and expected utility theory, such as the fourfold pattern (Tversky & Kahneman, 1992) and the Allais paradox (for an overview, see Camerer & Ho, 1994). Ironically, an inverse S-shaped curvature cannot explain the St. Petersburg paradox, the context from which the concept of probability weighting emerged (Blavatskyy, 2005).

Two main characteristics of cumulative prospect theory's weighting function, which are governed by separate parameters and by which the function can vary gradually, are its curvature and its elevation (see box 8.1 for a formal description or explore the shape of the probability weighting function yourself in interactive element 8.1 at https://taming-uncertainty.mpib-berlin.mpg.de/). As the left panel of figure 8.1b illustrates, the curvature reflects how sensitive a decision maker is to differences in probability; with a more pronounced curvature, the difference between, say, 30% and 50% is less strongly reflected in differences in decision weights than with a less pronounced curvature. As the right panel of figure 8.1b illustrates, the elevation governs the absolute magnitude of the decision weights. For risky gains, a higher elevation implies more optimistic (and thus more risk-seeking) choices than a lower elevation; for instance, a person with a higher elevation would be more willing to take a 10% chance of winning €1,000 (otherwise nothing). For risky losses, a higher elevation implies more pessimistic (and thus more risk-averse) choices. As is also shown in the left panel of figure 8.1b, the parameterization of the probability weighting function also allows it to take an S-shaped form (specifically, when the curvature parameter $\gamma > 1$; see box 8.1), such that small probabilities are underweighted, whereas intermediate and large probabilities are overweighted. This form would be consistent with the solution to the St. Petersburg paradox proposed by Nicolas Bernoulli (see van der Waerden, 1975), Buffon (1777), and Menger (1934).

Box 8.1
Formal description of cumulative prospect theory's weighting function.

To account for choices between options, each having outcomes $x_m > \ldots > x_1 \geq 0 > y_1 > \ldots > y_n$ and corresponding probabilities $p_m \ldots p_1$ and $q_1 \ldots q_n$, cumulative prospect theory assumes a rank-dependent transformation of the outcomes' probabilities into decision weights. More specifically, the weight π^+ (π^-) given to a positive (or negative) outcome is the difference between the probability of receiving an outcome at least as good (or bad) as x (or y) and the probability of receiving an outcome better (or worse) than x (or y):

$$\begin{aligned} \pi_m^+ &= w^+(p_m) \\ \pi_n^- &= w^-(q_n) \\ \pi_i^+ &= w^+(p_i + \cdots + p_m) - w^+(p_{i+1} + \cdots + p_m) \quad \textit{for } 1 \leq i < m \\ \pi_j^- &= w^-(q_j + \cdots + q_n) - w^-(q_{j+1} + \cdots + q_n) \quad \textit{for } 1 \leq j < n. \end{aligned} \tag{B1}$$

The probability weighting functions for gains and losses—w^+ and w^-, respectively—are typically assumed to have an inverse S-shaped curvature, embodying the overweighting of rare events and the underweighting of common events. Different types of weighting functions have been proposed (e.g., Prelec, 1998; Tversky & Kahneman, 1992; for an overview, see Stott, 2006). One of the most common versions features two parameters that separate the curvature of the weighting function from its elevation (e.g., W. M. Goldstein & Einhorn, 1987; R. Gonzalez & Wu, 1999):

$$\begin{aligned} w^+ &= \frac{\delta^+ p^{\gamma^+}}{\delta^+ p^{\gamma^+} + (1-p)^{\gamma^+}} \\ w^- &= \frac{\delta^- q^{\gamma^-}}{\delta^- q^{\gamma^-} + (1-q)^{\gamma^-}}. \end{aligned} \tag{B2}$$

The parameters γ^+ and γ^- (both > 0) govern the function's curvature in the gain and loss domains, respectively, and indicate how sensitive choices are to differences in probability (with smaller values of γ reflecting lower sensitivity). With $\gamma < 1$ the function has an inverse S-shaped form (indicating overweighting of rare events); with $\gamma > 1$ the function has an S-shaped form (indicating underweighting of rare events). The elevation of the weighting function is controlled by the parameters δ^+ and δ^- (both >0), respectively. As highlighted by R. Gonzalez and Wu (1999), the elevation reflects the degree of risk aversion (traditionally assumed to be captured by the curvature of the value function; but see Lopes, 1995; Wakker, 2010), with a lower (or higher) elevation in the gain (or loss) domain indicating higher risk aversion (or pessimism).

In sum, people's decisions under risk indicate that more probable events receive more subjective weight than less probable events, but that rare events are overweighted relative to their probabilities, whereas common events are underweighted. The most prominent weighting function formally characterizing the relationship between probabilities and subjective decision weights was proposed in the context of cumulative prospect theory. It can assume both an inverse S-shaped form (implying overweighting of rare events) and an S-shaped form (implying underweighting of rare events). It also permits different degrees of over- and underweighting. As we discuss next, the concept of probability weighting has also been used to conceptualize and measure differences between decisions under risk, where probabilities of outcomes are explicitly stated, and decisions under uncertainty, where probabilities are only vaguely known or unknown.

8.3 Probability Weighting in Decisions under Uncertainty

What determines the shape of the probability weighting function—that is, how sensitive people's decisions are to differences in probability (expressed in the curvature of the function) and how optimistic they are (expressed in its elevation)? Kahneman and Tversky (1979) speculated that the uncertainty of an outcome could play a role. For example, imagine you are offered the chance of winning €200 as a function of the probability of an epistemic event, such as the probability that the maximum daytime temperature in downtown San Francisco on April 1 next year is between 65 °F and 80 °F. How do you weight the possibility of winning €200 in this option relative to an option where the probability of winning is explicitly stated (e.g., 20%)? Tversky and Fox (1995) developed a formal account of probability weighting for the first kind of option, in which probabilities are uncertain (see also Tversky & Wakker, 1995). A crucial assumption is that before making a choice, decision makers first estimate the probability of an outcome (e.g., that the temperature will be between 65 °F and 80 °F) in such a situation. The decision weights are then inferred from their choices.

To illustrate how uncertainty impacts probability weighting, let us consider a study by Tversky and Fox (1995). Participants chose between options that could lead to different outcomes. In the risk condition, the probabilities of the outcomes were precisely described (e.g., "Receive $75 if the number on a single poker chip drawn from an urn containing 100 chips numbered consecutively from 1 to 100 is between 1 and 25."). In the uncertainty condition, the probabilities were not explicitly stated but participants could consult their memory of the past to estimate them (e.g., "Receive $75 if the maximum daytime temperature in downtown San Francisco on April next year is between 65 °F and 80 °F."). Tversky and Fox compared the weighting functions obtained from participants' choices in the two conditions with regard to their subadditivity, a robust property of empirical decision weights.[3] The greater the subadditivity, the lower the probability sensitivity; in other words, subadditivity implies reduced sensitivity to probabilities. Subadditivity of decision weights was found to be larger under uncertainty than under risk—people's decisions were less attuned to differences in probabilities when the probability information was fraught with uncertainty.

In Tversky and Fox's (1995) study, the probabilities were uncertain because they were expressed in terms of epistemic events (e.g., the temperature at a particular location on a particular day), where people's beliefs were informed by their knowledge stored in memory. We next turn to research that has studied probability weighting when probabilities are initially unknown and people could learn about them by sampling from the environment.

8.3.1 "As-If" Inferences about the Weighting of Objectively Rare Events in Decisions from Experience

The goal of the initial studies on the distinction between decisions from description and decisions from experience was to understand when and why people's choices deviate from expected value maximization—and to examine how this deviation differs between description and experience (Barron & Erev, 2003; Hertwig et al., 2004; E. U. Weber, Shafir, & Blais, 2004). Evidence from this and subsequent research (e.g., Erev, Ert, Plonsky, Cohen, & Cohen, 2017; Wulff, Mergenthaler-Canseco, & Hertwig, 2018) suggests that experience- and description-based choices differ systematically in how they deviate from expected value maximization. Wulff et al. (2018) conducted a meta-analysis of 33 datasets examining the difference

3. More precisely, a weighting function is subadditive when the decision weight for the sum of probabilities of various individual events is smaller than the sum of the decision weights for the probabilities of the individual events.

between description and experience in the average proportion of choices maximizing expected value. When choice problems involved a risky and a safe option—the type of problem commonly used to infer risk preference in economics and psychology—the size of the gap was rather large (on average, about 20 percentage points). Note that this gap in choice is not premised on any particular theory of choice (e.g., one that assumes probability weighting).

Another perspective on the description–experience gap was prompted by the way Hertwig et al. (2004), Barron and Erev (2003), and E. U. Weber et al. (2004) summarized their findings. These authors used an as-if probability-weighting terminology stating, for instance, that "observed choices indicated not overweighting of small-probability outcomes (henceforth, *rare events*), but rather the opposite: people made choices as if they underweighted rare events; that is, rare events received less weight than their objective probability of occurrence warranted" (Hertwig et al., 2004, p. 535). This and similar statements have invited the interpretation that the description–experience gap consists mainly of a reversal of the probability-weighting pattern. Consequently, numerous investigations have compared probability weighting in experience and description (e.g., Abdellaoui, L'Haridon, & Paraschiv, 2011; Glöckner et al., 2016; Hau, Pleskac, Kiefer, & Hertwig, 2008; Kellen et al., 2016; Lejarraga, Pachur, Frey, & Hertwig, 2016; Ungemach, Chater, & Stewart, 2009).

Like Kahneman and Tversky (1979), both Hertwig et al. (2004) and Barron and Erev (2003) made inferences about people's probability weighting directly from the observed choices. They did not formally estimate a weighting function, nor were their analyses conditioned on people's actual experience. In Hertwig et al. (2004), for instance, lottery problems were selected such that systematically different patterns of choices would result if rare events were accorded less weight in experience than in description. As a consequence, the weighting was meant in an as-if sense (i.e., people behaved as if rare events had less impact than they deserved). Further, the as-if weights referred to the objective probabilities of the outcome distributions—that is, the probabilities that governed the options' payoff distributions—not to the relative frequencies of the events that people had actually experienced.

To date, Regenwetter and Robinson (2017) have conducted the most systematic comparison of the weighting of objective probabilities in description and experience. After accounting for individual heterogeneity in preferences (their paramount concern), they found strong evidence for a description–experience gap in probability weighting and a choice pattern consistent with the original conclusion that, relative to the objective probabilities, rare events are overweighted in description and underweighted in experience (for more details, see table 1 in Hertwig & Pleskac, 2018).

8.3.2 Weighting of Rarely Experienced Events

However, researchers soon began, for good reasons, to measure probability weighting in decisions from experience based on people's actual samples of outcomes, rather than the objective probabilities. Is the shape of a probability weighting function estimated on the basis of the relative frequency with which events were actually experienced still different from that estimated for decisions from description? Are rare events still underweighted? Such underweighting could occur due to factors such as recency effects in memory (Ashby & Rakow, 2014; Wulff & Pachur, 2016). It is important to note that even if description-based and experience-based decision weights did not differ, choices in a given lottery problem could still systematically diverge: relative experienced frequencies (or the perception thereof) can deviate systematically from objective probabilities due to sampling error, recency, and other factors.

Table 8.1 (adapted from Wulff et al., 2018) shows the results of studies that have estimated probability weighting functions for decisions from experience. As can be seen, there is considerable heterogeneity, with some studies finding probability weighting functions consistent with the underweighting of rare events and others finding evidence for overweighting. However, several of these analyses have methodological limitations. Take, for instance, the study by Ungemach et al. (2009), who obtained evidence for underweighting. In their analyses, parameters were estimated based on a set of only six choice problems, and data from all participants were aggregated for the estimation. Both aspects are likely to compromise the robustness of the results (e.g., Broomell & Bhatia, 2014; Estes & Maddox, 2005; Regenwetter & Robinson, 2017).

In a more rigorous analysis, Kellen et al. (2016) asked each participant to make both experience-based and description-based decisions on a total of 114 choice problems from the gain, loss, and mixed domains. Each individual's decisions were modeled with cumulative prospect theory, separately

Table 8.1
Summary of parameter estimates on experience-based probability weighting in decisions from experience (adapted from Wulff et al., 2018).

Study	Description	Experience	Inferred weighting of rare events
Sampling paradigm			
Hau et al. (2008)		$\gamma=0.99$	Linear weighting
Ungemach et al. (2009)	—	$\gamma^+>1$ $\gamma^->1$	Underweighting
	—	$\gamma^+=[0, 2]$ $\gamma^-=[0, 2]$	—
	—	$\gamma^+>1$[a] $\gamma^->1$	Underweighting
	—	$\gamma^+>1$[a] $\gamma^->1$	Underweighting
Camilleri & Newell (2011b)	—	$\gamma=[0, 2]$	—
Camilleri & Newell (2013)	$\gamma=[0, 2]$	$\gamma>1$	Underweighting
	$\gamma=[0, 2]$	$\gamma>1$	Underweighting
Frey, Mata, & Hertwig (2015)	—	$\gamma^+=1.3$ $(\delta^+=1)$ $\gamma^-=1.35$ $(\delta^-=1)$	Underweighting
	—	$\gamma^+=1.03$ $(\delta^+=1)$ $\gamma^-=1.05$ $(\delta^-=1)$	Underweighting
Lejarraga et al. (2016)	$\gamma=0.89$ $(\delta=0.96)$	$\gamma=0.81$ $(\delta=0.87)$	Overweighting
	$\gamma=0.20$ $(\delta=4.33)$	$\gamma=0.53$ $(\delta=3.82)$	Overweighting
Glöckner et al. (2016)	$\gamma=0.73$ $(\delta=0.55)$	$\gamma=0.56$ $(\delta=0.55)$	Overweighting
	$\gamma=0.73$ $(\delta=0.32)$	$\gamma=0.55$ $(\delta=0.39)$	Overweighting
	$\gamma=0.96$ $(\delta=0.70)$	$\gamma=0.55$ $(\delta=0.48)$	Overweighting
	$\gamma=0.65$ $(\delta=0.80)$	$\gamma=0.42$ $(\delta=0.79)$	Overweighting
	$\gamma=0.59$ $(\delta=0.96)$	$\gamma=0.91$ $(\delta=1.04)$	Overweighting, but less pronounced relative to description

(continued)

Table 8.1 (continued)

Study	Description	Experience	Inferred weighting of rare events
Kellen et al. (2016)	$\gamma=0.66$ ($\delta^+=0.81$, $\delta^-=1.53$)	$\gamma=0.53$ ($\delta^+=0.71$, $\delta^-=1.66$)	Overweighting
Markant, Pleskac, Diederich, Pachur, & Hertwig (2015)	—	$\gamma=1.41$ ($\delta=1$)	Underweighting
	—	$\gamma=1.15$ ($\delta=1.61$)	Underweighting
	—	$\gamma=0.92$ ($\delta=1.3$)	Overweighting
Variants of the sampling paradigm			
Abdellaoui et al. (2011)	$\gamma^+=0.65$ ($\delta^+=0.70$) $\gamma^-=0.73$ ($\delta^-=0.78$)	$\gamma^+=0.66$ ($\delta^+=0.59$) $\gamma^-=0.74$ ($\delta^-=0.67$)	Overweighting, but less pronounced relative to description
	(Nonparametric estimation)		Overweighting, but less pronounced relative to description
Camilleri & Newell (2011b)	—	$\gamma>1$	Underweighting
	—	$\gamma>1$	Underweighting
Jarvstad, Hahn, Rushton, & Warren (2013)	(Qualitative evaluation; experienced frequencies)		Underweighting
	(Qualitative evaluation; experienced frequencies)		Overweighting
Zeigenfuse, Pleskac, & Liu (2014)	—	$\gamma=0.7$ ($\delta=0.3$)	Overweighting
Kemel & Travers (2016)	—	$\gamma=0.68$ ($\delta=0.68$)	Overweighting
	—	$\gamma=0.59$ ($\delta=0.78$)	Overweighting
	—	$\gamma=0.74$ ($\delta=0.66$)	Overweighting
	—	$\gamma=0.63$ ($\delta=0.83$)	Overweighting

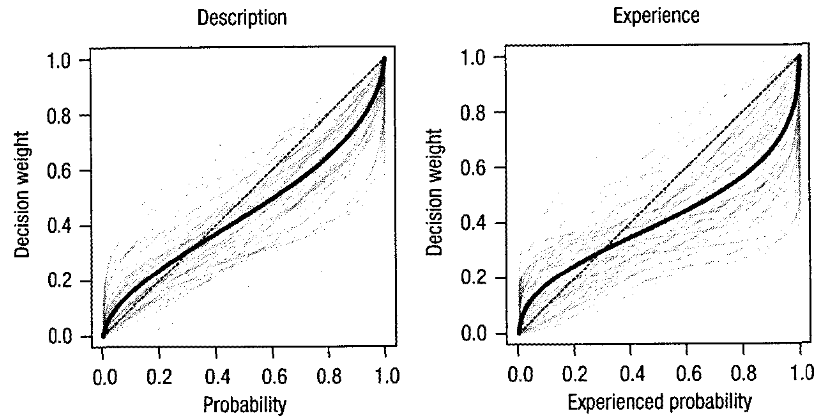

Figure 8.2
Individual-level probability-weighting functions (in gray), separately for decisions from description and decisions from experience in Kellen et al. (2016). The results are shown for the gain domain (differences between description and experience in the loss domain were similar). The black lines show the functions based on the group-level parameters.

for each learning mode. For experience-based decisions, the authors used the outcomes that each participant had actually experienced and, as probabilities, the relative frequencies of those outcomes in each participant's samples (thus taking sampling error into account). As figure 8.2 shows, the resulting probability weighting function for decisions from experience was inverse S-shaped, suggesting overweighting of rare events—the qualitatively same pattern observed for decisions from description. Importantly, however, the curvature was even more pronounced in experience than in description. This finding indicates, consistent with Tversky and Fox's (1995) analyses of decisions under uncertainty, reduced sensitivity to probabilities in decisions from experience (see also Glöckner et al., 2016; Lejarraga et al., 2016). At the same time, Kellen et al. (2016; see their table 1) obtained a choice pattern (on the respective choice problems) that was similar to the one that led Hertwig et al. (2004) to conclude that rare events are underweighted. Thus, although decision weights expressed relative to the objective probabilities suggest underweighting of rare events in decisions from experience—as in Hertwig et al.'s (2004) conclusions (see also Regenwetter & Robinson, 2017)—decision weights estimated using the probability information that people have actually experienced suggest overweighting.

Both patterns, over- and underweighting of rare events, can be obtained, depending on how probability weighting is estimated.

Another important factor that seems to systematically affect the type of probability weighting in decisions from experience is the type of choice problem. For choice problems containing two risky options (making up the large share of problems in Kellen et al., 2016), there is rather consistent evidence for a stronger overweighting of rare events in experience than in description. For problems containing a safe and a risky option, by contrast, the probability weighting function seems to be more linear in experience than in description (Glöckner et al., 2016; see figure 10 in Wulff et al., 2018). These differences between problem types suggest that the pattern of probability weighting is sensitive to the variability of the outcomes that people encounter during information search (see chapter 7)—and is therefore not easily generalizable. In sum, in investigations where probabilities are uncertain and people could reduce the uncertainty by sampling from memory, there is a regressive pattern. In addition, in investigations where probabilities are uncertain and people could reduce the uncertainty by sampling from the environment, there also seems to be a regressive pattern when both options are risky. Next, we discuss a possible explanation for this pattern in probability weighting—namely, that it represents a reasonable response to uncertainty.

8.4 Nonlinear Probability Weighting as a Rational Response to Uncertainty

From the perspective of expected value theory, that people's decision weights show a regressive, nonlinear distortion of the probabilities represents a clear violation of how a rational mind should respond to risk. So should nonlinear probability weighting be considered irrational? Fennell and Baddeley (2012) demonstrated that, far from being irrational, regressive probability weighting can reflect a rational response to uncertainty. They argued that people may internally "correct" probability information by integrating it with relevant background knowledge consistent with a rational Bayesian updating mechanism. Specifically, the stated probability of an event is viewed against the previously experienced probability distribution of similar events ("inference priors") as well as the expected probability distribution assuming a complete lack of knowledge about the class of events ("ignorance priors"). If the stated probability is compatible with the previous experience, the inference prior is used to correct the stated probability; otherwise the ignorance prior is used. Fennell and Baddeley showed that this approach leads to "corrected" probabilities that are regressed relative to the original probability information—in line with an inverse S-shaped weighting function.

To illustrate this point, Fennell and Baddeley (2012) demonstrated how probabilities for two classes of events—namely, positive events (e.g., birthdays, weddings) and negative events (e.g., earthquakes, house fires)—should be revised in light of one's knowledge about the frequency distribution of positive and negative events in the real world. In a first step, the authors analyzed Internet blogs to estimate the ecological distribution of the probability of these events occurring. This ecological analysis showed that in the blogs, both positive and negative events had an average (across different types of events) probability of occurring of less than 50%. Moreover, positive events were more likely than negative events, and the spread of the distribution for positive events was wider. In a second step, the authors used the Bayesian approach sketched earlier in this section as well as the results from their ecological analysis to derive, for different probability levels, posterior (i.e., "corrected") probability assessments. In other words, they determined how a Bayesian mind would assess probability information about positive and negative events against the background of the respective probability distributions in the world at large. When the original probabilities were mapped against the resulting posterior probabilities, the functions showed a regressive trend, with the probability assessments for lower probabilities being pushed upwards toward 50%, and the probability assessments for higher probabilities being pushed downwards. In addition, the regressive trend was more pronounced for positive than for negative events, reflecting the greater uncertainty associated with the former (i.e., the wider spread of the probability distribution).

The key insight from Fennell and Baddeley's (2012) results is that nonlinear weighting of probabilities is not necessarily a sign of irrationality. Instead, a regressive, inverse S-shaped probability weighting pattern is consistent with how a Bayesian mind would rationally respond to uncertainty (leaving aside the question of how exactly the mind may implement or approximate a Bayesian updating process). The amount of uncertainty is evidently higher in decisions from experience than in decisions from description. One interpretation of the regressive probability weighting pattern obtained by, for instance, Kellen et al. (2016) is thus that it represents a rational response to the uncertainty in decisions from experience.

8.5 Probability Weights as Reflections of Heuristics

Up to now, we have discussed several reasons why people might weight events differently than the probability of those events would imply. For instance, they may ignore events with small or very small probabilities because their experience tells them that they barely matter. Fennell and Baddeley (2012) demonstrated that a regressed, nonlinear probability weighting pattern is the natural consequence of a Bayesian mind dealing with uncertainty. Let us add another possible reason for the emergence of nonlinear probability weighting. It reconciles probability weighting—a notion rooted in the Bernoullian utility framework of decision making—with the Homo heuristicus (see chapter 1; Gigerenzer, Hertwig, & Pachur, 2011). Specifically, different kinds of nonlinear probability weighting may arise when decision makers rely on different boundedly rational heuristics.

Let us first turn to a heuristic that was proposed as a decision tool to be used in the face of uncertainty. Savage (1954) suggested that one way to deal with situations in which probabilities are unknown is to expect that the worst possible outcomes occur and to decide accordingly. The minimax heuristic, which implements this notion in games against a dispassionate nature, chooses the option whose worst outcome is more attractive. Importantly, probabilities play no role at all in the minimax heuristic. What form does cumulative prospect theory's probability weighting function take when decision makers apply this and, by extension, other heuristics? To answer this question, Pachur, Suter, and Hertwig (2017; see also Suter, Pachur, & Hertwig, 2016) fitted cumulative prospect theory to choices produced by five heuristics in the context of a computer simulation. The heuristics represent distinct policies in the face of risk and uncertainty: the minimax heuristic, the maximax heuristic, the priority heuristic (a lexicographic strategy), the least-likely heuristic, and the most-likely heuristic. Box 8.2 describes their policies in detail. The procedure was as follows: first, Pachur, Suter, and Hertwig determined the choices of each of the five heuristics for various types of choice problems (in the gain, loss, and mixed domain); second, they

Box 8.2

Definitions of the five heuristics tested in Pachur, Suter, and Hertwig (2017).

We illustrate each heuristic's policy and choice prediction with reference to the following choice problem with two options:

A 500 with a probability of .4 and 2000 with a probability of .6.

B 450 with a probability of .7 and 3500 with a probability of .3.

The *minimax heuristic* identifies the worst outcome of each option and selects the option with the more attractive worst outcome. If the options' worst outcomes are identical, minimax chooses randomly. It never considers probability information. Minimax chooses option *A*, because its worst outcome is higher than that of option *B* (500 vs. 450).

The *maximax heuristic* identifies the best outcome of each option and selects the option with the more attractive best outcome. If the options' worst outcomes are identical, maximax chooses randomly. It never considers probability information. Maximax chooses option *B*, because its best outcome is higher than that of option *A* (3500 vs. 2000).

The *priority heuristic* goes through the attributes in the following order: minimum gain, probability of minimum gain, and maximum gain. It stops examination if the minimum gains differ by 1/10 (or more) of the maximum gain; otherwise, it stops examination if the probabilities differ by 1/10 (or more) of the probability scale. The heuristic selects the option with the more attractive gain (probability). For options with more than two outcomes, the search rule is identical, apart from the addition of a fourth attribute: probability of maximum gain. For loss options, the heuristic remains the same except that "gains" are replaced by "losses." For mixed options, the heuristic remains the same except that "gains" are replaced by "outcomes." The priority heuristic sometimes considers probability information, depending on whether the minimum outcomes differ or not. In the example, the priority heuristic chooses option *A*, because the option has a lower probability than option *B* of leading to the minimum gain (.4 vs. .7).

The *least-likely heuristic* identifies each option's worst outcome and selects the option with the lowest probability of the worst outcome. It always considers probability information. Least-likely chooses option *A*, where the probability of the worst outcome (500) is .4, lower than in option *B* (.7).

The *most-likely heuristic* identifies each option's most likely outcome and selects the option with the more attractive most likely outcome. It always considers probability information. Most-likely chooses option *A*, where the most likely outcome is 2000, higher than in option *B* (450).

estimated the parameters of cumulative prospect theory for the set of choices produced by each heuristic.

As figure 8.3 shows, the heuristics produced distinctly shaped curves. For instance, the weighting functions estimated for the choices produced by minimax and maximax showed a strongly inverse S-shaped curvature, indicating low probability sensitivity. This finding echoes the previous result that people display lower sensitivity to probability information under uncertainty than under risk. Moreover, low probability sensitivity is consistent with the information processing architecture of the minimax and maximax heuristics, which are blind to probabilities. For the priority heuristic, the curvature was less pronounced, indicating somewhat higher probability sensitivity. Again, the shape of the weighting function is meaningfully related to the heuristic's policy: this heuristic sometimes relies on probabilities to make a choice—namely, when the options' worst possible outcomes are similar—but it sometimes ignores probabilities, depending on the characteristics of the choice problem (see box 8.2). For the most-likely and the least-likely heuristics, the weighting function was least strongly curved, reflecting that these heuristics always take probabilities into account when making a choice.

Furthermore, the elevations of the resulting weighting functions point to differences between the heuristics in the degree of "optimism" (i.e., risk attitude) they embody. For instance, whereas in the gain domain minimax resulted in a weighting function with a very low elevation, indicating highly pessimistic decision weights (and thus risk aversion), maximax produced a weighting function with a very high elevation, indicating highly optimistic decision weights (and thus risk-seeking). In the loss domain, this pattern was reversed. These results thus reveal another property of the heuristics' policies—namely, their risk attitude (see also Lopes, 1995). Whereas minimax aims to protect against the worst possible outcomes, maximax reaches for the stars and aims to maximize the best outcomes.

In summary, probability weighting as assumed, for instance, in cumulative prospect theory is agnostic with regard to the cognitive processes that shape the probability weighting function (but see Bordalo, Gennaioli, & Shleifer, 2012; Hogarth & Einhorn, 1990; Johnson & Busemeyer, 2016). Surprisingly, however, the probability weighting function is a construct at which theories of boundedly rational choice heuristics (see chapters 1 and 2) and neo-Bernoullian theories of choice can meet. Moving toward

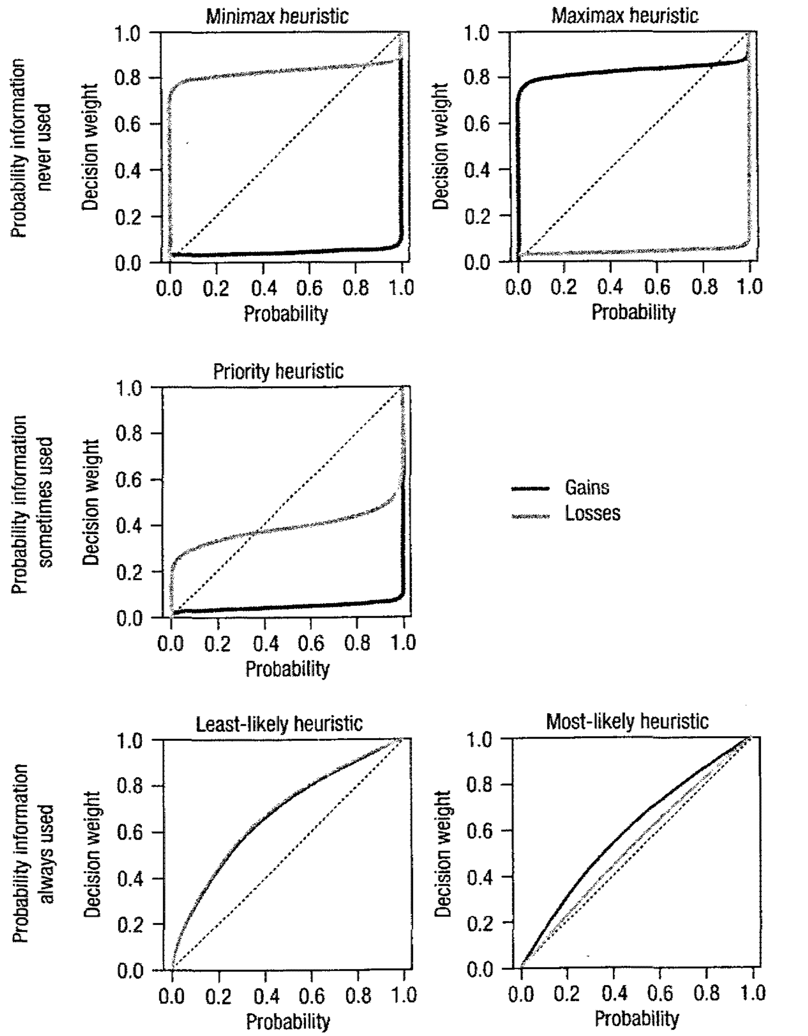

Figure 8.3

Cumulative prospect theory's weighting function estimated for the choices of five heuristics that differ in their consideration of probability information (from Pachur, Suter, & Hertwig, 2017). The functions show the relationship between the objective probability, p, and the transformed probability, $w(p)$, separately for gains (dark gray) and losses (light gray).

integration of theories for decisions under risk and uncertainty, Pachur, Suter, and Hertwig (2017) demonstrated that heuristics with distinct information-processing policies result in distinct shapes of the probability weighting function. In terms of their suggested probability sensitivity and optimism, these shapes are meaningfully related to differences in the heuristics' information processing policies. One practical implication of this finding is that analyses of probability weighting can be employed to track the operations of the Homo heuristicus as well as the underlying cognitive processes (e.g., Pachur, Schulte-Mecklenbeck, Murphy, & Hertwig, 2018; Suter et al., 2016).

8.6 Probability Weighting: A Window onto Uncertainty

Scholars' attempts to explain how the mind weighs risky or uncertain events date back to at least the 18th century and to perhaps the most influential game in the history of economics and psychology: the St. Petersburg game. Our historical account of work motivated by this game prompts an interesting thought experiment: What kinds of theories of choice would have evolved had Nicolas Bernoulli's (see van der Waerden, 1975) and Buffon's (1777) explanations of the St. Petersburg paradox won the day? Perhaps even more would be known about the causes and (adaptive) logic behind different kinds of nonlinear probability weighting in the face of risk and uncertainty. Yet what we have nevertheless begun to discern is that probability weighting offers a window onto how much uncertainty people perceive in the environment—and possibly onto the kind of simple strategies they recruit to deal with it.