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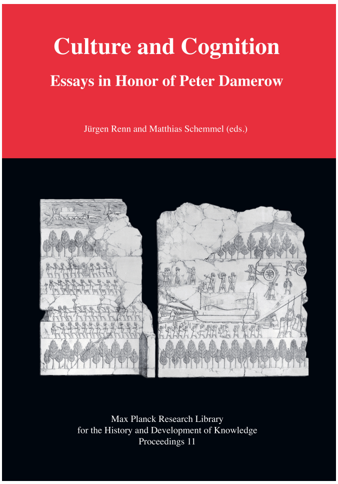
Ursula Klein:

Paper Tools



In: Jürgen Renn and Matthias Schemmel (eds.): *Culture and Cognition : Essays in Honor of Peter Damerow*
Online version at `http://mprl-series.mpg.de/proceedings/11/`

ISBN 978-3-945561-35-5
First published 2019 by Edition Open Access, Max Planck Institute for the History of Science.

Printed and distributed by:

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at `http://dnb.d-nb.de`

Chapter 13
Paper Tools
Ursula Klein

In *Rechenstein, Experiment, Sprache*—a book co-edited with Wolfgang Lefèvre—Peter Damerow published a long essay on the "Origin of Arithmetical Thinking," in which he studied the structural features of arithmetic in ancient Egypt and Babylonia (Damerow and Lefèvre 1981, 11–113). The key questions raised in this essay are the following: How can the peculiarities of arithmetic established in these two cultures be explained? Are they the result of practices involving particular "material tools" (*gegenständliche Mittel*) for calculating (Damerow and Lefèvre 1981, 14)? Typical examples of such material tools, Peter has pointed out in his essay, are the counting rods and counting boards used in ancient China.

Peter's questions relied on a broader theoretical background that was defined by the developmental psychology of Piaget and others. Our logical-mathematical structures of thinking, Piaget argued, ultimately result from the "reflection upon actions with material objects" (*Reflexion auf gegenständliche Handlungen* (Damerow and Lefèvre 1981, 13f)). If we extend this psychological argument, which refers to the *ontogenetic* development of cognitive structure, to the *historical* development of arithmetic, and more specifically to ancient Egyptian and Babylonian arithmetic, this implies that inventions of arithmetic techniques did not presuppose mathematical thinking. It was rather the other way around: mathematical concepts and thinking result from "operations with material tools" (*Operieren mit gegenständlichen Mitteln*) used for solving certain arithmetic problems (Damerow and Lefèvre 1981, 106).

A characteristic feature of arithmetic in ancient Egypt is the predominance of techniques of addition. Thus multiplication was performed by using techniques of addition. Likewise, division was carried out by means of additions of fractions. As "arithmetic begins with operative manipulations of representations of numbers," Peter has argued, these peculiar features of arithmetic relied on the Egyptian mode of representing numbers.

In ancient Egypt, arithmetic was the business of a particular social group: state officials who were concerned with the planning of production and administration in the centralized state. These men belonged to the upper social class, and they underwent a long professional education and training. Writing and script, Peter has pointed out, marked their professional life. And script was also a symbolic system they used for representing numbers. In Egyptian arithmetic, the characters of script were transformed into tools for representing numbers and calculating.

The ancient Egyptians used simplified hieroglyphic symbols for representing numbers.[1] A vertical line meant 1. Furthermore, there were distinct individual symbols for 10, 100, 1000, and so on. All other numbers were combined symbols, made up of a series of basic symbols arranged in a row. "In the most simple case," Peter has explained, "the series (*Reihung*) of one and the same symbol, namely that for one, engenders configurations that yield the number by counting [the individual symbols]" (Damerow and Lefèvre 1981, 29). He has designated this way of representing numbers a "constructive-additive" technique. We are familiar with this technique from the system of Roman numbers.

Based on this observation, Peter has argued as follows: "the *physical-geometrical* properties of the symbol are used here for constructing *material* models (*gegenständliche Modelle*) of numbers, and for designating numbers by means of the models" (Damerow and Lefèvre 1981, 29, my emphasis). And further: "Symbols of numbers that are generated in a constructive-additive way represent quantities of real objects, just like counting stones, and all operations of assembling and dividing possible with real objects can be performed in a similar way with their representations, which are *material* (*gegenständlich*) as well" (Damerow and Lefèvre 1981, 107). In this context, Peter has also spoken of "material tools of calculating" (*gegenständliche Rechenmittel*) (Damerow and Lefèvre 1981, 106).

It is certainly reasonable to argue that the Chinese counting rods are *material* tools of calculation. But it is perhaps less compelling to argue that symbols written on paper are comparable entities. Can symbols be reasonably designated "*material* tools of calculation" (*gegenständliche Rechenmittel*)? And what does Peter mean when he states that "the physical-geometrical properties of symbols are used for constructing material models of numbers"? What is the meaning of "material" in this context?

We may evade this question by saying that the German word *gegenständlich* does not necessarily mean "material." However, Peter's insistence on the analogy with the Chinese counting rods, which are common-sense material objects, bars this solution, as does the background of cognitive psychology, which refers to down-to-earth operations with material things (in "first-order representations"). It should be noted that this question is also crucial for Peter's attempt to connect

[1] It should be noted that the ancient Egyptians used two different systems of numbers. As this essay is concerned with Peter Damerow's argument, I concentrate on the hieroglyphic symbols.

Piaget's theory, which refers to the cognitive development of individuals, with studies of cognitive development in history, which refers to cultures and societies that change in history. The crucial link between the two kinds of development of cognitive structures resides in the similarity of people's reflections upon actions with real material objects and actions with symbolic devices.

A look at semiotics may be helpful in this context. In a later essay, published in 1999, Peter borrowed from semiotics as well (Damerow 1999).[2] Semioticians distinguish between the "semantics" of sign systems and their "syntax." While "semantics" refers to the meaning of signs, or the concepts they represent, "syntax" refers to the visual traces on paper (or another medium), the maneuverability of signs on the surface of paper, and the rules governing their manipulations.[3] Visibility and maneuverability are aspects of sign systems that they share with material objects, and more specifically with material tools used to achieve certain goals. We may designate these features their "material" dimension. Comparable to the materiality of ordinary tools, they define objective possibilities and constraints of work with a given type of sign system. Paper tools are material devices in the broader sense of being visible and maneuverable entities that are exterior to mental processes but help to generate mental processes.

I would like to briefly exemplify this argument by means of a different example, namely chemical formulae.[4] Chemical formulae such as H_2O for water or H_2SO_4 for sulphuric acid were introduced by the Swedish chemist Jöns Jacob Berzelius in 1813/14, and they proliferated in the chemical community in the 1830s. Their meaning was defined by a theory of chemical composition that went back to Lavoisier, Dalton, and some other chemists, including Berzelius himself. This chemical theory postulated that all chemical compounds consisted of discrete, quantitative units of elements. But it did not further define these chemical units in the light of atomism. Whereas all nineteenth-century chemists accepted Berzelian formulae, many of them were agnostic concerning the more far-going atomistic interpretation of the chemical units represented by this sign system. They restricted themselves to the postulation of discontinuous quantitative units of chemical elements, which was evinced (to some extent) by the empirical laws of stoichiometry. The semantics of Berzelian formulae presupposed the empirical

[2] It should be noted that in this later essay Peter has further elaborated his original argument. He has introduced, or further clarified, a number of additional concepts and distinctions, such as the distinction of first-order and second-order presentations, which are important for a full-fledged theory about the historical development of the concept of number. The goal of my short essay is not to recapitulate this particular theory but rather an element of it—the material dimension of operations on paper with sign systems—that has implications for the history of science and technology more broadly.

[3] It should be noted that I do not restrict syntax to cultural rules of working with symbols.

[4] For the following see Klein (2003).

laws of stoichiometry, and chemical formulae extended these laws hypothetically to all chemical compounds, including organic ones.[5] However, they neither necessitated the acceptance of the philosophical tradition of atomism nor that of the newer physical atomism and Dalton's chemical atomism.[6]

The material dimension of Berzelian formulae is even more interesting. Berzelian chemical formulae consisted of letters, taken from the initial letter (or two letters) of the Latin names of substances, and numbers. Letters are arbitrary signs, or "symbols," that did not resemble the entities they represented. A letter like S denoted a quantitative unit of sulphur in a completely arbitrary way. However, like the symbol for the Egyptian number one—the vertical line—the Berzelian letters had a certain "graphic suggestiveness" (Goodman) owing to the one-to-one correspondence between the letter and the unit of an element (Goodman 1976, 154). One letter denoted one elemental unit. The historian of art Rudolf Arnheim gave a cogent expression to what is meant here:

> In the strictest sense it is perhaps impossible for a visual thing to be nothing but a sign. Portrayal tends to slip in. The letters of the alphabet used in algebra come close to pure signs. But even they stand for discrete entities by being discrete entities: a and b portray twoness. Otherwise, however, they do not resemble the things they represent in any way […] (Arnheim 1969, 136)

In other words, the visible, "physical-geometrical" coherence of the symbol displayed the represented unit—a defined quantity of a chemical element—in a quasi-pictorial fashion. This is perhaps what Peter meant when he claimed that the Egyptians used the "physical-geometrical properties" of the symbol [for one] for constructing "material models" of numbers. The stroke, representing the number one, was a visible, geometrical unit, and the iteration of such units visibly constructed a bundle of strokes that stood for a certain number x.

The visual letters of Berzelian formulae carried the meaning of units of chemical elements, and no more than this. The material form of the paper tool was well suited to represent a kind of chemical theory that deliberately left open many additional questions asked in the tradition of philosophical atomism. It also facilitated purely additive constructions of chemical models. In their experimental studies of chemical reactions and molecular structure, nineteenth-century chemists used Berzelian formulae to model the outcome of a reaction, and they further combined various reaction models to construct a formula representing the invisible molecular structure of a chemical compound. In so doing, they moved

[5]Stoichiometry had been established in inorganic chemistry. Experimental studies of organic compounds actually questioned the stoichiometric laws.

[6]For the distinction of early nineteenth-century physical and chemical atomism, see Rocke (1984).

the letters denoting units of chemical elements around on the two-dimensional surface of paper, and they additively combined them in new ways that fit the outcome of their experiments. Comparable to some extent to the construction of Egyptian compound numbers, the nineteenth-century chemical models were built by what Peter called a "constructive-additive" technique.

An ordinary tool must be suitable to the object involved in labor. So, too, with sign systems used as tools on paper. Chemical formulae were well suited to their application as tools for investigating chemical reactions and structure in a constructive-additive way. Chemists' constructive-additive rules could be directly translated into a mechanical model of chemical compounds and reactions, which I have called the "building-block-model" of chemical compounds. If we analyze the concrete ways in which the nineteenth-century chemists used their formulae, it is reasonable to argue that the visibility of the Berzelian symbols and the manipulation of them on paper—that is, their "materiality"—helped to generate models of chemical structure and reactions.[7]

My comparison of constructive-additive representations of numbers in ancient Egypt with the use of Berzelian chemical formulae in nineteenth-century Europe has highlighted relatively simple forms of materiality, action, and thinking, which yield representations that can be directly related to real objects ("first-order representations"). The construction of such simple representations is accompanied by abstractions from local features and contexts of actions with objects. It thus yields insights into some general features of the objects under investigation. However, such kinds of (first-order) external representations are not yet mathematical concepts or scientific theories. If you want to know more about these issues, Peter has many interesting ideas to offer as well.[8]

References

Arnheim, Rudolf (1969). *Visual Thinking*. Berkeley: University of California Press.

Damerow, Peter (1999). *The Material Culture of Calculation: A Conceptual Framework for an Historical Epistemology of the Concept of Number*. Preprint 117. Berlin: Max Planck Institute for the History of Science.

Damerow, Peter and Wolfgang Lefèvre, eds. (1981). *Rechenstein, Experiment, Sprache: Historische Fallstudien zur Entstehung der exakten Wissenschaften*. Stuttgart: Klett-Cotta.

Goodman, Nelson (1976). *Languages of Art: An Approach to a Theory of Symbols*. 2nd ed. Indianapolis: Hackett.

Klein, Ursula (2003). *Experiments, Models, Paper Tools: Cultures of Organic Chemistry in the Nineteenth Century*. Writing Science. Stanford: Stanford University Press.

Rocke, Alan J. (1984). *Chemical Atomism in the Nineteenth Century: From Dalton to Cannizzaro*. Columbus: Ohio State University Press.

[7] For examples, see Klein (2003).

[8] See Damerow (1999).