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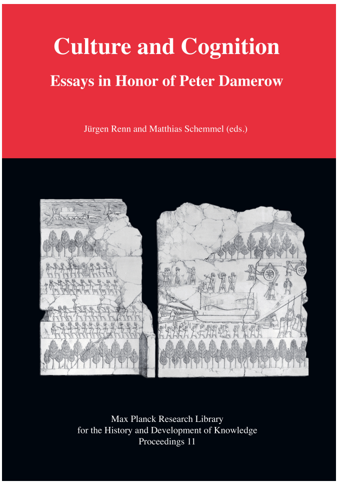
Wolfgang Lefèvre:

Drawing Instruments



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Chapter 14
Drawing Instruments
Wolfgang Lefèvre

Among Peter's writings there are quite a few that are unknown even to his friends and fans—either because they were written in German or published in edited books, which are notoriously apt hiding-places for texts. The text to which I want to draw your attention is one of those—it is written in German and, to complicate matters further, a commentary to a chapter in a reissue of an eighteenth-century book on mathematical instruments, namely of George Adams' *Geometrical and Graphical Essays* (1791). The commentary's title is "Die alltägliche Seite der Geometrie: Zum Kapitel über die Zeicheninstrumente" (*The mundane side of geometry: Regarding the chapter on drawing instruments*) (Adams 1985, 283–300).[1]

Since George Adams' chapter on drawing instruments deals with commonplace instruments such as pairs of compasses or rulers, some readers might be tempted to translate *alltägliche Seite* as the "banal" or "trivial side" of geometry. True, in some of the instruments described in this chapter—such as the pantograph, the pair of proportionable compasses, or the cyclograph—a geometry is embodied that is not that trivial. But this geometry is not Adams' topic and remains probably more often than not obscure to the addressees of the chapter—surveyors, captains, gunners, or architects. Thus, some readers may even wonder whether an employment of mathematical instruments by such practitioners should be called geometry at all. And the fact that Adams puts a list of Euclid's definitions at the head of his book,[2] although deductions of propositions or theorems cannot be found in it, will bring many a historian of science to consider this listing of Euclid's definitions as a mere rhetorical device by which the author tries to gain a higher reputation or social standing.

Now, it comes as no surprise that Peter did not belong to these historians of science. He took the obvious discrepancy between erudite geometry in the tradi-

[1] And, as if to make sure that nobody will ever find this text, this edition does not clarify which of the two editors authored the commentary to a particular chapter—Peter or the author of these lines.

[2] It is an almost complete listing of the definitions of book I of Euclid's *Elements*, albeit sometimes with slight deviations as regards the wording, and furthermore of definitions of geometrically conceived trigonometric subjects.

tion of Euclid and practical geometry of mathematical practitioners as a starting point for re-considering the relation between these two sides of geometry. And he developed this re-consideration by focusing on the functions the seemingly banal drawing instruments had for both practical and deductive geometry.

Before going a bit into his considerations, a few words about the whereabouts of this commentary might be in order. In the early 1980s, Peter and I came accidentally across a contemporary German edition of George Adams' *Geometrical and graphical essays*. Realizing that these essays by a renowned London instrument maker[3] amount to an extraordinarily rich and informative portrait of the realm of mathematical instruments and their employment before the industrial age and having a contemporary translation (Adams 1795) in our hands, we thought it worthwhile to edit a selection of it and furnish this edition with historical commentaries.

Let me now briefly sum up some of the main points of Peter's commentary to the chapter on drawing instruments. Peter started his considerations by recalling the double face of geometry—it is a deductive science and an empirical one, and these two sides cannot be completely mapped one upon another. As Albert Einstein put it in his essay *Geometry and Experience*: "As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality."[4] As an empirical science, namely as the science that explores real space, geometry is as dependent on drawing instruments as other natural sciences are upon observational and experimental instruments. Drawing instruments enable the application of geometrical knowledge in real space and reveal application limitations of geometrical ideas. Peter expected, therefore, that the importance of drawing instruments for the development of geometry is comparable to that of observational and experimental instruments for the development of natural sciences such as astronomy, physics, or chemistry.

Euclid's postulate that ruler and compass be admitted in geometry[5] is usually discussed in a philosophical mode as a means of ensuring the existence and objectivity of geometrical constructions without showing an interest in these instruments. In contrast, Peter highlighted various functions that drawing instruments have for geometry: They are means of exploration, means of demonstration, and means of representation and production (*Darstellungsmittel*). Whereas their employment for purposes of exploring and demonstrating pertains exclusively to theoretical geometry, their use as means of producing, that is, constructing geo-

[3] George Adams Jr. (1750–1795), the principal of an internationally known manufactory of mathematical and optical instruments, published several books on topics connected with these instruments.

[4] "Insofern sich die Sätze der Mathematik auf die Wirklichkeit beziehen, sind sie nicht sicher, und insofern sie sicher sind, beziehen sie sich nicht auf die Wirklichkeit" Einstein (1921, 3f); translation from Einstein (1922).

[5] Euclid *Elements* book I, postulates 1–3.

metrical figures and shapes was not restricted to theoretical geometry. As is well known, the practice of constructing forms and shapes with drawing instruments can be traced back to times long before the emergence of any theoretical geometry.

Being an expert in Babylonian mathematics, Peter knew of course that geometrical issues—first and foremost methods of determining the size of cultivable land—already got a theoretical treatment in Mesopotamian schools of scribes, namely a numerical or arithmetical treatment. However, these theories remained unconnected with the various methods of constructing shapes and patterns with ruler, compass, and other drawing instruments that were in use in the domain of ornamental arts (pottery, metal work, architecture, and so on). Against this background he realized the epoch-making significance of the fact that in ancient Greece a theoretical geometry came into being that reflected geometrical constructions. With this a completely new situation was created in which the practice of constructing geometrical figures and geometrical theorizing became related and mutually dependent on each other. In this Greek tradition of doing geometry the development of geometry became essentially a development of the relation between construction and theory.

Drawing instruments partook in this interconnected development. Becoming refined and diversified in this process, they offered new construction techniques and were at the same time indicative of the state this development had achieved at a certain point of time. The interconnection of construction and theory, once established, turned drawing instruments into embodiments of geometrical knowledge and, thus, into archaeological evidence for the historian. That's why they matter for a history of mathematics.

In this short summary of Peter's arguments, I cannot go into a very interesting and important point he made regarding Greek geometry, namely that the reflection on the action of constructing by means of ruler and compass used systematically ordinary language—more specifically: literary language—as its means of representation. This transformation of literary language into a specific means of representation and deduction capitalized on the contemporary sophists' dialectics, that is, techniques of argumentation, and particularly on their utilization of definitions, postulates, and axioms.

In the framework of Euclid's geometry, constructions play an essential, indispensable role, though also a servile, auxiliary one. They procure evidence on which the theorems base their deductions. Though absolutely subordinated to these theoretical purposes, the very fact that constructions were instrumental in a deductive theory caused offense. From idealistic philosophers like Proclos up to modern champions of pure mathematics, the constructions in Euclid's *Elements* were regarded as displeasing impurities in a deductive enterprise.

However, a complementary story could be told on the side of practical geometry. Out of the wealth of geometrical constructions employed in Greek practical geometry—just think of the refinements used in architecture—Euclid selected only a few constructions that were of use for the deductive purposes of his work. In other words, practical geometry, the geometry of practitioners, did not merge into theoretical geometry. It continued to have a life of its own although it did not remain completely untouched by or disconnected from theoretical geometry. Practitioners for their part did select and use results of theoretical geometry that were available for their purposes. But now it was the theorems' turn to play an auxiliary role.

Pursuing this productive and at the same time tense relation between practical and theoretical geometry, Peter eventually drew attention to a paradoxical situation characteristic of the early modern period. In this period, constructions lost their earlier significance for developments in the frame of learned Euclidean geometry and did so to an extent that their original role in this context was almost forgotten. At the same time, practical geometry and the art of geometrical constructing boomed in an unprecedented way exactly in this period. The traditional arsenal of geometric constructions was enormously extended and refined in the context of astronomy, cartography, surveying, and leveling, and particularly in the context of perspective and stereotomy. And so was the traditional arsenal of drawing instruments.

George Adams' *Essays* epitomize this prosperous realm of practical geometry at the close of the eighteenth century. And, by the seemingly odd listing of Euclid's definitions, they are also an indication of the bond between this realm and that of learned geometry. Summing up these considerations, Peter wrote:

> The relationship of the pure geometry that arose out of the re-appropriation of Euclid's Elements, and the practical geometry to which Adam's work must be assigned, is not that of a theory and its application. Rather, they relate in a complementary way to a shared origin in a reflective employment of drawing instruments. Pure geometry abstracts from those concrete figures whose exploration yielded geometrical insights; practical geometry abstracts from the geometrical knowledge embodied in or transmitted by the constructions of concrete figures.[6]

[6] "Die reine Geometrie wie sie aus der Wiederaneignung der Elemente des Euklid hervorgegangen ist, und die praktische Geometrie, der Adams Werk zuzurechnen ist, stehen also nicht einfach in der Beziehung einer mathematischen Theorie und ihrer Anwendung zueinander. Sie beziehen sich vielmehr komplementär auf einen gemeinsamen Ursprung im reflektierenden Umgang mit den Zeicheninstrumenten. Die reine Geometrie abstrahiert von den konkreten Figuren, an denen die geometrischen Kenntnisse gewonnen wurden, die praktische Geometrie abstrahiert von den geometrischen Kennt-

In concluding for my part, I don't hesitate to state that Peter's commentary on a chapter about drawing instruments amounts to no less than a general outline of the basic principles of a history of Euclidian geometry that captures the interplay of its different realms as the true motor of its development. To my knowledge, he never thought of writing such a history himself. And if I am right, such a desirable history has not been written up to the present day.

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nissen, auf denen die Konstruktionen konkreter Figuren beruhen oder die durch sie vermittelt werden" Adams (1985, 296f).