Gyrokinetics of electron-positron plasmas in a magnetic Z-pinch: towards a turbulence free plasma?

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1. Introduction

In the near future, the first experiment aiming for the creation and confinement of an electron-positron plasma will be constructed [6], using the magnetic field generate by a current carrying wire to confine the plasma.

It has been shown by Helander [1] that pair plasmas possess unique gyrokinetic stability properties due to the mass symmetry between the particle species. For example, drift instabilities are completely absent in straight geometry, provided that the density and temperature profiles of the two species are identical ("symmetric" pair plasmas). In contrast to slab geometry, a dipole magnetic field has finite curvature. In this case, the symmetry between the species is broken by curvature drifts and the plasma can be driven unstable by temperature and density gradients [1], even without ion contamination and for identical temperature profiles of the two species. The nonlinear stability of dipole pair plasmas has also been addressed by Helander [2] and it was found such plasmas ought to enjoy excellent confinement. More recently, Mishchenko et al. [5] performed a detailed study of the gyrokinetic stability of pure pair plasma in the dipole geometry, making use of both the Z-pinch and point-dipole limits. Again, it was found that such pair plasmas can be driven unstable by magnetic curvature, density and temperature gradients. Here we extend these results to the experimentally relevant geometry of the Z-pinch with a view towards numerical simulations the full magnetic dipole.

It is a general feature of many plasma systems that such instabilites can drive a diffusive spreading of the density and temperature profiles, driving heat and particles away from the ideal boundary and leading to a loss of confinement. A remarkable example of a geometry for which this does not occur is that of a magnetic dipole, of the type that will be used for pair plasma confinement in the upcoming APEX experiments. In these systems, it can be shown that such instabilites can transport plasma inwards in conventional electron-hydrogen plasmas, creating centrally peaked temperature and density profiles: a so called "inward pinch" effect. In this paper, we aim to explore these parameter regimes for the geometry of a magnetic Z-pinch, an asymptotic limit of the full dipolar geometry with the aim of numerically demonstrating, for the first time, inward particle pinch in electron-positron plasmas.

In this paper, we use the gyrokinetic code GENE [3] to study the linear and nonlinear stability of electron-positron plasmas in a Z-pinch magnetic configuration. In §2 we give a background of some results from the analytic theory of for electron-positron plasmas; with particular emphasis being placed on the testable predictions which we have investiaged using our gyrokinetic code. We also give a brief introduction to the magentic geometry of the ring dipole. In §3 we introduce the GENE code and the assumptions and modifications required to run simulations for electron positron plasmas. In §4 we show the results of linear simuations. In §5 we pave the way for future work, leading towards nonllinear simulations of pair plasmas in experientelly relevant geometries.

2. Magnetic field of a ring dipole

Magn. field of a current carrying ring with radius r_0 and total current I in cylindrical coordinates is given by $\boldsymbol{B}(r,z) = \nabla \psi(r,z) \times \nabla \alpha$ with

$$\psi(r,z) = \frac{C}{2}\sqrt{(r_0+r)^2+z^2}\left[\frac{r_0^2+r^2+z^2}{(r_0+r)^2+z^2}K(\kappa) - E(\kappa)\right], \quad C = \frac{\mu_0 I}{\pi}, \quad \kappa = \sqrt{\frac{4r_0 r}{(r_0+r)^2+z^2}}$$

This dipole geometry has now been implemented in the gyrokinetic code GENE through a field line tracing module.

3. Asymptotic Limits and the Z-pinch geometry

In the region close to the current loop, the field is approximately that of a Z-pinch.

$$\frac{r-r_0}{r_0} \sim \frac{z}{r_0} \sim \frac{\rho}{r_0} \ll 1.$$

One can take the field-line average of the linear drift kinetic equation and perform the veolicty space integrals analytically. One obtains

$$1 + k_\perp^2 \lambda_D^2 = \frac{1}{2}(D_+ + D_-), \quad D_\pm = \frac{1}{\sqrt{\pi}}\int \frac{\Omega \mp \Omega_\star^T}{\Omega \mp x_\perp^2/2 \mp x_\parallel^2}\exp(-x^2)x_\perp \, \mathrm{d}x_\perp \, \mathrm{d}x_\parallel, \quad (1)$$

where we have adopted the notation of Mishchenko et al. [5] and introduced $\omega_d = \hat{\omega}_d(x_\parallel^2 + x_\perp^2/2)$, $\Omega = \omega/\hat{\omega}_d$, $x = \sqrt{x_\parallel^2 + x_\perp^2}$, $\Omega_\star^T = \omega_\star^T/\hat{\omega}_d = \Omega_\star[1 + \eta(x^2 - 3/2)]$, and $\Omega_\star = \omega_\star/\hat{\omega}_d$.

In this paper we will be primarily interested limits of this equation where we are able to deduce regions of linear stability in $(\omega_\star, \omega_\star^T)$ space. We will then use these stability maps in $(\omega_\star, \omega_\star^T)$ space to select appropriate parameters for investigating the nonlinear stability of plasmas in Z-pinch geometry. At $k_\perp \lambda_D \lesssim 1$, the fluid limit $\Omega \gg 1$ can be applied to equation (1) and from this equation we can deduce the "fluid instability condition" which states that we

2

require

$$\frac{\omega_\star}{\hat{\omega}_d}(1+\eta) > \frac{7}{4} \tag{2}$$

for an instability.

One can also use equation (1) to derive the "resonant stability boundary" by taking $\Omega \to 0$ to obtain

$$\frac{\omega_\star}{\hat{\omega}_d}(1-\eta) = \frac{1+k_\perp^2\lambda_D^2}{\pi}. \tag{3}$$

An excellent check of our numerical implementation will be the first numerical stability diagram for electron-positron plasmas and will be a reproduction of these stability boundaries.

Numerical solution of the dispersion relation
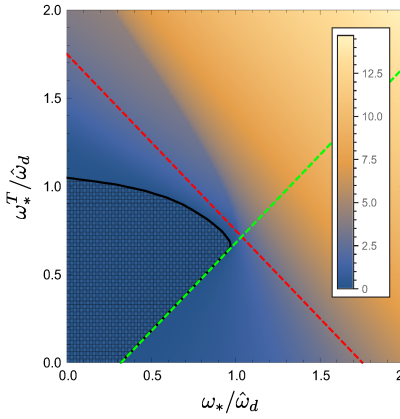
Figure 1. $k_\perp^2\lambda_D^2 = 0.01$
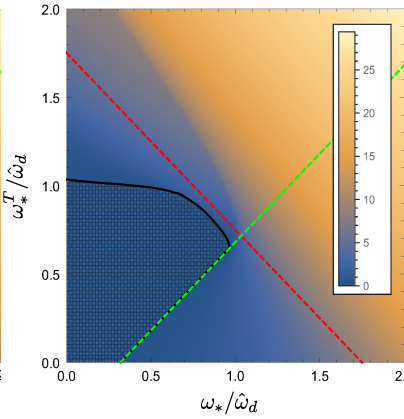
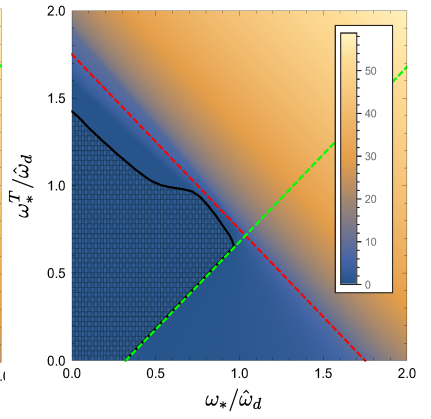Figure 2. $k_\perp^2\lambda_D^2 = 0.0025$

Figure 3. $k_\perp^2\lambda_D^2 = 0.000625$

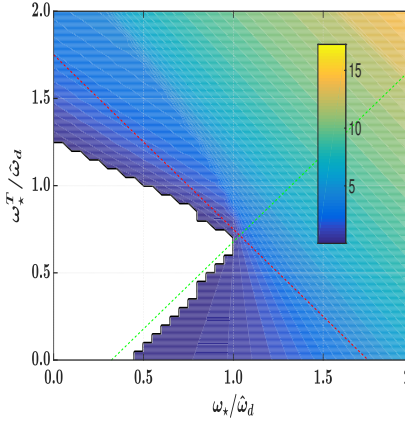### Gyrokinetic simulations using GENE.
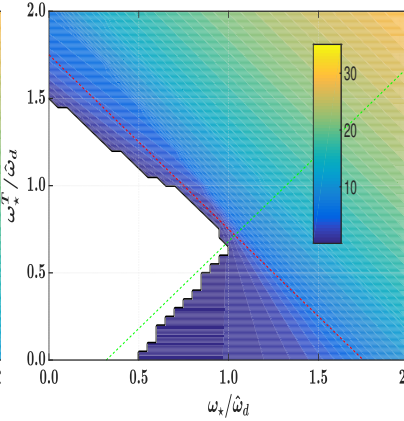

Figure 4. $k_\perp^2\rho_s^2 = 0.01$
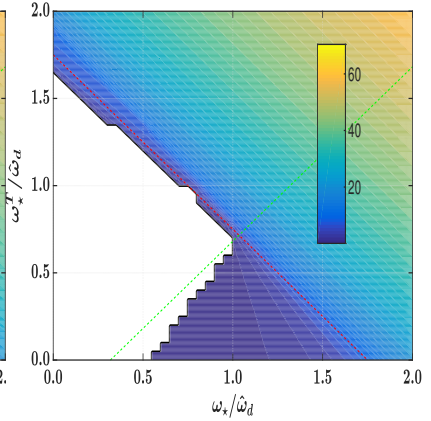
Figure 5. $k_\perp^2\rho_s^2 = 0.0025$

Figure 6. $k_\perp^2\rho_s^2 = 0.000625$

4. Linear results in Dipole geometry

We have performed the first linear simulations of electron-positron plasmas in dipole geometry with an intermediate variation of the magentic field along the flux tube.
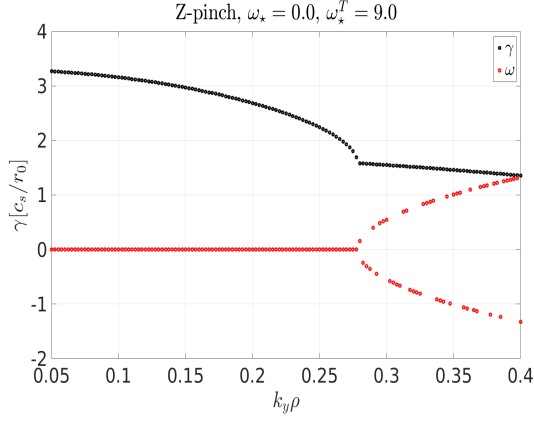
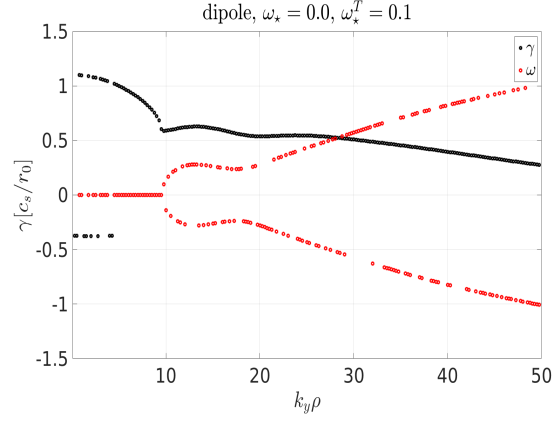Figure 7. $k_\perp \rho_s$ scan. Z-pinch $B_{max}/B_{min} = 1$.   Figure 8. $k_\perp \rho_s$ scan. Dipole $B_{max}/B_{min} \approx 10$.

We observe qualitatively similar results when comparing between the dipole and Z-pinch cases. This is similar to what was observed for conventional plasmas by Kobayashi et al. [4] using the gyrokinetic code GS2 and the numerical differences can be traced back to differences in normalisation.

5. Towards nonlinear simulations

We are now in a position, having explored the linear regime, to explore nonlinear simulations and transport in the dipole regime for electron-positron plasmas.

Nonlinear simulatons in dipole geometry and computationally very taxing due to the large number of gridpoints needed to resolve the low $k_\perp \rho_i$ features of the instability. This is the next stage of this investigation.

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