Towards A Bayesian Equilibrium Reconstruction using JET's Microwave Diagnostics

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Introduction

For the JET tokamak, standard equilibrium reconstructions find solutions of the Grad-Shafranov equation (GSE) given diverse constraints on plasma pressure and/or magnetic field [1]. The most basic reconstruction neglects plasma pressure entirely and constraints the magnetic field via measurements of pick-up coils surrounding the plasma. A more advanced reconstruction exploits polarimetry and motional-stark-effect diagnostics which provide information about the magnetic field inside the plasma. These conventional approaches provide different results, estimating to some extent the systematic uncertainties of the reconstruction. An alternative approach uses Bayes' theorem to estimate the flux surface geometry via current tomography without solving the GSE [2]. The results obtained are consistent with the underlying physics model and with uncertain data measured by magnetic and motional-stark-effect diagnostics. In preparation for a Bayesian approach involving the GSE, the work presented tests probabilistically results of the standard reconstruction of the axisymmetric equilibrium of an Ohmic JET plasma, relying on data measured with two broadband ECE diagnostics and an extra-ordinary (X-) mode reflectometer [3].

Bayesian Model

A probabilistic model (see its scheme in Figure 1), including submodels for an equilibrated axisymmetric plasma, for broadband ECE spectra, and for three microwave diagnostics, has been developed within the Bayesian framework Minerva [4]. With this model, the magnitudes of the magnetic field components B_Φ and B_θ, the spatial shifts ΔR and Δz applied to the position of the flux surface geometry, the electron temperature $T_e(\psi_N)$ and density $n_e(\psi_N)$ profiles and their length-scales, and wall reflection properties are inferred jointly given effective data supplied by two Martin-Puplett interferometers and one X-mode reflectometer. The implemented plasma equilibrium model relies on a solution for the GSE which holds during an Ohmic heating phase with low T_e and n_e. This solution provides the normalised flux surface geometry $\psi_N(R,z)$ (major radius R, height z) and the absolute fluxes ψ_{MA} (at the magnetic axis) and ψ_{LCFS} (at the last-closed flux surface). Due to the inferred change $\Delta\psi$ in the total absolute edge flux, B_θ is scaled. With the assumptions made, $B_\Phi(R,z) = B_{\Phi Vac}R_0/R$ follows with given vacuum field $B_{\Phi Vac}$ at a major radius R_0. The amplitude of this component is investigated via the parameter b_Φ, so that the rescaling becomes $B'_\Phi = B_\Phi(1+b_\Phi/100)$. The electron plasma pressure is assumed to be constant on a given flux surface, trivially fulfilled by letting $T_e = const.$ and $n_e = const.$ at a given ψ_N. The profiles $T_e(\psi_N)$ and $n_e(\psi_N)$ are represented each by 53 parameters located at $\psi_N = 0, 0.02, 0.04, ..., 1.04$. For each profile, a multi-variate normal prior with almost vanishing mean and with a generalised squared-exponential covariance. This covariance type allows to estimate profile length-scales present in distinct

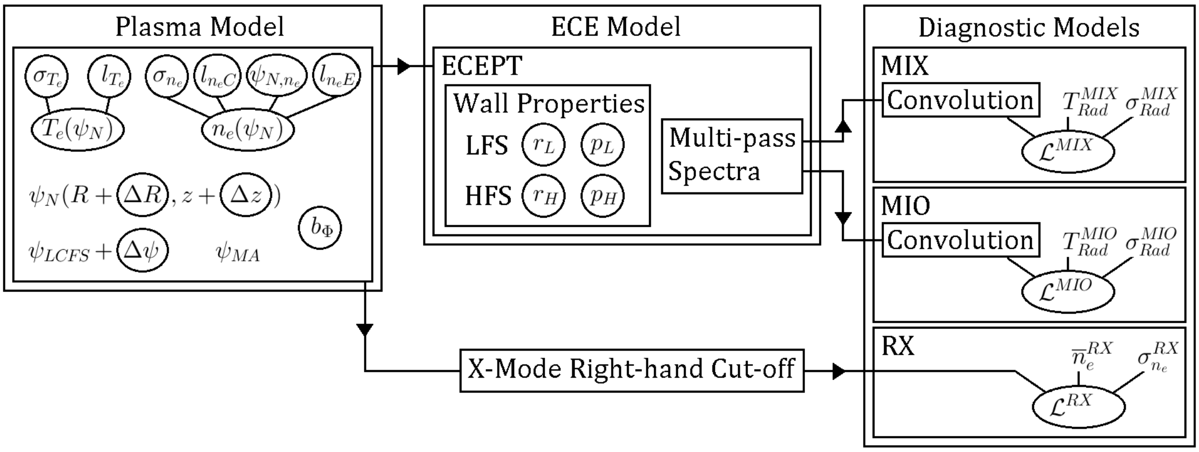

Figure 1: Scheme of probabilistic model implemented in Bayesian framework Minerva to infer electron temperature and density profiles and their length-scales, scalings to toroidal and poloidal magnetic field components, a two-dimensional spatial shift to the flux surface geometry, and reflection properties of the inner wall.

ψ_N domains. For the example below, the joint prior covariance for T_e has 2 parameters (scaling σ_{T_e}, length-scale l_{T_e}), and 4 parameters (σ_{n_e}, $l_{n_eC(E)}$ for core(edge) domain, separator ψ_{N,n_e} of core and edge domains) are considered for n_e. ECEPT predicts multi-reflection broadband ECE spectra in X- and O-mode polarisations at a diagnostic antenna for input parameters received from the plasma submodel. This developed predictor assumes a tenuous plasma (low n_e and T_e), standard diagnostic observation, and negligible refraction. At the moment, ECEPT uses the cold-resonance condition to evaluate local emission and absorption for X- and O-mode contributions at a given frequency and harmonic number [5]. The right-hand cut-off for the first harmonic range in X-mode polarisation is considered. Especially for optically thin spectral domains, multiple reflections and polarisation-scrambling at the inner wall are essential. These features are modelled by assuming parallel walls and allowing for different properties for the high-field (HFS) and low-field (LFS) side walls. For the HFS (LFS), $r_{H(L)}$ (reflection coefficient) and $p_{H(L)}$ (polarisation-scrambling coefficient) are inferred. In case, predicted X- and O-mode spectra (in terms of radiative temperature) are projected to the transmission direction of a diagnostic selective polariser placed close to the receiving antenna. At JET, the interferometers MIX and MIO probe broadband spectra mainly in X- and O-mode polarisation, respectively. Any spectrum $T^{MIX(MIO)}_{RadA}$ transmitted by the selective polariser undergoes the diagnostic principle which is given by a spectral convolution. A convoluted prediction and the effective data $T^{MIX(MIO)}_{Rad}$ enter into the Gaussian likelihood $\mathscr{L}^{MIX(MIO)}$ with uncertainty $\sigma^{MIX(MIO)}_{Rad}$ estimated thoroughly [6]. The basic diagnostic principle modelled for the X-mode reflectometer exploits the right-hand cut-off by which a launched wave is reflected at a known frequency. Prediction, i.e., n_e versus probing frequency are compared with effective data objectively via a Gaussian likelihood $\mathscr{L}^{RX}$ with uncertainties derived from the temporal variation of the effective data.

Results

At first, the maximum posterior MAP is found, which marks the "best fit to the data" by the 120 parameters, for the JET pulse 92436 at the time 1.9981 s (Ohmic heating phase). At the MAP, Markov-chain-Monte-Carlo sampling is carried out to explore the parameter space. From these samples, mean values, marginal distributions and correlations are evaluated (see some results in Fig. 2). The low T_e and n_e profiles have an uncertainty of about 10 eV and 10^{17} m^{-3}, increasing towards the edge by a factor of 4 to 5. The length-scale l_{T_e} peaks close to 0.51. l_{n_eC} and l_{n_eE} locate in well distinct domains in the vicinity of 0.76 and 0.28, respectively. The separator ψ_{N,n_e} is well estimated at 0.9 ± 0.01. According to these estimated length-scales, each core profile has a strong correlation over the scale of about $\psi_N = \pm\ 0.05$. The shift Δz has a marginal posterior with the maximum at about -0.8 cm and the standard deviaton of 1.1 cm. The radial shift has a maximum at -3 cm, and its slightly skewed distribution is alike to the distribution of the correction to the toroidal magnetic field (peak at 0.15%). Futhermore, ΔR and b_Φ have a very high correlation of 0.95. This is due to the fact that a radial shift of the flux surfaces is a synonym for changing the local toroidal magnetic field and vice versa. The change $\Delta\psi$ has a very broad marginal distribution. At least, the mean value 59 mWb (increase of B_θ by about 12%) seems reasonable, but the huge standard deviation of 430 mWb would allow B_θ to vanish at $\Delta\psi = -480$ mWb. This would be a clear violation of the assumed equilibrium. The posteriors for the wall properties peak at (r_L, p_L) = (0.33, 0.96) and (r_H, p_H) = (0.70, 0.77).

Conclusions

A Bayesian model was implemented and used to infer jointly plasma parameters, including spatial shifts to the flux surface geometry, an edge flux change, and wall reflection properties for an JET Ohmic plasma. The inference relies on effective data and its uncertainty determined with the standard microwave

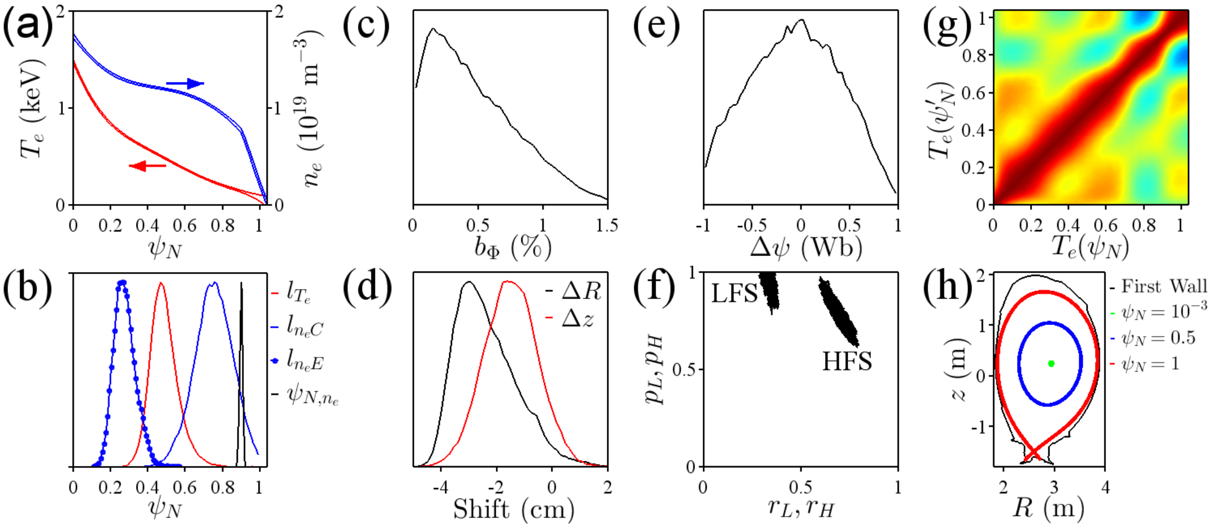

Figure 2: Some results obtained with Bayesian model for JET pulse 92436 at time 1.9981 s.

diagnostic system (two broadband ECE diagnostics, X-mode reflectometer) at JET. Smooth kinetic profiles could be inferred due to the estimation of length-scales. The core and edge electron density domains could be seperated in terms of normalised flux with high reliability. The estimated shifts in radial direction and in height point to a modified placement of the flux surface geometry (inward by 1-4 cm, downward by 0-3 cm, see uncertain flux surfaces in Fig. 2(h)). The inward radial shift is able to compensate the increase by more than 1%($>$ 25 mT) in the toroidal magnetic field component obtained when the inference is carried out at vanishing spatial shifts. The inferred scaling of the poloidal magnetic field seems unreliable. Most likely, the magnitude of the poloidal component is too small for the given uncertainty on the effective data, constraining the inference to few. The estimated reflection properties characterise the high-field side (ITER-like) wall to have much less losses in the microwave range than the low-field side wall (Inconel) has. The results have been obtained with ECEPT to predict broadband ECE spectra. This predictor is missing some relevant physics features like the relativistic resonance condition. Hence, the findings with ECEPT need to be compared or in case replaced with results, following from the usage of a much more accurate but also computationally demanding ray-tracer like SPECE. First SPECE studies confirm some of the presented results. Excluding the change of the edge flux, equilibrium related quantities like the local electron pressure, absolute position of the flux surfaces and the toroidal field correction could be estimated reliably. This gives confidence that the contributions from the electron pressure and the poloidal current function can be captured well probabilistically, at least in the plasma center with high electron pressure (small relative uncertainty). This would allow to retrieve the central flux surface geometry with good reliability once a GSE constraint is available in Minerva. Furthermore, these contributions and the GSE are expected to reduce the uncertainty in the poloidal magnetic field component much further. Towards the plasma edge, however, the pressure drops (high relative uncertainty), which would demand the inclusion of a model for the magnetic pick-up coils.

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