Shrinking of resonant manifold under flow shear at the stellarator TJ-K

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Introduction   In 2D fluids, shear flows can drive turbulent structures to larger scales as for example by the straining out process of vortices [1]. Hence, for plasma turbulence in the Hasegawa-Mima drift wave model [2], one is expecting a similar effect of shear flows on the nonlinear interaction term too [3]. This term describes three mode couplings which fulfill the resonance conditions in wavenumber ($\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3$) and frequency space ($\omega_1 + \omega_2 = \omega_3 + \Delta\omega$). However, those conditions are dependent from each other because of the drift wave dispersion relation $\omega_i(\mathbf{k}_i)$ and the set of possible couplings in the k-space is restricted to the so called resonant manifold [4]. In an extension, Gürcan modified the model by making the radial wavenumber dependent on time as subject to an introduced poloidal, constant shearing rate [3]. As a consequence the manifold in k-space shrinks in time excluding couplings with higher modenumbers i.e. smaller turbulent structures, in favour of couplings with larger structures. For this contribution, the Gürcan extension is tested experimentally on the basis of time-dependent, Gaussian shearing rates, which resemble the transient behavior of zonal flows in the stellarator experiment TJ-K. An effective modenumber, which represents the behavior of the coupling space, can be calculated from a time-dependent, wavelet wavenumber-frequency-bicoherence and is analyzed with respect to an upcoming zonal flow.

Shear dependent dispersion relation   The resonance conditions are coupled through the dispersion relation of drift waves ($\omega_i(\mathbf{k}_i) \propto k_{i,y}/(1 + \mathbf{k}_i^2)$). This limits the set of coupling possibilities as shown in fig.1, left. Here, for a constant mode $\mathbf{k}_1 = (0.6, 0.4)$ at $\tau = 0$ the coupling possibilities in $\mathbf{k}_3$ are shown as bandlike structure with a spectral width of $\Delta\omega = 0.001$. The time-dependent shear (v' see eq. 1) is implemented as Gaussian (see fig. 1, right) such that

$$\omega(\mathbf{k}) \propto \frac{k_y}{1 + k_y^2 + k_x^2} = \frac{k_y}{1 + k_y^2 + (k_x' - v'k_y\tau)^2} \quad \text{with} \quad v' = v'(\tau) \approx e^{-(\tau - 0.5)^2} \quad . \quad (1)$$

The effect of flow shear on the set of couplings is shown in fig.1 left for selected times. The set of couplings shrinks with higher shear. In order to quantify this behavior and to reduce the dimensions, the minimum and maximum of the set in $k_{3,y}$ are examined. Those are shown in fig. 1, right. With the occurance of the shear flow, the span of the coupling modes decreases. This

implies that the coupling partners at higher modenumbers are excluded and only the ones with lower values remain in the coupling process. Note that there are time-independent unaffected couplings with $\mathbf{k}_3 = (0,0)$ and $k\mathbf{k}_3 = \mathbf{k}_1$ of the set. Hence, for comparison with the experiment three sectors are defined: Sectors I (blue), II (yellow) and III (red), where a shrinking can only be detected in sector I & III.
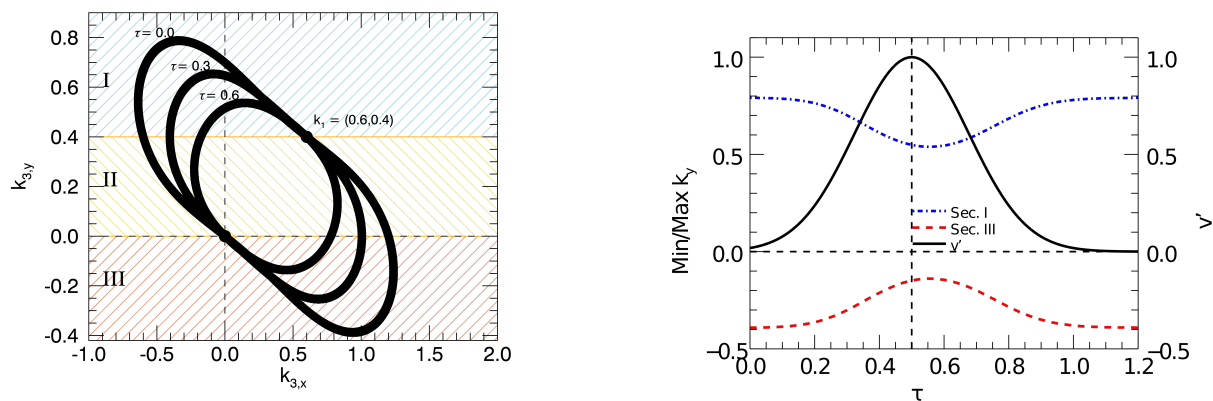

Figure 1: *Left: The set of couplings (resonant manifold) for a constant $k_1 = (0.6, 0.4)$ and selected realisations in time. The set shrinks in sectors I & III only. Right: The thick, black line showns the time dependent shear. The blue, dash-dotted line the maximum and red, dashed the minimum wavenumber for the set of couplings.*

Stellarator TJ-K The low-temperature plasma at the stellarator experiment TJ-K allows to analyze the plasma dynamics with Langmuir probes throughout the confinement region. An array of Langmuir probes, poloidally distributed on four neighbouring flux surfaces, measures the potential fluctuations in the confined region [5]. This provides a unique possibility to detect mesoscopic shear flows like the zonal flows and further to apply a kf-bicoherence analysis, which quantifies the nonlinear coupling. Using the array data, the poloidal wavenumber space is accessible for the coupling analysis.

Time-dependent kf-bicoherence Based on the idea of [6], a time-dependent wavelet kf-bicoherence analysis is carried out, which takes into account the dispersion relation in the resonance conditions. First, a wavelet transformation is applied for each probe, in order to transform the data into the frequency domain by simultaneously preserving the time resolution. At each point in time, the scale separated results are transformed into poloidal wavewnumber space. As ensembles for the bispectrum average, time points were taken conditionally, when the zonally

averaged potential takes maximum values after a threshold of two times the standard deviation is exceeded. Then the bicoherence can be applied as $b^2(k_1, k_2, \omega_1, \omega_2, \tau)$ with τ the delay time with respect to large trigger events in the zonal potential. This way the nonlinear coupling behavior can be compared with an upcoming zonal flow (ZF). The conditionally averaged ZF, normalized to its maximum, is shown in fig. 2 together with the theoretically expected evolution of the wavenumber limits of the coupling set similar to fig. 1.

Shrinking of the effective coupling space

The bicoherence is integrated over all frequencies in order to reduce the analysis to k-space. As in [3], the analysis is focused on one mode ($\rho_s k_1 \approx 0.48$) and its bicoherences amplitude with other modes in $\rho_s k_3$ are taken as coupling distribution. This distribution is used as (normalized) weighting function (w_i) for estimating an efficient wavenumber ($k_{eff} = \sum_i w_i \cdot k_i$) in sectors I & II. The two effective couplings are shown in fig. 3. One the left side, the effective modenumber

Figure 2: *Conditionally averaged zonal potential and extrema of the set of couplings corresponding to eq. 1.*

for the positive sector is shown. The corresponding extrema of the calculated set of couplings from fig. 2 are shown as square symbols. Except for the absolut values, both quantities show a quite remarkably behavior. With high shear the set of coupling decreases and the effective modenumber takes lower values. Such decrease of effective modenumber can also be observed in the negative sector III (fig. 3, right).
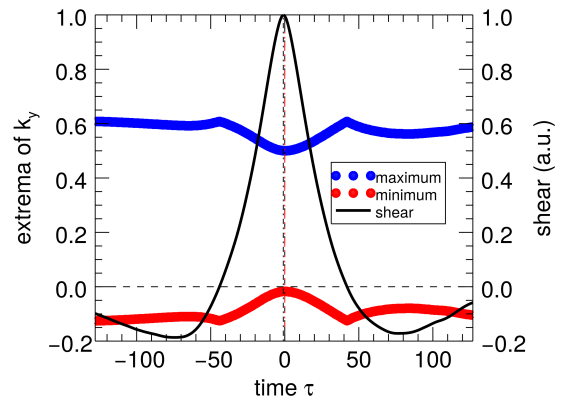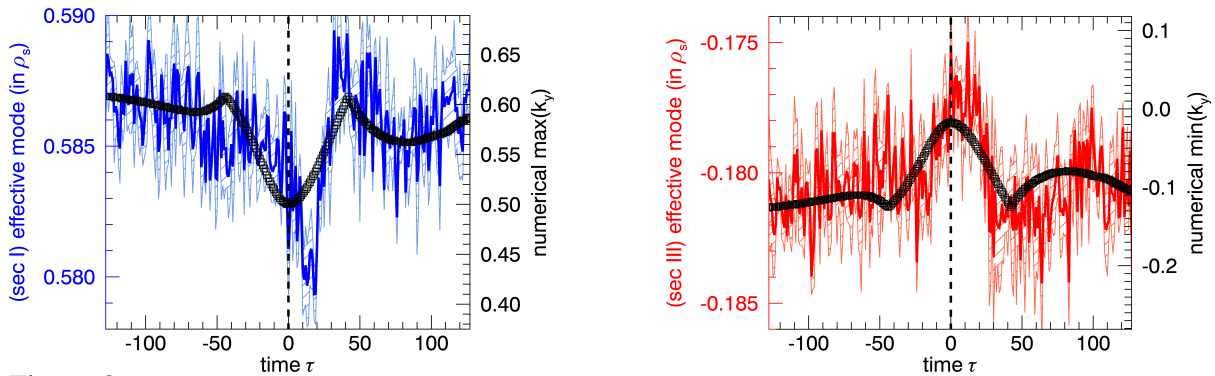
Figure 3: *The effective modenumber for sector I (left-, blue) and sector III (right, red) in fig. 1 and its corresponding evolution from fig. 2.*

Summary The effect of time-dependent flow shear on the set of three-wave couplings has been investigated. To this end, Gürcans model is utilized with time-dependent shear for which a reduction of the set of coupling possibilities is observed. A wavelet-wavenumber-frequency-bicoherence is employed in order to ensure that the resonance conditions for the nonlinear coupling term in the Hasegawa-Mima model are satisfied. By deducing an effective modenumber in the wavenumber-frequency domain an indicator for the width of the set is made available. Indeed, in the presence of the upcoming shear, the effective modenumber decreases. This is the first experimental evidence that the resonant manifold shrinks with the shear. By the exclusion of couplings with higher wavenumbers only couplings with lower wavenumbers remain and, thus, the relative amount of coupling possibilities with larger turbulent structures might increase as to render zonal flow drive more efficient.

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