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Mapping Heuristics and Prospect Theory:

A Study of Theory Integration

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Abstract

Efforts to model boundedly rational risky choice have generated two influential but separate lines of research: cumulative prospect theory (CPT) and heuristics. Each approach has pursued its own research questions, and both have made important contributions to the study of decision making under risk, but these contributions have hardly been connected to each other. In this chapter, I illustrate how the two approaches can be brought together. Specifically, I show how the choices produced by heuristics of risky choice are reflected in CPT's value and weighting functions, and how CPT can be used to reveal and measure how the properties of choices produced by some heuristics are contingent on the structure of the environment. Finally, I discuss how empirical choice phenomena (e.g., the fourfold pattern of risk attitudes) that have critically shaped key assumptions in CPT might result from heuristic information processing, and how insights into the boundary conditions of heuristic information processing can in turn suggest moderating conditions for those choice phenomena. These analyses highlight the value of elaborating the relationships between theories in order to pursue theory integration.

One of the most valuable methodological tools in the behavioral science repertoire is to build models of behavior. The predictions of the models can then be contrasted with data, either to refute a model or to refine it. Formal models often have adjustable parameters representing behaviorally relevant constructs, and variability on these constructs can be measured by fitting the models to empirical data.

Herbert Simon's (1956, 1990) idea of bounded rationality—according to which successful and adaptive behavior is shaped by the mind's natural limits with respect to information processing and computational power—has inspired two directions in the modeling of decision making. First, *prospect theory* (Kahneman & Tversky, 1979) and its subsequent elaboration and formal specification in *cumulative prospect theory* (CPT; Tversky & Kahneman, 1992) assume that people's sensitivity to differences in the outcomes and probabilities of risky options diminishes the further away those magnitudes are from natural reference points (such as zero, impossibility, or certainty) and that losses carry more psychological weight than gains. These notions are described algebraically with psycho-economic functions that translate objective magnitudes of outcomes and probabilities into subjective ones. CPT is typically considered as an "as-if" model, in the sense these functions are not meant to represent the actual cognitive processes underlying choices. The second modeling tradition rooted in bounded rationality, by contrast, has developed *heuristics*, that are intended as cognitive process models. Heuristics implement bounded rationality by describing choices as being based on simplifying information processing operations, such as lexicographic search, difference thresholds, stopping rules, and limited search (e.g., Brandstätter, Gigerenzer, & Hertwig, 2006; Payne, Bettman, & Johnson, 1993; Thorngate, 1980; Venkatraman, Payne, & Huettel, 2014).

Despite their common theoretical roots, CPT and heuristics thus rely on very different conceptual languages—algebraic functions vs. simple information processing operations—to model boundedly rational decision making. Potentially due to these differences, there have

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been only few attempts to systematically connect the two approaches.[1] As a consequence, theoretical concepts for characterizing risky choice—such as probability weighting, risk aversion, and loss aversion—exist side-by-side theoretical concepts characterizing heuristic cognitive processes—such as lexicographic search, difference thresholds, stopping rules, and one-reason decision making (see below for details), with only little understanding of how these concepts relate to each other (but see Willemsen, Böckenholt, & Johnson, 2011). Moreover, it is little understood how heuristics might relate to empirical choice phenomena that have critically shaped the development of CPT. Take, as an example, the fourfold pattern of risk attitudes (other phenomena include the common ratio effect, the certainty effect, and reflection effect). The fourfold pattern refers to the finding that whether people appear risk averse or risk seeking shifts depending on whether the options offer positive or negative outcomes and whether the probability of the risky outcome is high or low. CPT accounts for the fourfold pattern by assuming distorted probability weighting, where rare events are overweighted and common events are underweighted (relative to their objective probabilities). But there might be other, more process-oriented accounts (e.g., heuristics) that can explain the fourfold pattern—a possibility that has hardly been pursued.

In this chapter I will illustrate how CPT and heuristics can be brought together to their mutual benefit. The analyses presented contribute to theory integration (e.g., Gigerenzer, 2017) by clarifying the relationship between the different theoretical concepts of CPT and heuristic decision making. For instance, I show that the typical assumption in CPT of distorted probability weighting can arise from a hallmark of heuristic processing: limited search. Conversely, CPT's conceptual language can be used to isolate and measure how properties of the choices produced by heuristics change across choice ecologies.

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[1] Kahneman and Tversky (1979) proposed heuristic editing operations (e.g., cancellation, segregation, combination) that people might apply before evaluating lotteries, but these operations were barely investigated further. Moreover, whenever CPT and heuristics of risky choice have been considered within the same analysis, they have been treated as rival accounts (e.g., Brandstätter et al., 2006; Fiedler, 2010; Glöckner & Herbold, 2013; Glöckner & Pachur, 2012; Pachur, Hertwig, Gigerenzer, & Brandstätter, 2013; Su et al., 2013).

It should be emphasized that the primary goal of this theory integration is not to eventually merge the approaches in a unified theory. Instead, it aims to overcome the disintegration between alternative theoretical accounts of decisions under risk by contributing to a better understanding of how the frameworks overlap and how they complement each other in describing behavior. The field of risky choice is teeming with theories that together provide a rich conceptual repertoire to account for decisions under risk: in addition to heuristics and prospect theory, there are, for instance, the transfer-of-attention exchange model (e.g., Birnbaum, 2008), the security-potential aspiration model (Lopes & Oden, 1993), the proportional difference model (González-Vallejo, 2002), and decision field theory (Busemeyer & Townsend, 1993). Work on these theories, however, has mostly either focused on the performance of a single theory, or pitted some of these theories against each other. As a consequence, we know little of the extent to which the conceptual languages of these theories cover separate parts of the terrain or whether they overlap in their explanatory contribution. Importantly, the theory integration pursued here can also lead to new predictions. For instance, if heuristic choices can be reflected in nonlinear probability weighting, which in turn accounts for a particular choice phenomenon, insights into the boundary conditions of the use of the heuristics can used to predict moderating conditions for the emergence of the choice phenomenon. Eventually, a better understanding of the network between the theories might contribute to a unification of existing approaches, but this is not necessary for theory integration to be useful.

In the following, I first describe CPT and heuristics in more detail before presenting an analysis that shows how CPT accommodates the choices produced by various heuristics in the shapes of its weighting and value functions. I then illustrate how prominent choice phenomena that have shaped critical assumptions in CPT could arise from heuristic information processing. Finally, I highlight an important added value from this theory integration, namely, that insights about the contingent nature of heuristic information

processing (e.g., Payne, 1976) can be used to derive novel predictions regarding moderating conditions for the occurrence of the choice phenomena.

Two Modeling Approaches for Boundedly Rational Risky Choice

Cumulative Prospect Theory

CPT was developed as an attempt to account for empirical violations of expected value (EV) and expected utility (EU) theory, such as the common ratio effect (also known as the Allais paradox; Allais, 1953), the fourfold pattern of risk attitudes, and the certainty effect. In EV theory the valuation of a risky option A with outcomes $x_m > \dots > x_1 \geq 0 > y_1 > \dots > y_n$ and corresponding probabilities $p_m \dots p_1$ and $q_1 \dots q_n$ follows from summing the outcomes, each weighted by its probability (i.e., $EV = \sum_{i=1}^{N} x_i p_i$). In CPT, by contrast, it is assumed that outcomes and probabilities are subject to a nonlinear transformation. The transformation of outcomes is described by a value function that is defined as

$$
\begin{aligned}
v(x) &= x^\alpha \\
v(y) &= -\lambda(-y)^\alpha
\end{aligned} .
\tag{1}
$$

The parameter α governs how strongly differences in objective outcomes are reflected in differences in subjective value and thus indicates outcome sensitivity (with lower values indicating less sensitivity). Parameter λ reflects the relative weighting of losses and gains; with values of λ larger than 1, a higher weight is given to losses, indicating loss aversion. The left panel of Figure 1 depicts value functions for different values of α and λ.
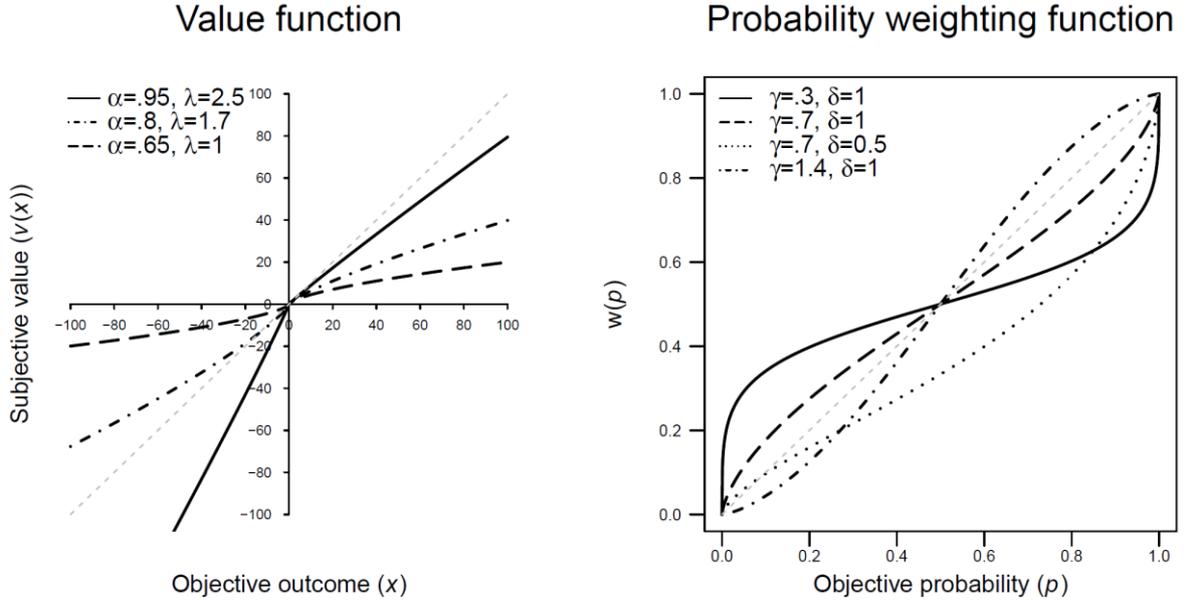
Figure 1. CPT's value function for different values of the outcome sensitivity (α) and loss aversion (λ) parameters (left); and the probability weighting function for different values of the probability-sensitivity (γ) and elevation (δ) parameters (right).

The transformation of the outcomes' probabilities (more precisely, the rank-dependent, cumulative probability distribution) is described with a probability weighting function. A commonly used parameterization of the weighting function (see Goldstein & Einhorn, 1987) separates the curvature of the probability weighting function from its elevation and is defined as follows:

$$w^+ = \frac{\delta^+ p^{\gamma^+}}{\delta^+ p^{\gamma^+} + (1-p)^{\gamma^+}} \quad for \quad x$$
$$w^+ = \frac{\delta^+ q^{\gamma^-}}{\delta^+ q^{\gamma^-} + (1-q)^{\gamma^-}} \quad for \quad y$$

(2)

with γ^+ and γ^- governing the curvature of the weighting function in the gain and loss domains, respectively (in the following analyses, a common γ for both domains is estimated). Lower values on γ^+ and γ^- indicate greater curvature and thus lower sensitivity to probabilities. The parameters δ^+ and δ^- govern the elevation of the weighting function for gains and losses, respectively, and are often interpreted as indicating the degree of optimism thus risk seeking (e.g., Gonzalez & Wu, 1999). Higher values on δ^+ indicate higher risk seeking in the gain

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domain, higher values on δ^- indicate higher risk aversion in the loss domain. The right panel of Figure 1 depicts probability weighting functions for different values of the γ and δ parameters. Assuming these transformations of the objective outcomes and probabilities of risky options, as formalized in the value function and the weighting function, CPT can account for the violations of people's choices of EV and EU theory mentioned above (for a more extensive discussion of CPT, see Wakker, 2010).

Heuristics

Whereas CPT's value and weighting functions are not meant to describe the cognitive processes leading to a choice, researchers following the heuristics approach use conceptual building blocks that acknowledge which mental operations actual decision makers are able to perform. These include lexicographic search (i.e., the sequential consideration of attributes, which in risky choice are the possible outcomes and their probabilities), within-attribute comparison of options, difference thresholds (i.e., boundaries indicating when two attribute values are to be treated as different), stopping rules (i.e., conditions under which information search is truncated), and one-reason decision making (i.e., basing a choice on a single attribute).

Table 1 gives an overview of five heuristics of risky choice. Some involve extremely simple processing and consider only a single attribute. Take, for instance, the minimax heuristic (e.g., Savage, 1954). In order to choose between options—each described by possible outcomes and their probabilities—minimax focuses exclusively on each option's worst possible outcome and chooses the option whose worst outcome is more attractive. All other attributes—that is, the other outcomes and all probabilities—are ignored. The least-likely heuristic (Thorngate, 1980) is somewhat more complex. Having likewise identified the worst possible outcome of each option in a first step, it bases its decision on the probability of that outcome, choosing the option whose worst possible outcome is least probable. The most-likely heuristic (Thorngate, 1980) also has a two-step structure: It first identifies each option's

most likely outcome and then chooses the option whose most likely outcome is more
attractive. Finally, some heuristics implement difference thresholds (also known as aspiration
levels) and involve conditional processing of attributes. The priority heuristic (Brandstätter et
al., 2006), for example, first considers the options' minimum gain or loss, depending on the
domain. If the two options differ sufficiently on that attribute (formalized by a difference
threshold, defined as 10% of the maximum gain or loss), it chooses the option with the more
attractive minimum outcome. If the difference between the options does not exceed the
difference threshold, the priority heuristic moves on to the next attribute, the probabilities of
the outcomes. If the probabilities differ by at least 10% (the difference threshold for the
probability attribute), the heuristic chooses the option with the lower probability of the
minimum outcome; otherwise, it moves on to the next attribute, the maximum outcome, and
chooses the option that is most attractive on that attribute.

Table 1

Five Heuristics of Risky Choice

Heuristic	Description
Minimax	Choose the option with the highest minimum outcome.[1]
Maximax	Choose the option with the highest outcome.[1]
Least-likely	Identify each option's worst outcome. Then choose the option with the lowest probability of the worst outcome.[1]
Most-likely	Identify each option's most likely outcome. Then choose the option with the highest most likely outcome.[1]
Priority heuristic	Go through attributes in the following order: minimum gain, probability of minimum gain, and maximum gain. Stop examination if the minimum gains differ by 1/10 (or more) of the maximum gain; otherwise, stop examination if the probabilities differ by 1/10 (or more) of the probability scale. Choose the option with the more attractive gain (probability). For options with more than two outcomes, the search rule is identical, apart from the addition of a fourth reason: probability of maximum gain. For loss lotteries, "gains" are replaced by "losses." For mixed lotteries, "gains" are replaced by "outcomes."

Note. [1]If the decisive reason does not discriminate, the heuristic choses randomly. Heuristics are from Thorngate (1980) and Brandstätter et al. (2006).

Mapping Heuristics onto CPT

How could CPT and heuristics—despite using very different modeling frameworks to describe people's decisions under risk—be related to each other? Gigerenzer (2017) has sketched out an approach to theory-integration. Rather than submitting theories to critical tests (e.g., Popper, 1934/1959) or pitting them against each other, the goal is to build networks between theories, integrating and connecting empirical phenomena and theoretical concepts, in order to achieve greater coherence in a field, identifying alternative routes to explain the same behavior or to see how they complement each other.

So how can heuristics and CPT be meaningfully integrated? Different heuristics make different assumptions about which attributes are considered, the order in which attributes are considered, and which attribute determines a choice. As a consequence, they also differ in the priority they give to probability information and to the minimum and maximum outcomes. The crucial idea is now to use CPT to measure theoretical properties of the heuristics' choices, such as the degree of probability sensitivity, risk aversion, and loss aversion. Differences between the heuristics on these theoretical properties can then be linked to differences in the heuristics' information processing architectures. Elaborating these connections therefore helps to illuminate how CPT's theoretical constructs, and key empirical phenomena (e.g., fourfold pattern) that have shaped CPT, might be related to information processing principles. I next describe such a theory-integration analysis of CPT and heuristics.

What Shapes of CPT's Weighting and Value Functions Do Heuristics Produce?

In order to understand how heuristics map onto the conceptual framework of CPT, Pachur, Suter, and Hertwig (2017) investigated how the choices produced by the heuristics described in Table 1 would be reflected in CPT's weighting and value functions. To that end, they estimated the shapes of functions that allowed to reproduce, as best as possible, the heuristics' choices. The results, shown in Figure 2, indicate that the five heuristics gave rise to

very distinct value and weighting functions.[2] For instance, the choices of the minimax and the maximax heuristics were accommodated in CPT by assuming very strongly curved weighting functions (upper row of Figure 2), indicating very low probability sensitivity. Importantly, this result is consistent with the heuristics' information processing architecture, which completely ignores probability information. For minimax, moreover, the elevation of the resulting weighting function was very low for gains and very high for losses, indicating generally high risk aversion. For maximax, the pattern was reversed, indicating generally high risk seeking. For the priority heuristic, the curvature of the weighting function was not quite as pronounced as for minimax and maximax, indicating higher probability sensitivity—again consistent with the fact that the heuristic considers probability information when the first-ranked attribute does not discriminate between the options (this was the case in around 20% of the choice problems used in the analysis). The weighting functions estimated for the choices of the least-likely and most-likely heuristics, finally, showed the least pronounced curvature, indicating high probability sensitivity and therefore accurately reflecting that these heuristics always consider probability information (though in different ways).

The lower row of Figure 2 shows CPT's value functions that best describe the choices produced by the heuristics. Here as well there were considerable differences between the heuristics, and these differences were consistent with the differences between the heuristics in terms of how their information processing architectures treat outcome information: the least-likely heuristic, which never bases its choices on outcomes, yields a very strongly curved value function, indicating very low outcome sensitivity. The minimax, maximax, and most-likely heuristics, for all of which outcomes are the decisive attribute, have a less curved value function, indicating high outcome sensitivity.

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[2] Unsurprisingly, CPT cannot mimic the heuristics' choices perfectly. Moreover, there are differences between the heuristics in how well CPT can capture their choices. For instance, CPT accommodated the choices by minimax and maximax heuristics better than those of the least-likely heuristic. See Pachur et al. (2017) for more details.
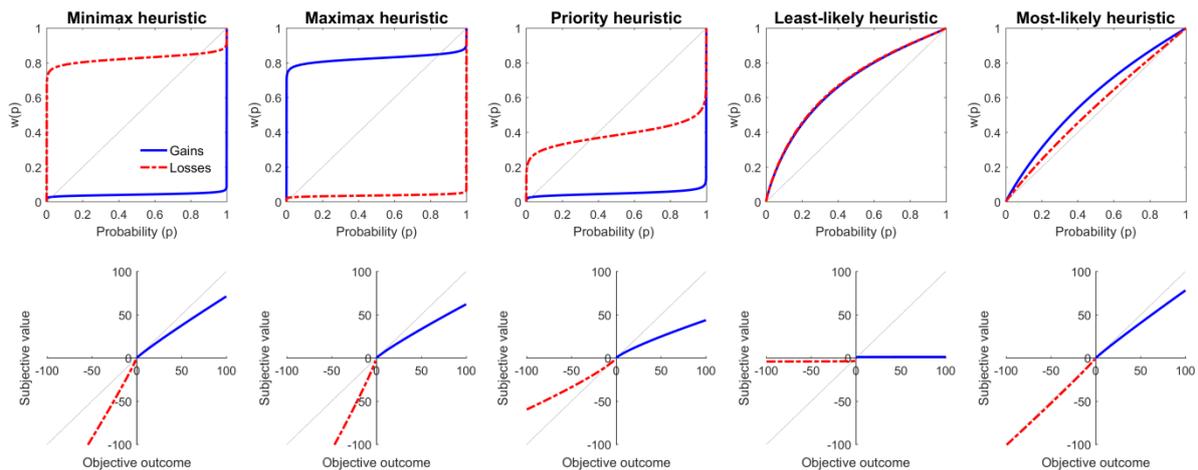
Figure 2. CPT's weighting function (upper row) and value functions (lower row) estimated for the choices of the five heuristics described in Table 1.

These results highlight two things. First, they demonstrate that CPT's weighting and value functions, though not meant to represent cognitive mechanisms, can meaningfully measure characteristics of the cognitive mechanism underlying choices: Choices produced by heuristics whose information processing architecture implement different degrees of attention to probabilities and outcomes are accommodated in CPT by systematic differences in the curvature of the weighting and value functions, respectively, with lower attention giving rise to stronger curvatures. Pachur et al. (2017) showed the CPT profiles produced by different heuristics are sufficiently distinct from each other to allow differentiating which heuristic produced a set of choices. Second, it is not necessary to assume differential distributions of attention at the high and low ends of the probability scale to produce overweighting of rare and underweighting of common events (e.g., Bordalo, Gennaioli, & Shleifer, 2012; Hogarth & Einhorn, 1990). Instead, this pattern can arise from a single cognitive mechanism, namely, reduced attention to *any* probability information (for other proposals, see Bhatia, 2014; Johnson & Busemeyer, 2016). This means that the apparent overweighting of low-probability events can, ironically, result from underattending to low probabilities.

Characterizing Changes in the Behavior of Heuristics Across Environments

In some heuristics, the character of the choices they produce depends on the structure of the environment. For instance, in the priority heuristic, whether a choice is made based on a probability attribute depends on whether the minimum outcomes in a decision problem differ sufficiently. In the most-likely heuristic, whether the option it chooses has a very attractive or a rather unattractive outcome depends on which of the two outcomes is more likely—and thus whether and how outcomes and probabilities are correlated. In the following, I show how the CPT framework can make visible and measure these interdependencies between environmental structures and the properties of heuristic choices.

One fundamental property of choice behavior is the strength of the tendency to pick or to avoid the more risky option—that is, how risk-seeking or risk-averse choices are. A common measure of an option's risk is the *coefficient of variation* (CV; e.g., Weber, Shafir, & Blais, 2004), with a higher CV indicating higher risk.[3] In CPT, the tendency to choose the more risky option is reflected in the elevation of the weighting function. To see how the apparent risk attitude—defined as the willingness to choose the option with a higher CV— produced by heuristics might differ across environments, let us first consider the minimax heuristic. As mentioned above, minimax produces strongly risk-averse (risk-seeking) choices in the gain (loss) domain; the opposite holds for maximax. The high level of risk aversion and risk seeking, respectively, generated by these heuristics is due to their focus on extreme outcomes (either the minimum or the maximum outcome). The option with the more attractive minimum outcome (which minimax chooses) is usually the less risky one, because (unless it is a dominating option) its maximum outcome is usually rather unattractive (and the range of outcomes of that option is usually smaller, yielding a lower CV). The option with the

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[3] The CV is a standardized measure of risk and expresses the amount of risk per unit of return. It is a function of the outcomes and probabilities of an option, and is larger the larger the range and the more skewed the distribution of possible outcomes. In addition, ceteris paribus, the CV is larger the more the distribution of probabilities is similarly skewed as the distribution of outcomes, but in the opposite direction. Specifically, the CV is defined as the standard deviation of an option, $\sigma = \sqrt{\sum_{i=1}^{N} x_i^2 \times p_i - \left(\sum_{i=1}^{N} x_i \times p_i \right)^2}$, with N outcomes x and probabilities p, divided by the absolute value of the option's expected value.

more attractive maximum outcome (which maximax chooses) is usually the more risky one, because its minimum outcome is usually rather unattractive (and the range of outcomes thus rather large, yielding a higher CV).

Minimax and maximax give rise to risk aversion and risk seeking, respectively, irrespective of the degree to which outcomes and probabilities are correlated, a key property of decision environments (e.g., Pleskac & Hertwig, 2014). But consider the most-likely heuristic. Recall that it first identifies each option's most likely outcome and then picks the option where that outcome is more attractive. In environments in which the magnitudes of outcomes and their probabilities are negatively correlated, the most likely outcome of the option chosen by the most-likely heuristic is usually also the minimum one in the gain domain and the maximum one in the loss domain. As a consequence, the heuristic should produce risk-averse choices in the gain domain and risk-seeking choices in the loss domain in such environments. Importantly, the regularity that the most likely outcome is also the minimum gain (or the maximum loss) does not hold in an uncorrelated environment, where the magnitude of the outcomes and their probabilities are not correlated; here, the heuristic should thus produce neither systematically risk-averse or risk-seeking, but risk-neutral choices. In other words, the risk attitude produced by the most-likely heuristic is contingent on the structure of the environment.

CPT can make visible such contingent risk attitudes of heuristics through the elevation of the probability weighting function, governed by the parameter δ (see Equation 2). In Pachur et al. (2017), we measured the dependency of the most-likely heuristic's degree of risk aversion on the structure of the environment by estimating CPT's weighting and value functions for choices produced by the heuristic, separately for an environment in which outcomes and probabilities were negatively correlated (within a decision problem) and an environment in which outcomes and probabilities were uncorrelated. Figure 3 shows the resulting weighting functions (the elevation parameter δ was estimated separately for the

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probabilities of gain and loss outcomes). As can be seen, the most-likely heuristic produced

distinct shapes in the two environments, mainly driven by differences in the elevation

parameter. In the uncorrelated environment, the weighting function was approximately linear

(and δ close to 1), indicating that the heuristic produced risk-neutral choices. In the correlated

environment, by contrast, the elevation was very low (and δ rather small), indicating strong

risk aversion in the gain domain and strong risk seeking in the loss domain. The other

heuristics in Table 1 did not show such contingency in the risk attitudes (see Pachur et al.,
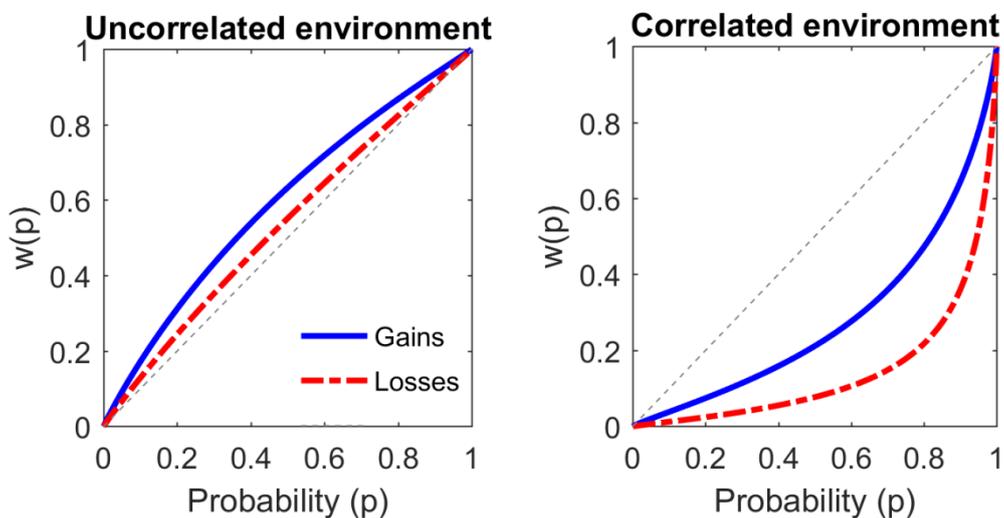
2017, for details).

Figure 3. Probability weighting functions estimated for choices produced by the most-likely
heuristic, separately for an environment in which the magnitudes of outcomes and
probabilities were uncorrelated (left panel) and an environment in which the magnitudes of
outcomes and probabilities were negatively correlated (right panel).

To summarize, the parametric measurement framework of CPT makes it possible to

track how the behavior of heuristics is affected (or not) by the structure of the environment.

Although the qualitative shift in the risk attitude of the most-likely heuristic across

environments that we observed in the present analysis could, in principle, also be derived

without invoking CPT, CPT has the advantage that it also captures more gradual changes in

behavior in response to more gradual changes in the structure of the environment (e.g.,

correlations between outcomes and probabilities of different strengths).

Connecting Phenomena

The previous section has shown how theory integration between CPT and heuristics can be achieved by mapping the two theoretical frameworks onto each other. Another route to theory integration is to connect empirical phenomena that are rooted in different theoretical traditions (Gigerenzer, 2017). In this section, I apply this approach for CPT and heuristics, considering the aforementioned fourfold pattern of risk attitudes (Tversky & Kahneman, 1992; Tversky & Fox, 1995). The fourfold pattern has played an important role in the development of CPT and is illustrated in Table 2. When the risky option offers a gain with a relatively high probability, people display risk aversion, with the majority choosing a safe option over a risky option. When the risky gain has a low probability, choices are more risk seeking, in that the willingness to choose the risky option is increased. In the loss domain, the opposite holds (i.e., people are risk seeking for low-probability losses and risk averse for high-probability losses).

Table 2

The Fourfold Pattern of Risk Attitudes: Typical Risk Attitude Observed for the Lottery Problems, Containing a Safe Option and a Risky Option with the Specific Combination of Probability Level and Domain. The Percentages in Brackets Indicate the Proportions of Choices of the Risky Option in Hertwig, Barron, Weber, and Erev (2004).

Probability level	Domain	
	Gains	Losses
Low	$32, .1; $0, .9 vs. $3, 1 Risk seeking (48%)[a]	−$32, .1; $0, .9 vs. −$3, 1 Risk aversion (36%)
High	$4, .8; $0, .2 vs. $3, 1 Risk aversion (36%)	−$4, .8; $0, .2 vs. −$3, 1 Risk seeking (72%)

Note. [a] Although this particular choice proportion observed in Hertwig et al. (2004) does not represent risk seeking, risk seeking is often observed for other choice problems of the same type—in particular, when risk attitudes are estimated with certainty equivalents (e.g., Tversky & Fox, 1995; Tversky & Kahneman, 1992; for a discussion, see Harbaugh, Krause, & Vesterlund, 2010).

CPT accounts for the fourfold pattern with its assumption of an inverse S-shaped weighting function, where low-probability events are overweighted and high-probability events are underweighted relative to their objective probabilities. As Brandstätter et al. (2006) showed, however, the fourfold pattern is also consistent with heuristic information processing. Specifically, it can follow from the heuristic principles of lexicographic processing, difference thresholds, stopping rules, and one-reason decision making implemented by the priority heuristic. Here is how, starting with the choice problem with a low-probability gain. Comparing a safe gain of $3 with a 1% chance of winning $32, the priority heuristic starts by comparing the options' minimum gains: $3 for the safe option and $0 for the risky option. Because the difference between these two outcomes does not exceed the difference threshold of 10% of the maximum gain ($3 < $32 * .1 = $3.2), the priority heuristic next compares the probabilities of the minimum gains. Because for the risky option this probability is lower than for the safe option (.9 < 1), the risky option is chosen, giving rise to risk-seeking behavior. For the choice problem with a high-probability gain, the choice is made on the first attribute, the minimum gain, because the difference between the options on that attribute exceeds the difference threshold ($3 > $4 * .1 = $0.4), and the safe option is chosen, giving rise to risk-averse behavior. In the problem with a low-probability loss, the priority heuristic first compares the minimum losses. Because this attribute does not discriminate between the options ($3 < $32 *.1 = $3.2), the heuristic next considers the probabilities of the minimum losses. The safe option has a higher probability (1 > .9) and is thus picked, leading to risk aversion. In the problem with a high-probability loss, the heuristic makes a decision on the first attribute, the minimum loss, because the difference between the options on that attribute exceeds the difference threshold ($3 > $4 * .1 = $0.4); the risky option is chosen because its minimum loss is smaller ($0 < −$3), leading to risk seeking.

This example exemplifies how empirical phenomena that have shaped key assumptions in CPT—such as an inverse S-shaped weighting function—could also arise from heuristic information processing. It thereby also points to the cognitive mechanisms behind these hallmark features in CPT. Specifically, the apparent overweighting of low-probability events and underweighting of high-probability events can result from key principles of heuristic information processing: lexicographic search, difference thresholds, stopping rules, and one-reason decision making. Brandstätter et al. (2006) illustrate how also the possibility effect, the common ratio effect, the reflection effect, and the certainty effect are consistent with the information-processing architecture of the priority heuristic.

One interesting implication of building bridges between heuristics and these empirical choice phenomena is that insights about the boundary conditions of heuristic information processing can also cast light on potential moderators of the choice phenomena, allowing respective hypotheses to be formulated. For instance, Pachur, Hertwig, and Wolkewitz (2014; see also Suter, Pachur, & Hertwig, 2016) showed that the neglect of probability information in risky choice is more pronounced when outcomes are affect-rich (e.g., averse medical side-effects) than when they are affect-poor (e.g., moderate amounts of monetary losses). Higher probability neglect is associated with a more pronounced curvature of CPT's weighting function, which is in turn related to the strength of the fourfold pattern. It follows that the fourfold pattern could be more pronounced when outcomes are affect-rich than when they are affect-poor—a prediction that only emerges once CPT and heuristics have been connected.

Conclusion

This chapter has illustrated the potential of bringing together CPT and heuristics—two models of boundedly rational decision making formulated in very different conceptual languages and with very different modeling goals. In addition to building a network between CPT and heuristics, a further benefit of this theory integration is that it forges connections between the influential parametric framework of CPT and process-tracing methods that

originated in work on heuristics (see Pachur, Schulte-Mecklenbeck, Murphy, & Hertwig, 2018). But theory integration can also accentuate aspects in which the two approaches diverge. For instance, whereas the algebraic framework of CPT, based on continuous functions, enforces transitive choice patterns, the conditional nature of the information search of lexicographic heuristics such as the priority heuristic can lead to intransitive choices (see also Tversky, 1969).

It is important to emphasize again that theory integration does not seek to replace one approach with the other—even if one approach may in some respects be descriptively superior to the other. Rather, theory-integration analyses are complementary, taking into account that each approach has its relative strengths and limitations. CPT has been shown to be often more versatile than heuristics in predicting choices across a large range of choice problems (e.g., Glöckner & Pachur, 2012; but see Mohnert, Pachur, & Lieder, in press). Conversely, process analyses have obtained clear evidence for heuristic processes (e.g., Mann & Ball, 1994; Pachur et al., 2013; Payne & Braunstein, 1978; Su et al., 2013; Venkatraman et al., 2014). So based on the respective merits of each approach, insights from integrating CPT and heuristics will eventually contribute to making both approaches more complete and guide further theory development.

In conclusion, although the development of new models is without doubt important, it is also crucial to take stock of existing models and to elaborate the conceptual network that connects them; as argued in this chapter, this can even lead to novel predictions, highlighting how different model can enrich each other. Such theory integration can target qualitatively different modeling frameworks, as presented in this chapter (see also Bhatia, 2017, for analyses relating heuristics with connectionist models), but is equally applicable and informative for relating models with similar roots (e.g., different types of diffusion models; Khodadadi & Townsend, 2015). It is time to dissolve the boundaries that have separated

accounts of bounded rationality and to progress toward a more comprehensive perspective of

how the mind, and models of it, work.

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