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24 Intelligence and Decision-Making

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There are two ways to study how people make decisions. Decision-making under *risk* deals with well-defined situations where all possible outcomes and their probabilities are known for certain. If you visit a casino in Las Vegas to seek your fortune by playing the roulette and contemplate whether to bet $1,000 on the number "7" or on "red," all possible outcomes and their probabilities are known. No skills other than calculation are needed. Decision-making under *uncertainty*, by contrast, deals with ill-defined situations where this certainty is not attainable for humans or machines. If you ponder how to invest your money, whom to vote for, or whom to marry, the exhaustive set of all possible future outcomes is not known for certain, nor are their probabilities, and surprises may happen. Many situations involve a mixture of risk and uncertainty. For instance, the owner of a Las Vegas casino was able to calculate the gambling odds and the profit that the house could expect to make in the long run. Yet the main losses occurred through unforeseeable events: The star artist, performing his famous tiger act, was attacked by the animal; a disgruntled former contractor attempted to dynamite the casino; and an employee caused the casino to be heavily fined by failing to file tax reports for a long period (Taleb, 2007, pp. 129–130).

The distinction between risk and uncertainty goes back to the economist Frank Knight (1921). In situations of risk, the laws of logic and probability are sufficient to determine which action can be expected to result in the best outcome. In situations of uncertainty, probabilistic reasoning does not suffice, and psychological tools such as heuristics, experience, and intuition are needed. The distinction between risk and uncertainty is reminiscent of that between *known unknowns* and *unknown unknowns* in the NASA terminology popularized by former US secretary of defense Donald Rumsfeld.

In this chapter, I provide an introduction to these two approaches to decision-making and end with the question of how to integrate them with the study of intelligence. Each part begins with a brief historical sketch. The history of ideas is indispensable for understanding why we are asking the questions we ask today, and for understanding what the alternatives are.

Decision-Making under Risk

The Origins: Gambling

In psychological experiments, many thousands of participants have been asked to choose between two or more monetary gambles. This intimate fondness for gambling may appear surprising. After all, one is unlikely to find hordes of psychologists in the machine gambling zone of Las Vegas. Yet for decision research, history is destiny.

Gambling became a craze in several European countries during the seventeenth and eighteenth centuries. People passionately bet their money on lotteries and the outcomes of dice rolls to seek their fortune (Daston, 1988). According to legend, quite a few rebellious citizens of Paris made their regular stop at the lottery stands to try their luck on the way to storming the Bastille at the start of the French Revolution. In England and France, life insurance was almost synonymous with gambling, being not insurance as we know it but rather a bet on the life of a *third party* (Daston, 1987). For instance, London underwriters issued policies on the lives of celebrities, such as the unfortunate Admiral Byng, who stood trial for failing to prevent the island Minorca from falling to the French. Byng was found guilty and shot by a firing squad, much to the satisfaction of those who had bought insurance on his life (Clark, 2002). Underwriters even offered bets on the lives of kings, such as 25 percent against King George II's returning alive from the Battle of Dettingen, Germany. The last British monarch to personally lead his troops on the field, George II survived the battle; in this case, the gamblers lost their money.

A huge shift in psychological perspective was required to eventually convince people that they should bet on their own death rather than on that of someone else. The increasing mobility of society in the nineteenth century helped; no longer could people expect to live their lives in a stationary community that would look after a widow and children if the father unexpectedly died. During this shift, books on probability emphasized that probability had outgrown its frivolous phase and could now serve more moral and sober pursuits, such as pricing insurance and annuities (Daston, 1987). Today, of course, life insurance is often perceived as a moral duty, and it no longer feels strange to bet on one's own death. Moral or immoral, gambling and lotteries have defined decision theory under risk.

Expected Value Theory

Following its historical origins, the staple task posed to participants in psychological experiments is a choice between lotteries, such as:

A. A 50 percent chance to win \$100, otherwise nothing.
B. \$40 for sure.

Which option would you choose? If you prefer A, you maximize the *expected value.* The expected value of gamble is the sum of all outcomes times their probabilities. For gamble A, the expectation is $0.5 \times \$100 + 0.5 \times \$0 = \$50$, while the sure option B pays only \$40. According to *expected value theory* it is prudent to choose the gamble with the maximum expected value, here option A.

Risk Aversion and Risk Seeking

If a person prefers a sure gain to a lottery with a higher expected value, that person is said to be *risk averse*. An example is to prefer B over A. If a person prefers a lottery to a sure gain despite the lottery having a smaller expected value, that person is classified as *risk seeking*. An example would be to prefer A over a sure gain of \$60. If the amounts at stake are *losses* rather than *gains*, then the person is said to be risk averse or risk seeking for losses.

How do actual people decide? They are divided. In a representative sample of about 1,000 Germans and 1,000 Spaniards, 55 percent were risk averse and preferred \$40 for sure, thereby violating expected value theory (Gigerenzer & Garcia-Retamero, 2017). As the historical association between gambling and insurance might lead one to expect, those who were risk averse more often bought life insurance, household insurance, and other insurances than did those who were risk seeking. Risk-averse people were also more likely to not want to know when they will die and from what cause, and whether their marriage will end in divorce.

Expected Utility Theory

Expected value theory was the brainchild of two famous French mathematicians, Blaise Pascal and Pierre Fermat, who in 1654 solved gambling problems posed by a notorious gambler. Yet expected value theory soon ran into troubles because it conflicted with the intuitions of educated people, as in the case of risk aversion. The most celebrated conflict was another gamble:

> A fair coin is tossed. If "heads" comes up on the first flip, the house pays the gambler \$1, and the game ends. If the first head is on the second flip, the house pays \$2, and the game ends. If the first head is on the third flip, the house pays \$4, and so on. In general, if the first head is on the nth flip, then the house pays $\$2^{n-1}$.

What is the fair entry price for this game? *Fair* means leaving open which side of the bet one takes. (The same principle of fairness is in play when a child cuts a piece of cake in two, and a second child gets to choose first.) If the fair price is the expected value, we get:

$$\text{Expected value} = 1/2 \times \$1 + 1/4 \times \$2 + 1/8 \times \$4 + \ldots + (1/2)^n \times \$2^{n-1} + \ldots = \$1/2 + \$1/2 + \ldots = \infty$$

Translated into words, the probability of winning \$1 is 1/2, that of winning \$2 is 1/4, that of winning \$4 is 1/8, and so on. Each term on the right side of the equation equals 50 cents, and since the number of terms is infinite, the expected value is also infinitely large. In keeping with the theory, we should thus wager everything we own to play this game. Yet no sensible person would be willing to pay more than a small sum. The discrepancy between expected value theory and people's intuitions was dubbed the *St. Petersburg Paradox*.

In 1738, the twenty-five-year-old Swiss mathematician Daniel Bernoulli published a solution in the annuals of the Petersburg Academy (1738/1954), after which the paradox was named. Bernoulli reasoned that winning \$200 (in modern currency) is not necessarily double the "utility" of winning \$100, and that the wealthier a player is to begin with, the more money he needs to win in order to experience an equal increase of utility. This reasoning later became known as the *principle of marginal decreasing utility.* Bernoulli assumed that the relation between dollar (x) and utility (u) is logarithmic, $u(x) = \log(x)$. By replacing the actual dollar values with their logarithms, the problem of the infinite "expected value" of the gamble was resolved. The "expected utility" of the gamble amounts to a small dollar value, in the region of what most people are willing to pay.

Today, a weaker version of Bernoulli's equation is called *expected utility theory*: it is "weaker" because, unlike Bernoulli's logarithmic function, the utility function u is not specified. Here, the expected utility of an option is defined as the sum of the utilities of its consequences, multiplied by their probabilities. Its central idea is that people decide between options as if it were possible to know beforehand all consequences of each option, then estimate their utilities, multiply these by their probabilities, add these terms up, and finally choose the option with the highest expected utility. Although this knowledge is unlikely to exist in situations of uncertainty, expected utility theory became the template for a large number of psychological theories, including consumer choice, health behavior, attitude formation, motivation, and intuition. The theory also single-handedly shaped entire disciplines such as economics and finance.

Axioms of Choice

Despite the mathematical appeal of expected utility theory, criticism was raised about the complete lack of evidence that people actually perform the sequence of calculations the theory entails. This criticism was countered by arguing that if people satisfy a small number of choice axioms, then they behave *as if* they maximized a utility function. In fact, one of the celebrated successes of decision theory is the proof by von Neumann and Morgenstern that if the following choice axioms hold, then the options can be represented as numbers on a line called *utility function*:

Axiom 1: Completeness: $A \succeq B$ or $B \succeq A$.
Axiom 2: Transitivity: if $A \succeq B$, $B \succeq C$, then $A \succeq C$.
Axiom 3: Archimedean property (assumes continuity).
Axiom 4: Independence: if $A \succ B$, then $pA + (1-p)C \succ pB + (1-p)C$ (for any C and probability p).

A, B, C are the options in the choice set; $A \succeq B$ means that one either prefers A over B or is indifferent between both ("weak preference"). Completeness means that one either prefers A weakly over B, or vice versa. Everything else is excluded, such as not having any preference or not making a choice. Transitivity means that if one prefers A over B, and B over C, then one also prefers A over C. The Archimedean property, named after the ancient Greek geometer Archimedes of Syracuse, roughly means that some kind of trade-off is always possible, which guarantees continuity of the number line. Independence means that if the same amount is added to each of two options, their order remains the same. In expected utility theory, preferences are simply inferred from choices, which is called the *principle of revealed preferences.*

As the axioms show, the theory is behavioristic: Preference does not mean "liking" or "deriving more pleasure" but only consistent choice.

Von Neumann and Morgenstern formulated these axioms in order to provide the mathematical conditions for a utility function, which are similar to the axioms of number theory. When doing so, they made no claims that these describe what people do or should do. Others proposed that the axioms – and, by implication, expected utility theory – describe how people actually make decisions or, at the very least, how they ought to make decisions. Both the descriptive and the prescriptive interpretation generated intense controversies.

Controversies

Do Choice Axioms Describe Behavior?

Two kinds of arguments have been levied against the descriptive interpretation. The first is theoretical. Consider the completeness axiom. It appears to be almost trivial to find out whether a person's choices fulfill this axiom, yet it is not. For instance, consider the decision of which websites to visit and in which order. According to Internet Live Stats, ten websites existed on the Internet in 1992. To order these according to preference, one had to make forty-five ($10 \times 9/2$) binary choices. At that time, checking for completeness was tractable. In the year 2016, the number of websites had increased to about 1,085,628,900, which would require in the order of 10^{18} checks. Here, checking for completeness is no longer tractable, neither for humans nor for machines. And without that, one cannot check transitivity and find out whether the choice axioms describe behavior. In general, if there is a large choice set, checking for completeness is computationally intractable. And even when it is tractable, checking may amount to a foolish loss of time.

The second argument is empirical. Beginning with the Allais paradox and the Ellsberg paradox, a number of psychological studies showed that people systematically violate expected utility theory in simple choice situations (e.g., Kahneman & Tversky, 1979). Allais (1953) constructed gambles in which people tend to violate the independence axiom, and Ellsberg (1961) showed that people's preferences for gambles are sensitive to whether the probabilities are known (risk) or ambiguous (a form of uncertainty), all of which result in behavior inconsistent with expected utility theory. This has led to revisions of expected utility theory by adding more free parameters to fit deviating behavior, such as prospect theory.

Thus, expected utility theory faced the same challenge as expected value theory. People sometimes behave *as if* they maximized expected utility, at other times not. Most important, there is little evidence that expected utility theory (or its modifications, such as prospect theory) describes how people actually reason, that is, that the postulated multiplications, additions, and transformations of values are performed by humans. Yet there is also an important difference. When expected value theory conflicted with educated people's intuition, the blame was placed on the theory, not on the people. When expected utility theory contradicted people's decisions, the blame was placed on people, not on the theory, which was maintained as being prescriptive.

Do Choice Axioms Prescribe Behavior?

Beginning in the 1970s, the *heuristics-and-biases program* documented that people's judgments systematically deviate from choice axioms and various other logical rules. In contrast to Bernoulli, Kahneman and Tversky (1974) attributed these discrepancies to flaws in the human mind rather than in the norms. These deviations were named *cognitive illusions* in analogy to visual illusions, suggesting that they are equally stable and stubborn, and people's intuitions were called "a multitude of sins," "ludicrous," "indefensible," and "self-defeating" (Tversky & Kahneman, 1971, pp. 107–110). Today, long lists of cognitive illusions exist, including violations of transitivity, the conjunction fallacy, and framing effects, with Wikipedia cataloguing some 175 of these. But again, two arguments, one theoretical and one empirical, have been mounted against the interpretation of logical axioms and rules as prescriptive in all situations and their violations as cognitive illusions.

First, when Maurice Allais and Daniel Ellsberg (the man who released the Pentagon papers) published their famous "paradoxes" in the 1950s and 1960s, demonstrating systematic discrepancies between people's intuitions and the choice axioms, they criticized the normative interpretation of the latter. Ellsberg (1961) urged distinguishing between risk and uncertainty. He concluded that the belief held by many researchers that choice axioms define rational behavior under uncertainty amounts to "bad advice" (p. 669), and that when he and other people systematically violate logical axioms, this is not irrational but in fact a sensible way to behave.

A related critique is that expected utility theory pays attention solely to the mean outcome, not to the variance (or to higher moments) of the outcomes. If one pays attention to the variance, a sure gain of $40 in alternative A is *not* necessarily inferior to an expected value of $50 in alternative B, because the first outcome has variance zero (it is certain) while the second has a variability between $0 and $100. Thus, looking at the expectation (the mean) alone is not a universal yardstick of rational decision-making.

The second argument is empirical. It is directed against the claim (Thaler & Sunstein, 2008) that violations of logical rules incur substantial real-world costs. Arkes, Gigerenzer, and Hertwig (2016) searched through more than 1,000 articles on so-called cognitive illusions, and could find little to no evidence that violations of logical rules are associated with less income, poorer health, lower happiness, inaccurate beliefs, shorter lives, or any other measurable outcome (see also Berg, Biele, & Gigerenzer, 2016). Similarly, when Stanovich and West (2008) investigated whether the biases discussed in the heuristics and biases literature were correlated with measures of ability such as SAT tests, they concluded "that a large number of thinking biases are uncorrelated with cognitive ability" (p. 672).

The Turkey Illusion

Many psychologists do not distinguish between risk and uncertainty but instead believe that all problems can be solved by probability theory, as assumed in Bayesian theories of mind or brain. This position has been criticized as being the "turkey illusion."

Imagine you are a turkey. It's the first day of your life. All of a sudden a man appears. In panic, you fear that he will kill you, but he kindly feeds you. On the second day, the man returns, and again you fear that he might kill you. But once more, he feeds you. On the third day, the same happens. According to Bayesian probability updating, the probability that he will feed rather than kill you increases each day. On day 100, it is higher than ever before – but it happens to be the day before Thanksgiving. You are dead meat.

The turkey missed a crucial piece of information: It was not in a situation of risk, where the past predicts the future. The turkey illusion goes back to the philosopher Bertrand Russell (who used a chicken in his account), and has been popularized by trader Nassim Taleb. Yet it appears to be committed more frequently by humans than turkeys, one example being the increasing confidence of financial institutions in the stability of the financial market in the years before the crisis of 2007–2008, up to shortly before the breakdown. Calibrating their models on the past years, the rating agencies – like the turkey – predicted that the future resembles the past.

All three points of critique – the descriptive, the prescriptive, and the turkey – are essential whenever decisions are evaluated as rational or irrational. At a minimum, these suggest that choice axioms, or similar logical rules, are of limited value for describing how people make decisions and inadequate as universal norms of how we should behave. Based on these limits, Herbert Simon (1955, 1979) asked for an extension and revision of the study of decision-making in two respects: (1) to study how people make decisions in the real world of uncertainty, as opposed to risk, and (2) to study the process of how people actually make decisions, as opposed to as-if models of expected utility maximization and its variants. Yet Simon's call was little heeded for decades, and it took several more decades before his program was finally fleshed out (Gigerenzer, Hertwig, & Pachur, 2011).

Decision-Making under Uncertainty

The Origins: Heuristic Decision-Making

As far as we can know, humans and other animals have always relied on heuristics to solve adaptive problems. Ants use a simple rule to estimate the area of a candidate nest cavity: Run around on an irregular path for a fixed period while laying down a pheromone trail, then leave; next return, move around again in an irregular path, and estimate the size of the cavity by the frequency of encountering one's old trail. This heuristic is remarkably precise: Nests half the area of others yielded reencounter frequencies 1.96 times greater.

To choose a mate, peahens similarly use a heuristic: Investigate only three or four of the peacocks in a lek (an assembly of males engaged in competitive displays to attract a mate) and choose the one with the largest number of eyespots (see Hutchinson & Gigerenzer, 2005). Roughly speaking, a heuristic is a simple rule that uses a minimum of the available information (in contrast to expected utility maximization) in order to make efficient decisions under uncertainty. The term *heuristic* is of Greek origin, meaning "serving to find out or discover."

One of the fathers of the study of heuristics is Herbert A. Simon (1916–2001). He also made seminal contributions to artificial intelligence, psychology, political science, and economics. In the mid-1930s, young "Herb" had taken a class on price theory at the University of Chicago, and then tried to apply what he had learned on utility maximization to real budget decisions in his native Milwaukee recreation department. To his surprise, experienced managers did not estimate utilities and probabilities but rather used heuristics and added incremental changes to last year's budget. This venture into the real world taught him that even experienced managers cannot know in advance all possible states of the world, their consequences, and probabilities. In an uncertain world, he concluded, utility maximization was hopeless.

Simon (1955, 1979, 1990) proposed a division of labor: Heuristic reasoning is necessary in situations of uncertainty, while probabilistic reasoning is necessary for situations of risk. Contrast this with the heuristics-and-biases program. First, Simon rejected choice axioms and expected utility theory as a universal principle, whereas Kahneman and Tversky accepted these and similar logical rules as universal yardsticks for rationality and attributed deviating behavior to flaws in the human mind. Second, while Simon insisted on formal models of heuristics that could be simulated and tested, the heuristics-and-bias program relied on vague one-word labels, such as availability and representativeness. By doing so, this program could not discover the power of heuristics. In this chapter, I will focus on models of heuristics, either formal or in the form of precise verbal descriptions of the steps of decision-making, as studied by Simon and subsequently by Payne, Bettman, and Johnson (1993) and Gigerenzer, Todd, and the ABC Research Group (1999). In what follows, I proceed by means of examples; a systematic treatment can be found in Gigerenzer and colleagues (2011).

The program of fast-and-frugal heuristics (Gigerenzer et al., 2011) is often perceived as being in opposition to the heuristics-and-biases program (Kahneman, 2011). In fact, it should be seen as a necessary extension of the latter by introducing formal models of heuristics, replacing logical with ecological rationality, and taking uncertainty seriously. The program has a descriptive, prescriptive, and engineering component: the study of the *adaptive toolbox* (which heuristics are in the repertoire of a person?), *ecological rationality* (which heuristic should be selected for a given problem?), and *intuitive design* (how to design intuitive expert systems based on the adaptive toolbox).

The Adaptive Toolbox

Through individual learning and social imitation, humans acquire during their lifetime a repertoire of heuristics. This repertoire is called the *adaptive toolbox* of a person, where the term *adaptive* signals that heuristics are tailored to classes of problems, just as a hammer is designed for nails and screwdrivers for screws. The key difference to decision-making under risk is that these tools can deal with uncertainty. To illustrate the difference, I begin with a decision similar to the choice between gambles, the stock-in-trade for decision-making under risk.

Satisficing

Consider an entrepreneur who is looking for an investment in real estate. She has discovered a potential site S to invest in and develop, and is faced with this decision:

C. Invest in site S.
D. Forgo S and continue to search for a better site.

Compare this choice to the one between A and B. First, the set of all possible outcomes (profits) of site S and their probabilities are no longer known for sure. Second, option D entails further uncertainty: New sites need to be searched for, without knowing ahead what, if anything, will be found. In other words, decision-making takes place in time; options are not presented simultaneously, as in the choice between A and B, but discovered sequentially. If the set of all possible sites, their outcomes, and their probabilities are not known, one cannot determine the site with the highest expected utility. Nevertheless, people can make good decisions. But how? Simon argued that people rely on a heuristic called *satisficing*, named after the Northumbrian word for "good enough."

> *Satisficing:* Set an aspiration level x, and choose the first object that satisfies x.

Consider again the decision between C and D. Berg (2014) studied forty-nine entrepreneurs in the Dallas-Fort Worth greater metropolitan area who developed commercial high-rises or residential areas. He reported that every single one of the professionals relied on satisficing:

> If I believe I can get at least x percent return within y years, then I take the option.

The entrepreneurs differed in their aspiration levels and the time horizon. The time horizon y was mostly one to three years, and x a *prominent number.* Prominent numbers are powers of ten, their halves, and their doubles (i.e., 1, 2, 5, 10, 20, 50, ...). For instance, convenience store and gas station investors required at least a 10 percent annual return on capital within one or two years. Most entrepreneurs considered only one, two, or three sites before making a decision (similar to the peahen's mate choice). Not a single one tried to determine the point where the marginal benefit of search equals its costs. Many expressed open skepticism that such utility calculations could be made in one-off decisions in high-stake and quickly changing environments.

Like all heuristics, satisficing is used in a variety of sequential search problems, from consumer choice to mate choice (Todd & Miller, 1999). Consider a mundane everyday decision: fast and frugal food choice. How do customers in a restaurant decide what to order for dinner? Unlike the entrepreneurs, customers differ in the heuristics they use. Yet again, one of these is satisficing.

Satisficing: First, pick a category from the menu (say, fish). Then read the first item in this category, and decide whether it is good enough. If yes, close the menu and order that dish. If no, read the second item and proceed in the same way.

In a representative study of 1,000 German adults, 34 percent reported that this fast-and-frugal rule is how they typically decide (Figure 24.1; Gigerenzer, 2014). Yet 17 percent reported an even faster rule:

Habit: Don't open the menu. Order your favorite dish.

This rule is reminiscent of risk aversion; it avoids disappointment and appears reasonable when one is familiar with the restaurant. Satisficing and habit are both individual heuristics, where the decision is made without social input. The next two, in contrast, are social heuristics.

Advice taking: Don't open the menu. Ask the waiter what they recommend and order it.

Advice taking is a social heuristic because it relies on and trusts the judgment of someone else. In situations where one has little experience, social heuristics are generally useful. Advice taking is a reasonable rule in a good restaurant where the waiter knows what is best and does not deceive the guest. Among the Germans

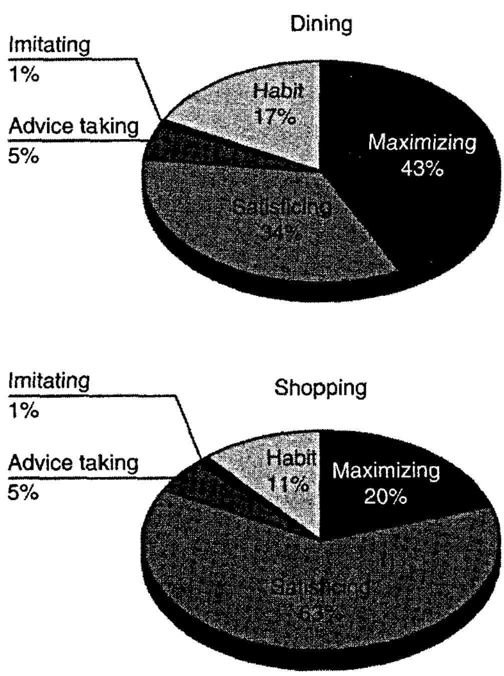

Figure 24.1 *Individual differences in heuristic decision-making (from Gigerenzer, 2014).*

Top: How to order in a restaurant? Bottom: How to shop for a pair of trousers? Shown are the responses of a representative sample of 1,000 German adults.

surveyed, however, it was not very popular, with only 5 percent relying on it. The least popular heuristic was also a social one:

> *Imitating:* Don't open the menu. Find out who among your friends at the table has most experience with this restaurant, and order the dish they order.

Imitating or copying appears reasonable when in a foreign country or an unfamiliar restaurant. Yet only 1 percent of the general public reported relying on it. Copying appears to be a taboo among Germans, unlike in more family-oriented countries where it is fine to eat what others eat. The largest group (43%), however, relied on a time-expensive rule:

> *Maximizing:* Study every item in the menu carefully and try to figure out the best option.

Maximizing is an individualistic rule and requires stamina in a restaurant where the menu resembles an encyclopedia. Yet for some people, making a decision without having inspected all options is emotionally unbearable. Maximizing resembles expected utility calculations, but there is an essential difference: No probabilities and utilities are estimated, multiplied, and added up.

When the same participants were asked how they proceed when buying a pair of trousers, the major difference was that more reported satisficing and fewer maximizing (Figure 24.1). Maximizing requires checking all trousers in a department store, then heading to another department store and on to boutiques to find the best pair. Maximizing can be a direct route to unhappiness: One wants the best, and nothing less. But even if happening on it right away, one would not know it and continue to look for something better. Studies indicate that people who rely on satisficing tend to be more optimistic and have higher self-esteem than those who rely on maximizing. The latter excel in perfectionism, depression, and self-blame (Schwartz et al., 2002). Food choice and shopping are not the biggest decisions we face. Imagine if everyone tried to maximize in mate choice, that is, to find the perfect partner and nothing less. That would be a recipe for disaster and divorce.

Once again, social heuristics – to take advice from a salesperson or buy the trousers others wear – were as rarely reported as in food choice. This reluctance to rely on others when making decisions, or to admit to doing so, may be particularly strong in individualistic Western societies (Hertwig & Hoffrage, 2013).

Do Animals and Humans Share Common Heuristics?

Both animals and humans rely on heuristics to deal with uncertainty, and sometimes even rely on the same heuristic. In an experiment, Norway rats had a choice between two kinds of food, one they recognized by smell from the breath of another rat, while the other was new. The far majority of rats choose the recognized food, even in situations where the fellow rat was sick (experimentally induced). This decision rule is known as the *recognition heuristic* (Table 24.1). Experiments with humans showed that they tend to rely on the same heuristic, specifically when it is *ecologically rational.* The recognition heuristic is said to be ecologically rational in

Table 24.1 *Twelve well-studied heuristics with evidence of use in the adaptive toolbox of humans (after Todd, Gigerenzer, & ABC Research Group, 2012, pp. 9–10)*

Heuristic	**Description**	**Counterintuitive results**
Recognition heuristic (Goldstein & Gigerenzer, 2002)	If one of two alternatives is recognized, infer that it has the higher value on the criterion.	Less-is-more effect.
Fluency heuristic (Schooler & Hertwig, 2005)	If both alternatives are recognized but one is recognized faster, infer that it has the higher value on the criterion.	Less-is-more effect.
Take-the-best (Gigerenzer & Goldstein, 1996)	To infer which of the two alternatives has the higher value, (1) search through cues in order of validity; (2) stop search as soon as a cue discriminates; (3) choose the alternative this cue favors.	Often predicts as accurately as or better than multiple regression, neural networks, exemplar models, and decision-tree algorithms.
Fast-and-frugal trees (Martignon et al., 2003)	To classify a person or object, (1) search through cues in order; (2) stop search when first cue is found that allows a decision; (3) choose the option at the decision node.	Often predicts as accurately as or better than logistic regression.
Tallying (Dawes, 1979)	To estimate a criterion, do not estimate weights but simply count the number of positive cues.	Can predict as accurately as or better than multiple regression.
Satisficing (Simon, 1955)	Search through alternatives and choose the first one that exceeds your aspiration level.	Aspiration levels can lead to substantially better choices than by chance, even if they are arbitrary.
Gaze heuristic (McBeath, Shafer, & Kaiser, 1995)	To catch a ball that is coming down from overhead, fix your gaze on it, start running, and adjust your running speed so that the angle of gaze remain constant.	Balls will be caught while running, possibly on a curved path.
1/*N* rule (DeMiguel, Garlappi, & Uppal, 2009)	Allocate resources equally to each of *N* alternatives.	Can outperform optimal asset allocation portfolios.
Default heuristic (Johnson & Goldstein, 2003)	If there is a default, follow it.	Explains cultural differences in organ donor registration; predicts behavior when trait and preference theories fail.
Tit-for-tat (Axelrod, 1984)	Cooperate first and then imitate your partner's last behavior.	Can lead to a higher payoff than "rational" strategies (e.g., backward induction).

Table 24.1 *(cont.)*

Imitate the majority (Boyd & Richardson, 2005)	Determine the behavior followed by the majority of people in your group and imitate it.	A driving force in bonding, group identification, and moral behavior.
Imitate the successful (Boyd & Richardson, 2005)	Determine the most successful person and imitate their behavior.	A driving force in cultural evolution.

situations where recognition is correlated with the criterion. For instance, human participants were presented pairs of Swiss cities, such as Aarau and Basel, and asked which of the two cities has the larger population (Pohl, 2006). Most had not heard of Aarau but of Basel, so using the recognition heuristic would lead to the inference that Basel is larger (which is correct). Across all pairs of cities, 89 percent of the inferences followed the recognition heuristic. When the question was changed from population to which city is nearer to the center of Switzerland, the percentage went down to 54 percent. This sensitivity of the participants is ecologically rational: Name recognition is a valid predictor for population but not for distance from the center of Switzerland, and the difference in validity was almost identical to the difference in use of the heuristic. People's adaptive use of the recognition heuristic has been documented across forty-three experiments, where the correlation between recognition validity and the percentage of cases in which people follow it is $r = 0.57$ (Gigerenzer & Goldstein, 2011). The similarities and differences in heuristic decision-making in animals and humans is the topic of Hutchinson and Gigerenzer (2005).

Do Animals, Humans, and Machines Share Common Heuristics?

Consider the *gaze heuristic*, which is used by living organisms and machines. When a hawk pursues a dove, it relies on the *gaze heuristic*, fixating its eyes on the target and adapting the direction of flight so that the angle of gaze always remains constant (Hamlin, 2017). The angle of gaze is the angle between the direction of the hawk's flight and the line between the position of the hawk and the dove. Using this fast-and-frugal heuristic, the hawk does not need to estimate the dove's trajectory in three-dimensional space and calculate the intersection point. Moreover, when the target tries to evade, the pursuer adjusts its course so that the angle of gaze remains constant. Dogs rely on the same heuristic to catch Frisbees, as do baseball outfielders to intercept fly balls and sailors to avoid collisions (Table 24.1). The heuristic was built into the Sidewinder A1M9 short-range air-to-air missile, one of the most successful guided modern weapon systems and still in use, whose "gaze" is directed at a source of heat, which is the target.

The gaze heuristic is a prime example of a simple, robust decision system that has been discovered by animals, humans, and controllers of fighter planes and missiles. Moreover, it illustrates how a heuristic can travel from animal to human to machine, providing a simple solution to complex problems in nonstationary environments.

Like the recognition heuristic, it relies on a single input; both are members of the class of one-good-reason heuristics (Gigerenzer & Gaissmaier, 2011).

Ecological Rationality

The term *rationality* has at least two different meanings: logical consistency and attainment of one's goals. Logical rationality means that behavior should conform to logical rules, such as the choice axioms. Ecological rationality, in contrast, means that behavior should lead to successful performance, as measured by accuracy of prediction, speed of decision-making, or efficiency in reaching one's goals. As mentioned before, consistency and accuracy of beliefs appear largely unrelated, and violations of logical consistency do not seem to have demonstrable consequences for performance. The rationality of heuristics is not logical but ecological. In the words of Simon (1990), rational behavior "is shaped by a scissors whose two blades are the structure of task environments and the computational capabilities of the actor" (p. 7).

The study of ecological rationality asks the question, when should people rely on a given heuristic rather than a complex strategy to make better decisions? While the study of the adaptive toolbox is descriptive, relying on experiment and observation, that of ecological rationality is prescriptive, relying on mathematical proof and computer simulation. As mentioned above, the recognition heuristic is ecologically rational in situations where the recognition validity (the correlation between recognition and the criterion) is substantially above chance. The study of ecological rationality delves deeper into the analysis of environmental structures, beyond the scope of this introduction. The conditions for the ecological rationality of the heuristics in Table 24.1 can be found in Todd and colleagues (2012), pp. 9–10, and in Gigerenzer (2016).

Intuitive Design

Expert systems can be classified into those that aim at efficiency and those that additionally value transparency and simplicity so that users can understand how the system works. Machine learning focuses mainly on accuracy and efficiency. Its techniques range from logistic regression, which many experts, such as most physicians and judges, have difficulties understanding, to deep neural networks whose inner workings are opaque even to its creators. Intuitive design, in contrast, aims at expert systems that are both efficient and transparent. It is called *intuitive* because it applies the results of the study of the adaptive toolbox and ecological rationality to design systems that mirror people's psychological processes and thus can be easily learned, remembered, and used.

To illustrate intuitive design, I take a class of heuristics called *fast-and-frugal trees* (Table 24.1). A fast-and-frugal tree embodies three principles of human decision-making under uncertainty: ordering, limited search, and one-reason decision-making. In contrast, a full decision tree does not order cues (reasons), uses exhaustive search, and combines all reasons to make the final decision. In more formal terms, a fast-and-frugal tree asks only a few questions (cues) and allows for making a decision after each question. If n is the number of questions with yes/no answers, then a fast-and-frugal tree has $n + 1$ exits, whereas a full tree has 2^n exits. Figure 24.2 (a) shows a fast-and-frugal tree. It is a model of how magistrates at London courts decide whether to grant bail or subject a defendant to a punitive measure such as jail. The tree has three building blocks:

Search rule: Look through cues in order.
Stopping rule: Stop search when the first cue is found that allows a decision.
Decision rule: Choose the option at the exit.

For instance, if the prosecution requests conditional bail or opposes bail, then magistrates follow suit and make a punitive decision such as jail. If the prosecution does not, then magistrates respond to a second question in the same way, and so on. This fast-and-frugal tree predicted the actual decisions of British magistrates better than linear models that used more cues. In experimental research, this form of sequential decision-making has been often reported. It differs from the prescriptions of expected utility theory, where all information should be searched for and integrated. Intuitive design begins with this structure rather than a logistic regression or

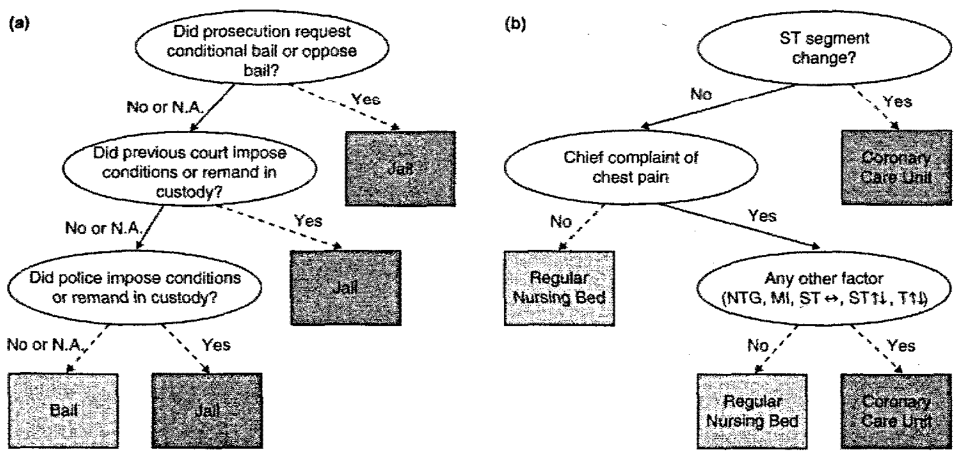

Figure 24.2 *Fast-and-frugal trees for bail decisions in London courts (a) and for coronary care unit (CCU) allocation in emergency medicine (b).*
The bail-and-jail tree is a descriptive model of how London magistrates decide whether to grant bail or subject a defendant to a punitive measure such as jail (Dhami, 2003). The CCU tree is a prescriptive model of how physicians should make decisions, embodying intuitive design. It led to fewer errors in allocation decisions compared to a complex logistic regression and the defensive decisions of physicians (Green & Mehr, 1997). ST segment change = electrocardiogram shows ST segment with elevation or depression of 1mm or more; NTG = history of nitroglycerin use for chest pain; MI = history of heart attack; the other measures are based on the electrocardiogram.

similar statistical models; the idea is to build expert systems that mirror human psychology.

Based on these principles of intuitive decision-making, the fast-and-frugal tree on the right side of Figure 24.2 has been designed by medical researchers for a medical emergency situation: when a patient is rushed into a hospital with severe chest pains. The emergency physicians have to decide quickly whether the patient suffers from acute ischemic heart disease and should be assigned to an intensive unit, the coronary care unit (CCU), or a regular bed with telemetry. The tree asks only three questions, and allows for a decision after each one. For instance, if there is an anomaly in the ST segment of the electrocardiogram, then the patient is immediately assigned to the CCU. No further questions are asked.

The tree was developed in a Michigan hospital in response to the problem that doctors used to send about 90 percent of patients into the CCU, although only 25 percent of these actually had a myocardial infarction (Green & Mehr, 1997). The result of this defensive decision-making was an overly crowded coronary care unit, decrease in quality of care, increase in cost, and a risk of serious infection among those who were incorrectly assigned ("false alarms"). The fast-and-frugal tree reduced both false alarms and misses considerably compared to doctors' decisions and to a logistic regression expert system for patient allocation. The tree is intuitive because, unlike a logistic regression, doctors can understand and memorize it, and also adapt it to new patient populations.

How to Balance False Alarms and Misses

In both bail and care unit decisions, one can make two kinds of errors. The first is a *false alarm*: classifying someone wrongly as positive, such as sending a defendant to jail who would have not committed a crime or a patient to the CCU who has no heart disease. The second error is called a *miss*: classifying someone wrongly as negative, such as granting bail to a defendant who will then commit a crime or allocating a patient who will have a heart attack to a regular bed. The design of the tree determines the balance of the two error rates. In the bail-and-jail tree, all exits are "jail" except one, which reduces the rate of misses at the cost of increasing the rate of false alarms. In the CCU tree, the exits – and, accordingly, the two possible errors – are more balanced.

These two examples illustrate a general principle for constructing fast-and-frugal trees. With three ordered cues, there are four possible trees with differing exits ("S" for signal and "N" for noise). The top of Figure 24.3 depicts signal detection theory with two hypotheses, noise and signal, and a decision criterion that sets the balance between the two errors. If the data fall to the left of the criterion, the conclusion is "noise"; if they fall to the right, the conclusion is "signal." The bottom of Figure 24.3 shows how the four possible fast-and-frugal trees map into this scheme (Luan, Schooler, & Gigerenzer, 2011). The tree on the far left side corresponds to a decision criterion set at the far left arrow, which reduces misses at the cost of more false alarms. This tree has the structure of the bail-and-jail tree. The CCU tree has the structure of the second tree from the left, striking more of a balance between misses

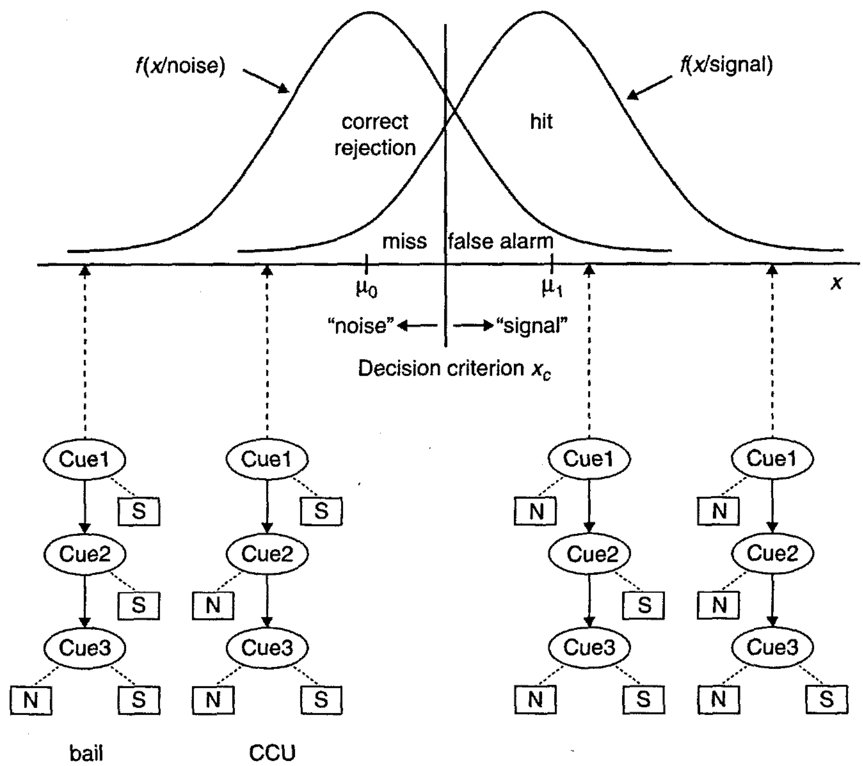

Figure 24.3 *Balancing misses and false alarms (after Luan et al., 2011).*
In signal detection theory, the balance between misses and false alarms is set by a decision criterion. In fast-and-frugal trees, the balance is set by the exit structure, where "S" stands for "signal" and "N" for "noise." A fast-and-frugal tree with three ordered cues has four possible exit structures. The two on the left correspond to the two trees in Figure 24.2.

and false alarms. To minimize false alarms over misses, however, one would have to choose one of the two trees on the right.

In sum, intuitive design honors transparency and usability. Moreover as illustrated by the CCU tree, it can also lead to equally good or better classification than complex, nonintuitive statistical decision models.

Controversies

The Rationality Debate

What is known as the *rationality debate* concerns the question whether humans are rational or not, and what rationality means in the first place. Kahneman and Tversky (1996) argued that the rules of logic and probability universally define rationality, that people lack rationality, and that people are hardly educable out of their "cognitive illusions." Others added that deviations from logical rules would be associated with substantial costs, from obesity to HIV to the financial crisis, and called on governments to "nudge" their citizens into proper behavior (Thaler & Sunstein, 2008). Governments across the globe increasingly follow this "libertarian paternalism."

In contrast, Gigerenzer (1996, 2015) and others conjectured that logical rules cannot provide universal yardsticks for rationality in real-world situations of uncertainty, and that human behavior should be evaluated in terms of whether it reaches its goals (ecological rationality). Moreover, many so-called cognitive illusions (including the base-rate fallacy, conjunction fallacy, and overconfidence) have been shown to reflect reasonable judgments under uncertainty (Gigerenzer, 2018), and automatically following logical rules is not necessarily an intelligent strategy. Furthermore, experiments show that education and training can help to improve reasoning, while there is a general lack of evidence that violations of logical rules are associated with substantial costs (Arkes et al., 2016).

The status of heuristics is essential to this debate. In the heuristics-and-biases program, heuristics grew to be associated with biases and irrationality, and logic and probability theory are taken as a universal definition of rationality both in situations of risk and under uncertainty. In contrast, when labels such as availability were replaced by formal models of heuristics (Table 24.1), it could be shown that simple heuristics often make more accurate predictions than complex strategies that use more information and calculation. This is called a *less-is-more effect*, an example of which is the higher accuracy of the CCU tree over the logistic regression decision system. In general, the two views differ in whether they take heuristics and uncertainty seriously.

Decision-Making and Intelligence: Toward Theory Integration

Open a book on intelligence and you will likely find little if anything on decision-making. By the same token, pick up a book on decision-making and you will likely look in vain for the term *intelligence*. This mutual ignorance is surprising, given that making good decisions should in some way involve intelligence.

The Two Disciplines

During the twentieth century, psychology evolved into two separate disciplines, which Lee J. Cronbach (1957) called the "Tight Little Island" of experimental psychology and the "Holy Roman Empire" of correlational psychology. The study of intelligence traces back to the nineteenth-century work of Francis Galton and Karl Pearson on individual differences in "natural ability," later named *intelligence*, and relies heavily on correlational methods developed by Galton. The psychological study of decision-making, however, became associated with experimental psychology and emerged mainly in the second half of the twentieth century. This historical split is one reason for the surprising lack of interaction, but there is also a psychological reason. Many psychologists show strong in-group behavior, creating and defending subdisciplines, which leads to ignoring what others write on similar topics. As Walter Mischel (2008) put it, "Psychologists treat other people's theories like toothbrushes – no self-respecting person wants to use anyone else's."

Yet one of psychology's vital goals is *theory integration*, that is, to link the available theories, concepts, and phenomena into a common network. In 2017, the journal *Decision* published two issues to launch this integration program. The integration between signal detection theory and fast-and-frugal trees is one successful example (Figure 24.3). Integration between research on intelligence and decision-making, however, has rarely been considered. Here are a few thoughts.

Theory Integration

Let us begin with decision-making under risk. What is the notion of intelligence underlying this program? It appears to be logical and calculative intelligence: complying with logical axioms, checking and maintaining coherence, and maximizing expected utility. Expertise and knowledge plays little to no role, and individual differences are not the focus, apart from exceptions such as the distinction between risk averse and risk seeking. A test analogous to the IQ test for decision-making under risk would measure the degree to which people exhibit coherence in their choices, similar to how coherence is measured in Bayesian judgments (Berg et al., 2016). This would lead to research questions such as, is IQ associated with coherence?

For decision-making in situations of uncertainty, intelligence has a broader function than maintaining coherence in a world of certainty. In the words of the late psychologist Jerome Bruner, intelligence means going beyond the information given. The heuristics described in this chapter go beyond this information because what is known is not sufficient. In decisions under uncertainty, intelligence entails inference, that is, making informed bets using heuristics.

Bröder (2012) studied the question of whether people with higher IQs rely on more complex strategies when making inferences under uncertainty. The answer is no, which is consistent with the finding that experts often rely on simple heuristics because they know what to ignore. A second question is whether people with higher IQs better know what heuristic to choose in what situation, that is, have better intuitions about the ecological rationality of heuristics. Here, the answer is yes; there appears to be a correlation between the adaptive use of heuristics and IQ (Bröder, 2012). Thus, intelligence could be understood as the ability to find the right heuristic for a given problem.

The integration between decision research and intelligence research could start by exploiting the specific strengths of each program. Research on intelligence focuses on individual differences; decision-making research could adopt this perspective to acquire a better understanding of the heterogeneity in people's (adaptive) use of heuristics. These individual differences exist, as illustrated earlier: Entrepreneurs differ in the aspiration level they set when deciding where to invest and customers differ in the heuristics they use for food choice or consumer choice in general.

One strength of decision-making research is that it builds and tests models about processes, such as search rules, stopping rules, and decision rules in fast-and-frugal trees. Research on intelligence could consider building process models into their theories. The transparency and usability of these models can add a practical dimension to intelligence. Like the adaptive toolbox with its multiple heuristics, theories of intelligence often postulate multiple intelligences, which provides another point of integration. What decision-making research can learn is to take individual differences seriously. What intelligence research can learn is to take the difference between risk and uncertainty and the analysis of heuristic processes seriously.

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