

Two-neutrino double electron capture on ^{124}Xe based on an effective theory and the nuclear shell model

E. A. Coello Pérez,^{1,2,*} J. Menéndez,^{3,†} and A. Schwenk^{1,2,4,‡}

¹*Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany*

²*ExtreMe Matter Institute EMMI, Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany*

³*Center for Nuclear Study, The University of Tokyo, Tokyo 113-0033, Japan*

⁴*Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*

We study the two-neutrino double electron capture on ^{124}Xe based on an effective theory (ET) and large-scale shell model calculations, two modern nuclear structure approaches that have been tested against Gamow-Teller and double-beta decay data. In the ET, the low-energy constants are fit to electron capture and β^- transitions around xenon. For the nuclear shell model, we use an interaction in a large configuration space that reproduces the spectroscopy of nuclei in this mass region. For the dominant transition to the ^{124}Te ground state, we find half-lives $T_{1/2}^{2\nu\text{ECEC}} = (1.3 - 18) \times 10^{22}$ y for the ET and $T_{1/2}^{2\nu\text{ECEC}} = (0.43 - 2.9) \times 10^{22}$ y for the shell model. The ET uncertainty leads to a half-life almost entirely consistent with present experimental limits and largely within the reach of ongoing experiments. The shell model half-life range overlaps with the ET, but extends less beyond current limits. Our findings thus suggest that the two-neutrino double electron capture on ^{124}Xe has a good chance to be discovered by ongoing or future experiments. In addition, we present results for the two-neutrino double electron capture to excited states of ^{124}Te .

Introduction.—Second-order weak processes give rise to extremely rare decay modes of atomic nuclei. They have been observed in about a dozen nuclei with the longest half-lives in the nuclear chart of about $10^{19} - 10^{21}$ years [1]. All these are two-neutrino double beta ($\beta\beta$) decays, where the emission of two electrons is accompanied by two antineutrinos. An even rarer decay can occur if no neutrinos are emitted, the neutrinoless $\beta\beta$ decay. This process is particularly intriguing, because neutrinoless $\beta\beta$ decay is not allowed in the Standard Model, does not conserve lepton number, and can only happen if neutrinos are their own antiparticles (Majorana particles) [2].

Due to its unique potential for neutrino physics, beyond the Standard Model physics, and the understanding of the matter-antimatter asymmetry of the Universe, neutrinoless $\beta\beta$ decay searches are increasingly active [3–9]. The planning and interpretation of these experiments relies on a good understanding of the decay half-life, which depends on a nuclear matrix element. However, these are poorly known as neutrinoless $\beta\beta$ decay matrix-element calculations disagree by at least a factor two [10].

Second-order weak processes with neutrino emission are ideal tests of neutrinoless $\beta\beta$ decay matrix-element calculations. The initial and final states are common, and the transition operator is also very similar, dominated by the physics of spin and isospin. In addition to $\beta\beta$ decay, a related mode is the two-neutrino double electron capture ($2\nu\text{ECEC}$). Here, two K- or L-shell orbital electrons are simultaneously captured, rather than β emitted. This mode is kinematically unfavored with respect to $\beta\beta$ decay, and at present only a geochemical measurement of ^{130}Ba [11, 12], and a possible detection in ^{78}Kr [13, 14] have been claimed. Moreover, a resonant neutrinoless ECEC could be fulfilled in selected nuclei [15–17]. For both ECEC modes limits of $10^{21} - 10^{22}$ years have been

set in various isotopes [11, 18–23].

^{124}Xe is one of the most promising isotopes to observe $2\nu\text{ECEC}$ due to its largest Q -value of 2857 keV [24]. Large-volume liquid-xenon experiments primarily designed for the direct detection of dark matter such as XMASS [25], XENON100 [26], or LUX [27] are sensitive to ECEC and $\beta\beta$ decays in ^{124}Xe , ^{126}Xe and ^{134}Xe [28, 29]. Enriched xenon gas detectors are also very competitive [30, 31]. Recent searches have reached a sensitivity comparable to the half-lives expected by most theoretical calculations [32, 33]. Moreover the latest limits set by the XMASS collaboration [34] exclude most theoretical predictions.

Calculations of $2\nu\text{ECEC}$ and two-neutrino $\beta\beta$ decay are challenging because they involve the quantum many-body problem of heavy nuclei with even-even and odd-odd numbers of neutrons and protons. The methods of choice are the quasiparticle random-phase approximation (QRPA) [35, 36], extensively used for ^{124}Xe $2\nu\text{ECEC}$ [37–39], and the large-scale nuclear shell model [40], which predicted successfully the ^{48}Ca $\beta\beta$ half-life before its measurement [41, 42]. In this mass region, shell model studies have covered ^{124}Sn , $^{128,130}\text{Te}$ and ^{136}Xe [43–47], but no shell-model calculation exists for ^{124}Xe $2\nu\text{ECEC}$. In addition to QRPA, other more schematic approaches have also been applied to ^{124}Xe [48–51].

Further state-of-the-art ^{124}Xe $2\nu\text{ECEC}$ calculations are thus required given the tension between theoretical predictions and experimental limits. In this Letter, we calculate the corresponding nuclear matrix elements using an effective theory (ET), introduced in Ref. [52], which describes well Gamow-Teller and two-neutrino $\beta\beta$ decays of heavy nuclei, including $^{128,130}\text{Te}$. One of the advantages of the ET is to provide consistent theoretical uncertainties. Similar ETs have been used

to study electromagnetic transitions in spherical [53, 54] and deformed [55–61] nuclei. In addition, we present the first large-scale nuclear shell model calculation for ^{124}Xe $2\nu\text{ECEC}$. We focus on transitions to the ^{124}Te 0_{gs}^+ ground state, but also consider $2\nu\text{ECEC}$ to the lowest excited 0_2^+ and 2_1^+ states. The relation between the calculated nuclear matrix element $M^{2\nu\text{ECEC}}$ and the $2\nu\text{ECEC}$ half-life is given by

$$(T_{1/2}^{2\nu\text{ECEC}})^{-1} = G^{2\nu\text{ECEC}} g_A^4 |M^{2\nu\text{ECEC}}|^2, \quad (1)$$

where $G^{2\nu\text{ECEC}}$ is a known phase-space factor [62] and $g_A = 1.27$ the axial-vector coupling constant.

Effective theory.—We use an ET that describes the initial ^{124}Xe and final ^{124}Te nuclei, both with even number of protons and neutrons, as spherical collective cores. The intermediate nucleus, ^{124}I , has odd number of protons and neutrons. The ET describes its lowest 1_1^+ state as a double-fermion excitation of a 0^+ reference state that represents the ground state of either ^{124}Xe or ^{124}Te , $|1_1^+; j_p; j_n\rangle = (n^\dagger \otimes p^\dagger)^{(1)} |0^+\rangle$. Depending on the reference state, n^\dagger (p^\dagger) creates a neutron (proton) particle or hole in the single-particle orbital j_n (j_p). At leading order, higher 1^+ states are described as multiphonon excitations, with energies with respect to the reference state $E(1_{n+1}^+) = E(1_1^+) + n\omega$, where ω is the excitation energy and n is the number of phonon excitations.

The effective spin-isospin ($\sigma\tau$) Gamow-Teller operator is systematically constructed as the most general rank-one operator. At leading order it takes the form [52]

$$\begin{aligned} O_{\text{GT}} &= C_\beta (\tilde{p} \otimes \tilde{n})^{(1)} \\ &+ \sum_\ell C_{\beta\ell} \left[(d^\dagger + \tilde{d}) \otimes (\tilde{p} \otimes \tilde{n})^{(\ell)} \right]^{(1)} \\ &+ \sum_{L\ell} C_{\beta L\ell} \left[(d^\dagger \otimes d^\dagger + \tilde{d} \otimes \tilde{d})^{(L)} \otimes (\tilde{p} \otimes \tilde{n})^{(\ell)} \right]^{(1)} \\ &+ \dots, \end{aligned} \quad (2)$$

where the tilde denotes well defined annihilation tensor operators, and the phonon (\tilde{d} , d^\dagger) and nucleon operators are tensor coupled. The low-energy constants C must be fitted to data, and the expansion above is truncated after terms involving more than two phonon creation or annihilation operators. The first term in Eq. (2) couples the reference state to the lowest 1_1^+ state of the odd-odd nucleus, so that C_β can be extracted from the known $\log(ft)$ value of the corresponding β decay or EC:

$$\langle 0^+ | O_{\text{GT}} | 1_1^+ \rangle = \sqrt{\frac{3\kappa}{g_A^2 10^{\log(ft)}}}, \quad (3)$$

where $\kappa = 6147$ is a constant. The power counting of the ET [52–54] relates the Gamow-Teller matrix elements from the lowest and higher 1^+ initial states to

the common final reference state by $\langle 0^+ | O_{\text{GT}} | 1_{n+1}^+ \rangle \sim (\omega/\Lambda)^{n/2} \langle 0^+ | O_{\text{GT}} | 1_1^+ \rangle$, where $\Lambda \sim 3\omega$ is the breakdown scale of the ET. This allows us to estimate the values of $C_{\beta\ell}$ and $C_{L\beta\ell}$ with consistent theoretical uncertainties.

The $2\nu\text{ECEC}$ matrix element from the ground state of the initial nucleus to a 0^+ state of the final one is

$$M^{2\nu\text{ECEC}} = \sum_j \frac{\langle 0_f^+ | O_{\text{GT}} | 1_j^+ \rangle \langle 1_j^+ | O_{\text{GT}} | 0_{\text{gs},i}^+ \rangle}{D(1_j^+)/m_e}, \quad (4)$$

where j sums over all 1_j^+ states of the intermediate nucleus. The electron mass m_e keeps the matrix element dimensionless, and the energy denominator is $D(1_j^+) = E(1_j^+) - E(0_{\text{gs},i}^+) + [E(0_f^+) - E(0_{\text{gs},i}^+)]/2$, neglecting the difference in electron binding energies. The expression for the $2\nu\text{ECEC}$ to a final 2^+ state is similar [39], but the energy denominator appears to the third power.

Because the ET is designed to reproduce low-energy states, we calculate the $2\nu\text{ECEC}$ matrix elements within the single-state dominance (SSD) approximation:

$$M^{2\nu\text{ECEC}} \approx \frac{\langle 0_f^+ | O_{\text{GT}} | 1_1^+ \rangle \langle 1_1^+ | O_{\text{GT}} | 0_{\text{gs},i}^+ \rangle}{D(1_1^+)/m_e}, \quad (5)$$

which implies that only the matrix elements involving the lowest 1_1^+ state contribute. The advantage is that the ET can fit these using Eq. (3). The contribution due to omitted higher intermediate 1^+ states is estimated within the ET and treated as a theoretical uncertainty [52]:

$$\frac{\Delta M^{2\nu\text{ECEC}}}{M^{2\nu\text{ECEC}}} = \frac{D(1_1^+)}{\Lambda} \Phi\left(\frac{\omega}{\Lambda}, 1, \frac{D(1_1^+) + \omega}{\omega}\right), \quad (6)$$

where $\Phi(z, s, a) = \sum_{n=0}^{\infty} z^n / (a+n)^s$ is the Lerch transcendent. The ET describes very well the experimentally known two-neutrino $\beta\beta$ decay half-lives once the ET uncertainties, including from Eq. (6), are taken into account [52]. This agreement includes $^{128,130}\text{Te}$ among other heavy nuclei.

The ET $2\nu\text{ECEC}$ matrix element calculation thus requires the known ground-state energies and the lowest 1_1^+ excitation energy to calculate the energy denominator, as well as the Gamow-Teller β -decay and EC matrix elements from the 1_1^+ to the initial and final states of the $2\nu\text{ECEC}$ to fit the low-energy constants. In addition, the collective mode ω sets the ET uncertainty. Unfortunately, for ^{124}Xe there are no direct measurements for Gamow-Teller β decay or EC from the lowest 1_1^+ state in ^{124}I (the ground state is 2^-), or alternatively zero-angle charge-exchange reaction cross sections involving the nuclei of interest. The 1_1^+ excitation energy in ^{124}I is also unknown.

Therefore, we adopt the following strategy. First, we set $\log(ft)^{\text{EC}} = 5.00(10)$ for the EC on the lowest 1_1^+ state in ^{124}I , based on the experimental range of known EC on iodine isotopes with nucleon number

$A = 122 - 128$. This quantity varies smoothly for nuclei within an isotopic chain. For the β^- decay, we set $\log(ft)^{\beta^-} = 1.06(1) \log(ft)^{\text{EC}}$, based on the systematics of odd-odd nuclei in this region of the nuclear chart. Guided by the known spin-unassigned excited states of ^{124}I and the systematics in neighboring odd-odd nuclei, we set the excitation energy of the first 1_1^+ state in ^{124}I as 105 – 170 keV. Because this range is much smaller than the energy differences in $D(1_1^+)$, the associated uncertainty in the matrix elements is only a few percent. From the above considerations we obtain a range for the ^{124}Xe $2\nu\text{ECEC}$ matrix element based on our choice of parameters entering the ET. Finally, we set the excitation energy to $\omega = 478.3$ keV, the average of the excitation energies of the lowest 2_1^+ states in the corresponding even-even nuclei [52]. This allows us to estimate the ET uncertainty associated to the SSD approximation, Eq. (6).

Nuclear shell model.—Next we perform large-scale shell model calculations to obtain the nuclear matrix element using the full expression Eq. (4). We solve the many-body Schrödinger equation $H|\psi\rangle = E|\psi\rangle$ for ^{124}Xe , ^{124}Te , ^{124}I , using a shell model Hamiltonian H in the configuration space comprising the $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$ single-particle orbitals for neutrons and protons. To keep the dimensions of the shell model diagonalization tractable, especially for the largest calculation ^{124}Xe , we need a truncated configuration space. In a first truncation scheme, similar to the one used in Ref. [63], we limit to two the number of nucleon excitations from the lower energy $0g_{7/2}$, $1d_{5/2}$ orbitals to the higher lying $1d_{3/2}$, $2s_{1/2}$, $0h_{11/2}$ orbitals. Second, we adopt a complementary truncation scheme that keeps a maximum of two neutron excitations but does not limit the proton excitations from lower to higher lying orbitals (a maximum of six nucleon excitations are permitted in ^{124}Xe). This keeps the $0g_{7/2}$ orbital fully occupied. A third scheme with the $1d_{5/2}$ orbital fully occupied gives results within those of the other two truncations.

We use the shell model interaction GCN5082 [64, 65], fitted to spectroscopic properties of nuclei in the mass region of ^{124}Xe . The shell model interaction has been tested against experimental data on Gamow-Teller decays and charge-exchange transitions in this region, showing a good description of data with a renormalization, or “quenching”, of the $\sigma\tau$ operator $q = 0.57$ [43]. For the two-neutrino $\beta\beta$ decay of ^{128}Te , ^{130}Te , and ^{136}Xe , however, this interaction fits data best after a larger renormalization $q = 0.48$ [43]. An extreme case is the very small $\beta\beta$ ^{136}Xe matrix element, which is only reproduced with $q = 0.42$ [43]. The renormalization of the spin-isospin operator is needed to correct for the approximations made in the many-body calculation, such as unaccounted correlations beyond the configuration space or neglected two-body currents [10, 66]. A full understanding of its origin would require an ab initio study that is currently possible only for nuclei lighter than

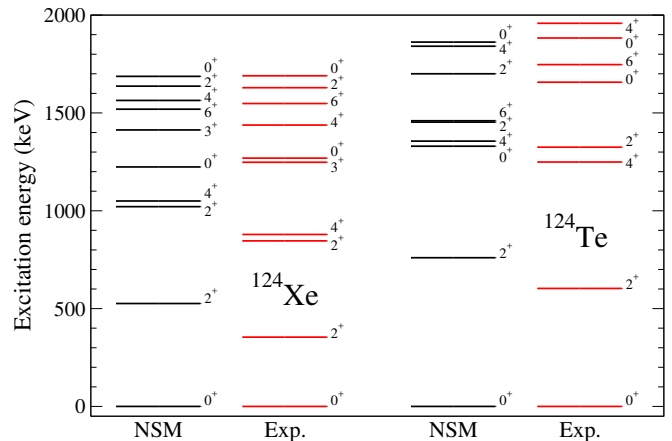


FIG. 1. ^{124}Xe and ^{124}Te excitation spectra obtained by the nuclear shell model (NSM) compared to experiment [71].

xenon [67–70]. Here we follow the strategy of previous shell model $\beta\beta$ decay predictions [41, 42] and include the above “quenching” factors phenomenologically to predict the half-life of ^{124}Xe .

The low-energy excitation spectra of the three isotopes are well reproduced. Figure 1 compares the experimental and calculated spectra for ^{124}Xe and ^{124}Te , obtained with the first truncation scheme described above. The spectra corresponding to the second truncation scheme is of similar quality. When additional excitations to the higher lying orbitals are permitted, the first excited 0_2^+ in ^{124}Te is raised to 1.6 MeV, in much better agreement with experiment. However, such extended truncation yields too large dimensions for ^{124}Xe , and cannot be used in our $2\nu\text{ECEC}$ calculations. The spectra of the intermediate ^{124}I is not well known besides the ground state and few tentative spin assignments. The GCN5082 interaction reproduces correctly the spin and parity of the 2^- ground state, although with a lowest 1_1^+ state at only about 10 keV, below any measured level. For the $2\nu\text{ECEC}$, as in the ET calculation, we consider the lowest 1_1^+ state at 105 – 170 keV excitation energy. All shell model calculations have been performed with the codes ANTOINE and NATHAN [40, 72].

Results and discussion.—The calculated nuclear matrix elements are common for the capture of K- or L-shell electrons. However, the presented half-lives correspond to the ^{124}Xe $2\nu\text{ECEC}$ of two K-shell electrons, as this is the mode explored in recent experiments [30–34].

Table I summarizes our main results. The ET predicts a smaller central value for the ^{124}Xe $2\nu\text{ECEC}$ matrix element than the NSM, even though both results are consistent when taking uncertainties into account. The ET uncertainty results from combining the uncertainty associated to the SSD approximation, Eq. (6), with the range of the parameters used as input for the ET. Both contributions are of similar size. For the NSM, one part of the

TABLE I. Nuclear matrix elements ($M^{2\nu\text{ECEC}}$) and half-lives ($T_{1/2}^{2\nu\text{ECEC}}$) calculated with the ET and the nuclear shell model (NSM). Results are given for the ^{124}Xe $2\nu\text{ECEC}$ of two K-shell electrons into the ^{124}Te ground 0_{gs}^+ and excited 0_2^+ and 2_1^+ states. The phase-space factors $G^{2\nu\text{ECEC}}$ in y^{-1} are from Refs. [39, 62].

$2\nu\text{ECEC}$	$0_{\text{gs},i}^+ \rightarrow 0_{\text{gs},f}^+$	$0_{\text{gs},i}^+ \rightarrow 0_2^+$	$0_{\text{gs},i}^+ \rightarrow 2_1^+$
$G^{2\nu\text{ECEC}}$	1.72×10^{-20}	1.67×10^{-22}	1.38×10^{-23}
	$M^{2\nu\text{ECEC}}$		
ET	0.011 – 0.041	0.002 – 0.050	$(0.8 - 9.0) \times 10^{-4}$
NSM	0.028 – 0.072	0.005 – 0.010	$(1.1 - 2.3) \times 10^{-4}$
	$T_{1/2}^{2\nu\text{ECEC}}$		
	[10^{22}y]	[10^{25}y]	[10^{30}y]
ET	1.3 – 18	0.092 – 57	0.034 – 4.3
NSM	0.43 – 2.9	2.3 – 9.3	0.57 – 2.5

theoretical uncertainty is given by the range of results obtained with different truncation schemes. The dominant part, however, is given by the three “quenching” values considered: the average $q = 0.57$ and $q = 0.48$, corresponding to the best description of Gamow-Teller transitions and $\beta\beta$ decays, respectively, plus the additional conservative $q = 0.42$ needed in the ^{136}Xe $\beta\beta$ decay. The NSM ranges in Table I cover the results obtained with the two truncations and three “quenching” values.

Table I also shows our predictions for the $2\nu\text{ECEC}$ into excited states of ^{124}Te . For both final 0_2^+ and 2_1^+ states, the ET and NSM matrix elements are consistent, even though the central values predicted by the shell model are about one third of the ET ones. The suppressed NSM matrix element to the final 0_2^+ state with respect to the transition to the ground state is consistent with the results on neutrinoless $\beta\beta$ decay in $^{128,130}\text{Te}$ and ^{136}Xe , using the same interaction [64]. While the shell model uncertainties are somewhat smaller than in the $2\nu\text{ECEC}$ to the ground state, the ET ones are much larger, because of the limitations of the SSD approximation when the energy denominator $D(1_1^+)$ is small [52]. The ET and NSM half-lives are in general shorter than the QRPA ones for the 0_2^+ $2\nu\text{ECEC}$ [38, 39], while for the 2_1^+ $2\nu\text{ECEC}$ the NSM and QRPA [39] predictions are very similar. Transitions to excited states are extremely suppressed because of the reduced Q -value and corresponding phase-space factor. The $2\nu\text{ECEC}$ to the final 2_1^+ state, which requires the capture of K- and L-shell electrons, is further suppressed because of the small nuclear matrix element.

Figure 2 compares our theoretical predictions for the $2\nu\text{ECEC}$ on ^{124}Xe to the ^{124}Te ground state with the most advanced QRPA results from Refs. [38, 39] and the most recent experimental $2\nu\text{ECEC}$ limits [31–34]. Theoretical half-lives are shown as black bars. The predictions from the ET, NSM, and QRPA are consistent. However, the ET shows a clear preference for longer half-lives than

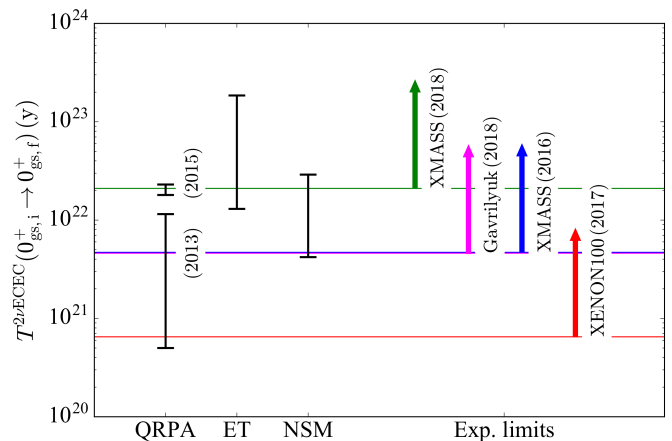


FIG. 2. ^{124}Xe half-life for the $2\nu\text{ECEC}$ of two K-shell electrons. The black bars show the theoretical predictions from the effective theory (ET) and the nuclear shell model (NSM), as well as most recent QRPA calculations [38, 39], in comparison to the horizontal lines that indicate the experimental lower limits set by the XENON100 [33] (red) and XMASS [32, 34] (blue, green) collaborations as well as Gavriluk et al. [31] (purple).

the NSM. On the other hand, the QRPA spans much shorter half-lives than those predicted by the ET or NSM.

Figure 2 shows that the theoretical predictions are consistent with the lower half-life limits established by the first results of the XMASS [32] and XENON100 [33] collaborations and with Ref. [31], shown as red, blue and purple horizontal lines in Fig. 2, respectively. However, the most recent limit established very recently by XMASS [34] (green line) excludes most of our NSM results, but a part of the predicted range remains permitted. Note that since the shell model configuration space had to be truncated, we could not obtain the exact nuclear matrix element without “quenching”. On the other hand, the ET half-life is almost fully consistent with the current XMASS limit. The ET central half-life is only about five times longer, and the range predicted by the ET lies largely within the sensitivity of ongoing experiments [33]. The QRPA predictions are mostly excluded including error bars, except the very recent results from Ref. [39], just at the border of the permitted region. Most other older theoretical calculations are also in tension with the XMASS limit [34]. Overall, our results suggest that the ^{124}Xe $2\nu\text{ECEC}$ could very well be discovered in ongoing or upcoming experiments in the near future.

Summary.—We have calculated the nuclear matrix elements for the $2\nu\text{ECEC}$ on ^{124}Xe using an ET and the large-scale nuclear shell model, two of the nuclear many-body approaches best suited to describe β and EC transitions in heavy nuclei. The ET results are based on β decay and EC on neighboring nuclei, while the shell model uses an interaction that describes well $\beta\beta$ decays of neighboring nuclei. The ET provides consistent the-

oretical uncertainties set by the order of the ET calculation, while the shell model uncertainty is dominated by the range of “quenching” considered for the 2ν ECEC operator. The ET predicts a half-life consistent and up to several times longer than current experimental limits, while the shell model prediction extends less beyond current limits. When all uncertainties are taken into account, the ET and NSM results are consistent, as well as with the most advanced QRPA results.

Future directions include higher-order calculations in the ET to reduce the uncertainties, and improved NSM studies with a better understanding of the “quenching” of the operator, and limiting truncations in the configuration space. Our findings suggest that the ^{124}Xe 2ν ECEC has a good chance to be discovered by ongoing or future experiments, so that these predictions can be tested by upcoming analyses of ongoing experiments and can further stimulate future searches.

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* tcoello@theorie.ikp.physik.tu-darmstadt.de

† menendez@cns.s.u-tokyo.ac.jp

‡ schwenk@physik.tu-darmstadt.de

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