Gravitational wave stochastic background from cosmological particle decay

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We assume that cosmological dark matter is composed of massive neutral scalar particles that decay into two massless particles. The decay produces a stochastic background of gravitational waves because of the "memory effect". We calculate the spectrum of this background and discuss its potential observability. Penrose has proposed a cosmological model for which these particles have the Planck mass and decay into two gravitons [1]. For these, the spectrum has an additional "direct" contribution from the decay products, which we also estimate and discuss.

INTRODUCTION

There is strong evidence that approximately 25% of the cosmological fluid is composed of "dark matter" [2]. This behaves as pressureless dust with a mass density

$$\rho_{\rm DM} \approx 2.1 \times 10^{-30} {\rm g/cm}^3$$
,

but its composition is unknown.

This paper was motivated by Penrose's "conformal cyclic cosmology" [1], which suggests that dark matter is composed of "erebons". These have Planck mass

$$M_{\rm P} = \sqrt{\frac{\hbar c}{G}} = 2.2 \times 10^{-5} \text{g},$$

and only interact gravitationally. Erebons are not stable, but have a lifetime τ which is longer than the Hubble time $T_{\rm H}=H^{-1}=4.6\times 10^{17}\,{\rm s}.$

Since erebons decay into gravitons, they should leave behind a stochastic cosmological background of gravitational waves (GW). Penrose conjectured that these might produce a detectable background in the Laser Interferometer Gravitational-wave Observatory (LIGO) GW detectors. In Section II we calculate the spectrum of that GW background, and show that it is far too weak to observe with LIGO or other instruments.

Surprisingly, even if dark matter decays into massless particles that are *not* gravitons (for example, photons) a GW background is produced by an effect called "GW memory". In Section III we calculate that contribution to the GW spectrum, which dominates the previous part at the low frequencies accessible to current GW detectors. Our results apply to any neutral scalar dark matter particle which decays into two massless one.

In this paper, c denotes the speed of light, G is Newton's gravitational constant, and \hbar is Planck's constant.

I. SPECTRAL FUNCTION

In the literature, the stochastic background of GWs is often characterized with a dimensionless spectral function $\Omega gw(f)$, which describes how the GW energy is distributed in frequency [3, 4]. If $d\rho_{\rm GW} c^2$ is the energy density in GWs in the frequency interval [f, f+df], then

$$\Omega_{\rm gw}(f) = \frac{f}{\rho_{\rm c}} \frac{d\rho_{\rm GW}}{df},\tag{1}$$

where $\rho_{\rm c}=3H^2/8\pi G=8.6\times 10^{-30}{\rm g/cm^3}$ is the closure/critical density for which the Universe is spatially flat. We use the Planck satellite value $H=67.4\,{\rm km\,s^{-1}\,Mpc^{-1}}$ [2] for the current Hubble expansion rate.

Current observations with advanced LIGO, combining O1 and O2 data, place the best constraints at around 40 Hz, as shown in Fig. 3 of [5]. These limits, $\Omega_{\rm GW}(40\,{\rm Hz}) < 6\times 10^{-8}$, can be compared directly with model predictions. There are additional low-frequency limits from pulsar timing arrays, and broadband limits from big bang nucleosynthesis modeling.

II. THE "DIRECT" SPECTRUM

We assume that the decay of the dark matter particle of mass m results in two massless particles with equal and opposite momenta. If those massless particles are gravitons, then these gravitons themselves provide a spectrum of GWs. Here we calculate that direct graviton spectrum. For particles which decay into other massless particles (for example photons) please skip to Section III.

In the rest frame of the dark matter particle, the two gravitons resulting from the decay have equal energy and opposite linear momentum. The mean frequency of the two particles f_0 follows from energy conservation:

$$f_0 = \frac{mc^2}{4\pi\hbar} = 1.5 \times 10^{42} \left(\frac{m}{M_{\rm P}}\right) \text{Hz}.$$

In the rest frame of the decaying particle, the gravitons have a frequency close to f_0 and an energy close to $mc^2/2$. The distribution around these values has a width determined by the particle lifetime τ .

Since we are assuming that these particles compose the dark matter, which has survived to the present, the lifetime τ must be greater than the Hubble time $T_{\rm H}$. This means that the width $\Delta f = 1/2\pi\tau$ of the emission line

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is very narrow, $\Delta f \ll f_0$. Nevertheless there is some small probability that the gravitons will be produced at a frequency low enough to detect with LIGO.

In this narrow-line case, the normalized probability distribution of the graviton frequencies f is described by a Lorentzian distribution

$$L(x) = \frac{1}{\pi(1+x^2)},\tag{2}$$

where $x = 2(f - f_0)/\Delta f$. In a time interval dt, the contribution to the GW stochastic background energy density from decaying particles in a frequency band [f, f + df] is

$$c^2 d\rho_{\rm GW} = 2 \times 2\pi \hbar f L \left(\frac{2(f - f_0)}{\Delta f}\right) \frac{2df}{\Delta f} n \frac{dt}{\tau},$$

where $n = \rho_{\rm DM}/m$ is the number of particles per unit volume, and the leading factor of two arises because each decay produces two gravitons.

Integrating this over the Hubble time, and expressing it in terms of the dimensionless spectral function Eq. (1) yields

$$\Omega_{\rm GW}(f) = 2 \frac{f^2}{f_0 \Delta f} \left(\frac{\rho_{\rm DM}}{\rho_{\rm c}}\right) \left(\frac{T_{\rm H}}{\tau}\right) L\left(\frac{2(f - f_0)}{\Delta f}\right). \quad (3)$$

By integrating over frequency it can be easily verified that the total energy density in GWs is

$$c^2 \int d\rho_{\rm GW} = c^2 \rho_{\rm c} \int \frac{1}{f} \Omega_{\rm GW} df = \rho_{\rm DM} c^2 \frac{T_{\rm H}}{\tau}.$$

This is what we expect from energy conservation, since in this model the energy in GWs comes from the decay of the dark matter; in the Hubble time the fraction that has decayed is $T_{\rm H}/\tau$.

The predicted $\Omega_{\rm GW}(f)$ at low frequency can be compared with LIGO sensitivity. At low frequencies one finds $L(x) \approx 1/\pi x^2 \approx \Delta f^2/4\pi f_0^2$, so

$$\Omega_{\rm GW}(f \ll f_0) = \frac{1}{4\pi^2} \frac{f^2}{\tau f_0^3} \left(\frac{\rho_{\rm DM}}{\rho_{\rm c}}\right) \left(\frac{T_{\rm H}}{\tau}\right).$$
(4)

For the erebon, since the lifetime τ must be greater than the Hubble time, we find $\Omega_{\rm GW}(40\,{\rm Hz}) < 7\times 10^{-144}$. This is below detectability in LIGO by about 136 orders of magnitude.

III. GW "MEMORY" EFFECT

There is another source of GWs, which puts more of the energy at low frequencies. The decay of a massive particle into two massless particles produces a sudden change in the gravitational field, which propagates outwards at the speed of light. Similar effects were first described by Zel'dovich and Polnarev [6] in the slow motion approximation, in the context of massive sources such as supernovae. Braginskii and Thorne [7] called such sources "GW bursts with memory". Later work by Christodoulou [8] showed that there was an additional effect related to the loss of energy by the source.

For massive particle decays into two point particles, one of which is massless, the gravitational waveform was calculated exactly in the weak-field limit by Tolish, Bieri, Garfinkle, and Wald [9], building on earlier work by Tolish and Wald [10]. If the initial (massive) particle is at rest at distance r from it, an interferometric GW detector registers a step function (filtered through the response of the detector) at time r/c after the decay.

The magnitude of the step is easily determined. Place the origin of coordinates at the massive particle and align the z-axis along the path of the massless decay products. Then Eq. (12) of [9] (set E=m/2, since both decay products are massless) gives the displacement ΔL^a in a detector arm L^a , located at distance r from the particle, as

$$\Delta L^a = \frac{Gm}{rc^2} (\theta^a \theta_b - \phi^a \phi_b) L^b, \tag{5}$$

where θ^a and ϕ^a are orthonormal spatial vectors tangent to the sphere of radius r at the location of the detector, with θ^a pointing along a line of constant longitude and ϕ^a pointing along a line of constant latitude. Detector arms based on optical delay lines or Fabry-Pérot cavities sense length changes in the parallel directions. So contracting both sides of this equation with L^a gives the observed GW strain $\Delta L/L$ in one arm as

$$\frac{\Delta L}{L} = \frac{Gm}{rc^2} \left[(\hat{L}^a \theta_a)^2 - (\hat{L}^a \phi_a)^2 \right],$$

where \hat{L}^a is a unit-length vector along the arm. A detector arm which points in the radial direction is unaffected, a detector arm which points along the longitudinal direction gets longer, and a detector arm which points along a line of latitude gets shorter.

In a LIGO-like detector with two equal-length perpendicular arms M and N, the differential strain is given by

$$h = \frac{\Delta M}{M} - \frac{\Delta N}{N} = \frac{Gm}{rc^2} \left[(\hat{M}^a \theta_a)^2 - (\hat{N}^a \theta_a)^2 - (\hat{M}^a \phi_a)^2 + (\hat{N}^a \phi_a)^2 \right].$$

Squaring this quantity and averaging over detector orientations gives a mean-squared strain $\langle h^2 \rangle = 4G^2m^2/5r^2c^4$. This is equivalent to a root-mean-square (rms) strain

$$h_{\rm rms} = \frac{2}{\sqrt{5}} \frac{Gm}{rc^2}.$$
 (6)

This is easy to check. Although the source is axisymmetric, the average effect on a GW detector (after rotation through all possible orientations of the detector) is independent of θ and ϕ and only depends upon r. This follows because Eq. (5) has no θ or ϕ dependence. Examination of Eq. (33) of [9] shows that the angular dependence of the two null particles $1 \pm \cos \theta$ cancels.

A. GW "memory" spectrum

If the dark matter is composed of particles which decay in this way, then they will act as GW source and give rise to a uniform and isotropic background of stochastic GWs. To find its spectrum, we calculate the auto-correlation function $C(\mathcal{T}) = \langle h(t)h(t+\mathcal{T})\rangle$ of the strain h(t) in one detector.

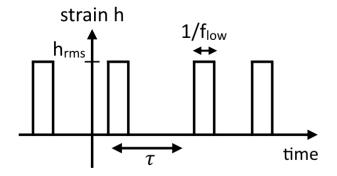


FIG. 1. The "memory effect" waveform in a GW detector from a set of particles decaying at fixed distance r with mean time τ between events. The amplitude is given in Eq. (6); the duration of the steps is the inverse of the detector's effective low-frequency cutoff f_{low} . The physical signal would consist of the sum of these events and similar ones with lower amplitude and higher rate, coming from larger distances.

We begin by looking at the strain signal that would arise from dark matter particles decaying at a fixed distance r from the detector, with mean time τ between events. Assume that the detector has a low-frequency cutoff $f_{\text{low}} = 20$ Hz, so that its strain response h(t) is a series of step functions as illustrated in Fig. 1. Overlapping this waveform with a time-delayed copy and averaging gives the auto-correlation for sources at radius r:

$$C_r(t) = \begin{cases} h_{\rm rms}^2 (1 - |t| f_{\rm low}) / \tau f_{\rm low} & \text{if } |t| \le 1 / f_{\rm low} \\ 0 & \text{otherwise.} \end{cases}$$

Fourier transforming this triangular auto-correlation function into the frequency domain gives

$$\tilde{C}_r(f) = \int_{-\infty}^{\infty} C_r(t) e^{2\pi i f t} dt = \frac{1 - \cos(2\pi f / f_{\text{low}})}{2\pi^2 \tau f^2} h_{\text{rms}}^2.$$

For an ideal detector $(f_{\text{low}} \to 0)$ the oscillating term on the right averages to zero, giving

$$\tilde{C}_r(f) = \frac{h_{\text{rms}}^2}{2\pi^2 \tau f^2} = \frac{2G^2 m^2}{5\pi^2 r^2 c^4 \tau f^2}$$
 (7)

for the auto-correlation function of sources at distance r. Since the sources are incoherent, we can sum their contributions out to the Hubble radius $cT_{\rm H}$ to get the frequency-domain auto-correlation function of the detector strain:

$$\tilde{C}(f) = \int_0^{cT_{\rm H}} \tilde{C}_r(f) n \, 4\pi r^2 dr = \frac{8G^2 m^2 n T_{\rm H}}{5\pi \tau c^3 f^2}, \quad (8)$$

where $n = \rho_{\rm DM}/m$ is the number of sources per unit volume.

The frequency domain correlation function of the strain in one detector is related to the spectral function $\Omega_{\rm GW}(f)$ using Eq. (3.59) of [4], with overlap reduction function $\gamma=1$:

$$\Omega_{\rm GW}(f) = \frac{20\pi^2}{3H^2} f^3 \tilde{C}(f) = \frac{2}{\pi} \frac{\rho_{\rm DM}}{\rho_{\rm c}} \frac{T_{\rm H}}{\tau} \frac{m}{M_{\rm P}} \frac{f}{f_{\rm P}},$$
(9)

where we have defined the Planck frequency as $f_{\rm P} = \sqrt{\frac{c^5}{\hbar G}}/2\pi = 3.0 \times 10^{42}\,{\rm Hz}$. This spectrum is "white": it describes a flat (uniform) distribution of energy as a function of frequency. In Subsection III B we discuss the high-frequency cutoff of this expression.

For a Planck-mass erebon $m=M_{\rm P}$, at LIGO frequencies $f\approx 40$ Hz, this "indirect" contribution to the GW stochastic background is 100 orders of magnitude larger than the "direct" contribution given in Eq. (4). Unfortunately it is still very small. For example at LIGO frequencies $\Omega_{\rm GW}(40~{\rm Hz}) < 2.2 \times 10^{-42}$ misses detectability by 34 orders of magnitude.

Reference [1] argues that this "memory" effect can be calculated for particle decay using the "impulsive Vaidya metric" [11], in which a spherically-symmetric Schwarzschild space-time converts itself to Minkowski space-time along an outgoing spherically-symmetric null hypersurface. We believe this is not correct. For example Eq. (2.43c) of [12] shows that for an isotropic source N_l makes the integral vanish. Here, we obtain a memory effect specifically because the two outgoing null particles define an axis so that the spacetime has axial rather than spherical symmetry.

B. Total energy converted to GWs

The GW energy spectrum we have found for a single idealized decay (and for the resulting stochastic background) is white, meaning that the energy $c^2d\rho_{\rm GW}$ in a frequency band df is independent of frequency f. Physically, the spectrum is cut off at high frequency because the processes associated with the decay have a shortest characteristic time-scale. Here we examine this more closely.

It follows from Eq. (5) that the transverse-traceless metric perturbation takes the form

$$h_{ab} = \frac{2Gm}{rc^2} (\theta_a \theta_b - \phi_a \phi_b) W(t - r/c), \qquad (10)$$

where in the rest frame of the decaying particle t is time after the decay, r is radial distance from the decay, and W(x) is the "profile" of the expanding circular wavefront: a smoothed dimensionless step function which vanishes for negative argument and approaches unity for large positive argument.

In this gauge the energy-density in GWs is $c^2\dot{h}_{ab}\dot{h}^{ab}/32\pi G$ where $\dot{}=d/dt$. Integrate this over a spatial slice at time t>0, assuming that t is large enough

to ensure that the outgoing radiation has formed a shell away from the origin. One obtains a total energy

$$E_{\rm GW} = \frac{Gm^2}{c} \int_0^\infty \left[\dot{W}(t) \right]^2 dt, \tag{11}$$

where $\dot{W}(t) = dW(t)/dt$.

To evaluate this integral we use a simple linear model waveform W(t) which enforces causality [13], so vanishes for t < 0:

$$W(t) = \begin{cases} 0 & \text{for } t < 0, \\ t/\Delta t & \text{for } 0 \le t < \Delta t, \text{ and} \\ 1 & \text{for } \Delta t \le t. \end{cases}$$
 (12)

Here Δt is the time-duration of the outgoing pulse for an observer at fixed radius r.

The integral over time that appears in Eq. (11) yields $1/\Delta t$, so the outgoing GW carries energy $E_{\rm GW} = Gm^2/c\Delta t$. If the time-duration Δt of the pulse is related to the rest-mass energy mc^2 of the decaying particle via the Heisenberg uncertainty principle, then $\Delta t \geq \hbar/mc^2$ and the energy carried away is bounded by

$$E_{\rm GW} \le \frac{c \, Gm^3}{\hbar} = \left(\frac{m}{M_{\rm P}}\right)^2 mc^2. \tag{13}$$

Note that for particles of mass smaller than Planck mass, GWs carry away only a small fraction of the total energy, but for particles close to the Planck mass, a significant fraction of the energy could be lost to GWs.

CONCLUSIONS

We have calculated the spectrum of GWs produced by the decay of massive dark matter particles into two massless ones. Because of the "memory effect", the decay produces an "indirect" stochastic background of GWs with a flat spectrum, and spectral function $\Omega_{\rm GW}(f)$ given in Eq. (9). If the massless particles are gravitons, then the decay also produces a "direct" GW spectrum. The spectral function for these rises with frequency, and is given in Eq. (4), assuming that the gravitons produced by the decay have a frequency which is high compared to the detection band. For massive particles, the indirect part of the spectrum dominates.

Current technology offers no prospects for constraining or detecting these backgrounds. While our estimates do not fully account for cosmological expansion and redshift, those should not change the order-of-magnitude of the results.

More conventional sources do not produce high frequency GWs. So although it is weak, the background from particle decay may nevertheless dominate the high frequency GW spectrum.

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