

Proposal for a clumsiness-free test of macroscopic realism

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We propose a test of macrorealism that exploits the contextuality of two-time correlation functions to escape the so-called “clumsiness loophole” that plagues Leggett-Garg inequalities. The non-contextuality of reduced joint probability distributions is proven to be an unequivocal criterion to guarantee that measurements are carried out in the ideally-weak measurement regime of a class of generalized von Neumann measurements. In this regime, testing the so-called “no-signaling in time” condition allows to uncontextually ascertain whether a property of a given system is macrorealistic or non-macrorealistic. Interestingly, the resulting protocol allows for tests of macrorealism in situations where Leggett-Garg inequalities and ideal negative measurement cannot be used at all.

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Ever since the birth of quantum mechanics, theoretical works have deepened our understanding of its conceptual and mathematical structure. In this respect, any list of highlights should definitively include Bell’s [1] and Leggett-Garg [2] inequalities to disprove local [3, 4] and macroscopic realism [5, 6] respectively. In contrast to the violation of local realism [7–9], however, an inarguable violation of macrorealism has remained elusive to date [10, 11]. The reason is that whilst special relativity can be used to close the “communication loophole” in a Bell test of local realism [12–15], no such defence exists for a Leggett-Garg test of macrorealism.

Macrorealism does not assert that it is impossible to affect a physical system by measurement and therefore a violation of Leggett-Garg inequalities can only be a proof that system’s properties are either (i) non-macrorealistic or (ii) macrorealistic but subjected to a measurement technique that happens to disturb the system. This problem is known as the “clumsiness loophole” [10], and can always be exploited to refute the implications of a Leggett-Garg test of macrorealism. While a number of works have addressed this problem by making the explanation of Leggett-Garg inequalities violations in terms of experimental clumsiness so contrived as to be doubtful [13, 16], whether a loophole-free Leggett-Garg protocol can be constructed remains an open question [11].

In this Letter we propose a clumsiness-free test of macrorealism that relies on the notion of contextuality introduced by Bell [1], Kochen and Specker [17], which is known to yield observable effects at the level of time-correlation functions [18–20]. Measuring an observable A at time t and correlating the outcome, $y_A(t)$, with the measured value of B , $y_B(\tau)$, at a later time $\tau \geq t$, represents an unequivocal way of representing the dynamics of classical systems in terms of joint probabilities, i.e., $P(y_A, y_B) \leftrightarrow \text{system dynamics}$. In quantum mechanics, however, the unavoidable backaction

of the measurement process [21, 22] precludes such a clear-cut connection. Even using the best technological means, different measurement schemes, $\{\sigma_A\}$, can yield different probability distributions, i.e., $P(y_A, y_B)_{\sigma_A} \leftrightarrow \text{system} + \text{apparatus dynamics}$. This property of quantum mechanics can result in contradictions among tests of macrorealism that are based on different experimental set-ups.

To avoid contextuality, we will first identify the *ideally-weak measurement regime*: the regime where, for pure states, two-time correlation functions distinctively unravel either (i) the expectation value of two-time Heisenberg operators for non-macrorealistic properties or (ii) the product of the expectation values of two independent events for macrorealistic ones. We will then show that this regime can be experimentally identified by witnessing the non-contextuality of a reduced probability distribution. Finally, we will prove that for general (mixed) states, assessing the so-called “no-signaling in time” criterion [23, 24] under ideally-weak measurement conditions makes it possible to unambiguously distinguish between macrorealistic and non-macrorealistic properties.

We consider a generalized von Neumann measurement [25, 26], where the expectation value of a property A , associated to the operator $\hat{A} = \sum_i a_i |a_i\rangle\langle a_i|$ (with a_i and $|a_i\rangle$ being the corresponding eigen-values and -states), of a quantum system $|\psi(t)\rangle$ is determined by repeatedly reading-out the pointer position of the meter over a large ensemble of identically prepared experiments:

$$\langle y_A(t) \rangle = \int_{-\infty}^{\infty} dy_A y_A P(y_A), \quad (1)$$

where $P(y_A)$ is the probability of finding a value y_A of the pointer position at time t . According to Born’s rule, $P(y_A)$ can be expressed in terms of the system degrees

of freedom as $P(y_A) = |\psi_A(t)|^2$, where

$$|\psi_A(t)\rangle = \sum_i \Omega_{y_A - \lambda a_i} c_i |a_i\rangle \quad (2)$$

is the state of the system right after measuring y_A at time t [27]. In Eq. (2) we have defined the coefficients $c_i = \langle a_i | \psi(t) \rangle$, and the displaced (by an amount λa_i) wavepacket of the pointer, $\Omega_{y_A - \lambda a_i}$, with λ being a macroscopic parameter with units of $[L][A]^{-1}$ that hereafter is assumed to be $\lambda = 1$ [28]. In order to ensure that Eq. (1) always yields the correct expectation value $\langle y_A(t) \rangle = \langle \hat{A} \rangle$, it is enough to make the pointer wavepacket to be well normalized and obeying $\int y_A |\Omega_{y_A - a_i}|^2 dy_A = a_i$ [26, 29, 30].

A second, subsequent, measurement of a property B , associated to the operator $\hat{B} = \sum_i b_i |b_i\rangle \langle b_i|$ (with b_i and $|b_i\rangle$ being the corresponding eigen-values and -states) can be easily accommodated into the above scheme by simply reading-out the pointer position of a second measuring apparatus at time $\tau \geq t$. The two-time correlation function $\langle y_A(t) y_B(\tau) \rangle$ can be then evaluated as:

$$\langle y_A(t) y_B(\tau) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dy_A dy_B y_A y_B P(y_A, y_B), \quad (3)$$

where $P(y_A, y_B)$ is the joint probability of reading-out the values y_A and y_B at times t and τ respectively. Using Born's rule, this probability can be written as $P(y_A, y_B) = |\psi_{A,B}(\tau)|^2$, where

$$|\psi_{A,B}(\tau)\rangle = \sum_{i,j} \Omega_{y_A - a_i} \Omega_{y_B - b_j} c_i c_{j,i} |b_j\rangle \quad (4)$$

is the state of the system right after the two-time measurement process [31]. In Eq. (4) we have defined $c_{j,i} = \langle b_j | \hat{U}_\tau | a_i \rangle$, and $\hat{U}_\tau = \exp(i\hat{H}\tau/\hbar)$ describes the unitary evolution of the system between the two measurements.

Without the loss of generality, we can now restrict the meter wavepacket to be represented by a gaussian Krauss operator [32, 33], i.e., $\Omega_{y - a_j} = \mathcal{A} \exp[-(y - a_j)^2/4\sigma^2]$, where \mathcal{A} is a normalization constant. The dependence of Eq. (4) on the measuring apparatus can be then effectively characterized by the coupling-strength parameter $\sigma = \sigma_{A/B}$ and thus Eq. (3) reads [34]:

$$\langle y_A(t) y_B(\tau) \rangle_{\sigma_A} = \frac{1}{2} \sum_{i,j} a_i \mathcal{E}_{j,i} \mathcal{B}_{j,i}(\tau) + c.c., \quad (5)$$

where $\mathcal{B}_{j,i}(\tau) = \langle a_j | \hat{B}(\tau) | a_i \rangle$ are the matrix elements of the Heisenberg operator $\hat{B}(\tau) = \hat{U}_\tau^\dagger \hat{B} \hat{U}_\tau$, and we have defined $\mathcal{E}_{j,i} = c_j^* \exp[-(a_i - a_j)^2/4\sigma_A^2] c_i$. The expectation value in Eq. (5) now bears a subscript σ_A that reinforces the idea that this result depends on the measurement scheme. That is, two-time expectation values are generally contextual [19]. Concerning the assumption of the meter wavepacket to be represented by a Gaussian

operator, it is shown in Ref. [34] that Eq. (5) can be derived for non-specific meter wavefunction shapes.

The result in Eq. (5) can be generalized to systems made of N interacting particles. For that, we consider a general (non-separable) state $|\psi(t)\rangle = \sum_{i_1, \dots, i_N} c_{i_1, \dots, i_N} |a_{i_1}, \dots, a_{i_N}\rangle$, where $c_{i_1, \dots, i_N} = \langle a_{i_1}, \dots, a_{i_N} | \psi(t) \rangle$. We define also the many-body intensive operator $\hat{A} = \sum_{\xi=1}^N \hat{I} \otimes \dots \otimes \hat{A}_\xi \otimes \dots \otimes \hat{I} / N$, where the index ξ only denotes the degree of freedom that the single-particle operator, \hat{A}_ξ , acts on. Then, the analogous of Eq. (5) for a many-body system reads [35]:

$$\langle y_A(t) y_B(\tau) \rangle_{\sigma_A} = \frac{1}{2N} \sum_{\xi=1}^N \sum_{\substack{i_1, \dots, i_N \\ j_1, \dots, j_N}} a_{i_\xi} \mathcal{E}_{i_1, \dots, i_N} \mathcal{B}_{i_1, \dots, i_N} + c.c., \quad (6)$$

where we have defined the matrix elements:

$$\mathcal{B}_{i_1, \dots, i_N} = \langle a_{j_1}, \dots, a_{j_N} | \hat{B}(\tau) | a_{i_1}, \dots, a_{i_N} \rangle, \quad (7a)$$

$$\mathcal{E}_{i_1, \dots, i_N} = c_{j_1, \dots, j_N}^* \exp \left[- \frac{(\sum_{\nu=1}^N a_{i_\nu} - a_{j_\nu})^2}{4\sigma_A^2 N^2} \right] c_{i_1, \dots, i_N}. \quad (7b)$$

For $N = 1$ Eq. (6) trivially reduces to Eq. (5). For $N \neq 1$, the backaction of the measurement of A can induce entanglement among particles [36, 37].

At this point we want to address the question of whether there exists a specific measurement regime where the result in Eq. (6) becomes non-contextual, viz., $\langle y_A y_B \rangle_{\sigma_A} \approx \langle y_A y_B \rangle$. For that, we define the effective dimension of the system, d_{eff} , as a measure of the average width of the relevant spectrum of the system with respect to \hat{A} , i.e: $d_{\text{eff}} := \sum_{\nu=1}^N \max(\Delta A_\nu) / N$, where $\max(\Delta A_\nu)$ is the maximum distance between the occupied upper and lower bounds of the spectrum of \hat{A}_ν . Then, a simple inspection of the matrix elements in Eq. (7b) shows that for any coupling σ_A fulfilling the condition $d_{\text{eff}} \ll \sigma_A$, one always measures [38]:

$$\langle y_A(t) y_B(\tau) \rangle = \langle \psi(t) | \hat{A}(t) \hat{B}(\tau) | \psi(t) \rangle + c.c. \quad (8)$$

The condition $d_{\text{eff}} \ll \sigma_A$ defines what we call the ideally-weak measurement (IWM) regime: the regime where one always measures the same expectation value independently of σ_A [39]. This is the case even if the joint probability distribution of measuring $y_A(t)$ and $y_B(\tau)$ depends on σ_A . This result adds to previous findings [18-20, 40] by showing that, while quantum backaction is needed for correlation functions to be contextual, viz., $\langle y_A y_B \rangle_\sigma \Rightarrow P(y_B, y_A)_{\sigma_A}$, the contrary is not true for a general class of experiments, viz., $P(y_B, y_A)_{\sigma_A} \not\Rightarrow \langle y_A y_B \rangle_{\sigma_A}$.

Yet, the limit $d_{\text{eff}} \ll \sigma_A$ implies a strict cancellation of the sigma-dependence of the joint probability

$P(y_A, y_B)_{\sigma_A}$ when integrated over all possible values of y_A for σ_A s larger than a given threshold σ_{A_o} [41], i.e.:

$$\text{IWM: } \frac{d}{d\sigma_A} \int dy_A P(y_A, y_B)_{\sigma_A} = 0, \quad (\forall \sigma_A > \sigma_{A_o}). \quad (9)$$

Thus, by assessing the validity of Eq. (9) for a reasonable number of distinct measurement set-ups (with different system-meter coupling-strengths), an experimentalist can assert whether or not he/she is working in the IWM and hence whether the measurements conducted in the laboratory are generalized von Neumann measurements of the type described here.

Making sure that one is operating in the IWM regime, however, does not guarantee that the measurement of A is non-invasive. This is a crucial point that can be appreciated by rewriting the final state of the system in Eq. (4) using a first order Taylor expansion of $\Omega_{y_A - a_i}$ and $\Omega_{y_B - b_j}$ around y_A and y_B in the limit of $\sigma_{A/B} \rightarrow \infty$ [42]:

$$|\psi_{A,B}\rangle = \Omega_{y_B} \Omega_{y_A} \left(\mathbb{I} - \frac{y_A}{\sigma_A^2} \hat{B} \right) \left(|\psi(\tau)\rangle - \frac{y_B}{\sigma_B^2} |\tilde{\psi}(\tau)\rangle \right), \quad (10)$$

where we have defined $|\psi(\tau)\rangle = \hat{U}_\tau |\psi(t)\rangle$ and $|\tilde{\psi}(\tau)\rangle = \hat{U}_\tau \hat{A} |\psi(t)\rangle$. Expression (10) tells us that the state of the system right after two ideally-weak measurements can be written as a superposition of two states, and that only the first one, i.e., $\Omega_{y_B} \Omega_{y_A} \left(\mathbb{I} - \frac{y_A}{\sigma_A^2} \hat{B} \right) |\psi(\tau)\rangle$, contains information about the system having evolved freely from t to τ . Generally, the second term in Eq. (10) is not proportional to $\hat{B} \hat{U}_\tau |\psi(t)\rangle$ and hence it represents the non-negligible backaction of the first measurement on the subsequent evolution of the system.

Only when the state of the system $|\psi(t)\rangle$ can be approximated by an eigenstate of the operator \hat{A} , i.e.: $\hat{A} |\psi(t)\rangle \approx \langle \hat{A} \rangle |\psi(t)\rangle$, then the backaction of the first measurement is avoided, and hence property A is said to be macrorealistic. In short, Eq. (8) reduces to $\langle y_A(t) y_B(\tau) \rangle = \langle y_A(t) \rangle \langle y_B(\tau) \rangle$, which is the definition of macrorealism for a pure state, i.e., $P(y_B, y_A)_{\sigma_A} = P(y_A)_{\sigma_A} P(y_B)_{\sigma_A}$.

Let us recapitulate. While an IWM of a non-macrorealistic property does induce a backaction on the system (see Eq. (10)), the resulting effects at the level of the reduced probabilities $\int dy_A P(y_A, y_B)_{\sigma_A}$ in Eq. (9) are independent of the properties of the measuring apparatus. This is a very interesting result, valid also for general mixed states, that can be exploited to verify that a given experimental set-up can be effectively represented by a generalized von Neumann measurement model. This is precisely the type of “good” measuring apparatus that, as shown in the above paragraph, happen to be non-invasive for macrorealistic properties. Therefore, as it will be shown in the following, the use of the IWM conditions in combination with a given test of macrorealism can be used to close the clumsiness loophole.

For general (mixed) states, macrorealism can be defined as [23]:

$$\text{MR: } P(y_A, y_B) = \sum_\lambda \rho(\lambda) P_\lambda(y_A) P_\lambda(y_B), \quad (11)$$

where λ specifies all properties of the system. Due to the mixedness of the initial state, the violation of macrorealism can be hidden in the statistics of the experiment. A test of macrorealism can then be based on the statistical version of the “non-invasive measurability” condition, also referred to as “no-signaling in time” (NSIT) [23]:

$$\text{NSIT: } P(y_B) = \int dy_A P(y_A, y_B). \quad (12)$$

This condition, originally proposed as an alternative characterization of macrorealism, differs from the Leggett-Garg inequalities [23, 43]. However, while $\text{MR} \Rightarrow \text{NSIT}$, the violation of NSIT can only indicate either (i) that the system is non-macrorealistic or (ii) that the system is macrorealistic but subjected to a measurement technique that happens to disturb the system [44]. To discard (ii) above, we propose the following:

- (S1) Make sure that the measurement of A at time t is carried out in the IWM regime by testing Eq. (9) for a reasonable number of measurement set-ups $\{\sigma_A\}$.
- (S2) Equate the resulting reduced probability distributions as in Eq. (12). Property A is macrorealistic if NSIT is fulfilled and non-macrorealistic otherwise.

Note that $\text{NSIT} \Rightarrow \text{IWM}$, and therefore under the fulfillment of Eq. (12) the condition Eq. (9) is trivially fulfilled. Whenever NSIT is violated, however, being under the IWM regime will be the only warranty that the experimental set-up represents a “good” measuring apparatus (i.e. non-invasive for macrorealistic properties).

Assessing the IWM condition in Eq. (9) requires to design a number of different measurement set-ups. The larger the number of measurement set-ups that are compared one to each other, the more trustworthy the test of macrorealism will be. Put differently, the probability that Eq. (9) is fulfilled simultaneously by a number of classically invasive measurement apparatuses (different from the generalized von Neumann measurements described here) decreases with the number of experimental set-ups itself. Escaping this test would simply be too conspiratorial a loophole to take seriously.

Let us mention that the protocol described by (S1) and (S2) only assesses macrorealism at time t and with respect to an intensive property A. In a test of genuine macrorealism the validity of Eq. (12) should be proven for any observable at any time. This is obviously a prohibitive experimental task, and hence it is common to associate macrorealism only to a given observable of interest [16, 45–47]. Anyhow, genuine macrorealism is not

expected in general, not at least for operators representing extensive properties such as, e.g., the angular momentum or magnetization. Yet, examples of macrorealism for general intensive properties of the type considered here, far from being atypical, can be common for large systems made of weakly-interacting particles. Consider, e.g., a system defined by separable wavefunctions $|\psi(t)\rangle = |\psi_1(t)\rangle \otimes \dots \otimes |\psi_N(t)\rangle$ where $|\psi_i(t)\rangle$ are all identical single-particle states. To determine whether the state $|\psi(t)\rangle$ is an eigenstate of an intensive property \hat{A} , i.e., $\hat{A}|\psi(t)\rangle \approx \langle \hat{A} \rangle |\psi(t)\rangle$ with $\langle \hat{A} \rangle = \sum_{\xi}^N \langle \psi_{\xi}(t) | \hat{A}_{\xi} | \psi_{\xi}(t) \rangle / N$, we check the soundness of the identity $\langle \hat{A}^2 - \langle \hat{A} \rangle^2 \rangle = 0$. By writing $\hat{A}^2 = N^{-2} \sum_{\xi}^N (\hat{A}_{\xi} \hat{A}_{\xi} + \sum_{\nu \neq \xi}^N \hat{A}_{\xi} \hat{A}_{\nu})$, it is easy to realize that the expectation value $\langle \psi(t) | \hat{A}^2 | \psi(t) \rangle$ reads: $\langle \hat{A}^2(t) \rangle = N^{-2} \sum_{\xi=1}^N [\langle \hat{A}_{\xi}^2(t) \rangle + \sum_{\nu \neq \xi}^N \langle \hat{A}_{\xi}(t) \rangle \langle \hat{A}_{\nu}(t) \rangle]$. Therefore, in the limit $N \rightarrow \infty$ we get $\langle \hat{A}^2 \rangle = \langle \hat{A} \rangle^2$, so we conclude that $\hat{A}|\psi(t)\rangle = \langle \hat{A} \rangle |\psi(t)\rangle$. That is, even if individually $|\psi_{\xi}(t)\rangle$ are not eigenstates of \hat{A}_{ξ} , in the limit $N \rightarrow \infty$ one could arguably speak of macrorealism of any intensive property A [48]. This is in contrast with the quantumness of the system itself, which, being preserved, would prevent us to talk about realism at the microscopic level [11].

To illustrate the proposed test of macrorealism, we consider a simple numerical experiment. We will evaluate the autocorrelation function of the center-of-mass position operator, $\hat{X} = \sum_{\xi}^N \hat{X}_{\xi} / N$, for a number N of uncoupled one-dimensional double-well oscillator (see the top panel of Fig. 1). Hereafter we use atomic units, $\hbar = m = 1$, and define the single-particle oscillator's Hamiltonian as $\hat{H} = \hat{P}^2 / 2 + \omega_0^2 \hat{X}^2 / 2 + \cosh^{-2} \alpha \hat{X}$, where \hat{P} is the momentum operator, and the natural frequency of the underlying harmonic oscillator is $\omega_0 = 4.3 \cdot 10^{-3}$ a.u. The characteristic width of the barrier between the two wells is set to $\alpha = 5 \cdot 10^{-2}$ a.u. We choose $t = 0$ such that the only relevant time in the discussion is τ . We consider that the oscillators are all initially prepared in the ground state. Then, by taking the non-interacting limit of Eq. (6), we find (for arbitrary initial conditions see [49]):

$$\langle y_A y_B \rangle_{\sigma} = \frac{1}{2N} \sum_{i,j}^{\infty} \mathcal{E}_{j,i} \mathcal{B}_{j,i} (a_i + (N-1) \langle \hat{A}(t) \rangle) + c.c. \quad (13)$$

which in the limit of $N \rightarrow \infty$ reduces to $\langle y_A(t) \rangle \langle y_B(\tau) \rangle$.

The dynamics of a single oscillator for different values of σ_X is shown in Fig. 1. For a projective measurement, i.e., $\sigma_X \rightarrow 0$, the dynamics presents a central resonance peak at ω_0 (in dashed red line). This is due to the strong perturbation induced by the projective measurement at $t = 0$, which yields a subsequent dynamics characterized by a large amplitude (over-the-barrier) oscillation. Contrarily, in the limit $\sigma_X \rightarrow \infty$ the measurement produces only a small perturbation to the initial state and yields

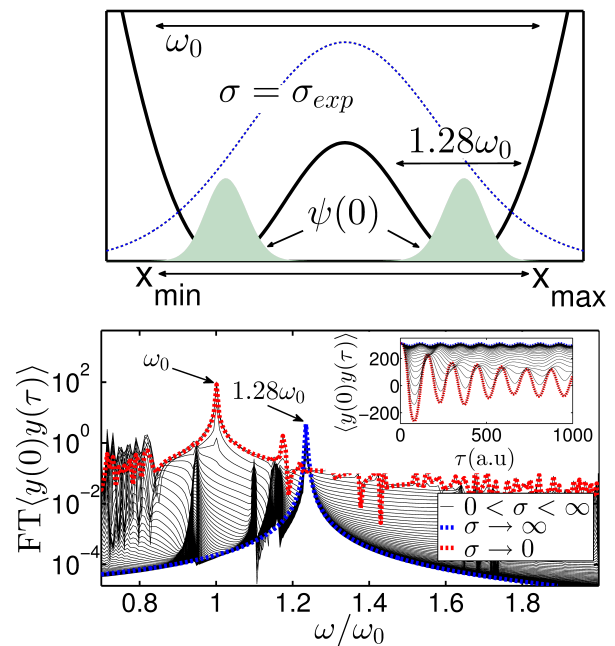


Figure 1: Top panel: schematic picture of the double-well oscillator. The potential energy curve is plot in solid black line. The initial state of the system (area in green) is taken to be the ground state of the system. Two main frequencies are involved in the dynamics of the system, viz., ω_0 and $1.28\omega_0$, related respectively with the inter-well and intra-well dynamics. The relevant upper and lower bounds of the spectrum of \hat{X} are denoted by x_{\max} and x_{\min} respectively, and the exponential function defined in Eq. (7b) is depicted for a particular value of σ_X in dashed blue line. Bottom panel: semi-log plot of the Fourier transform of the autocorrelation function in Eq. (5) as a function of σ_X and τ (solid black lines). The limits of $\sigma_X \rightarrow 0$ and $\sigma_X \rightarrow \infty$ are shown respectively in dashed red and blue lines. In the inset: the same results but for the autocorrelation function.

an ensuing dynamics confined in the wells with a characteristic frequency $\omega = 1.28\omega_0$ (in dashed blue line). In between these two regimes, an infinite number of dynamics can be inferred depending on the system-meter coupling strength (in black solid lines).

To conclude whether the position of a single oscillator is macrorealistic, we first need to ensure that the measurement of X at time $t = 0$ is carried out in the IWM regime (i.e., S1), and then compare the expectation values $\langle y(0)y(\tau) \rangle$ and $\langle y(0) \rangle \langle y(\tau) \rangle$ (i.e., S2). Note that since our example only considers pure states, the condition in Eq. (11) can be replaced by the simpler one $P(y_B, y_A) = P(y_A)P(y_B)$. We address (S1) and (S2) in a compact way using the quantity

$$\Delta(\sigma_X, N) = \frac{d\langle y_X(0)y_X(\tau) \rangle}{d\sigma_X} - \Delta_{QC} \quad (14)$$

where $\Delta_{QC} = \langle y_X(0)y_X(\tau) \rangle - \langle y_X(0) \rangle \langle y_X(\tau) \rangle$. Whenever $\Delta(\sigma_X, N)$ becomes constant, Eq. (9) is fulfilled, and

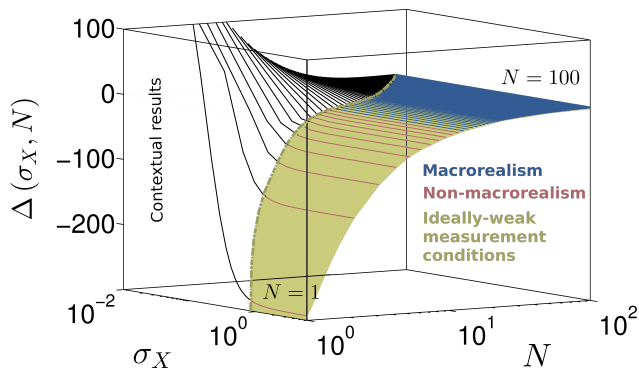


Figure 2: $\Delta(\sigma_X, N)$ as a function of σ_X and the number N of oscillators for $\tau = 33.3\pi$. Non-contextual results are differentiated from contextual results by an underlying yellow surface. Non-macrorealism and macrorealism results are shown in red and blue respectively.

whether the center-of-mass position is macrorealistic or not can be checked by simply assessing $\Delta(\sigma_X, N)$ in the asymptotic region. That is, X is macrorealistic if $\Delta(\sigma_X, N)$ vanishes in the asymptotic region and non-macrorealistic otherwise. In Fig. 2 we plot the quantity $\Delta(\sigma_X, N)$ as a function of σ_X and the number N of oscillators. A single oscillator is non-macrorealistic as $\Delta(\sigma_X, 1)$ asymptotically converges to a non-zero value. For a large enough number of oscillators, however, the dynamics of \hat{X} becomes independent of σ_X which is a clear signature of macrorealism as defined in Eq. (11). In general, the N oscillators become entangled right after the first measurement process and this allows a smooth transition (exponential decay with N) between the non-macrorealistic and macrorealistic results.

Conclusion. — Quantum dynamics is ambiguous unless it goes along with a proper discussion of the system-meter interaction. This applies also to the great majority of tests of macrorealism, where contextuality appears in the form of a clumsiness loophole.

In this Letter we have proven a sufficient condition for the non-contextuality of reduced one-time probability densities for a family of, classically non-perturbative, generalized von Neumann measurements. This condition, named IWM regime, can be assessed according to Eq. (9), which in turn requires to design a number of different experimental set-ups. For a large enough sample of set-ups, probably implemented at different laboratories, falsifying Eq. (9) would require a loophole too conspiratorial to be taken seriously.

Based on this result we have proposed a test of macrorealism that consists on witnessing the so-called non-signaling in time condition, Eq. (12), under the fulfillment of the IWM regime, Eq. (9). The resulting protocol allows for tests in situations (e.g., unbounded and non-dichotomic properties) where Leggett-Garg inequalities and ideal negative measurement cannot be used at all.

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