

The next decade of black hole spectroscopy

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Gravitational wave observations of the ringdown of the remnant black hole in a binary black hole coalescence provide a unique opportunity of confronting the black hole no-hair theorem in general relativity with observational data. The most robust tests are possible if multiple ringdown modes can be observed. In this paper, using state-of-the-art Bayesian inference methods and the most up-to-date knowledge of binary black hole population parameters and ringdown mode amplitudes, we evaluate the prospects for black hole spectroscopy with current and future ground based gravitational wave detectors over the next 10 years. For different population models, we estimate the likely number of events for which the subdominant mode can be detected and distinguished from the dominant mode.

I. INTRODUCTION

The remnant black hole (BH) formed after the coalescence of two compact objects emits gravitational radiation while settling down to a Kerr BH. This stage is known as the ringdown. Perturbation theory predicts that, at late enough times, the ringdown consists of a superposition of exponentially damped sinusoids called quasinormal modes (QNM) [1, 2] (see also [3, 4]). The QNMs are characterized by a set of complex frequencies $\Omega_{\ell mn}$ labeled by three integers; ℓ, m are angular quantum numbers while $n = 0, 1, 2, \dots$ is the overtone index. According to the no-hair theorem in standard general relativity (GR), $\Omega_{\ell mn}$ is uniquely defined by the BH mass and spin. The measurement of multiple QNMs in a BH ringdown, known as BH spectroscopy, is crucial for robust observational tests of the no-hair theorem with gravitational waves based only on the ringdown signal [5, 6].

The excitation of different QNMs depends on the nature of the perturbation, i.e. on the properties of the binary progenitor [7–11]. Thus, for aligned spin systems, the amplitude of the different modes are determined by the spins of the initial compact objects and the mass ratio $q = m_1/m_2 \geq 1$, with $m_{1,2}$ the mass of each object. The ringdown signature is dominated by the fundamental $(\ell, m) = (2, 2)$ mode [12]. For non-spinning binaries with equal masses ($q = 1$), odd ℓ modes vanish and the loudest subdominant mode is the $(\ell, m) = (4, 4)$ mode [7, 8]. As the mass ratio increases, the $(\ell, m) = (3, 3)$ mode becomes the loudest subdominant mode, reaching amplitude ratios as large as $A_{330}/A_{220} \simeq 0.3$. Hence, coalescences of two unequal-mass BHs or one neutron star (NS) and one BH are the most promising sources for measurability of subdominant modes in the ringdown. For still higher mass ratios, the relative amplitude of the modes can also tell us about the alignment of the orbit relative

to the BH spin during the inspiral phase [9–11].

In general, two main conditions are necessary to test the no-hair theorem: (i) the detectability of at least two modes, and (ii) the resolvability of the frequencies and/or damping times of each mode. Theoretical estimates of the necessary ringdown SNR for each of these conditions can be found in the literature [6, 13]. The Generalized Likelihood Ratio Test (GLRT) provides an estimate for detectability, while a natural criterion à la Rayleigh has been used for resolvability. As can be seen in Fig. 9 of [13], very loud SNRs are required to resolve both the frequency and damping time of different modes (ρ_{both}), although it is possible to resolve just one of these two parameters at lower SNRs (ρ_{crit}). Based on GLRT estimates, [14, 15] have predicted that Advanced LIGO should observe several ringdown events at design sensitivity, but will not be able to detect subdominant modes from the coalescence of stellar-mass BBH for BH spectroscopy.

At least 10 binary black-hole (BBH) coalescences have been observed in the first two observing runs of Advanced LIGO and Virgo [16–21]. The loudest BBH event is still the first detection, GW150914 [22], with a ringdown signal-to-noise ratio (SNR) $\rho \simeq 8.5$ [23] at 3 ms after merger. This event has not provided significant evidence for the presence of measurable subdominant modes with $\ell \neq 2$ [24]. However, recent work suggests that the inclusion of higher overtones of the dominant $\ell = 2$ mode allows for the modeling of the ringdown immediately after the merger, hence obtaining higher SNR in ringdown signatures [25]. The analysis of the GW150914 ringdown using the fundamental mode and its first overtone provides the first constraints to date of deviations of the no-hair theorem using two QNMs [26].

In this paper we study the prospects for accurate BH spectroscopy with the next decade of LIGO detectors. In general, asymmetric binaries are more likely to produce higher amplitudes for the subdominant ringdown modes. However, based on the BBH detections to date, more asymmetric systems are also likely to be much fewer in

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number. In addition, the orientation of a source relative to the detectors also has an important effect on the observed amplitudes. Systems where the angular momentum is aligned with the line-of-sight to the source are more luminous, but these orientations are not favorable for observing the subdominant modes. It is clear that to realistically understand the prospects of observing subdominant modes, it is necessary to generate realistic populations of BBH systems. We generate a population of BBH candidates based on population models derived from gravitational-wave observations by the LIGO Scientific and the Virgo Collaborations [27] and we use the most recent estimates of ringdown mode amplitudes calculated in [8]. Our analysis uses the state-of-the-art `PyCBC Inference` toolkit [28, 29] for estimating ringdown parameters.

While recent public alerts from the third observing run of Advanced LIGO and Virgo suggest possible detections of neutron-star black-hole binaries (NSBH) [30, 31], results are not yet available to the outside community and population models including NSBH are largely uncertain. Hence, we ignore NSBH mergers here, although inclusion of a NSBH population could change the results in this paper. Furthermore, we restrict this study to the resolvability of subdominant QNMs ($\ell \neq 2$) for two reasons: (1) the excitation amplitudes of overtones on the general parameter space of the binary’s properties are not yet well-understood and we lack predictions to model ringdown signatures that include overtones for a large population of BBH mergers, and (2) the frequencies of the overtones are very similar to each other, hence accurate resolvability of an overtone is more challenging than of a subdominant mode. We use the Bayesian inference and model selection frameworks to establish the measurability and resolvability of two ringdown QNMs, providing rate estimates for constraining the no-hair theorem to within $\pm 20\%$ at the 90% credible level. The ringdown analysis follows the methods developed in [32, 33], expanding on [34].

This manuscript is organized as follows. Section II introduces the Bayesian inference and model selection frameworks, as well as the ringdown model used. Section III describes the details on the BBH population considered. In Section IV we report the rates on measurable subdominant modes and prospects for resolvability of the necessary parameters to perform tests of the no-hair theorem. Finally, we conclude our findings in Sec. V.

II. BAYESIAN FRAMEWORK

We use Bayesian methods to infer the properties of the remnant BH from our data, $d(t)$, and to determine the presence of a measurable subdominant mode in the ringdown signature. Given a model hypothesis of the ringdown signal, H , parametrized by the source properties, $\vec{\vartheta}$, Bayes’ theorem defines the posterior probability

distribution:

$$p(\vec{\vartheta}|d, H) = p(\vec{\vartheta}|H) \frac{p(d|\vec{\vartheta}, H)}{p(d|H)}, \quad (1)$$

where $p(\vec{\vartheta}|H)$ is the prior knowledge based on astrophysical populations or theoretical models, the likelihood $p(d|\vec{\vartheta}, H)$ is the conditional probability of observing the data $d(t)$ given the model H with parameters $\vec{\vartheta}$, and the evidence $p(d|H)$ is a normalization constant that only depends on the data and the chosen model. Calculating the evidence requires marginalization over the entire parameter space, which can become computationally challenging. While this computation can be avoided for Bayesian parameter estimation, model selection between two competing models requires accurate estimates of the evidence.

In Bayesian model selection, the Bayes factor weighs the evidence provided by the data in support of one model versus another [35, 36]:

$$\mathcal{B}_{AB} = \frac{p(d|H_A)}{p(d|H_B)}. \quad (2)$$

The larger \mathcal{B}_{AB} is, the stronger the data supports hypothesis H_A over H_B . Following the nomenclature of [35], a Bayes factor > 3.2 indicates “substantial” support for H_A over H_B ; $\mathcal{B}_{AB} > 10$ indicates “strong” support, while $\mathcal{B}_{AB} > 100$ is “decisive”. A Bayes factor between $1/3.2$ and 3.2 is “not worth mentioning”; i.e., the data is inconclusive as to whether H_A or H_B is favored.

A. The likelihood function

For a GW detector network with uncorrelated stationary Gaussian noise, the likelihood is given by

$$p(d|\vec{\vartheta}, H) \propto \exp \left[-\frac{1}{2} \sum_{a=1}^N \langle d_a - h_a(\vec{\vartheta}), d_a - h_a(\vec{\vartheta}) \rangle \right], \quad (3)$$

where N is the number of detectors, d_a is the data for each detector, and $h_a(\vec{\vartheta})$ is the waveform model evaluated for a set of parameters $\vec{\vartheta}$ as observed by detector a . The noise-weighted inner product is defined as

$$\langle x, y \rangle = 4\Re \int_0^\infty \frac{\tilde{x}^*(f)\tilde{y}(f)}{S_n(f)} df, \quad (4)$$

with $S_n(f)$ being the one-sided power spectral density (PSD) of the detector’s noise, $\tilde{x}(f)$ the Fourier transform of $x(t)$, and $*$ indicating the complex conjugate.

In this paper we use the `PyCBC Inference` [28, 29] toolkit to compute the likelihood function and estimate posterior probability distributions. Accurate marginalization for evidence estimation is achieved using the nested sampling algorithm `cpnest` [37].

B. The ringdown model

The strain $h(t)$ produced by a gravitational wave at the detector is given by

$$h(t) = F_+(\alpha, \delta, \Psi)h_+(t) + F_\times(\alpha, \delta, \Psi)h_\times(t), \quad (5)$$

where $F_{+,\times}$ are the antenna pattern functions determined by the relative orientation between the detector frame and the wave frame [38], i.e. the sky location of the source (right ascension α and declination δ in a geocentric coordinate system) and the polarization angle Ψ that defines the relative orientation of the wave frame with the geocentric coordinate system. For short transient signals, these orientation angles (and hence $F_{+,\times}$) are assumed to be time independent. For future generation of observatories with improved low frequency sensitivity, it might become necessary to account for the time dependence of $F_{+,\times}$. However, the ringdown itself will be short enough that for our purposes we do not need to consider this effect here.

The ringdown signal of a Kerr BH consists of a sum of exponentially damped sinusoids:

$$h_+ + ih_\times = \frac{M}{D_L} \sum_{\ell,m,n} {}_{-2}S_{\ell m}(\iota, \varphi) A_{\ell mn} e^{i(\Omega_{\ell mn} t + \phi_{\ell mn})}, \quad (6)$$

where M is the mass of the BH in the detector frame and D_L is the luminosity distance to the source. The functions ${}_{-2}S_{\ell m}(\iota, \varphi)$ are the spin-weighted spheroidal harmonics, which depend on the inclination angle ι between the BH spin and the line-of-sight from the observer to the source, and the azimuth angle φ between the BH and the observer. The complex QNM frequencies $\Omega_{\ell mn}$, determined from the Teukolsky equation [39, 40], define the frequency and damping time of the damped sinusoid, $\Omega_{\ell mn} = \omega_{\ell mn} + i/\tau_{\ell mn}$. The amplitudes $A_{\ell mn}$ and $\phi_{\ell mn}$ depend on the initial perturbation and take different values for different (ℓ, m, n) modes.

Assuming that the ringdown begins at $t = 0$, the two gravitational-wave polarizations are given by

$$\begin{aligned} h_+(t) &= \frac{M}{D_L} \sum_{\ell,m} {}_{-2}Y_{\ell m}^+(\iota) A_{\ell m} e^{-t/\tau_{\ell m}} \cos(\omega_{\ell m} t + \phi_{\ell m}), \\ h_\times(t) &= \frac{M}{D_L} \sum_{\ell,m} {}_{-2}Y_{\ell m}^\times(\iota) A_{\ell m} e^{-t/\tau_{\ell m}} \sin(\omega_{\ell m} t + \phi_{\ell m}), \end{aligned} \quad (7)$$

where we restrict ourselves to the $n = 0$ overtone and drop the overtone index n for simplicity. Here we have approximated the spheroidal harmonics ${}_{-2}S_{\ell m}$ by spin-weighted spherical harmonics ${}_{-2}Y_{\ell m}$ [13, 41]:

$$\begin{aligned} {}_{-2}Y_{\ell m}^+(\iota) &= {}_{-2}Y_{\ell m}(\iota, 0) + (-1)^\ell {}_{-2}Y_{\ell -m}(\iota, 0), \\ {}_{-2}Y_{\ell m}^\times(\iota) &= {}_{-2}Y_{\ell m}(\iota, 0) - (-1)^\ell {}_{-2}Y_{\ell -m}(\iota, 0). \end{aligned} \quad (8)$$

In this paper we use two different waveform models, a (i) *Kerr model* where we assume the remnant object to be

a Kerr BH, hence the ringdown QNM frequencies $\Omega_{\ell mn}$ are uniquely determined by the mass M and the spin χ of the BH, and an (ii) *agnostic model* where we assume the nature of the remnant object to be unknown, hence the ringdown is parameterized by each individual QNM frequency $\Omega_{\ell mn}$ and we drop the factor M/D_L in Eq. (7). The Kerr model (i) is our starting point for determining the measurability of a subdominant mode. Resolvability of the subdominant mode for testing the no-hair theorem is determined using the agnostic model (ii).

III. POPULATIONS

We construct populations of candidate BBH ringdown signals based on the observational population model B of [27]. In this model, the component-mass and mass-ratio distributions follow power laws with exponents $-\alpha$ and β_q , respectively (see Eq. (2) in [27]). For the component-mass distribution, the measured median value is $\alpha = 1.6$, with masses in the range $[5.4, 57]M_\odot$ (we use the lowest m_{min} and the largest m_{max} values, to account for uncertainties in the mass bounds of BHs). For the mass-ratio distribution we use two different exponent values: the measured median value $\beta_q = 6.7$, and a uniform distribution $\beta_q = 0$ (which is used in model A of [27]). Mass ratios are restricted to be within the range $[1, 8]$. We assume the individual BHs to be non-spinning prior to the merger, which is consistent with the population of BBHs observed by LIGO/Virgo thus far. Sources are distributed uniformly in co-moving volume; we choose a maximum luminosity distance dependent on the considered detector network. The inclination angle ι is distributed uniformly in $\cos \iota \in [-1, 1]$, and the polarization angle ψ uniformly $\in [0, 2\pi)$.

From the initial BBH masses and spins, we obtain an estimate of the remnant's source frame mass $M^{(src)}$ and dimensionless spin χ using the fitting formulae to numerical relativity [42, 43][44]. Assuming that the remnant object is a Kerr BH, the mass and spin determine the ringdown frequencies $\Omega_{\ell m}$ [45]. To obtain the ringdown frequencies as measured in the detector, we calculate a redshift from the luminosity distance by assuming a standard Λ CDM cosmology [46] and use the relation $M = (1+z)M^{(src)}$ between source frame mass $M^{(src)}$ and detector frame mass M . The excitation amplitudes $A_{\ell m}$, which depend on the mass ratio q of the binary, are determined using the fitting formulae in [8] at $t = 10M$ after the merger. The phases $\phi_{\ell m}$ of the modes are distributed uniformly in $\phi_{\ell m} \in [0, 2\pi)$, in contrast to previous work in the literature where both phases were fixed for simplicity [13].

The BBH parameters for each candidate are drawn randomly from their respective distributions to generate two-mode ringdown signals with the dominant $(\ell, m) = (2, 2)$ mode and either the $(\ell, m) = (3, 3)$ or the $(\ell, m) = (4, 4)$ subdominant mode. We consider a three-detector LIGO network consisting of the observatories in Han-

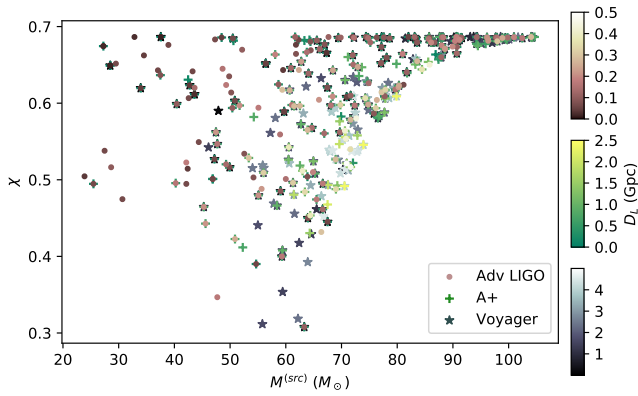


FIG. 1. Source frame mass, $M^{(\text{src})}$, and spin, χ , of the BHs obtained using the observational population models of [27] with optimal SNR $\rho_c \geq 2.5$ in the subdominant mode (either the (3,3) or the (4,4) mode). The colors represent the luminosity distance of the source, where the maximum allowed distance was $D_L = (1, 3, 5)$ Gpc for Adv. LIGO, A+ and Voyager, respectively.

ford (H1), Livingston (L1) and India (I1). We use three different sensitivities for these detectors [47]: Advanced LIGO design sensitivity (Adv. LIGO), A+ and Voyager. We do not consider here the complete third generation detectors, which include the Einstein Telescope [48–50] and Cosmic Explorer [51], or the space based LISA mission [52], since this would take us beyond the 10-year timeframe.

For each candidate, we calculate the optimal SNR of the subdominant mode in each detector, $\rho_{\text{det}} = \sqrt{\langle h, h \rangle}$, where h is the ringdown signal of the subdominant mode projected into the detector (see Eqs. (5) and (7)), and we have used the integral limits $f_{\text{low}} = 20$ Hz and $f_{\text{high}} = 2048$ Hz in Eq. (4). To avoid a large number of sources with no measurable subdominant mode, we reject candidates with combined optimal SNR

$$\rho_c = \sqrt{\sum_{\text{det}} \rho_{\text{det}}^2} < 2.5 \quad (9)$$

in the subdominant mode. For the same reason, the maximum D_L considered is limited to different values for different sensitivities, namely $D_L = (1, 3, 5)$ Gpc for Adv. LIGO, A+ and Voyager, respectively. The number of draws required to find a sample population of 100 signals with $\rho_c \geq 2.5$ in the subdominant mode yields the fraction of interesting candidates out of all BBH signals. Figure 1 shows the resulting populations for each detector network considered.

IV. ANALYSIS AND RESULTS

A. Rates of measurable subdominant modes

We add the population of accepted candidate ringdown signals (shown in Fig. 1) into different Gaussian noise realizations colored with the PSD of the desired detector. To determine the measurability of the subdominant mode, we use the Kerr ringdown model and perform two separate Bayesian parameter estimation analyses using: (H_A) templates with the fundamental (2,2) mode plus the corresponding (ℓ, m) subdominant mode, and (H_B) templates with only the fundamental (2,2) mode. The Bayes factor is then calculated as the ratio of the evidences for model H_A versus model H_B . Those sources with $\mathcal{B}_{AB} > 3.2$ are further analysed to determine the resolvability of the subdominant mode.

The parameters $(M, \chi, A_{\ell m}, \phi_{\ell m}, \iota, \psi)$ are estimated from the data, which represents a set of 8 parameters in the two-mode ringdown H_A , and 6 parameters in the single-mode ringdown H_B . The priors used in the parameter estimation analysis are uniform in all parameters: BH mass $M \in [10, 200]M_\odot$, BH spin $\chi \in [-0.99, 0.99]$, log-amplitude of the fundamental mode $\log_{10}(A_{22}) \in [-4, 4]$, relative subdominant mode amplitude $\hat{A}_{\ell m} = A_{\ell m}/A_{220} \in [0, 0.5]$, ringdown phases $\phi_{\ell m} \in [0, 2\pi]$, polarization angle $\psi \in [0, 2\pi]$, and inclination angle $\cos(\iota) \in [-1, 1]$. We fix the start time of the ringdown, the (ℓ, m) of the subdominant mode, the sky location and the distance to the source to the injected values. While the start time of the ringdown is not uniquely defined in the literature, we do not explore the issue in this paper and assume that this can be determined by other means [24, 33, 53]. Further, we can safely assume that we have some knowledge from the inspiral part of the signal regarding the mass ratio of the binary to determine which is the loudest subdominant mode to look for. Since we are using a network of three detectors, the sky location should be relatively well known from the analysis of the full gravitational-wave signal. Finally, while the distance might not be accurately measured, fixing this parameter to a wrong value will only affect the measurement of the fundamental amplitude A_{22} and not affect our conclusions.

To get the number of events per year with detectable subdominant mode, we multiply the fraction of interesting candidates (ringdown signals with SNR $\rho_c \geq 2.5$ in the subdominant mode) from the simulations by the BBH merger rate given in [27] ($R = 53.2_{-28.8}^{+58.5} \text{ Gpc}^{-3} \text{ yr}^{-1}$) and the co-moving volume up to D_L . Table I lists the rate of events per year with substantial ($\mathcal{B}_{AB} > 3.2$), strong ($\mathcal{B}_{AB} > 10$), and decisive ($\mathcal{B}_{AB} > 100$) support for the presence of a subdominant mode. These rates are the combination of both the (3,3) and the (4,4) modes. While we have made the simplifying assumption that only one subdominant mode will be measurable, some of the considered BBH systems might have two subdominant modes with SNR $\rho_c \geq 2.5$. However, studying the

	$\beta_q = 0$			$\beta_q = 6.7$		
	$\mathcal{B}_{AB} > 3.2$	$\mathcal{B}_{AB} > 10$	$\mathcal{B}_{AB} > 100$	$\mathcal{B}_{AB} > 3.2$	$\mathcal{B}_{AB} > 10$	$\mathcal{B}_{AB} > 100$
Adv. LIGO	$0.036^{+0.039}_{-0.019}$	$0.028^{+0.031}_{-0.015}$	$0.011^{+0.012}_{-0.006}$	$0.008^{+0.009}_{-0.004}$	$0.006^{+0.007}_{-0.003}$	$0.003^{+0.003}_{-0.001}$
A+	$0.46^{+0.51}_{-0.25}$	$0.28^{+0.31}_{-0.15}$	$0.14^{+0.15}_{-0.07}$	$0.08^{+0.09}_{-0.04}$	$0.06^{+0.06}_{-0.03}$	$0.03^{+0.03}_{-0.02}$
Voyager	$2.63^{+2.89}_{-1.42}$	$1.83^{+2.01}_{-0.99}$	$0.89^{+0.97}_{-0.48}$	$0.30^{+0.33}_{-0.16}$	$0.21^{+0.24}_{-0.12}$	$0.11^{+0.12}_{-0.06}$

TABLE I. Rates of BBH ringdown signals per year (yr^{-1}) with a detectable subdominant (3, 3) or (4, 4) mode for a population with uniform mass-ratio distribution ($\beta_q = 0$) and for a population with $\beta_q = 6.7$. The Bayes factors in each column indicate substantial support ($\mathcal{B}_{AB} > 3.2$), strong support ($\mathcal{B}_{AB} > 10$), and decisive support ($\mathcal{B}_{AB} > 100$) for the presence of a second mode.

performance of a three-mode ringdown analysis is beyond the scope of this paper.

B. Resolvable subdominant modes for testing GR

Detecting the presence of a subdominant mode is only the first step towards BH spectroscopy. Resolvability of the $\Omega_{\ell m}$ frequencies is also necessary to establish the Kerr nature of the remnant BH. However, QNMs of rotating BHs in modified theories of gravity have not been calculated [54], and Kerr-like exotic compact objects can have the same or similar QNM spectrum as Kerr BHs [55]. While it might be challenging to disprove all BH alternatives, accurate measurements of the QNM spectrum will be crucial to constrain deviations from a Kerr BH. It has been shown for non-rotating alternative models that GR deviations are more significant in the QNM frequencies than in the damping times [56]. Hence, we focus here on constraining deviations from the subdominant mode's frequency.

We perform the same parameter estimation analysis as in the previous section on events with $\mathcal{B}_{AB} > 3.2$, now using the agnostic model defined in Sec. IIB to estimate the ringdown $\Omega_{\ell m}$ frequencies of the two QNMs. Hence, 10 parameters ($\omega_{\ell m}$, $\tau_{\ell m}$, $A_{\ell m}$, $\phi_{\ell m}$, ι , ψ) are estimated from the data. The priors are uniform in the frequencies $f_{\ell m} = \omega_{\ell m}/2\pi \in [50, 1024]$ Hz and damping times $\tau_{\ell mn} \in [0.45, 30]$ ms, excluding parameters that yield masses and spins outside of the ranges used in the previous section with the Kerr model. The amplitudes of the (ℓ, m) modes have different orders of magnitude than in the Kerr model, because of the missing factor M/D_L when dropping the Kerr assumption. Hence, the prior in log-amplitude of the fundamental mode is now $\log_{10}(A_{22}) \in [-25, -17]$. The prior in the remaining parameters is the same as in the previous section. We apply an additional set of constraints on the subdominant frequency and damping time to be within $\pm 25\%$ of the GR expectation. These constraints are only acceptable as long as the obtained posterior distribution does not rail against the prior range.

Using the fitting formulae in [45], we can compare the mass and spin measurement obtained from the (2, 2) parameters and from the subdominant (ℓ, m) parameters. Furthermore, based on the measurement of the (2, 2)

mode, we can infer the measured deviation on the frequency of the subdominant (ℓ, m) mode, $\delta f_{\ell m}$. In Table II, we report the rates of BBH ringdown signals per year that constrain GR within $\delta f_{\ell m} \pm 20\%$ at the 90% credible level. The results are summarized in Fig. 2.

Network	$\delta f_{\ell m} \leq \pm 20\%$
Adv. LIGO	$0.02^{+0.02}_{-0.01}$
A+	$0.16^{+0.17}_{-0.08}$
Voyager	$0.84^{+0.92}_{-0.46}$

TABLE II. Rates of BBH ringdown signals per year (yr^{-1}) with strong support for the presence of a second mode ($\mathcal{B}_{AB} > 3.2$) where deviations of the GR frequencies are constrained to within $\delta f_{\ell m} \leq \pm 20\%$ at the 90% credible level. We only show the rates for the population with uniform mass-ratio distribution ($\beta_q = 0$), since we know from the previous section that rates for a population with $\beta_q = 6.7$ will be lower.

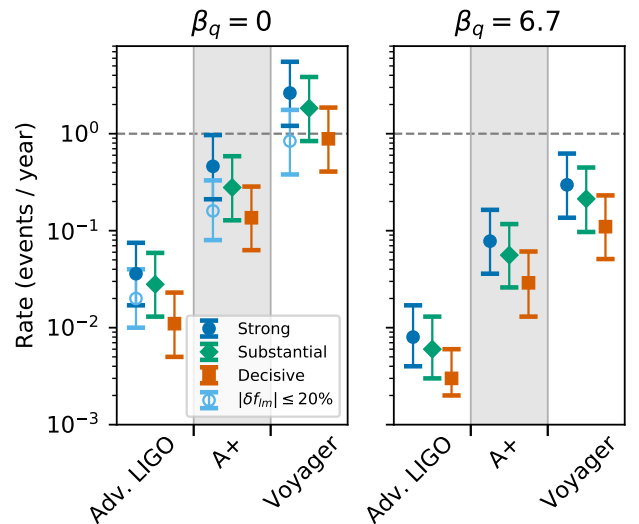


FIG. 2. Expected rates of BBH mergers for which two ringdown modes can be observed and resolved. As before, we consider two population models corresponding to $\beta_q = \{0, 6.7\}$, and different criteria.

V. CONCLUSIONS

In this paper we have applied for the first time the full Bayesian inference framework to a population of BH ringdowns derived from the observational population models published by the LIGO Scientific and Virgo Collaborations. Furthermore, we have allowed for completely variable ringdown phases, inclination angles, polarization angles and sky locations, contrary to previous works that have fixed one or more of these parameters for simplicity.

With the current population models, rates for detection of subdominant QNM modes in the context of Bayesian model selection are discouraging with Advanced LIGO design sensitivity, but promising with future generations of LIGO detectors within the next decade. This result is in agreement with previously published works [14, 15] (note that here we have not considered a population of intermediate mass BHs). However, resolvability of the subdominant frequencies is technically challenging, and accurate tests of the no-hair theorem will only be possible in very few cases.

Observational population models are still largely uncertain. The third observing run of Advanced LIGO and Virgo is uncovering a new population of NSBH, which could boost the rates of measurable and resolvable subdominant modes. Furthermore, current BBH population

models will become more complete with the new BBH observations. Hence, the rates obtained in this work might turn out to be pessimistic as more gravitational-wave detections are made available.

VI. ACKNOWLEDGEMENTS

We are thankful to Swetha Bhagwat, Duncan Brown, Evan Goetz, Scott Hughes, Alexander H. Nitz, Paolo Pani and Frans Pretorius for useful comments and discussions. We are especially grateful to Ssohrab Borhanian and Bangalore Sathyaprakash for providing us with the fits for the ringdown mode amplitudes in [8]. MC acknowledges support from NSF grant PHY-1607449, the Simons Foundation, and the Canadian Institute For Advanced Research (CIFAR). Computations have been performed on the Atlas cluster of the Albert Einstein Institute (Hannover).

VII. APPENDIX

Results for the injection with the largest Bayes factor in the (3, 3) population using the Voyager sensitivity. This BBH is located at a distance $D_L \simeq 250$ Mpc.

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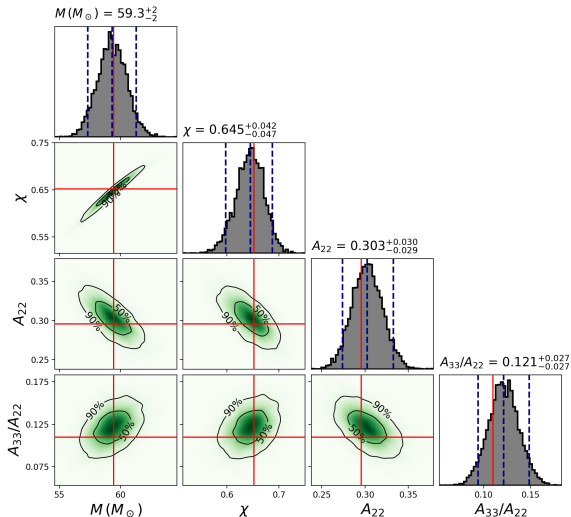


FIG. 3. Posterior distributions from the analysis with a Kerr ringdown. The parameters of interest are the BH mass M , BH spin χ , amplitude of the (2,2) mode, A_{22} , and amplitude ratio of the (3,3) mode, A_{33}/A_{22} . The red crosses indicate the injected parameters, and the dashed lines in the histograms correspond to the median value and the 90% credible level.

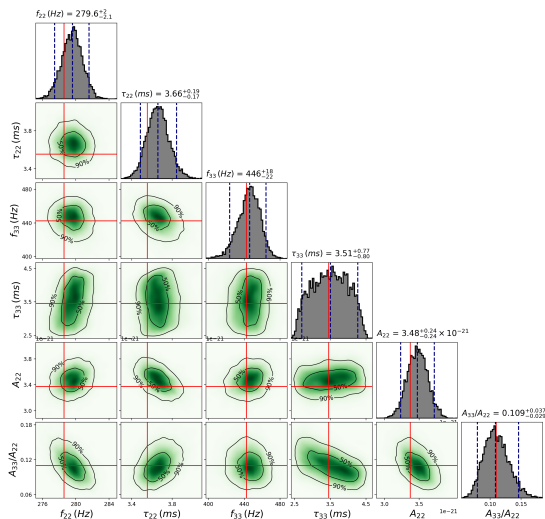


FIG. 4. Posterior distributions from the analysis with an agnostic ringdown. The parameters of interest are the ringdown frequencies, f_{lm} , and damping times, τ_{lm} , the amplitude of the (2,2) mode, A_{22} , and the amplitude ratio of the (3,3) mode, A_{33}/A_{22} . A_{22} has a different value because of the missing M/D_L factor in the approximant. The red crosses indicate the injected parameters, and the dashed lines in the histograms correspond to the median value and the 90% credible level.

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