

Impact of subdominant modes on the interpretation of gravitational-wave signals from heavy binary black hole systems

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Over the past year, a handful of new gravitational wave models have been developed to include multiple harmonic modes thereby enabling for the first time fully Bayesian inference studies including higher modes to be performed. Using one recently-developed numerical relativity surrogate model, NRHybSur3dq8, we investigate the importance of higher modes on parameter inference of coalescing massive binary black holes. We focus on examples relevant to the current three-detector network of observatories, with detector-frame masses and signal amplitudes consistent with plausible candidates for the next few observing runs. We show that for such systems the higher mode content will be important for interpreting coalescing binary black holes, reducing systematic bias, and computing properties of the remnant object. For asymmetric binaries with mass ratios $q > 1$, higher modes are critical, and their omission usually produces substantial parameter biases. Even for comparable-mass binaries and at low signal amplitude, the omission of higher modes can influence posterior probability distributions. We discuss the impact of our results on source population inference and self-consistency tests of general relativity.

I. INTRODUCTION

During their first and second observing runs, the Advanced LIGO [1] and Virgo [2] ground-based gravitational wave (GW) detectors have identified several coalescing compact binaries [3–9]. GW detectors are exceptionally sensitive to very massive objects [10], and the majority of compact binaries observed to date are pairs of $\mathcal{O}(30M_{\odot})$ binary black hole (BBH) systems [9]. The early analysis of these signals used phenomenological approximations to general relativity [11–13]. More recently, better approximations to general relativity have been developed [14–18], which include more of the available physics such as higher-harmonic modes.

Previous investigations have demonstrated that neglecting some of the physics present in real signals produces biased inferences for compact binaries; conversely, including full physics enables sharper inferences. For instance, studies [19–27] have shown that the non-quadrupole modes, while being subdominant, can play a nonnegligible role in detection and parameter estimation, particularly for high signal-to-noise-ratio (SNR), large total mass, high mass ratio, or systems favoring an edge-on orientation. In addition, nonquadrupole modes can help break the degeneracy between the binary inclination and distance, which is present for quadrupole-mode-only models (see e.g. [14, 28–30]).

For the first set of gravitational-wave observations, the massive binary black holes which dominate current observations produce short signals of modest SNRs. For

the first event, GW150914 [3], where detailed followups were done, the systematic errors due to the quadrupole-mode-only approximation is generally smaller than the statistical errors [31, 32], although higher modes may lead to modest changes in some of the extrinsic parameter values [30]. A recent study [31] considering GW150914-like events of near-equal mass and modest amplitude has concluded that neglecting sub-dominant waveform modes did not lead to bias and quadrupole-only models will suffice to characterize the observationally-accessible parameters of astrophysical binary black holes in the immediate future. However, as pointed out in Ref [31], at the time of that study there were no recovery models including higher modes and the systems considered were $q \approx 1.2$ and a detector-frame total mass of 74. Recently, Chatzioannou et al. [33] have reanalyzed GW170729, using IMRPhenomHM [14], SEOBNRv4HM [15], and NR-Sur7dq2 [18] and found that despite weak evidence for higher-order harmonic modes their inclusion in the analysis leads to increased support for unequal masses.

With newly developed multi-mode models it is now possible to revisit these questions. We can now compute, for example, the true posteriors using recovery models with multiple harmonic modes that can then be compared to posteriors recovered with dominant modes only. Such comparisons will allow us to precisely quantify the information gained by using subdominant modes. For example, even for an equal-mass system, we observe that the posterior produced without subdominant modes will experience a noticeable shift towards (incorrectly) favor-

ing lighter binary systems with more negative χ_{eff} values (cf. Figures 1 and 4). In fact this preferential bias appears to be a common feature across many of the cases we have considered.

In this paper, we use concrete examples of end-to-end parameter inference to quantify how much approximations that neglect subdominant modes can impact the interpretation of gravitational-wave events. Unlike previous studies, which typically used either a single detector, low signal-to-noise ratios (SNRs), or a Fisher matrix analysis, our fully Bayesian study uses a three-detector network with SNRs typical of detections expected in the near future. We demonstrate these inference biases occur even at moderate signal amplitude for some configurations, growing extreme at amplitudes expected for some sources when LIGO reaches design sensitivity [34].

We also explore additional physics that can be extracted with non-quadropoles modes using a spin-aligned model, such as improved measurability of individual spin components, final mass and spin properties of the remnant, black hole kicks [35], source population inference, and self-consistency tests of general relativity. For example, in the context of non-spinning BBH systems, Ref. [36] has demonstrated that when higher-modes are omitted from the recovery model, its effect can mimic deviations from General Relativity.

Our examples target sources with detector-frame masses $M_z \simeq 120M_\odot$, comparable to the detector-frame masses expected for typical near-future binary black hole observations (e.g., pairs of $35M_\odot$ BHs at moderate redshift). For comparison, as ground-based detector networks approach design sensitivity and regularly detect sources near $z \simeq 1$, a merging pair of BHs near the pair-instability mass-gap ($50M_\odot$) observed at $z \simeq 1$ would have a detector-frame mass of $M_z \simeq 200M_\odot$ [37]. We also consider target sources with mass ratios in the range $1 \leq q \leq 7$. To date all LIGO/Virgo events show support only for systems with mass ratios less than 2 [9]. Nevertheless, one should expect to observe larger mass ratio systems in the future. For example, unequal mass systems are generically expected for BBH mergers within the accretion disks of active galactic nuclei [38]. Furthermore, the first and second observing runs [9] have already observed compact objects over a mass range of $1.3M_\odot$ to $85M_\odot$ suggesting combinations involving mass-ratios as large as 7 are not unreasonable for LIGO/Virgo to observe.

This paper is organized as follows. In Section II we introduce the GW signal model and parameter inference techniques used in this work. In Section III we survey the results of parameter inference on a sequence of synthetic high-mass binary black holes with systematically-varied mass ratio, spin, and signal amplitude. We specifically address how higher modes impact inference, comparing parameter inferences performed with the full NRHyb-Sur3dq8 model and with a model truncated to include only $\ell = 2$ modes. In Section IV we discuss some consequences of our analysis. We conclude in Section V with

some brief remarks and future work.

II. PRELIMINARIES

A. Gravitational Wave Model

A coalescing compact binary in a quasicircular orbit can be completely characterized by eight intrinsic parameters, namely the individual masses, m_i , and spin vectors, \mathbf{S}_i , of each compact object. Gravitational waveform models and inference codes often employ parameterizations involving the system's total mass, $M = m_1 + m_2$, the mass ratio,

$$q = m_1/m_2, \quad (1)$$

where $m_1 \geq m_2$, the dimensionless spins,

$$\chi_i = \mathbf{S}_i/m_i^2, \quad (2)$$

on the individual black holes (BHs), and an effective spin parameter [39–41],

$$\chi_{\text{eff}} = (\mathbf{S}_1/m_1 + \mathbf{S}_2/m_2) \cdot \hat{\mathbf{L}}/M, \quad (3)$$

which is a weighted combination of the spins projected along the normalized orbital angular momentum vector $\hat{\mathbf{L}}$. We will express the dimensionless spins in terms of Cartesian components $\chi_{i,x}, \chi_{i,y}, \chi_{i,z}$, expressed relative to the source frame. We define this frame such that the z -axis is along the orbital angular momentum direction, which is constant for nonprecessing BBH systems. Since our focus is on the impact of higher-harmonic modes, we restrict ourselves to the 4-dimensional space of nonprecessing BBHs where non-quadropole, inspiral-merger-ringdown (IMR) models are more well developed. Such systems are characterized by $\chi_{i,x} = \chi_{i,y} = 0$ and $|\chi_{1z}|, |\chi_{2z}| \leq 1$.

When discussing waveform models, it is common practice to introduce a complex gravitational-wave strain

$$\begin{aligned} h_+(t; t_c, \iota, \phi_c, \vec{\lambda}) - ih_\times(t; t_c, \iota, \phi_c, \vec{\lambda}) \\ = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} h^{\ell m}(t - t_c; \vec{\lambda}) {}_{-2}Y_{\ell m}(\iota, \phi_c), \end{aligned} \quad (4)$$

which is subsequently decomposed into a basis of spin-weighted spherical harmonics ${}_{-2}Y_{\ell m}$. Here $\vec{\lambda} \equiv (q, M, \chi_{1z}, \chi_{2z})$ is used to denote the signal's dependence on the intrinsic parameters, ι is the inclination angle between the orbital angular momentum of the binary and line-of-sight to the detector, t_c is the coalescence time, and ϕ_c is the orbital phase at coalescence. Most gravitational waveform models make predictions for the modes $h^{\ell m}(t)$, from which the gravitational-wave strain detected

by a ground-based interferometer,

$$h(t; \vec{\Lambda}) = \frac{1}{r} F_+ (\text{ra}, \text{dec}, \psi) h_+(t; t_c, \iota, \phi_c, \vec{\Lambda}) + \frac{1}{r} F_\times (\text{ra}, \text{dec}, \psi) h_\times(t; t_c, \iota, \phi_c, \vec{\Lambda}), \quad (5)$$

is readily assembled. The signal's dependence on four additional extrinsic parameters are the polarization angle (ψ), the luminosity distance to the source's center-of-mass (r), and sky location determined by the right ascension (ra) and declination (dec). The antenna patterns $F_{(+,\times)}$ project the GW's $+$ - and \times -polarization states, $h_{(+,\times)}$, into the detector's frame. We shall use $\vec{\Lambda} \equiv (\text{ra}, \text{dec}, \psi, r, t_c, \iota, \phi_c, \vec{\Lambda})$ to denote the signal's dependence on all 11 parameters defining the problem.

Until recently, all spinning IMR models had set $h^{\ell m} = 0$ except for the dominant $h^{2,\pm 2}$ quadrupole modes. The expectation had been that higher modes won't substantially affect parameter inference for the O2 gravitational-wave observations, which are characterized by low SNRs and mostly face-on events of near-equal mass [22, 25, 31].

Over the past year or so, three new aligned-spin IMR models have been built to include non-quadrupole modes: (i) a phenomenological frequency-domain model, IMRPhenomHM [14], includes the $(\ell, |m|) = (2, 2), (3, 3), (4, 4), (2, 1), (3, 2), (4, 3)$ modes; (ii) an effective-one-body time-domain model, SEOB-NRv4HM [15], includes a similar set of $(\ell, |m|) = (2, 2), (3, 3), (4, 4), (5, 5), (2, 1)$ modes; (iii) a time-domain surrogate model for hybridized nonprecessing numerical relativity waveforms, NRHybSur3dq8 [16], includes all of the $\ell \leq 4$ and $(5, 5)$ spin-weighted spherical harmonic modes but not the $(4, 1)$ or $(4, 0)$ modes.

Our study will use NRHybSur3dq8 as it both includes the most modes and is expected to be more accurate when evaluated within its training region (cf. Fig 6 from Ref. [16]) of mass ratio $q \leq 8$, and $|\chi_{1z}|, |\chi_{2z}| \leq 0.8$. For the 20 Hz starting frequency considered here, this model is valid for the entire LIGO band for stellar mass binaries with total masses as low as $2.25 M_\odot$. We evaluate the model through the Python package GWSurrogate¹ [43, 44]. The GWSurrogate package provides direct access to the GW's harmonic modes $h^{\ell m}(t)$ appearing in the sum (4).

By comparing to NR, Ref. [16] has computed the NRHybSur3dq8 model's mismatches (averaged over many points on the sky) as a function of total mass using the Advanced LIGO design sensitivity noise curve. For the $120 M_\odot$ total mass systems predominantly used in our studies, the single-detector mismatches have a median value of 1×10^{-5} . A sufficient condition for two waveform models (in this case NR and NRHybSur3dq8)

to be considered indistinguishable is [31, 45–47]

$$\mathcal{M} < \frac{D}{2\rho^2}, \quad (6)$$

where \mathcal{M} is the mismatch and ρ is the signal-to-noise ratio (SNR). Here D is an unknown constant that is sometimes associated with number of model parameters [48], with $D = 4$ for our spinning BBH model. Using this value for D and a typical mismatch value quoted above, we find that the NRHybSur3dq8 model will give robust parameter estimates so long as $\rho \gtrsim 890$. Even using pessimistic values ($D = 1$ and the 95th percentile of mismatch errors 7×10^{-5}) we find that NR and our model will be indistinguishable according to Eq. (6) so long as $\rho \gtrsim 85$.

For context, we note that in the first and second observing runs most BBH signals had a network SNR of about 15 and spanning a range of 10 to 30. In the upcoming observing run we would expect typical BBH SNRs to be between 10 and roughly 40, based on the cumulative distribution of the loudest SNR ρ among n identified events ($[1 - (\rho/10)^3]^n$ using a fiducial value $n = 30$). We caution the reader that in practice the condition in Eq. (6) should only be taken as a rough estimate. For instance, it features an unknown constant D while the NR waveforms themselves have small, systematic sources of error that would prevent any model to claim indistinguishability from general relativity beyond estimates of this systematic error [49]. Finally, the definition of “indistinguishable” is not synonymous with “identical posterior distributions”. Indeed, Fig. 4 shows that even for simple systems at low SNR, which easily satisfy Eq. (6), there can be noticeable discrepancies between the recovered posteriors.

Due to the absence of higher-mode models for spinning BBH systems until recently, previous parameter-inference studies that have focused on the information content available higher modes have either used quadrupole-only (recovery) models or leveraged the Fisher matrix framework. For high-accuracy, high-SNR scenarios involving the 3-detector network neither of these are fully sufficient. For example, with the quadrupole-only model the reference (“true”) posterior will not be possible to compute in principle. Additionally, some of these models may have modeling errors in the dominant mode that could become noticeable at high SNR [50–52].

B. Bayesian Inference

The likelihood of GW data in Gaussian noise has the form (up to normalization),

$$\ln \mathcal{L}(\boldsymbol{\lambda}, \theta) = -\frac{1}{2} \sum_k \langle h_k(\boldsymbol{\lambda}, \theta) - d_k | h_k(\boldsymbol{\lambda}, \theta) - d_k \rangle_k - \langle d_k | d_k \rangle_k, \quad (7)$$

¹ We use GWSurrogate version 0.9.{4,5}, which exactly agrees with the lalsimulation [42] implementation of the NRHybSur3dq8 model.

where h_k are the predicted response of the k^{th} detector due to a source with parameters $(\boldsymbol{\lambda}, \theta)$ and d_k are the detector data in the k^{th} instrument; $\boldsymbol{\lambda}$ denotes the combination of redshifted total mass M_z and the remaining intrinsic parameters needed to uniquely specify the binary’s dynamics; θ represents the seven extrinsic parameters (4 spacetime coordinates for the coalescence event and 3 Euler angles for the binary’s orientation relative to the Earth); and $\langle a|b \rangle_k \equiv \int_{-\infty}^{\infty} 2df \tilde{a}(f)^* \tilde{b}(f) / S_{h,k}(|f|)$ is an inner product implied by the k^{th} detector’s noise power spectral density (PSD) $S_{h,k}(f)$. In practice we adopt both low- and high- frequency cutoffs $f_{\text{max}}, f_{\text{min}}$ so all inner products are modified to

$$\langle a|b \rangle_k \equiv 2 \int_{|f|>f_{\text{min}}, |f|<f_{\text{max}}} df \frac{[\tilde{a}(f)]^* \tilde{b}(f)}{S_{h,k}(|f|)}. \quad (8)$$

The joint posterior probability of $\boldsymbol{\lambda}, \theta$ follows from Bayes’ theorem:

$$p_{\text{post}}(\boldsymbol{\lambda}, \theta) = \frac{\mathcal{L}(\boldsymbol{\lambda}, \theta) p(\theta) p(\boldsymbol{\lambda})}{\int d\boldsymbol{\lambda} d\theta \mathcal{L}(\boldsymbol{\lambda}, \theta) p(\boldsymbol{\lambda}) p(\theta)}, \quad (9)$$

where $p(\theta)$ and $p(\boldsymbol{\lambda})$ are priors on the (independent) variables $\theta, \boldsymbol{\lambda}$. Following most previous work [9, 53, 54], we adopt uninformed separable priors for parameter inference.

C. RIFT

To construct the posterior distribution, we use the RIFT algorithm [54], which iteratively constructs and refines an approximation to the marginal likelihood

$$\mathcal{L}_{\text{marg}} \equiv \int \mathcal{L}(\boldsymbol{\lambda}, \theta) p(\theta) d\theta, \quad (10)$$

which appears in Bayes’ theorem for the marginal posterior distribution for $\boldsymbol{\lambda}$. We use an existing program (ILE, which Integrates the Likelihood over Extrinsic parameters) to perform the necessary marginalization, for each fixed source [32, 55–57], by marginalizing the likelihood of the data over the seven parameters characterizing the spacetime coordinates and orientation of the binary relative to the earth; see [55, 58] paper for technical details.

To achieve rapid turnaround times, we use the new GPU-accelerated implementation of ILE [58]. Working on the CARNiE cluster, which includes 15 NVIDIA Tesla V100 GPU-enabled nodes, our current configuration completes each of the binary black hole analyses presented in this work in about 15 to 20 hours. When using all 15 GPUs, a single ILE step for an SNR=30 case takes about 1 hour to finish.

Following the RIFT algorithm [54], we iteratively construct an approximation to the likelihood by generating and drawing from approximate posterior distributions, until our posterior distribution converges. At each

iteration, the likelihood is approximated using Gaussian process regression with a squared-exponential kernel, with hyperparameters tuned to the likelihood evaluations available at that iteration.

III. INTRINSIC-PARAMETER BIASES

In this section, we present parameter estimation (PE) results from sources listed in Table I. All synthetic datasets use PSDs generated from data near GW170814 [6], when all three detectors were operational, and are created with zero noise realizations. Specifically the synthetic detector data is exactly equal to the expected response due to our GW source. Since detector noise is assumed to be colored Gaussian noise with zero mean, using zero noise with the likelihood defined in Eq. (7) makes our analysis equivalent to an average over an ensemble of analyses which use infinitely many noise realizations [31].

Each synthetic dataset includes an injected signal from the expected response at each detector due to our GW source using the NRHybSur3dq8 model and including all of the surrogate’s available $\ell_{\text{max}} = 5$ modes (see Sec. II A for the exact modes, which, for example, only includes (5, 5) among the $\ell = 5$ modes). The model generates a waveform such that the instantaneous initial frequency of the (2, 2) mode has a frequency of 8 Hz, which ensures the (5, 5) mode’s instantaneous initial frequency is out-of-band. We taper the beginning and end portions of the waveform to avoid artificial oscillations in the Fourier domain. In particular, since NR waveforms (and therefore the NRHybSur3dq8 model) do not go to zero by the end of the simulation, we have found it necessary to taper the last portion of the ringdown signal.

We adopt conventional mass and distance priors, uniform in detector-frame mass and in the cube of the luminosity distance. For our nonprecessing spins, we adopt a uniform prior for $\chi_{i,z} \in [-0.9, 0.9]$. Sec. IV A considers the effect of using an alternative spin prior in the context of high SNR events.

Each of the following subsections describe a set of related runs, keeping all of the problem’s parameters fixed while varying one of them. For each source configuration, we present parameter estimates recovered using all of the available higher modes $\ell_{\text{max}} = 5$ (we may sometimes refer to this as the “true” or reference posterior) and compare with posteriors recovered using the same model restricted to only the $\ell_{\text{max}} = 2$ modes (using $|m| = \{2, 1\}$). In subsections III A ($q = 1$), III B ($q = 4$), and III C ($q = 7$) we vary the spin configurations of $\chi_{1z} = \chi_{2z} = \{-0.8, -0.5, 0.0, 0.5, 0.8\}$ while keeping the network SNR fixed at 30². For this sequence of runs, our choice of inclination angle, $\iota = 3\pi/4$,

² Given a fixed starting frequency, systems with their BH component spins (anti-)aligned with the orbital angular momentum

is neither face-on nor edge-on, but rather constitutes a “general” configuration. As such, we expect our observed biases to be larger (smaller) when compared to an edge-on (face-on) system at the same network SNR value. In subsection III D we consider varying the SNR to explore its effect on marginalized posterior distributions.

A. $q=1$

We first look at a set of equal mass runs with the different spin configurations mentioned above. It is well known that the relative power of subdominant harmonic modes are minimized for equal mass BBH systems, so these cases are expected to minimize bias. Previous studies [19, 22, 25, 31] have either found negligible bias (for face-on systems), small bias (for edge-on systems), or quoted results averaged over the source orientation where again only very small biases were found. At the time of these studies, however, there were no recovery models for spinning BBH systems including subdominant modes so these results were only suggestive. Here we confirm the general expectation of smaller bias at $q = 1$, while also making more precise the nature of the bias by comparing the true posterior to the approximate one found with $\ell_{\max} = 2$ modes only. For example, in all cases the true posterior’s peak is located at $q = 1$, while some of the biased posteriors have a non-negligible offset often peaking closer to $q \sim 1.25$. From Fig. 1 we also observe noticeable shifts in the posteriors 90% confidence region for anti-aligned configurations.

Figure 1 shows the posterior distributions of the intrinsic parameters for all the different spin configurations. The solid lines represent runs that were done with $\ell_{\max} = 2$ modes, and the dashed lines represent runs that include all available $\ell_{\max} = 5$. For each run, there is some degree of difference between the $\ell_{\max} = 2$ and $\ell_{\max} = 5$ runs. As anticipated by Ref. [19], which used a non-Bayesian approach and a single detector, this discrepancy between the two distributions become more extreme as the spins increase toward negative spin. For example, for negative spins there are noticeable shifts in the M vs χ_{eff} posteriors. We emphasize that even for the simplest case (equal mass and zero spin), differences between the two results are visible. Although parameter recovery is not biased in the sense that all of the injection values lie within their 90% confidence regions, it is also clear from the figure that the median recovered using all subdominant modes is almost always closer to the injection value. This is contrary to the general expectation that subdominant modes are largely irrelevant for equal-mass systems [22, 25, 31]. Section III D explores

how different SNRs affect the bias for these systems; Appendix A follows up on the curious differences seen in the simplest case of zero spin, equal mass.

B. $q=4$

We next increase our set of sources to $q = 4$. Similar to the $q = 1$ case, as far as we are aware, the existing literature for parameter estimation is comprised of results for non-spinning recovery models [22], results for near-equal mass without multi-mode recovery models [31], or Fisher matrix-based studies [19, 25]. None of those studies consider the 3-detector network configuration and a multi-modal recovery model with fully Bayesian inference. At larger mass ratios, our study confirms the general expectations described in Ref. [19], although the observed bias is often even larger than expected; compare to the typical errors indicated by corresponding green, red, and blue curves in Figure 6 of Ref. [19] for our fiducial mass. We also are able to more carefully quantify the nature of the bias by comparing to the true posteriors. In particular, similar to the $q = 1$ systems just considered, neglecting subdominant modes consistently shifts the posterior towards more extreme anti-aligned spin configurations with lighter total mass.

Figure 2 shows the posterior distributions for χ_{eff} vs q and χ_{eff} vs M for all the different spin configurations. The solid lines again represent runs that were done with $\ell_{\max} = 2$ modes, and the dashed lines represent runs that include all available $\ell_{\max} = 5$. Similar to Section III A, we again see that the differences become more extreme as the spins increase toward negative spin. Comparing the same spin configurations between $q = 1$ and $q = 4$ runs, it is clear that increasing the mass ratio dramatically increases the bias between the non-HM and HM runs. In particular, there are now many cases where parameter estimates recovered with $\ell_{\max} = 2$ modes do not lie within their 90% confidence regions. Looking at the two-dimensional posteriors, for example, shows many cases where either the $\ell_{\max} = 2$ posterior either does not contain the injection value or it is noticeably shifted from the true posterior. One notable exception is $\chi_{\text{eff}} = -0.8$, where the true parameters seem to just about lie in just inside 90% confidence region of the marginalized posteriors, whereas recovery for other cases is almost perfect. We suspect this is due to a combination of (i) the injection being very close to the boundaries of the prior and (ii) the posterior for a $\chi_{\text{eff}} = -0.8$ injection is much wider than the corresponding $\chi_{\text{eff}} = 0.8$ value, which does not show this unexpected behavior.

C. $q=7$

Finally, we analyze sources with $q = 7$. Figure 3 shows the posterior distributions of the intrinsic parameters for all the different spin configurations. The solid lines again

will be (shorter) longer. As a result, to achieve a fixed SNR the spin (anti-)aligned systems must be placed (closer) farther as compared to a reference non-spinning system.

ID#	ι	q	$M (M_\odot)$	χ_{1z}	χ_{2z}	SNR
1	$\pi/4$	2.267	127.1	0.72	0.0	30
2	$3\pi/4$	1.00	120.0	-0.80	-0.80	30
3	$3\pi/4$	1.00	120.0	-0.50	-0.50	30
4	$3\pi/4$	1.00	120.0	0.0	0.0	10,30,70
5	$3\pi/4$	1.00	120.0	0.50	0.50	30
6	$3\pi/4$	1.00	120.0	0.80	0.80	30
7	$3\pi/4$	4.00	120.0	-0.8	-0.8	30
8	$3\pi/4$	4.00	120.0	-0.5	-0.5	10,30,70
9	$3\pi/4$	4.00	120.0	0.0	0.0	30
10	$3\pi/4$	4.00	120.0	0.5	0.5	30
11	$3\pi/4$	4.00	120.0	0.8	0.8	30
12	$3\pi/4$	7.00	120.0	-0.8	-0.8	30
13	$3\pi/4$	7.00	120.0	-0.5	-0.5	30
14	$3\pi/4$	7.00	120.0	0.0	0.0	30
15	$3\pi/4$	7.00	120.0	0.5	0.5	30
16	$3\pi/4$	7.00	120.0	0.8	0.8	30

TABLE I. **Parameters of synthetic sources:** This table shows the parameters of all the synthetic sources used in this paper. ι is the inclination angle between the line of sight of the observer and the total angular momentum vector, q is the mass ratio defined with $q > 1$ (see Eq. 1), M is the detector-frame total mass, and χ_* are the components of the normalized spins (see Eq. 2). As we use a non-precessing model, we set all of the in-plane spin components to 0. All luminosity distances are set such that the network signal-to-noise ratio achieves the value specified under the SNR column. Other extrinsic parameters are fixed to the following values: right ascension is RA=0.0, declination is DEC=1.5707963, and the polarization angle is $\psi = \pi/4$.

represent runs that were done with $\ell_{\max} = 2$ modes, and the dashed lines represent runs that include all available $\ell_{\max} = 5$. As expected and consistent with the trend seen in the previous two subsections, we see substantial biases are often introduced in M , q and χ_{eff} if higher modes are omitted, especially for systems with large negative spin. Only the higher-mode model is able to make reliable parameter estimates, except for the large, positive spin configurations where a quadrupole-only model continues to do reasonably well. In some cases the the biased posterior doesn't even overlap with the true one, which would be problematic for likelihood-reweighting techniques [59], which require similar posterior distributions.

Somewhat unexpectedly, however, is that the $\chi_1 = \chi_2 = 0.8$ system's posterior shows almost no effect from neglecting subdominant modes; any effect that is present is smaller than the corresponding equal-mass system with $\chi_1 = \chi_2 = -0.8$. We believe this can be explained by the orbital hangup effect [60], whereby given two otherwise identical systems the one with larger aligned spin will experience more orbits before merger. Consequently, the $\chi_1 = \chi_2 = 0.8$ configuration will have more in-band cycles, and subdominant modes are known to be suppressed during the inspiral phase.

D. Effect of SNR on Biases

In the previous subsections, it was shown that a significant bias exists at SNR=30, even for the simplest systems. This subsection is dedicated to investigating how the SNR affects the bias. Here we use all the different SNR runs from ID4 and ID8 in Table I. Figures 4 and 5 show the posterior distributions for ID4 and ID8 respectively. As the SNR increases, the posteriors become more precise for both the non-HM and HM results. However, the HM results converge on the true parameters while the non-HM results converge to a point offset from the true parameter. As GW detectors get more sensitive, the need for HM will become paramount even for the simplest of events.

One particularly challenging configuration was the loudest $q = 4$, $\chi_{\text{eff}} = -0.5$ system shown in Fig. 5 (solid blue). In particular, the posterior recovered with the $\ell_{\max} = 2$ model shows evidence for a secondary peak widely separated from the primary one. We checked this unexpected feature by directly comparing the values of the likelihood in a small neighborhood around both peaks. The presence of these two widely-separated peaks proved to be challenging for the current implementation of the ILE/RIFT algorithm, which uses a single interpolant of the log-likelihood surface. As a result, running this case took a significantly longer time while also achieving a comparatively lower accuracy, where the accuracy is quantified by the effective number of

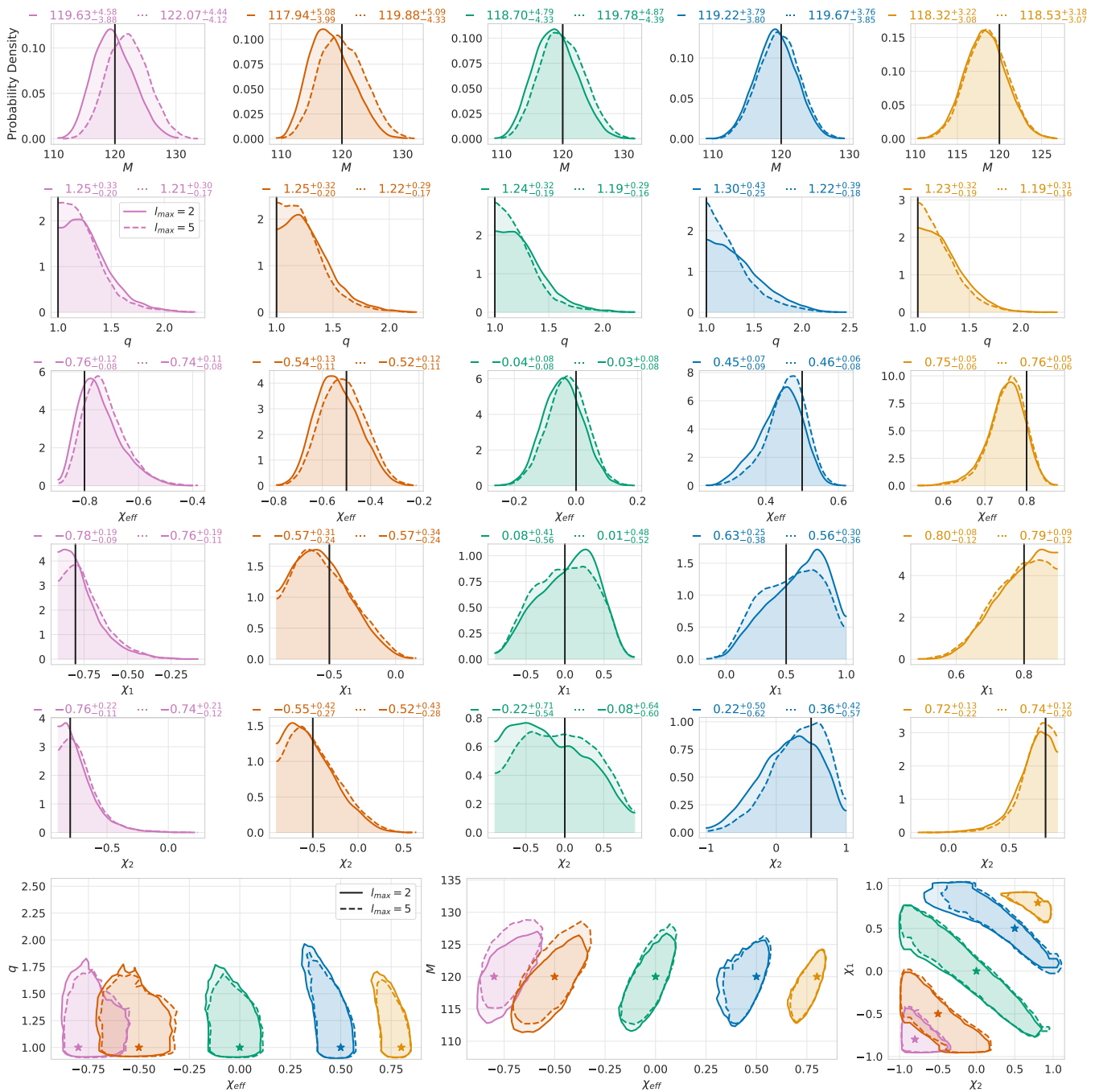


FIG. 1. **Non-HM and HM runs for $q=1$ spin set, with $\text{SNR}=30$ and $M=120$:** The first five rows show the M , q , χ_{eff} , χ_{1z} , χ_{2z} one-dimensional marginal distributions, where among this set of figures each column corresponds to a different synthetic source recovered with either all $l_{\text{max}} = 5$ modes (dashed line) or $l_{\text{max}} = 2$ modes (solid line). Our figures are organized such that the injected spin is systematically increased from left to right, where the synthetic source runs are ID2 ($\chi_{\text{eff}} = -.8$), ID3 ($\chi_{\text{eff}} = -.5$), ID4 ($\chi_{\text{eff}} = 0$), ID5 ($\chi_{\text{eff}} = .5$), and ID6 ($\chi_{\text{eff}} = .8$). A solid black vertical line denotes the true parameter value. The final bottom row corresponds to the joint distributions for q vs χ_{eff} , M vs χ_{eff} , and χ_{1z} vs χ_{2z} for all five injections.

adaptive Monte Carlo samples. This case underscores that for high SNR events the omission of subdominant modes can introduce highly complex likelihood surfaces, and prove challenging to explore accurately. Within the RIFT framework, a recently implemented Gaussian Mix-

ture Model sampler is expected to more efficiently sample from complicated likelihood surfaces. This case also demonstrates how incorrect models can accidentally yield good recovery of some parameters: the marginalized posterior for χ_2 (solid blue curve) looks remarkably accurate

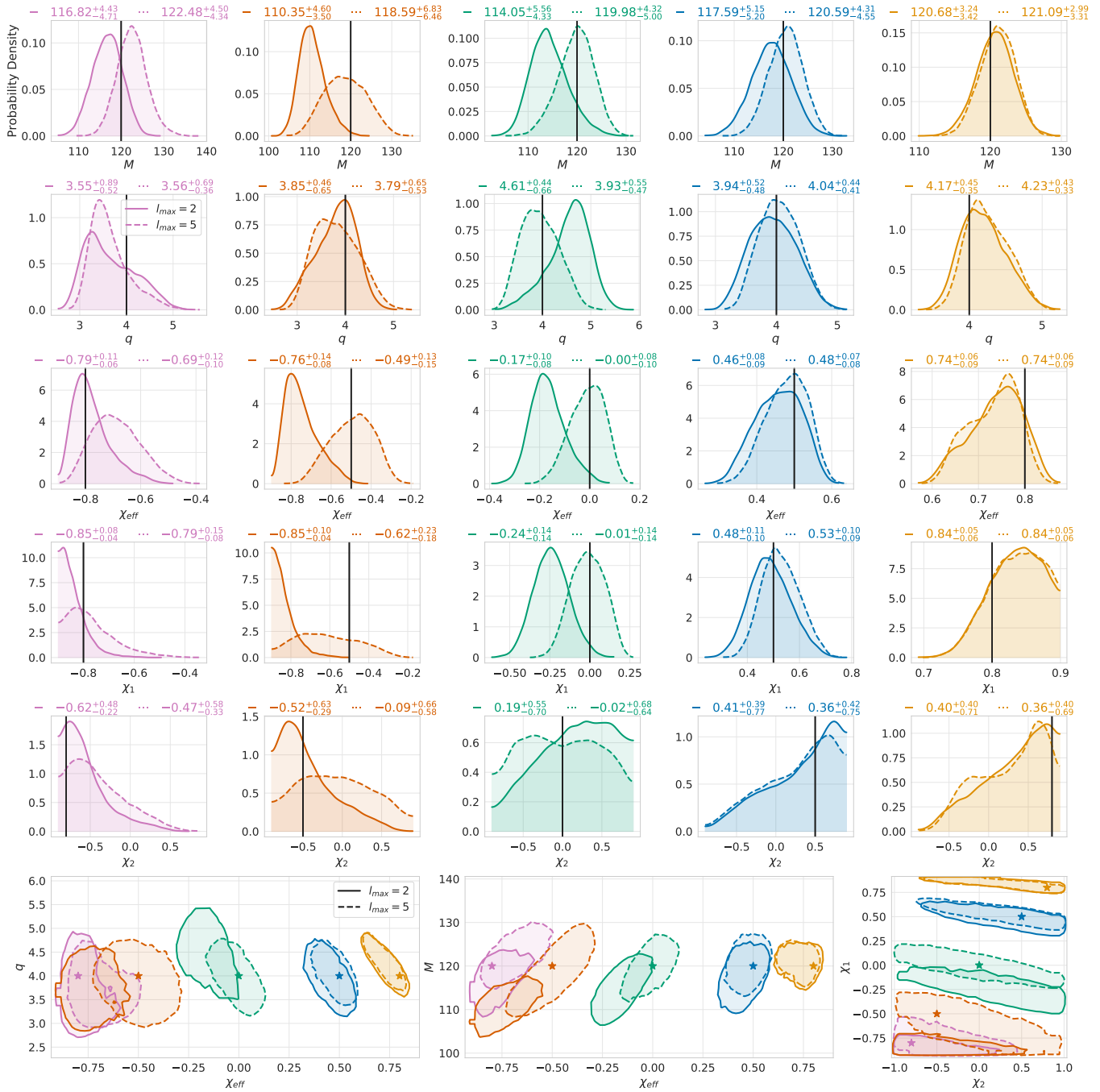


FIG. 2. **Non-HM and HM runs for $q=4$ spin set, with $\text{SNR}=30$ and $M=120$:** The first five rows show the M , q , χ_{eff} , χ_{1z} , χ_{2z} one-dimensional marginal distributions, where among this set of figures each column corresponds to a different synthetic source recovered with either all $l_{\text{max}} = 5$ modes (dashed line) or $l_{\text{max}} = 2$ modes (solid line). Our figures are organized such that the injected spin is systematically increased from left to right, where the synthetic source runs are ID7 ($\chi_{\text{eff}} = -.8$), ID8 ($\chi_{\text{eff}} = -.5$), ID9 ($\chi_{\text{eff}} = 0$), ID10 ($\chi_{\text{eff}} = .5$), and ID11 ($\chi_{\text{eff}} = .8$). A solid black vertical line denotes the true parameter value. The final bottom row corresponds to the joint distributions for q vs χ_{eff} , M vs χ_{eff} , and $\chi_{1,z}$ vs $\chi_{2,z}$ for all five injections.

around the primary peak despite the joint posterior (bottom right panel) being nowhere near the true value.

To quantify the bias between the non-HM and HM runs, we consider two commonly used measures of bias: (i) classifying the recovery of a particular parameter as

biased if the injected parameter value is outside of the 90% confidence region and (ii) the Jensen-Shannon divergence (JSD) between the different parameter distributions. Given two probability distributions $p(x)$ and

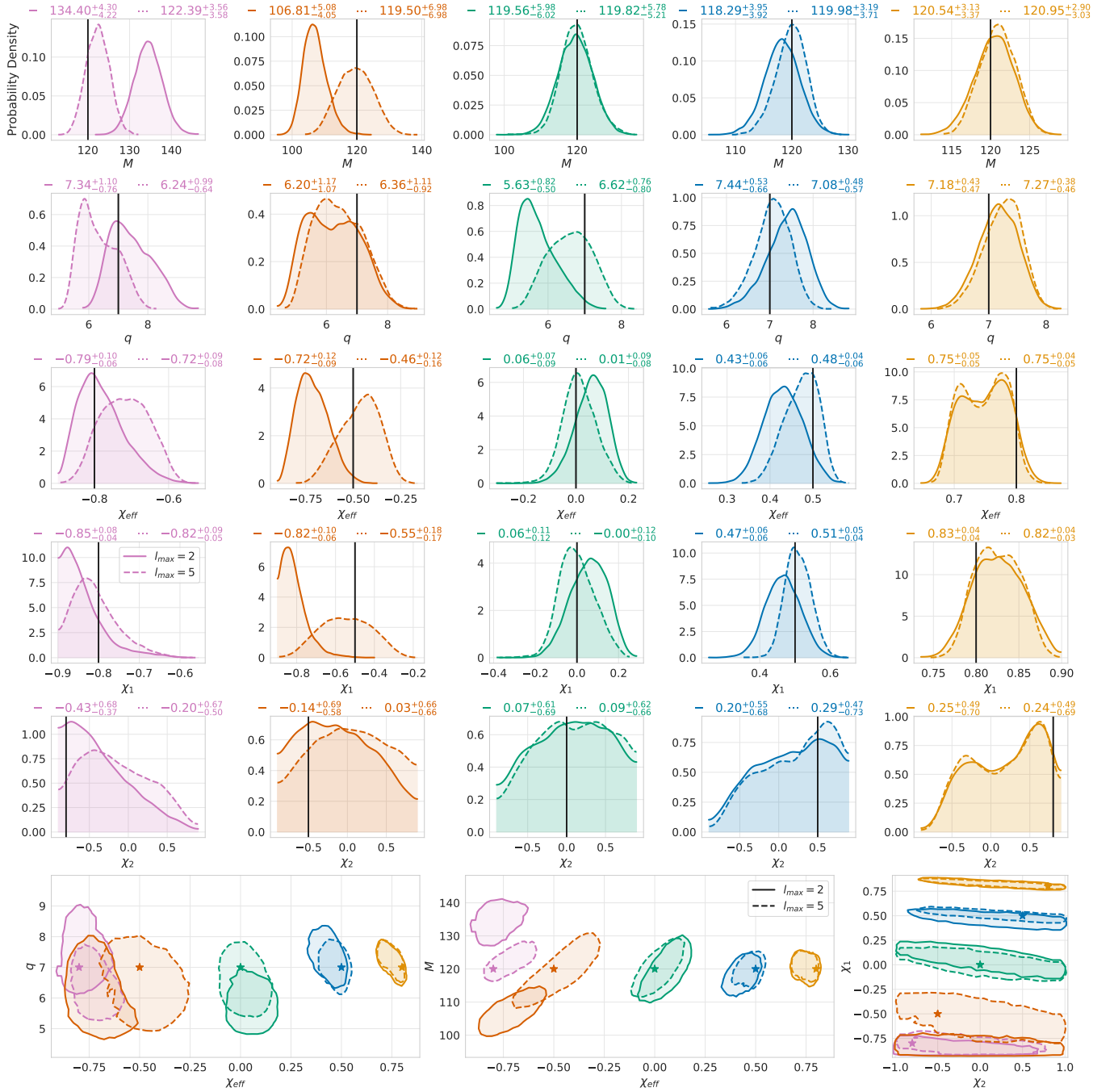


FIG. 3. **Non-HM and HM runs for $q=7$ spin set, with $\text{SNR}=30$ and $M=120$:** The first five rows show the M , q , χ_{eff} , χ_1 , χ_2 one-dimensional marginal distributions, where among this set of figures each column corresponds to a different synthetic source recovered with either all $\ell_{\text{max}} = 5$ modes (dashed line) or $\ell_{\text{max}} = 2$ modes (solid line). Our figures are organized such that the injected spin is systematically increased from left to right, where the synthetic source runs are ID12 ($\chi_{\text{eff}} = -0.8$), ID13 ($\chi_{\text{eff}} = -0.5$), ID14 ($\chi_{\text{eff}} = 0$), ID15 ($\chi_{\text{eff}} = 0.5$), and ID16 ($\chi_{\text{eff}} = 0.8$). A solid black vertical line denotes the true parameter value. The final bottom row corresponds to the joint distributions for q vs χ_{eff} , M vs χ_{eff} , and χ_1 , χ_2 for all five injections.

$g(x)$, the JSD is defined as

$$D_{\text{JS}}(p|g) = \frac{1}{2} \left(D_{\text{KL}}(p|s) + D_{\text{KL}}(g|s) \right), \quad (11)$$

where $s = 1/2(p + g)$ and

$$D_{\text{KL}}(p|g) = \int p(x) \log_2 \left(\frac{p(x)}{g(x)} \right) dx, \quad (12)$$

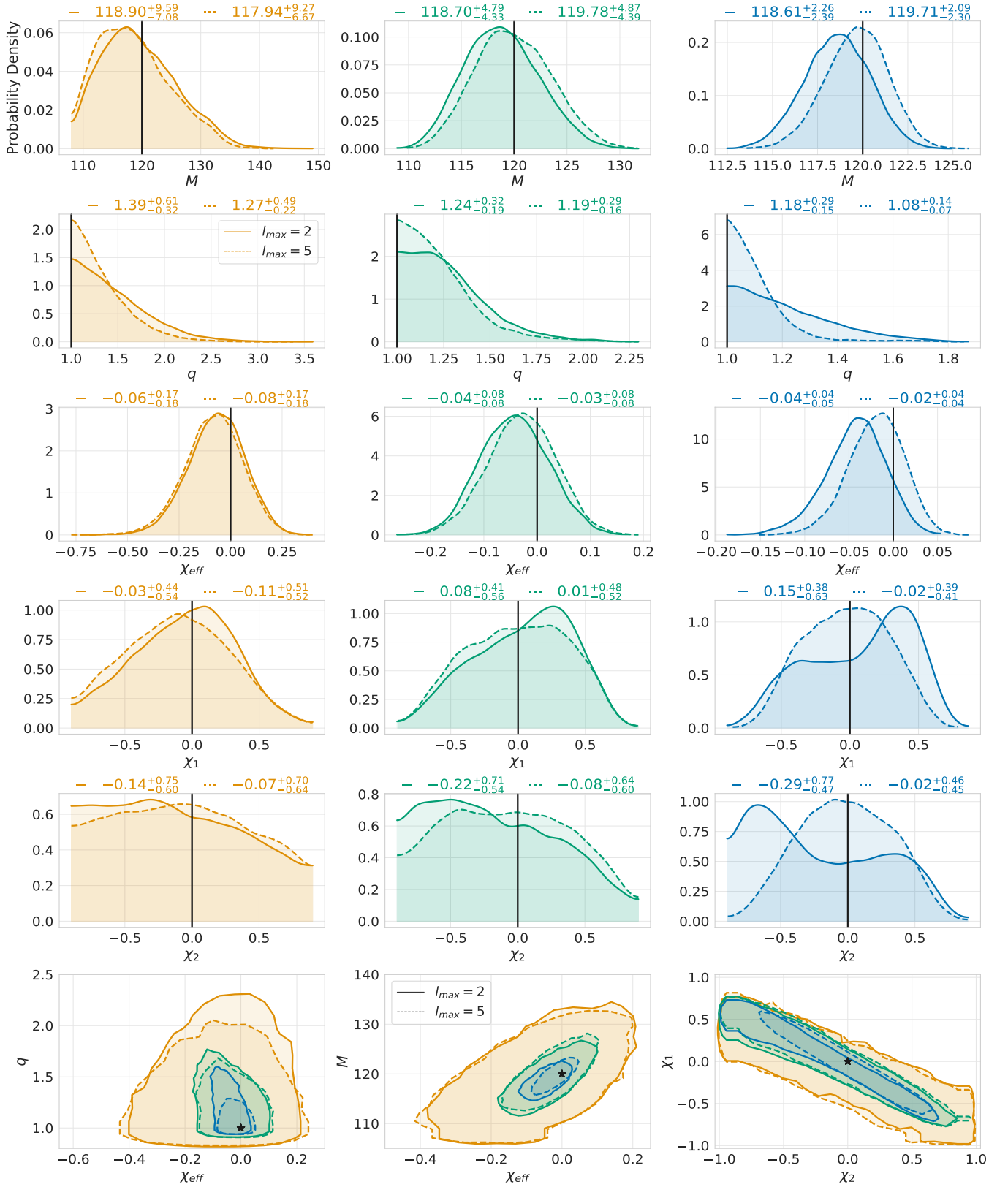


FIG. 4. **Non-HM and HM runs for a $q=1$, $M=120$, and zero-spin source (ID4), for different SNRs:** The first five rows show the $M, q, \chi_{\text{eff}}, \chi_{1z}, \chi_{2z}$ one-dimensional marginal distributions, where among this set of figures each column corresponds to a different synthetic source recovered with either all $\ell_{\text{max}} = 5$ modes (dashed line) or $\ell_{\text{max}} = 2$ modes (solid line). Our figures are organized such that the signal's network SNR is systematically varied as 10 (orange), 30 (green), and 70 (blue), corresponding to the left, middle, and right columns, respectively. A solid black vertical line denotes the true parameter value. The final bottom row corresponds to the joint distributions for q vs χ_{eff} , M vs χ_{eff} , and χ_{1z} vs χ_{2z} for all three injections.

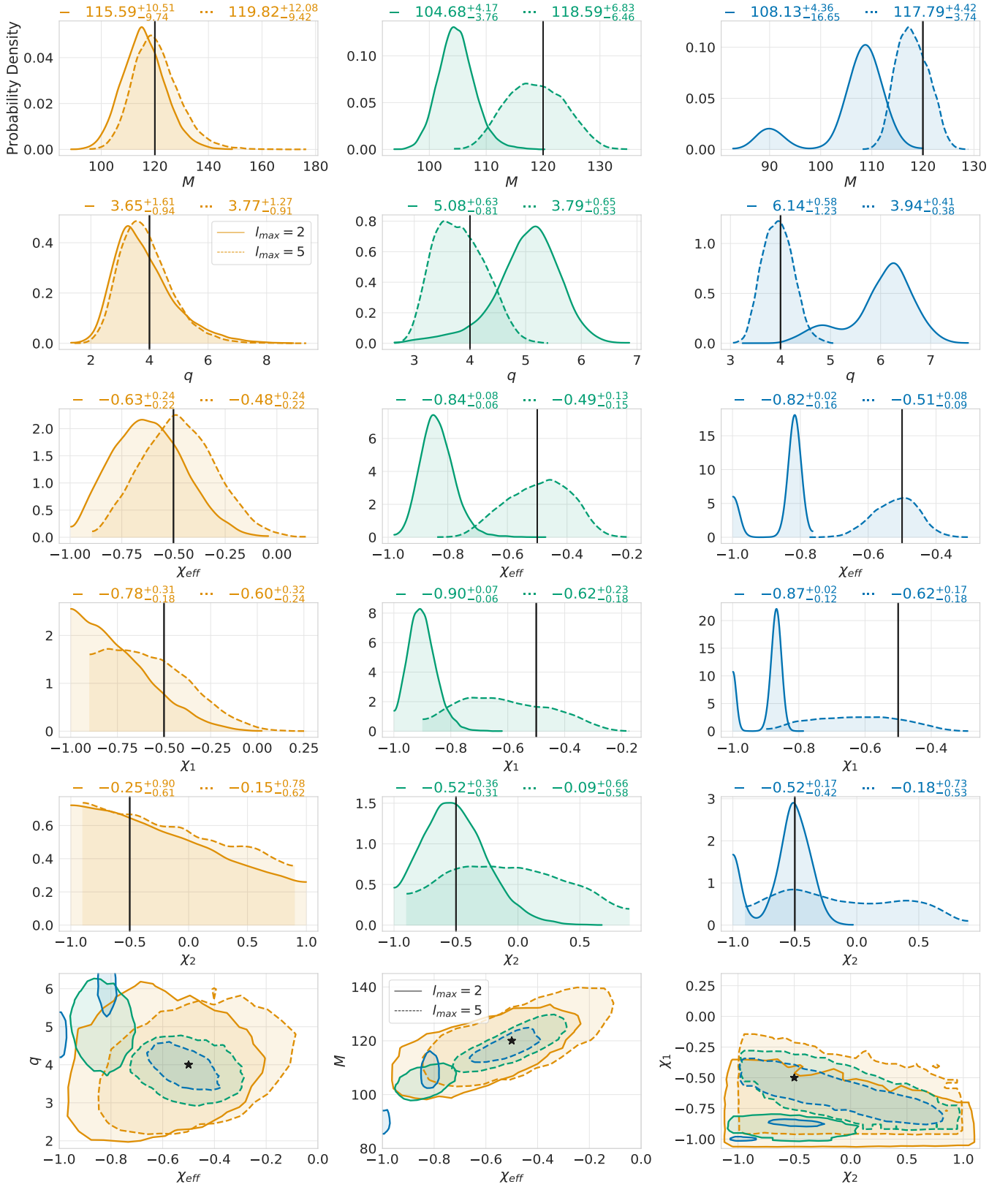


FIG. 5. Non-HM and HM runs for a $q=4$, $M=120$, and $\chi_{\text{eff}} = -0.5$ source (ID8), for different SNRs: The first five rows show the M , q , χ_{eff} , χ_{1z} , χ_{2z} one-dimensional marginal distributions, where among this set of figures each column corresponds to a different synthetic source recovered with either all $l_{\text{max}} = 5$ modes (dashed line) or $l_{\text{max}} = 2$ modes (solid line). Our figures are organized such that the signal's network SNR is systematically varied as 10 (orange), 30 (green), and 70 (blue), corresponding to the left, middle, and right columns, respectively. A solid black vertical line denotes the true parameter value. The final bottom row corresponds to the joint distributions for q vs χ_{eff} , M vs χ_{eff} , and $\chi_{1,z}$ vs $\chi_{2,z}$ for all three injections.

is the Kullback-Leibler divergence (KLD) between the distributions p and g , measured in bits. For context, this is the same calculation the LVC performed in [9] to quantify the agreement between different models. When measured in bits, the JSD is bounded below by 0. For a sense of scale, the KL divergence between two one-dimensional Gaussians with identical standard deviations but differing means μ_1, μ_2 is $(\mu_1 - \mu_2)^2 / 2\sigma^2 \ln 2$; inverting, $\text{JSD} = 0.2$ corresponds to $\mu_1 - \mu_2 \simeq 0.5\sigma$.

Figure 6 shows the JSD vs SNR and the simple “bias classifier” for both the ID4 and ID8 runs, respectively. Following the discussion in the LSC’s recently published Gravitational-Wave Transient Catalog [9] (cf. Appendix 2.B), we consider two marginalized posteriors to be sufficiently different (i.e. biased) if the JSD is greater than ≈ 0.15 . This number corresponds to a SNR $\simeq 30$ for non-spinning, equal-mass binaries; SNR $\simeq 10$ at $q = 4$ and $\chi_{1z} = \chi_{2z} = -0.5$. Since subdominant modes become more important at larger mass ratios and more negative values of χ_{eff} , the quoted SNRs provide convenient lower bounds for similar systems. For example, we expect HMs will also affect the posterior for systems with SNRs ≥ 30 and $q > 1$, $\chi_{\text{eff}} \leq 0$ (similar to ID4); for systems with SNRs ≥ 10 and $q > 4$, $\chi_{\text{eff}} \leq -0.5$ (similar to ID8).

IV. DISCUSSION

A. Effect of Different Spin Priors

Besides the impact of sub-dominant modes, the ability to accurately measure the spin parameters is also influenced by the choice in spin prior [61], which is not well-informed by astrophysical observations or source population models. In our study, we have used a prior which is uniform in χ_z (P1). However, many of the LVC’s analysis assume a prior that is uniform in spin magnitude, $|\vec{\chi}|$, and on the 2-sphere, which, for our non-precessing model, would induce a prior by projection of $\vec{\chi}$ along the orbital angular momentum vector (P2). This spin prior assumes the peak of the PDF of the individual component spins will strongly favor zero. To see how these two significantly different priors affect the ability to measure the spins, we compare posteriors for two runs ID2 and ID6 with SNR= 30 assuming the two different priors. Figure 7 shows the individual χ_{*z} spins as well as the effective spin χ_{eff} for each spin prior. Despite using a strong source, all spin parameters are significantly perturbed by the prior choice, similar to results found in previous work [61].

B. Consequences of biases for remnant properties and consistency tests

Using the posterior distributions of the BBH system’s component masses and spins one can compute the remnant mass, M_f , and spin, a_f of the final (merged) black

hole. The values of (M_f, a_f) are interesting in their own right as they can be used to infer a population of astrophysical compact objects that formed through the merger of a BBH system. Another use of (M_f, a_f) is to test the consistency of general relativity by predicting these remnant values found from (i) the post-merger portion of the signal which is described by a ringdown signal characterized entirely by (M_f, a_f) and (ii) the inspiral portion of the signal where we compute the BBH system’s component values and, using numerical relativity, arrive at an alternative estimate of (M_f, a_f) . If general relativity correctly describes the system’s entire evolution, we should expect the remnant values found through each to be mutually consistent [62]. A closely related test uses the remnant values computed with the inspiral-only portion of the signal to infer the expected quasi-normal mode (QNM) of ringdown signal, and then comparing this predicted QNM spectrum with the QNMs estimated directly from the ringdown-only portion of the data [62].

All of these studies require accurate measurement of the system’s remnant masses and spins. For example, unacceptably large bias in these quantities could provide misleading evidence for failed GR consistency tests.

In this subsection we explore bias in the remnant properties implied by the posterior distributions computed in Sec. III as the SNR increases. We compute the remnant mass and spin magnitude by evaluating the high-accuracy fitting formula provided by the surfinBH Python package [63] on the posteriors computed using $\ell_{\text{max}} = 5$ and $\ell_{\text{max}} = 2$ recovery waveform models.

As the first example, where we expect minimal bias, we consider the $q = 1$, zero-spin source system (ID4) whose posterior distributions for SNRs= {10, 30, 70} are reported in Fig. 4 from which we compute remnant posteriors in Fig. 8 (left set of figures). While the true remnant values are contained within all of the joint posteriors’s 90% credible region, we begin to see modest bias indicating impact from the higher-modes when the signal’s strength reaches an SNR value of 70. This is quantified in Fig. 6 which shows the Jensen-Shannon divergence for M_f and a_f are 0.24 and 0.17, respectively. For context, values above 0.15 are typically considered to reflect non-negligible bias [9]. At all values of the SNR, we find the $\ell_{\text{max}} = 5$ posterior more tightly constrains the true values.

Fig. 8 also shows a similar sequence for the $q = 4$, $\chi_{\text{eff}} = -0.5$ source (ID8) where now the true remnant values are no longer contained within the 90% credible intervals by SNR=30. As seen from Fig. 6, the JS divergence is already close to, or great than, 0.15 at SNR=10. This suggests that higher modes are very important when estimating the remnant values from such systems, and neglecting them would incorrectly lead to a failure of the IMR consistency test for essentially any event we might conceivably observe similar to ID8.

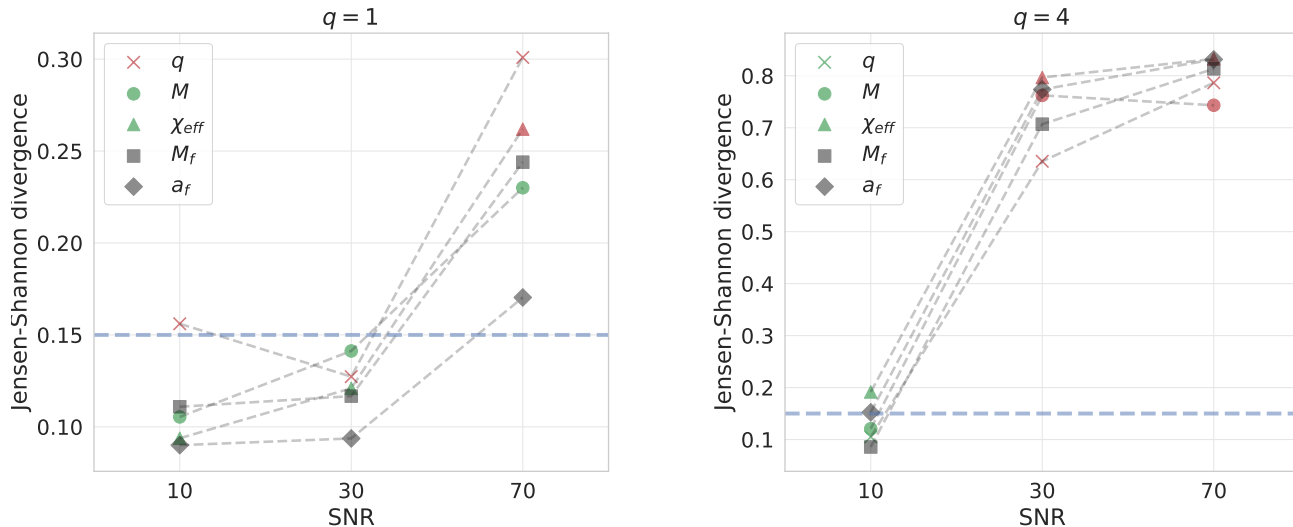


FIG. 6. **The importance of higher modes for loud signals: bias vs SNR:** These panels show the JSD vs SNR for source ID4 (left panel) and ID8 (right panel). Different markers indicate which one-dimensional marginal distribution was used to evaluate the JSD, which are depicted in Figures (4) and (5) for ID4 and ID8, respectively. The dashed horizontal blue line demarcates a commonly used threshold for unacceptably large bias. Markers colored in red indicate that the true value falls outside the 90% credible interval region for the $\ell_{\max} = 2$ case (significant bias in the recovered parameter value), while those colored in green indicate the opposite. For $\ell_{\max} = 5$, the true value is almost always within the 90% credible interval region except the parameter q in the $q = 1$ case, where the true value lies at the edge; despite not being in the 90% credible interval the marginalized distribution for q obtains its maximum value at $q = 1$ (cf. row 2 of Fig. 4). Markers in gray indicate the JSD for the final remnant masses and spins.

C. Consequences of biases on population reconstruction

In a second and more qualitative example of the impact of parameter biases due to neglect of physics, we consider astrophysical inference for the mass, mass ratio and spin distribution of coalescing BHs. For example, consider an SNR=30, zero-spin BBH event with $q = 4$. As illustrated by the green curves in Figure 2, inferences which neglect HMs would deduce negative effective spin (and a more extreme mass ratio). A single source with definitively negative χ_{eff} would be interpreted as a strong indication for dynamical formation in samples of less than several hundred mergers. Such biased inferences for high-amplitude sources could thus be misinterpreted to support qualitatively different formation channels (e.g., dynamical formation) than supported by the true parameters, which are well-characterized by multimodal PE.

More typically, parameter biases due to model incompleteness enter more insidiously into astrophysical inference, since population inference relies on combining information from multiple sources and since systematic biases impact all sources at a similar level. Following [64], we estimate that parameter biases $\Delta x = x^{\text{true}} - x^{\text{median}}$ will be significant for a population of N sources if the bias can be identified in the population mean by stacking observations: in other words, if $\Delta x \gtrsim \sqrt{\sigma_{\text{stat}}^2 + \sigma_{\text{astro}}^2} / \sqrt{N}$ where σ_{stat} and σ_{astro} are the statistical error in x and

the width of the astrophysical distribution of x , respectively. In terms of the JSD we anticipate that systematic differences in waveforms must produce a change in posteriors less than $\text{JSD} = 0.15/N$ to have no effect on population inference. Our examples show that even for zero-spin (but unequal-mass) binaries, inferences about the mass ratio, total mass, and effective spin in moderate-SNR sources can be significantly biased by the lack of HM. If a population of unequal mass-ratio binaries exist and has a spin distribution qualitatively similar to the seemingly low-spin BH population identified in O2, even inferences drawn from a handful of observations could be noticeably biased about BH masses and spins.

D. A GW170729-like source

While much of our focus has been on fiducial BBH systems, it is also interesting to consider sources that are similar to events from the most recent observing run. In this subsection, we analyze a synthetic source that has parameters (cf. ID1 in Table I) similar to GW170729, one of the more interesting events from O2. As mentioned in [9] and [65], the SNR of GW170729 was ~ 12 . However, to better highlight the importance of HMs for this event, we instead consider a GW170729-like event located at a distance such that the SNR is 30. For consistency with other synthetic events analyzed throughout this paper,

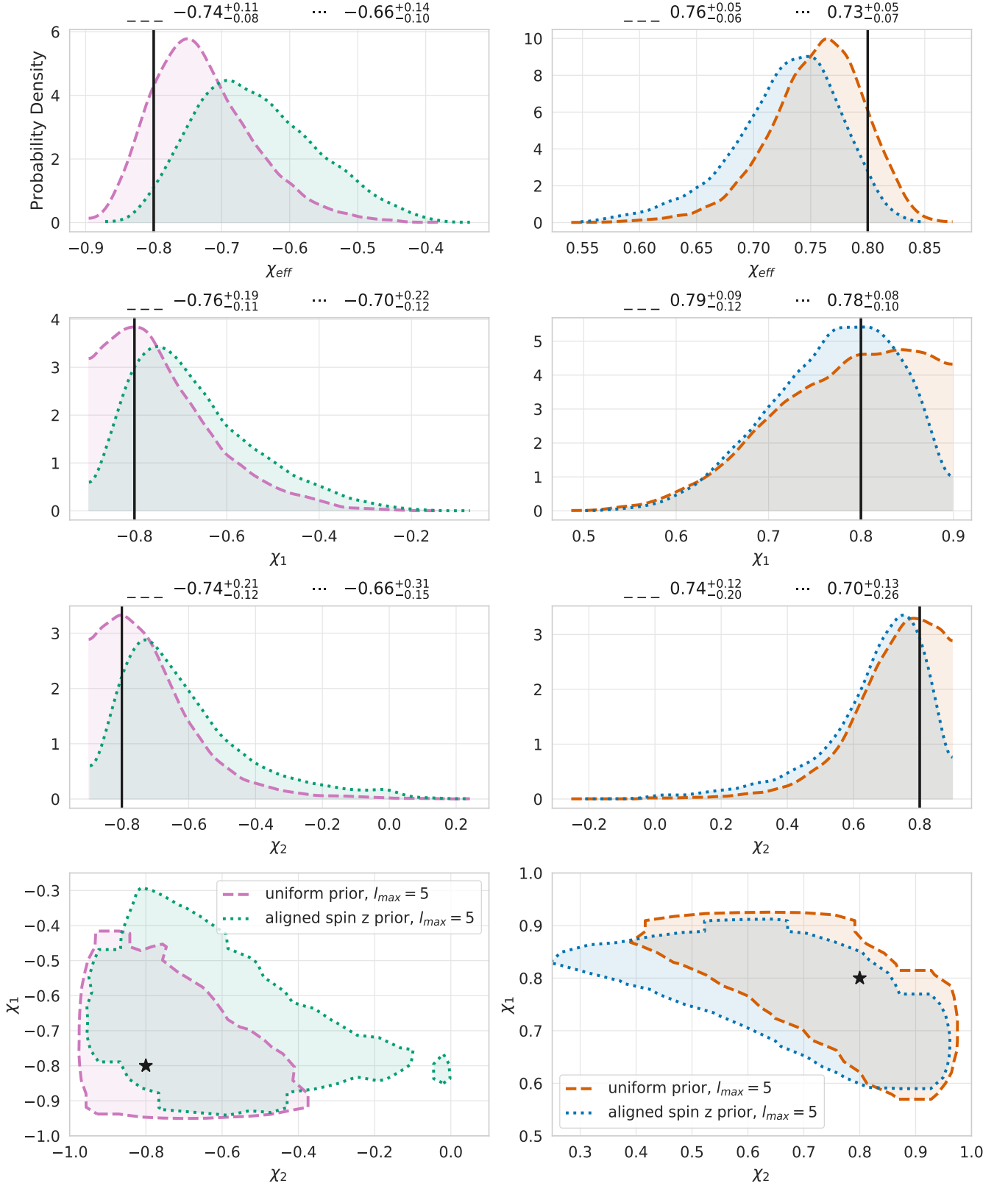


FIG. 7. **The effect of priors on spin measurability:** Individual and effective spin parameter recovery assuming two different priors, using synthetic datasets ID2 ($q = 1, \chi_{\text{eff}} = -0.8$) and ID6 ($q = 1, \chi_{\text{eff}} = 0.8$) with $\text{SNR} = 30$. The dashed curve represents the results using a prior that assumes uniform spin magnitudes in χ_z (P1; uniform prior), and the dotted curve represents the results using a prior that assumes uniform spin magnitudes in $\bar{\chi}$ (P2; aligned spin z prior). Despite the high value of SNR used here, we observe that the choice of prior has a significant influence on the recovered posteriors.

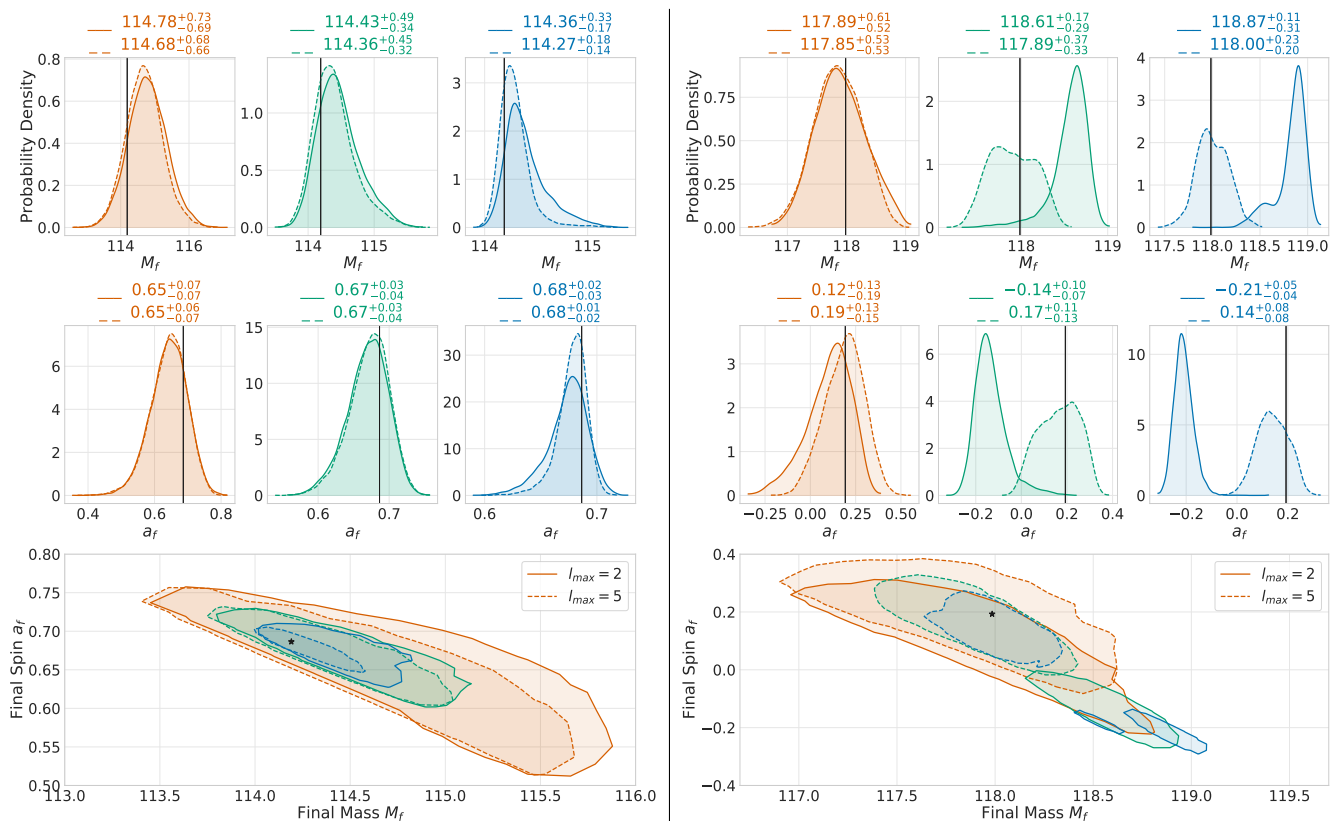


FIG. 8. **Effect of higher-order modes on remnant values and IMR consistency tests:** These panels show marginal distributions for remnant properties of the redshifted mass, M_f , and spin, a_f , for a non-spinning, $q = 1$ source (ID4; left panels) and $\chi_{\text{eff}} = -0.5, q = 4$ source (ID8; right panels). Our figures are organized such that the signal’s network SNR is systematically varied as 10 (orange), 30 (green), and 70 (blue), corresponding to the left, middle, and right columns of each panel.

we set $\chi_{\text{eff}} = 0.5$ as its true value, which is near the upper end of the 90% credible interval t [33]. Note that although $\chi_{\text{eff}} = 0.5$, we now have $\chi_{1z} \neq \chi_{2z} = 0$. We continue using a uniform spin magnitude in χ_z as our spin prior.

Figure 9 shows the posterior distributions for the runs that include only $\ell_{\text{max}} = 2$ (solid lines) and include all the $\ell_{\text{max}} = 5$ (dashed lines). As with all the results in Section III, we see a significant bias between the two runs in all the parameters. For example, we see that the $\ell_{\text{max}} = 5$ model does a much better job at recovering the individual spin components as well as placing somewhat tighter constraints on the spin of the larger BH, χ_{1z} . Interestingly, we see a similar shift in q and χ_{eff} that was observed in a recent re-analysis of the actual GW170729 event [33]. As our detectors continue to get more sensitive, we will increasingly see events with parameter and SNR values similar to the synthetic source ID1 considered here.

V. CONCLUSIONS

In this work, using the recently-developed NRHyb-Sur3dq8 model, we systematically investigate the impor-

tance of higher modes on the interpretation of gravitational wave signals from coalescing binary black hole systems. We have primarily focused on heavy systems with masses and spins similar to the detector-frame masses of near-future gravitational-wave observations while using current detector network sensitivities. Previous studies [19–27, 30, 31, 65] have also explored this question in various approximate contexts, either using a single detector, relying on Fisher information matrix approximations, or restricted to non-spinning BBH models. Here we perform coherent Bayesian inference on the combined datasets from the current three-detector network of observatories, which is the same setup used in the recent analysis of gravitational wave observations [9]. We confirm many of the general expectations of previous works, while providing a more direct quantification of the bias within this realistic setup.

As expected, we find that higher modes are very important for interpreting asymmetric binaries with $q > 1$. More surprisingly, we find noticeable differences even when the injected signal mass ratio is $q = 1$, when subdominant modes are expected to be suppressed (See Appendix A for a small follow up analysis). Also as expected, we find that the biases introduced by neglecting

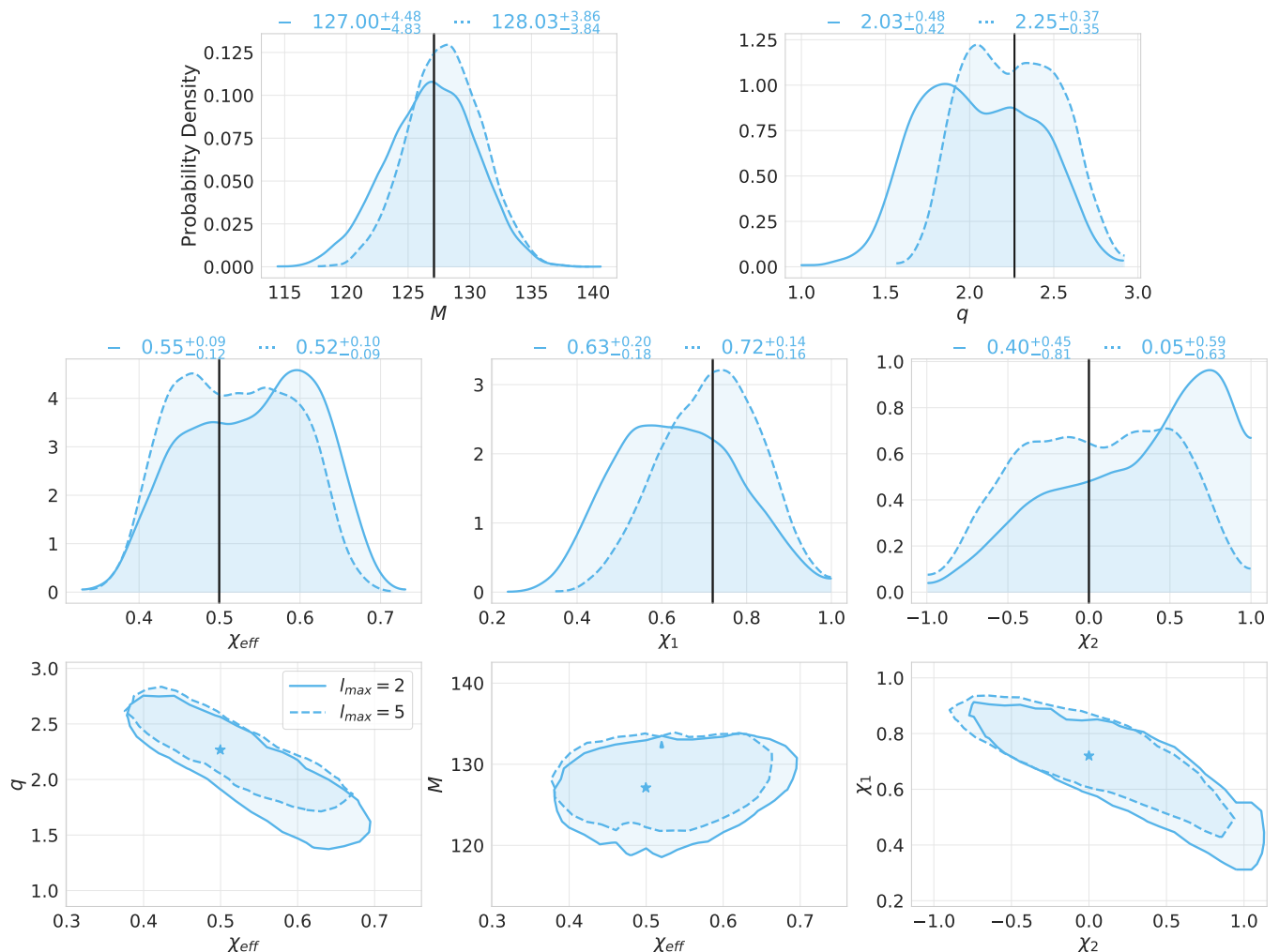


FIG. 9. **GW170729-like event**: Posterior plots for the ID1 run: $q = 2.267$, $M(M_{\odot}) = 127.1$, $\chi_{1z} = 0.72$, $\chi_{2z} = 0.0$, $\text{SNR} = 30$. The solid and dashed lines represent the $\ell_{\max} = 2$ and $\ell_{\max} = 5$ runs respectively. When including HM, we are able to improve the recovery of individual spin components. We also see a significant shift in the q and χ_{eff} distributions.

higher-modes are very important for $q > 1$ and $\text{SNR} \geq 30$ [19, 26, 57]. However, in our examples we also find that inference without higher modes has a significant impact on the interpretation of *low-SNR* sources, particularly by influencing our knowledge of the binary’s mass ratio.

Finally, consistent with previous work, we find that configurations with $M_z \simeq 120$ and large aligned spins have almost no parameter bias [19] even at high SNRs. Such systems with large aligned spin exhibit the orbital hangup effect and have more in-band cycles. Since the impact of higher modes is inversely correlated with the length of the signal, we suspect that the orbital hangup effect serves to suppress the importance of higher modes.

In our examples, parameter inference of spinning BBH systems without higher modes are frequently biased. These consistent systematic biases may accumulate in population inference calculations, as described in Sec. IV C (see also Ref. [64]). We anticipate that any population inferences of asymmetric, high-mass black hole bi-

naries will require significant attention to waveform systematics. Sec. IV B also demonstrated that neglecting higher-order modes in the analysis of GW observations leads to biased estimates of the remnant object’s mass and spin. For instance, Fig. 8 shows that higher-modes provide significantly better constraints on the remnant values, while the computed Jensen-Shannon divergence indicates a tension between the remnant mass and spin posteriors recovered by the $\ell_{\max} = 2$ model and the true one (cf. Fig. 8) over a range of SNRs and mass ratios. As the remnant values feature prominently in IMR consistency tests of general relativity, our study suggests that neglecting higher-modes could incorrectly trigger failed tests of GR, for example when carrying out consistency tests between the strong-field merger and ringdown portions of the signal.

Finally, we have found that posteriors using an incomplete waveform model are often significantly offset from the full-model posterior, typically towards (incorrectly)

favoring lighter binary systems with more negative χ_{eff} values. For example, a significant fraction of the probability for the $\ell_{\text{max}} = 5$ posterior is not contained within the high-probability boundaries of the $\ell_{\text{max}} = 2$ posterior. This suggests that it may be difficult to apply the likelihood-reweighting techniques advocated in [59], which require similar posterior distributions in all binary intrinsic and extrinsic parameters for the two models being applied (i.e., a simplified $\ell_{\text{max}} = 2$ model and a model including higher modes).

Looking ahead, we anticipate that aligned-spin IMR models including higher modes [14–16] will become standard in the analysis gravitational wave observations. Indeed, as shown here, the inclusion of subdominant modes will improve the interpretation of most events, and in some cases substantially so. A very recent study by Kalaghatgi et al. [65], using lighter BBH systems and a IMRPhenomHM recovery model, has also concluded that higher modes significantly reduces bias. Using the most physically-complete models will also remove the need for ad hoc regions-of-validity that depend on both the source parameters as well as the scientific questions under consideration. However, to enable our model to fully encompass the range of likely events, our models must also allow for generic precessing sources. Recent modeling of precessing binaries will allow for improved analysis of generic precessing sources [66]. Indeed, as already indicated by Ref. [36], we expect that many tests of general relativity could be biased unless they account for *both* higher modes and precession.

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Appendix A: Follow up on the significance of higher modes for equal mass, zero spin, SNR= 10 case

As pointed out in Sections III D and V, there seems to be significant differences between the $\ell_{\text{max}} = 5$ and

$\ell_{\text{max}} = 2$ runs for the equal mass, zero spin, SNR= 10 case, which runs contrary to several previous studies that had implied that HM would have minimal impact at low SNR for comparable-mass binaries.

To better understand our results, we perform a complementary analysis under the assumption of zero spin (i.e. lay out a grid only in M_{tot}, q), allowing us to directly evaluate the marginal likelihood versus the two remaining binary parameters. Figure 10 shows the results of both the $\ell_{\text{max}} = 5$ and $\ell_{\text{max}} = 2$ results. We continue to observe notable differences between the two posteriors even when restricted to two dimensions (i.e. only mass parameters). It is certainly surprising to see any difference given that this is a low SNR, equal mass event. One possibility is that due to the broadness of the posterior in mass ratio, a significant fraction of the posterior needs to be evaluated at values of $q \gtrsim 2$ where higher modes begin to play an increasingly important role.

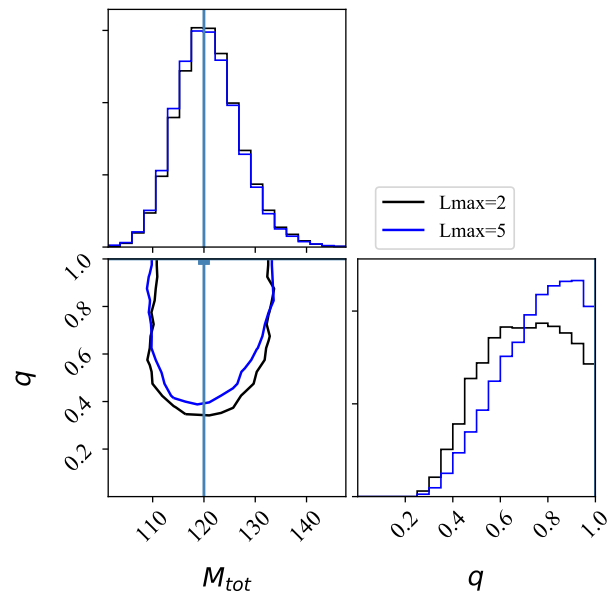


FIG. 10. **Reanalysis of equal mass, zero spin, SNR= 10:** This corner plot shows the reanalyses of a equal mass, zero spin, SNR= 10 source using $\ell_{\text{max}} = 2$ (black) and $\ell_{\text{max}} = 5$ (blue) mode but only on a grid in mass parameters (i.e. assuming zero spin). As first shown in Figure 4, there are noticeable differences between the two different distributions.

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