Direction Concepts in Wayfinding Assistance Systems

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Abstract

We report new findings about the mental representation of direction concepts and how these findings may revise formal models of spatial reasoning and navigation assistance systems. Research on formal models of direction concepts has a long tradition in AI. While early models where designed for unstructured space, for example, reasoning about cardinal directions, research on the influence of context has questioned the universal applicability of these models; mental direction concepts in city street networks differ from those in sea or air navigation. We investigated direction concepts at intersections in city street networks by using methods from cognitive psychology for eliciting conceptual knowledge. The results are used to modify the direction concepts employed in our wayfinding assistance framework. Within this framework it is possible to use abstract conceptualizations and to externalize them in different formats, for example, verbal or pictorial. Hence, this research may influence both, verbal and pictorial route directions and, additionally, the transfer from one into the other.

Keywords

direction concepts, research cycle, wayfinding assistance system, conceptual modeling

Introduction

Concepts of directions have a long tradition in AI research and various models have been proposed for different areas of application, for example, in qualitative spatial reasoning. Directions (orientations) are viewed as basic spatial relations (Habel, Herweg, and Pribbenow, 1995). Common to all models is that equivalence classes are used to represent a category of directions, following the general approach of AI to reduce and structure the information available. Early direction models partitioned space homogenously. Applications to cardinal directions and to egocentric reference systems can be found in Frank (1992) or Hernandez (1994). Different levels of granularity were achieved, for instance, by bisecting sectors, i.e. 4 sector models were transformed to 8 sector models and so on.

Besides using sectors to represent direction categories, some models use axes as well as sectors. The double cross calculus by Freksa (1992) (see also Freksa & Zimmermann, 1992), or the cardinal direction model by Ligozat (1998) show examples of their application. In models that use axes two options can be differentiated: axes that are true axes, i.e. they represent an equivalence class of their own, and axes that are prototypical instances of a sector and are taken as the representation of a sector. In the mentioned approaches by Freksa, Zimmermann, and Ligozat, the axes are 'true' axes. Other approaches, like the smart environment approach by Baus, Breihof, Butz, Lohse, and Krüger (2000) or the wayfinding choreme approach by Klippel (2003), use axes as prototypical instantiations of sectors to bridge, for example, the gap between underspecified expressions found in natural languages and the graphic representation of direction concepts.

The early homogenous direction models have been extensively criticized by Montello and Frank (1996). They ran various simulations to explain data that Sadalla and Montello (1989) collected and found that direction models with differently sized sectors fit the behavioral data best. This example shows how important behavioral research is and how it can be employed to modify existing models. This research in cycles is not only valid for Human-Centered-Design (e.g., ISO 13407) but also for basic research as approached in this paper.

Direction Concepts in Route Directions

The processing and representation of angular/direction information is essential for human spatial cognition and especially for wayfinding (e.g., Sholl, 1988; Montello et al., 1999; Waller et al., 2002). A growing number of experimental results indicate that route directions and wayfinding basically consist of making direction choices at decision points (e.g., Denis et al., 1999). Pursuing this line of thought, wayfinding can be characterized as following a route segment up to a decision point, making a directional choice, following the next route segment up to the next decision point, making a directional choice, and so on. Decision points can be operationalized as belonging to two main categories: decision points with a direction change (DP+) and decision points without a direction change (DP-). The question arises, how do humans conceptualize directions at decision points, especially at DP+? What are prototypical direction (turning) concepts and what do their graphical externalizations look like?

For most situations, qualitative information of direction—in the sense of a small number of equivalence classes—is sufficient. Especially in city street networks, which constrain the environment, directional choices of exact angular information are rarely necessary. Various studies show that angular information in city street networks—as well as in geographic space in general—is conceptualized and remembered qualitatively by humans (e.g., Byrne, 1979; Tversky, 1981; Moar & Bower, 1983). Verbal route directions reflect this qualitativeness: precise, i.e. very fine grained, direction information is rather an exception that is hardly ever given (e.g., Denis et al., 1999; Allen, 2000; Klippel & Montello, submitted). If we take the perspective of conceptual spatial primitives (e.g., Golledge, 1999; Klippel, 2003) the question arises how many different categories of directions are necessary, and how many categories humans employ? Additionally, we can pose the question whether there are prototypical turning concepts at all and how their graphic or verbal externalizations may look like.

Evans (1980) reported three major strategies that occur in representing directional information in city street networks mentally. These aspects are:

- straightening curved paths,

- squaring oblique intersections, and

- aligning nonparallel streets.

The second observation is strongly supported by recent approaches on cognitive adequate route directions (e.g., Tversky & Lee, 1998, 1999). But, especially for European style city street networks, this observation has to be researched in greater detail since these are often not regularly shaped.

Direction Concept Experiment

To explore how many categories have to be assumed for directions in city street networks and what the relation between sectors and axes is in this domain, we used an experimental method from cognitive psychology. We chose the grouping task paradigm, which is traditionally one of the most important methods to investigate conceptual knowledge in psychology (e.g., Cooke, 1999). The main idea of such tasks is that conceptual knowledge plays the central role in rating the similarity of given stimuli: stimuli are assessed as similar if they are instances of the same concepts. They are assessed as dissimilar if they are instances of different concepts. If other aspects of presentation are controlled, like in our experiment, such grouping experiments can provide important insight into the internal structure of conceptual knowledge. To realize the experiment, we used an experimental tool that has been developed by Knauff, Rauh, and Renz (1997). The tool realizes a method that is comparable to card sorting but helps to generate the experimental materials, presents the stimuli, and collects the relevant data. In contrast to other card sorting/grouping tools (e.g., Harper et al., 2003), it is especially designed to use pictorial stimuli and is therefore well suited for spatial and map related research.

Methods

Participants
Twenty-five students of the University of Bremen were paid for their participation (9 female, 16 male).

Design and Procedure
The experiments took place in a lab space at the University of Bremen. The grouping tool was adapted to the requirements of the present study. 108 icons where used to depict different possibilities to 'make a turn' at an intersection. We designed the icons according to the following criteria: using 1 degree increments would have resulted in 359 different icons (excluding the 'direct back'), which seemed infeasible. Instead, we set off from the results of Klippel (2003) and added bisection lines incrementally. In other words, starting with the prototypical direction concepts (in degrees: 45, 90, 135, 180, 225, 270, 315), we added four times bisecting lines for the resulting sectors. This resulted in increments of 5.625 degrees. We only used two branches of an intersection; the participants were advised to imagine the pictures as representations of possible turns at an intersection. The back sector (corresponding to angles between 315 and 45 degrees) was excluded for graphical reasons. The items were doubled to test whether the same items were placed in the same groups.

The icons were integrated in the grouping tool. Figure 1 shows a screenshot of an ongoing experiment. The grouping tool divides the screen in two parts. On the left side, the stimulus material, i.e. all icons depicting possibilities to 'make a turn' at an intersection are placed in random order. The large number of icons requires scrolling for accessing all items. This is a common procedure in interact-

ing with computer interfaces; no problems were expected nor found during the experiments. The right side of the screen is empty at start; here, groups of icons are created during the experiment. The actions the participants can perform were kept simple and the interface shows no unnecessary other features. Participants could perform the following actions:

- Create a new group: the grouping tool allows participants to create as many groups as they want and regard as suitable for the task at hand. For each group a new box is created on the right side of the screen. In case more items are placed in a group than fit the width of the original box, the box extends and scrolling is required to access all items in one group.

- Delete a group: participants were allowed to delete groups. The grouping tool requires a group to be empty before it can be deleted.

- Rearrange: The items on the left part of the screen can be newly arranged.

- Done: indicates that the task is completed.

Finally, the participants were asked to verbally label the groups that they had created.
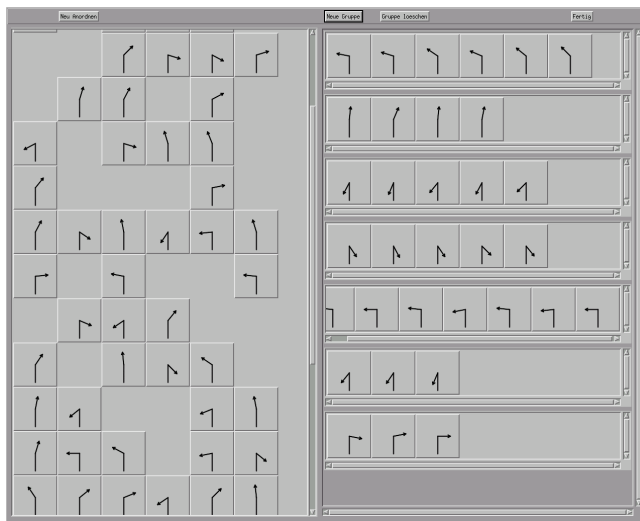
Figure 1. The grouping tool (snapshot from an ongoing experiment). On the left side the icons representing turns at intersections are presented in random order. On the right side a participant has started to group icons according to her categories of turning actions.
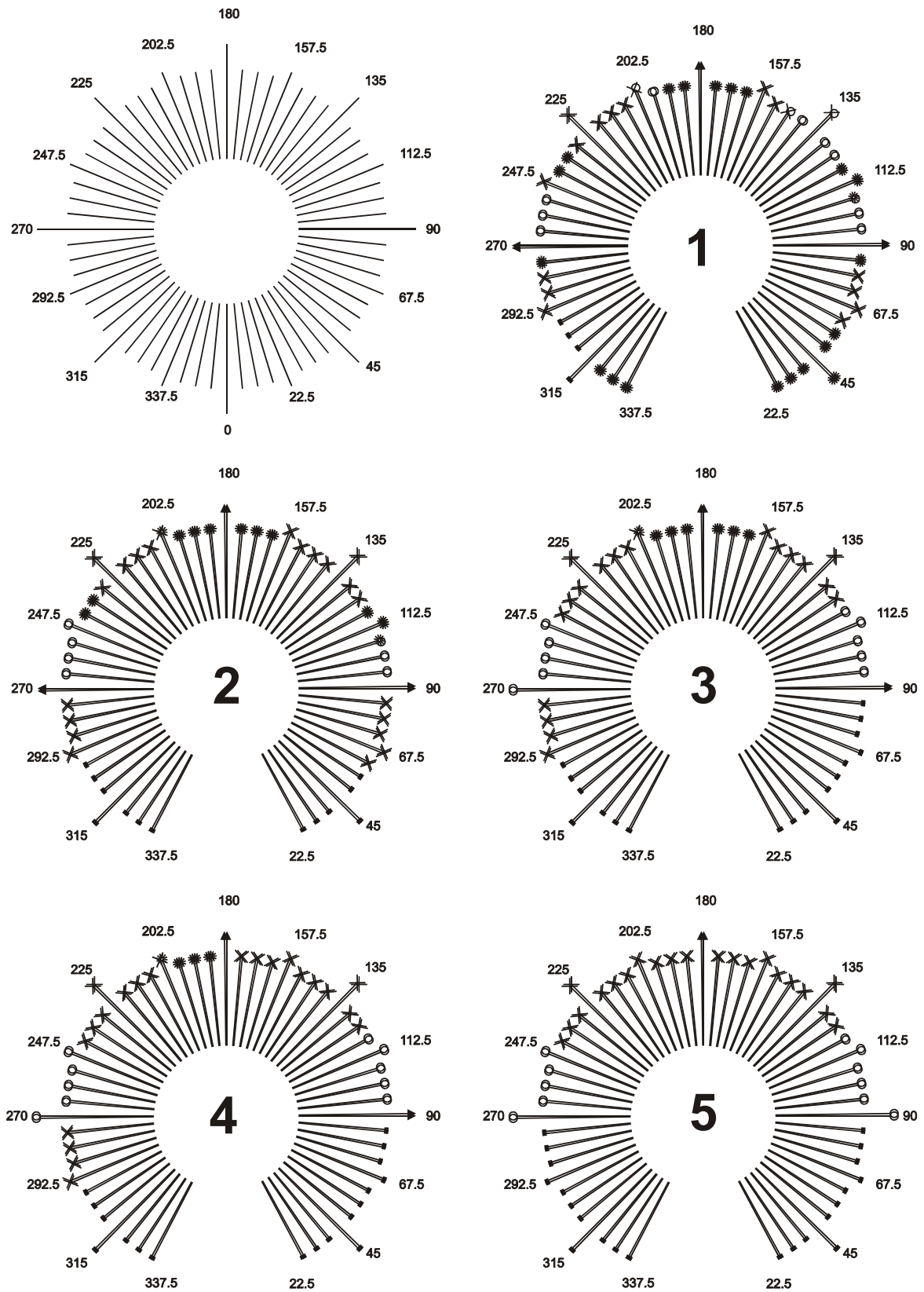
Results

A hierarchical cluster analysis has been used to analyze the data. The procedure is an exploratory tool designed to reveal natural groupings within a data set. It identifies relatively homogeneous groups of items (cases), using an algorithm that starts with each case in a separate cluster and combines clusters until only one is left. There are different possibilities to compute the clusters. We used the "linkage

between groups" method, as it provides a low variance within groups. Additionally, we chose squared Euclidean distance to enhance the grouping procedure. The output of a cluster analysis is usually a dendrogram in which the grouping of the individual items is stepwise provided, i.e. for each new calculation step it is shown which items fall into the same group and which groups go together, respectively. However, instead of a dendrogram, we visualized the data as rays corresponding to the directions depicted by the icons used in the experiment. Each ray represents one icon; the rays are doubled since two identical icons exist. This way, it is possible to visualize the different steps in the grouping algorithm. Since in the end, each cluster analysis groups all items in one single group, we defined a finishing criterion: as soon as in two consecutive calculation steps no groups were combined, the clustering stops. We briefly discuss the individual steps as they highlight interesting aspects of mental direction concepts, too. Figure 2 illustrates the following discussion.

The first level of clustering (Figure 2, part 1) does not show much more than that some directions start grouping together—indicated by the little geometric figures at the end of each ray—while others do not. On the second level (Figure 2, part 2), however, a clearer picture starts shaping: while most directions are placed in a group, three of them remain ungrouped—these are the rays that could be labeled 'straight', 'exact left', and 'exact right'. It is noteworthy that indeed all other icons (directions concepts) are already grouped. To some groups it is already possible to assign verbal labels while others may require more complicated expressions. The next levels of clustering show that there seems to be a relation between the persistence of a group and the simplicity (or complexity) of the potential verbal label. The back plane—from 270 over 0 to 90 degrees— seems to form less groups and seems to be clearer structured than the front plane. Yet, this result may be biased by leaving out the 'back' sector in the study design. In step (3), the front plane becomes more structured and the back plane starts to form one big group on the right side. From the three axes of step (2) only two remain: the 'straight' and the 'right' axis. In step (4) more groups of the front plane go together and step (5) is the last step in this analysis.

The results of the clustering show 7 clearly distinguishable groups; one of them is an axis. The left and the right plane are symmetric, the front and the back plane are not. Interestingly, the front and the back plane are clearly separated by 90 degrees left and right turns. The direction sectors differ in their size. As mentioned before, cluster analysis is not necessarily designed to verify (or falsify) hypotheses but to find clusters. Additionally, not only the end result is of interest, but also the individual steps. Regarding earlier steps of the analysis, it seems to be the case that a sector is not necessarily prototypically represented by the bisecting lines of that sector (i.e., left and right).

Figure 2. Results of the clustering analysis. Each ray represents one of the icons used in the grouping tool. Stars 1-5 show the first 5 steps of the clustering algorithm. The geometric figures at the end of each ray indicate groups of icons.

Discussion and Application to Mobile Systems

Based on these findings we propose a revised model of direction concepts applicable to the generation of verbal route directions and the schematization of maps, especially in mobile systems and electronic route planners (see Figure 3). In general, the abstract conceptual characterization and the automatic generation of route directions require a formal model to decide when a change in direction is considered, for example, a 'left turn' or a 'veer left'. The proposed model is the **basic** model for direction concepts at intersections in city street networks. Changing situations and changing contexts—T-intersections or circles, transportation modalities or traveling speed—do require further investigation.
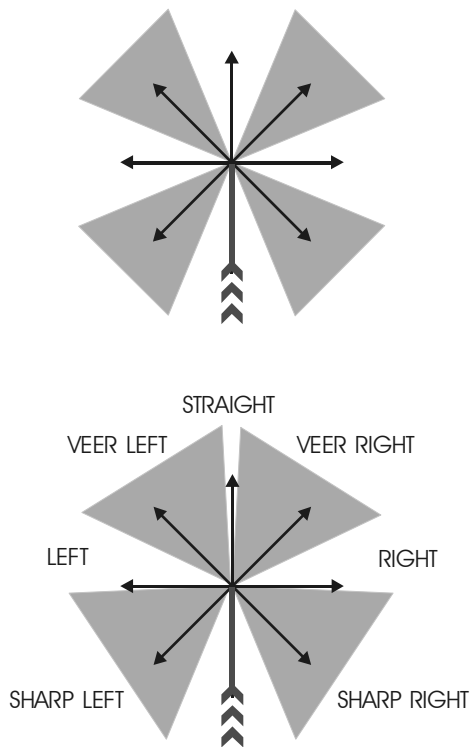
Figure 3. Original (upper part) and revised (lower part) direction model for the generation of verbal route directions and the schematization of maps. Black lines represent prototypical directions (e.g., Klippel, 2003).

On the last level of the presented cluster analysis our model would comprise the following features (see Figure 3): It consists of 7 sectors with **different** sizes; more precisely, 6 differently sized sectors plus 1 axis (plus the not further examined 'back' sector). As mentioned above, the formal specification of direction concepts is necessary for the following three application areas:

- the abstract conceptual characterization of route knowledge,

- the assignment of natural language expressions to turning concepts, and

- the schematic presentation of route maps.

The first area of application is ongoing research in the SFB/TR 8 MapSpace Project and the CRC SI (e.g., Klippel et. al., preliminarily accepted; Richter, Klippel, and Freksa, 2004). The aim is to advise a conceptual specification language for route knowledge. The abstract representation format is chosen to allow for situation and context adapted provision of route information in different modalities and externalization formats, for example, verbal or graphical.

To use this model in a wayfinding assistance system, the sectors and the axis can be assigned to natural language expressions, for example, *turn right*, *turn left*, *go straight* (see Figure 3; see Outlook for a discussion of work on a corpus of verbal externalizations of directions concepts). On this basic level, our results challenge the assumption of homogenous direction models and render the specification of conceptual structures underlying directions in city street networks more precise. Yet, for the assignment of proper natural language expressions a couple of open questions remain:

- How can we explain the rather sharp demarcation of front and back plane?

- What would be a proper label for the SHARP LEFT sector?

- What are proper labels for the other sectors?

- How can we account for results gained by Dale, Geldorf, and Prost (2003) and Klippel, Tappe, and Habel (2003) that show that aggregation/chunking is one of the key elements in the conceptualization of route elements, i.e. a term *turn right at the post office* or *turn right at the third intersection* is preferred over '*go straight*, *go straight*, *turn right*' (see also Wahlster et al., 1998).

Some answers can be provided based on methodological issues and need a more detailed analysis of the existing data, for example, an account for individual differences and the juxtaposition and discussion of the results of different clustering methods. Others are left for future work, like the further specification of the 'back' sector.

Mobile wayfinding assistance systems not only provide verbal instructions but communicate in a map-like manner, too. How can the results of our study be used to advise graphic route directions? One of the greatest issues in route map design—Webmapping, mobile services, etc.—is the definition of suitable schematization algorithms (e.g., Agrawalla & Stolte, 2001) or, from a more theoretical perspective, the question of aspectualization (e.g., Freksa, 1999). The approach taken here is to set out from prototypical representations for basic actions in following a route and from communications of these actions, respectively. This idea originates in work by Tversky and Lee (1998, 1999) on toolboxes for verbal and graphical route directions that has already inspired the approach by Agrawalla and Stolte (e.g., 2001). While the latter abandoned the idea of primitives to come up with an excellent

technical solution, we stick to the idea of having prototypical graphical representations of (mental) conceptual primitives of route direction elements (e.g., Klippel and Richter, 2004).

The prototypical direction concepts (black lines in Figure 3) needed for creating route maps or parts thereof originate in the model by Klippel (2003). Yet, although there seem to be prototypical direction concepts, not all represent sectors and they cannot easily be computed as bisecting lines. In the case of the concept LEFT and RIGHT they seem to demarcate the lower boundary of a sector. Likewise in other sectors they do not seem to be the bisecting lines. However, in this basic case left and right can be treated equally, which keeps the model computationally feasible.

If we stick to the idea of prototypical graphical representational elements for (mental) conceptual elements or routes, the newly proposed model allows for the assignment of different turning angles to the corresponding prototypical graphical representations. In opposition to the natural language expressions we do not have the luxury of an inherently underspecified representation but are forced to decide for exactly one instantiation (see also Habel, 1998). Maps are bounded, often temporarily and spatially fixed media that make it necessary to commit to one of many different alternative representations at a given point in time. The prototypical elements are therefore taken as the representations of the sectors (the axis) found in the results of this study.

The new model corresponds to various results in behavioral cognitive science (e.g., a higher differentiated front plane; for an overview cf. van der Zee and Slack, 2003). The rather unusual demarcation of the front and the back plane can additionally be explained by the visual characteristics of traveling through a city street network: One can look into the streets in the front plane (resulting in a greater differentiation) but not into the streets in the back plane. This poses an interesting question on the difference between overview (birds-eye) perspective and route (field) perspective (e.g., Herrmann et al., 1995) and can be exploited in the difference between maps and real world (or VR) interfaces (e.g., Klippel & Montello, submitted).

The interesting theoretical question is where do our direction concepts come from? Are they persistent phenomena or do they change with respect to the interaction with different interfaces? Are direction concepts embodied (e.g., Wilson, 2002), i.e. is our body with its physical characteristics the driving force behind direction concepts, or do we have to assume other factors? Plenty of research relates our body axis to the direction concepts we developed (e.g., Bryant, 1992). To which degree these results are applicable to city street networks is an open question. Additionally, for the embodiment explanation there are two alternative perspectives: the first is that the environment as such is responsible for shaping our concepts. This could imply that

North American city dwellers have different concepts than Europeans who are exposed to irregular street grid patterns to a greater extend (e.g., Davies & Pederson, 2001). Second, natural language could have—to a certain degree—an influence on the direction concepts. The latter topic is discussed under the term 'linguistic relativity' (e.g., Gumperz & Levinson, 1996). Proof for the influence of language can also been found in our experiment as some participants reported that if they had known in advance that the groups had to be labeled they would have created different groups. This question is under ongoing research in our lab and it might lead to modifications in the conceptual model assumed for verbal in opposition to graphic route directions.

The distinction by Klippel (2003) between standard directions concepts (LEFT, RIGHT and STRAIGHT) and modified directions (e.g. VEER LEFT)—as a concept, not literally as a verbal expression—has or can be extended to super-modified concepts like VEER SLIGHTLY LEFT. This becomes obvious from the discussion of the individual steps in the cluster analysis as well as from language analyses (e.g., Klippel & Montello, submitted). These concepts may not be as dominant as the standard and modified turning concepts but are present until the third and forth step in the cluster analysis, respectively (see Figure 2).

One remaining problem is the flexibility of clustering methods. This flexibility makes it a very valuable tool for exploratory data analyses as different clustering methods may be used to reveal the structure in the data. As such it was used in the present work. A word of caution, however, is that the method provides several degrees of freedom so that it is not suitable for more detailed statistical analysis or even for a test of specific hypotheses. Having said that, we still believe that the present findings help to develop more specific hypothesis and to gain more insight into the mental representation of direction concepts.

Conclusions and Outlook

The work undertaken here is part of a greater research effort on specifying the (mental) conceptual structure underlying direction concepts in route directions. As the change in direction at decision points is the most pertinent information in wayfinding and route directions, the focus on this knowledge should provide: first, valuable insight in basic research questions on cognitive processes underlying wayfinding and route directions; second, a challenge on existing assumptions on the formal specification of direction concepts; and third, an alternative to existing directions models. The results presented here are exploratory in nature but nonetheless allow for the modification of existing formal direction models.

Based on this initial research we are undertaking further research efforts on three basic research questions: the (mental) conceptual structure of directions concepts, the verbal externalization of mental directions concepts, and the applications of the findings to (graphic) schematization principles.

Acknowledgements

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