Critical behavior of charged dilaton black holes in AdS space

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We revisit critical behaviour and phase structure of charged anti-deSitter (AdS) dilaton black holes for arbitrary values of dilaton coupling α, and realize several novel phase behaviour for this system. We adopt the viewpoint that cosmological constant (pressure) is fixed and treat the charge of the black hole as a thermodynamical variable. We study critical behaviour and phase structure by analyzing the phase diagrams in $T-S$ and $q-T$ planes. We numerically derive the critical point in terms of α and observe that for $\alpha=1$ and $\alpha\geq\sqrt{3}$, the system does not admit any critical point, while for $0<\alpha<1$, the critical quantities are not significantly affected by α. We find that unstable behavior of the Gibbs free energy for $q<q_c$ exhibits a *first order* (discontinuous) phase transition between small and large black holes for $0\leq\alpha<1$. For $1<\alpha<\sqrt{3}$ and $q>q_c$, however, a novel first order phase transition occurs between small and large black hole, which has not been observed in the previous studies on phase transition of charged AdS black holes.

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I. INTRODUCTION

Phase transition is certainly one of the most intriguing and interesting phenomena in the thermodynamic description of black holes which may shed some light on the nature of quantum gravity. In particular, the investigations on the black hole phase structure/transition in an asymptotically AdS spacetime have received considerable attentions in the past years. This is mainly due to the remarkable duality between gravity in an $(n+1)$-dimensional AdS spacetime and the conformal field theory living on the boundary of its n-dimensional spacetime, (AdS/CFT) correspondence. Perhaps, one of the earliest studies in this direction was done by Hawking and Page [1], who disclosed that there is indeed a first order phase transition, latter named Hawking-Page phase transition, between the thermal radiation and the stable large schwarzschild black hole with spherical horizon in the background of AdS spacetime. Later, Witten discovered [2] that this phase transition can be interpreted in the AdS/CFT duality as the confinement/deconfinement phase transition in the strongly coupled gauge theory. Recently, it has been shown that, for charged AdS black hole, a second and first order phase transition occurs between small and large black holes in an extended phase space which resembles the liquid-gas phase transition in the usual Van der Waals liquid-gas system [3, 4]. In an extended thermodynamic phase space, the cosmological constant (AdS length) is considered as a thermodynamic pressure which can vary, and its corresponding conjugate quantity is the thermodynamic volume of the black hole. Taking into account the variation of pressure in the first law, one observes that the mass of AdS black hole is equivalent to the enthalpy [5]. In the recent years, thermodynamic phase transitions in an extended phase space have been explored for various types of black holes in AdS space (see Refs. [6–22] and references therein). The studies on phase structure of charged AdS dilaton black hole have been carried out from both thermal and dynamical point of view [23], where the cosmological constant appears as a thermodynamical variable. They found that for small dilaton coupling, $\alpha\approx 0.01$, the system resembles the Van der Waals fluid behaviour, while for $\alpha>1$, the $P-v$ diagram of the system deviates and new phenomena beyond the Van der Waals liquid-gas-like appears [23]. Recently, a novel phase behaviour represents a small/large black hole *zeroth-order* phase transition, in an extended phase space with varying cosmological constant, has been observed for charged dilaton black holes, where the geometry of spacetime is not asymptotically AdS [24].

Another possible approach to study thermodynamic phase structure of black hole is to consider the variation of electric charge of the black hole, while the cosmological constant (AdS length) is kept fixed. With regard to this perspective, the critical behavior and phase transition of charged AdS black holes were investigated in a fixed AdS geometry, indicating that it exhibits the small/large black hole phase transition of Van der Waals type [25, 29]. Interestingly enough, it has been realized in [25] that the phase transition of charged AdS black hole can occur in $Q^2-\Psi$ plane, where $\Psi=1/2r_+$ is the conjugate of Q^2, without extending the phase space. Indeed, in this alternative perspective, the relevant response function clearly signifies the stable and unstable region. Also there still exist a deep analogy between critical phenomena and critical exponents of the system with those of Van der Waals liquid-gas system [25]. The advantages of this new approach is that one do not need to extend the phase space by treating the cosmological constant as a thermodynamical variable which may physically not make sense [25]. It has been confirmed that this new approach also works in other gravity theories [26, 27] as well as in higher spacetime dimensions [28]. Recently, the universality class and

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critical properties of any AdS black hole, independent of spacetime metric, via an alternative phase space has also been explored [30]. It was shown that the values of critical exponents for generic black hole are the same with the Van der Waals fluid system [30]. For the Born-Infeld AdS black holed in four dimensional spacetime, we analytically calculated the critical point by studying the behavior of specific heat in a fixed AdS geometry [31]. Furthermore, the interesting reentrant phase transition of Born-Infeld AdS black hole has been investigated in the thermodynamic phase space [31]. The structure of charged black holes has been studied by employing the Ruppeiner geometry [36], in which the charge is allowed to vary.

In this paper, we study thermodynamic properties of charged AdS dilaton black hole in $(3+1)$-dimensional spacetime. Our work differs from [23], in that we keep the cosmological constant as a fixed quantity and treat the charge of the black hole as the thermodynamic variable, while the authors of [23] extended the phase space by treating cosmological constant as a variable. Besides, we analyze the phase structure in $T-S$ and $q-T$ planes and observe a novel first order phase transition, which has not been reppored in the previous investigations on phase transition of charged AdS black holes. As we shall see, the behavior of black hole temperature crucially depends on the dilaton-electromagnetic coupling constant (α) for the small horizon radius. When $\alpha=1$ and $\alpha\geq\sqrt{3}$, we cannot realize any critical point in the system. Besides, for $0\leq\alpha<1$, we observe a small/large first order phase transition for $q<q_c$, while similar behaviour is seen for $1<\alpha<\sqrt{3}$ provided $q>q_c$, where q_c is the critical charge of the black hole.

This paper is structured in the following manner. In Sec. II, a brief overview on thermodynamics of the four-dimensional charged dilaton black hole in the AdS background is given. In Sec. III, we investigate critical behavior of charged dilaton AdS black holes by studying the specific heat at constant electric charge in T-S plane. In Sec. IV, we use the Gibbs free energy to determine the possible phase transition in the system. Finally, we summarize the main results Sec. V.

II. CHARGED DILATON BLACK HOLES IN ADS SPACE

We start with a brief review on charged AdS black holes in dilaton gravity and calculate the associated conserved and thermodynamic quantities. The four-dimensional action of Einstein-Maxwell gravity coupled to a dilaton field is [32, 33]

$$\mathcal{S} = -\frac{1}{16\pi}\int d^4x\sqrt{-g}\Big(\mathcal{R}-2(\nabla\varphi)^2-V(\varphi) -e^{-2\alpha\varphi}F_{\mu\nu}F^{\mu\nu}\Big), \tag{1}$$

where $\mathcal{R}$ is the Ricci scalar curvature, φ is the dilaton field and $V(\varphi)$ is the dilaton potential. Herein, the electromagnetic field tensor $F_{\mu\nu}$ is defined in terms of the gauge field A_μ via $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$. For an arbitrary value of the dilaton coupling strength α in AdS space, the dilaton potential is chosen to take the following form [34, 35]

$$V(\varphi) = \frac{2\Lambda}{3\left(\alpha^2+1\right)^2}\Big[8\alpha^2 e^{(\alpha^2-1)\varphi/\alpha}-\left(\alpha^2-3\right)e^{2\alpha\varphi} +\alpha^2\left(3\alpha^2-1\right)e^{-2\varphi/\alpha}\Big], \tag{2}$$

where Λ is the cosmological constant that relates to the AdS radius l as $\Lambda=-3/l^2$. The potential given in Eq. (2) shows that the cosmological constant Λ is coupled to the dilaton field φ in a non-trivial way. When the coupling constant $\alpha=\pm1/\sqrt{3},\pm1,\pm\sqrt{3}$, the dilaton potential in Eq.(2) is indeed the SUSY potential of string theory. Note that in the absence of the dilaton field, i.e. $V(\varphi=0)=2\Lambda$, the action Eq.(1) reduces to the usual Einstein-Maxwell theory with cosmological constant. In $3+1$ dimensions, the line element of a static spherically symmetric spacetime is written

$$ds^2=-f(\rho)dt^2+\frac{d\rho^2}{f(\rho)}+\rho^2R^2(\rho)d\Omega^2, \tag{3}$$

where $d\Omega^2$ is the metric of the 2-dimensional unit sphere with volume $\omega=4\pi$ and the metric functions $f(\rho)$ and $R(\rho)$ are given by [32]

$$f(\rho)=\left(1-\frac{b}{\rho}\right)^\gamma\left[\left(1-\frac{b}{\rho}\right)^{1-2\gamma}\left(1-\frac{c}{\rho}\right)+\frac{\rho^2}{l^2}\right], \tag{4}$$

$$R^2(\rho)=\left(1-\frac{b}{\rho}\right)^\gamma, \tag{5}$$

where b and c are integration constants and $\gamma=2\alpha^2/(\alpha^2+1)$. Also, the dilaton field and the only non-vanishing component of the gauge field A_μ are obtained as [32]

$$\varphi(\rho)=\frac{\sqrt{\gamma\,(2-\gamma)}}{2}\ln\left(1-\frac{b}{\rho}\right), \quad A_t=-\frac{q}{\rho}, \tag{6}$$

where q, an integration constant, is the charge parameter which is related to b and c via the following relation

$$q^2=\frac{bc}{\alpha^2+1}. \tag{7}$$

For $\alpha\neq0$, these solutions become imaginary in the range of $0<\rho<b$, so this region should be excluded from the spacetime. One may also have a close look on the expansion of $V(\varphi)$. Given $\varphi(\rho)$ at hand, it is a matter of calculation to show that for small α,

$$V(\varphi)=2\Lambda+4\Lambda\alpha^2\left\{\frac{b(\rho-7b/6)}{\rho^2(1-b/\rho)^2}+\ln(1-b/\rho)\right\}+\mathcal{O}(\alpha^4), \tag{8}$$

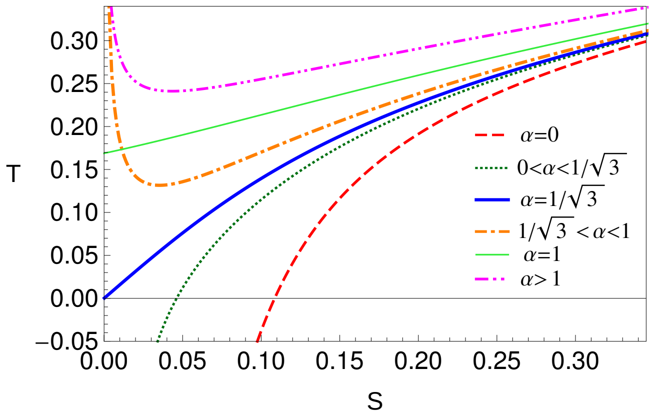


FIG. 1: T-S diagram of charged dilaton AdS black hole. This figure shows the remarkable influence of the coupling constant α on the temperature. Here, we have set $l = 1$ and $q = 1$.

which implies that, in the presence of dilaton field, the leading correction term to the cosmological constant is of order α^2. The black hole event horizon is located at $\rho = \rho_+$ which is determined by the largest real root of $f(\rho_+) = 0$. The mass and electric charge of the dilaton AdS black hole per unit volume ω are [32]

$$M = \frac{1}{8\pi}\left(c - b\frac{\alpha^2-1}{\alpha^2+1}\right), \quad Q = \frac{q}{4\pi}. \tag{9}$$

Also, the other associated thermodynamic quantities, such as the Hawking temperature T, entropy S and electric potential U, are

$$T = \frac{1}{4\pi\rho_+}\left(1 - \frac{b}{\rho_+}\right)^{1-\gamma}\left\{1 + \frac{\rho_+}{l^2}\left[3\rho_+ + 2b(\gamma-2)\right] \times\left(1-\frac{b}{\rho_+}\right)^{2(\gamma-1)}\right\}, \tag{10}$$

$$S = \frac{\rho_+^2}{4}\left(1 - \frac{b}{\rho_+}\right)^{\gamma}, \qquad U = \frac{q}{\rho_+}, \tag{11}$$

where entropy S is written per unit volume ω. It is easy to verify that the first law of black hole thermodynamics

$$dM = TdS + UdQ, \tag{12}$$

is satisfied on the event horizon [32].

It is worthwhile to mention that in the absence of the dilaton filed ($\alpha = 0$), the solutions reduce to the well-known four-dimensional Reissner-Nordstrom (RN)-AdS black hole. It is also notable to mention that these solutions are even functions in α. In the next section, we study the critical behavior of dilaton AdS black hole in the phase space.

III. CRITICAL BEHAVIOR OF CHARGED DILATON ADS BLACK HOLE

In this section we are going to investigate the effects of the dilaton field on the critical behavior of charged dilaton AdS black hole. To end this, we analyze behavior of the specific heat at constant charge

$$C_q = T\left(\frac{dS}{dT}\right)_q, \tag{13}$$

where l and α are also fixed. The sign of this quantity determines the local thermodynamic stability, i.e. the stability (instability) is accompanied by $C_q > 0$ ($C_q < 0$). To see the influence of the dilaton field (α) on C_q, we plot the behavior of the temperature as a function of entropy in Fig. 1 for different values of α and $q = 1$. It is obvious from Fig. 1, that the behavior of the black hole temperature significantly depends on α for small S. Accordingly, we expand the temperature of the charged dilaton AdS black hole for small entropy as follows:

- For $0 < \alpha < 1/\sqrt{3} \approx 0.58$,

$$T = \frac{\left(3\alpha^2-1\right)\left(\alpha^2+1\right)^{1/(\alpha^2+1)-1} q^{2/(\alpha^2+1)}}{\pi 2^{4/(\alpha^2+1)+1} l^{2-2/(\alpha^2+1)} S^{2/(\alpha^2+1)-1/2}} + \mathcal{O}\left(S^{2/(\alpha^2+1)-5/2}\right), \tag{14}$$

  the black hole is "*Reissner-Nordstrom-AdS*" (RN) type in which with decreasing entropy, the temperature goes over zero.
- For $\alpha = 1/\sqrt{3}$,

$$T = \frac{3\left(3l^2 + 12q^2 - l\sqrt{48q^2+9l^2}\right)}{2\pi l^4 q^2 (\sqrt{9+48q^2/l^2}-3)^{3/2}} \times\sqrt{3l^2+8q^2+l\sqrt{48q^2+9l^2}}S + \mathcal{O}\left(S^3\right), \tag{15}$$

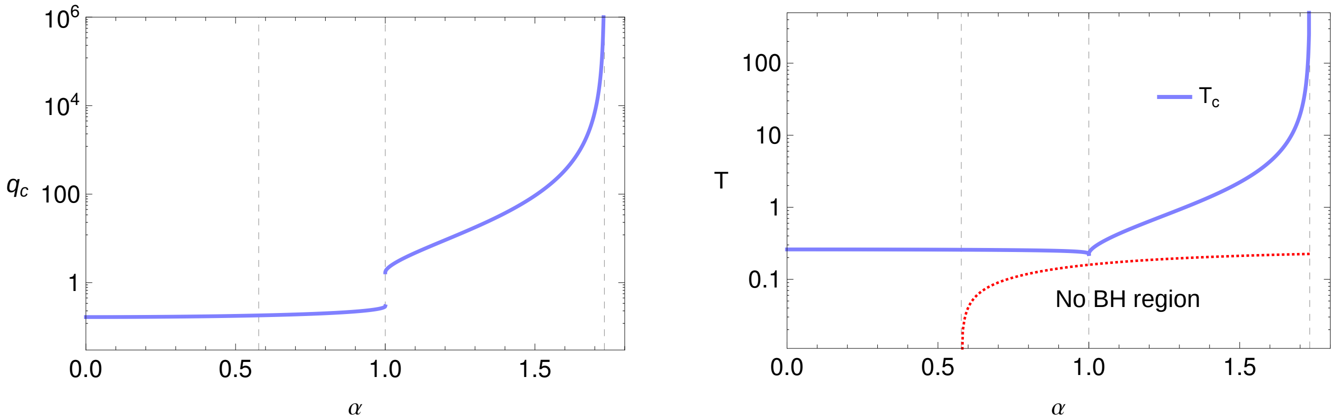
  where the dilaton black hole has zero temperature at the vanishing entropy limit. This α may be called the "*marginal coupling constant*" (α_m).
- For $1/\sqrt{3} < \alpha < 1$,

$$T = \frac{\left(3\alpha^2-1\right)\left(\alpha^2+1\right)^{1/(2\alpha^2)-1}}{\pi 2^{1/\alpha^2+1} l^2 q^{-1/\alpha^2} S^{1/(2\alpha^2)-1/2}} + \mathcal{O}\left(S^{1/(2\alpha^2)-1/2}\right), \tag{16}$$

  black hole is "*Schwarzschild-AdS*" (Schw)-type. In this case, black hole solution does not exist in the low-temperature regime.
- For $\alpha = 1$,

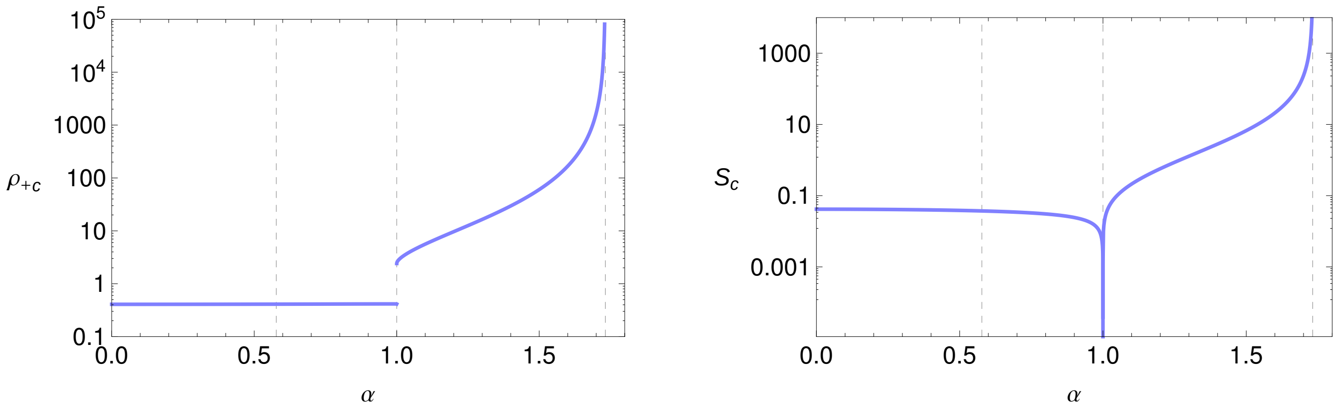
$$T = \frac{l^2+2q^2}{4\sqrt{2}l^2\pi q} + \mathcal{O}(S), \tag{17}$$

  which is the "*spacial*" case where the dilaton black hole has finite temperature at $S = 0$.



(a) q_c versus α (b) T_c versus α

FIG. 2: The behaviors of the critical electric charge (q_c) and critical temperature (T_c) versus α. The no BH region corresponds to no BH solution. The vertical dashed lines mark the values of $\alpha = 1/\sqrt{3}$, $\alpha = 1$ and $\alpha = \sqrt{3}$. We use the logarithmic scales on the vertical axis and set $l = 1$.



(a) ρ_{+c} versus α (b) S_c versus α

FIG. 3: The behaviors of the critical event horizon radius (ρ_{+c}) and critical entropy (S_c) versus α. The vertical dashed lines mark the values of $\alpha = 1/\sqrt{3}$, $\alpha = 1$ and $\alpha = \sqrt{3}$. We use the logarithmic scales on the vertical axis and set $l = 1$.

- For $\alpha > 1$,

$$T = \frac{2^{1/\alpha^2-3}\left(\alpha^2+1\right)^{-1/(2\alpha^2)}}{\pi q^{1/(\alpha^2)} S^{1/2-1/(2\alpha^2)}} + \mathcal{O}\left(S^{1/2-1/(2\alpha^2)}\right), \quad (18)$$

black hole is Schw-type again. As can be seen from Fig. 1, the right branch of isocharge for Schw-type black hole is locally stable, i.e., the specific heat at constant charge is positive. On the other hand, the large entropy limit of the temperature is

$$T \approx 3\frac{\sqrt{S}}{2\pi l^2} \quad \Rightarrow \quad C_q = 2S > 0, \quad (19)$$

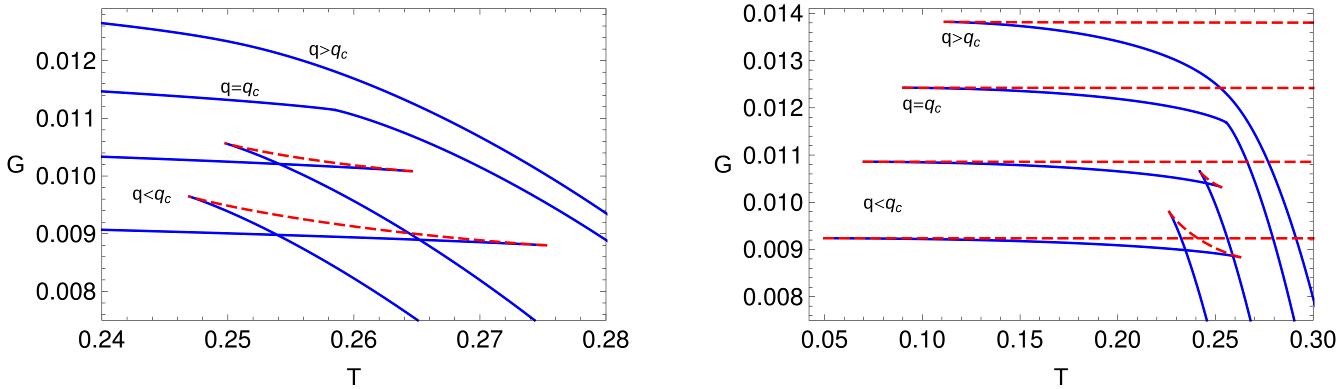
which is independent of the charge and dilaton coupling constant and always yields a thermal stable system.

In what follows, we are going to obtain the critical point, which corresponds to a second order phase transition, for various type of charged dilaton AdS black holes. For fixed q and l, the value of the critical point is characterized by the inflection point

$$\left.\frac{\partial T}{\partial S}\right|_{q_c} = 0, \qquad \left.\frac{\partial^2 T}{\partial S^2}\right|_{q_c} = 0. \quad (20)$$

To calculate the above expressions from Eq. (10) and (11), the following relation is used

$$\left.\frac{\partial T}{\partial S}\right|_q = \frac{\left.\frac{\partial T}{\partial \rho_+}\right|_{b,q} + \left.\frac{\partial T}{\partial b}\right|_{\rho_+,q}\left.\frac{\partial b}{\partial \rho_+}\right|_q}{\left.\frac{\partial S}{\partial \rho_+}\right|_b + \left.\frac{\partial S}{\partial b}\right|_{\rho_+}\left.\frac{\partial b}{\partial \rho_+}\right|_q},$$



(a) $\alpha = 0.5 \in [0, 1/\sqrt{3}]$ (b) $\alpha = 0.7 \in (1/\sqrt{3}, 1)$

FIG. 4: Gibbs free energy as a function of temperature for $l = 1$ and various values of q. For $q < q_c$, the system undergoes a first order phase transition between SBH and LBH. The positive (negative) sign of C_q is identified by the blue solid (dashed red) line. The curves are shifted for clarity.

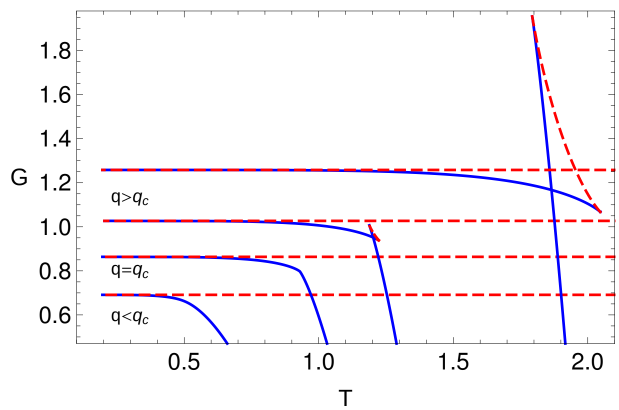


FIG. 5: Gibbs free energy as a function of temperature for $l = 1$, $\alpha = 1.3 \in (1, \sqrt{3})$ and various values of q. For $q > q_c$, the system undergoes a first order phase transition between SBH and LBH. The positive (negative) sign of C_q is identified by the blue solid (dashed red) line. The curves are shifted for clarity.

where

$$\left.\frac{\partial b}{\partial \rho_+}\right|_q = -\frac{\left.\frac{\partial f(\rho_+)}{\partial \rho_+}\right|_{b,q}}{\left.\frac{\partial f(\rho_+)}{\partial b}\right|_{\rho_+,q}}.$$

Due to the complicated form of Eqs. (10) and (20), it is almost impossible to obtain the critical values, analytically. Hence, we numerically solve the set of Eq. (20) for a given value of α. The calculated values of the critical quantities, such as q_c, T_c, ρ_{+c} and S_c, for various α are illustrated in Figs. 2 and 3. We observe that there is no critical point for charged dilaton AdS black hole in cases where $\alpha = 1$ and $\alpha \geq \sqrt{3} \approx 1.73$. As one can see from Figs. 2 and 3, for $0 < \alpha < 1$, the dilaton coupling parameter (α) does not significantly affect the critical quantities, except entropy which abruptly decreases close to 1. Fig. 2(b) shows that the critical point occurs in the RN-type of black hole when $0 \leq \alpha < 1/\sqrt{3}$, whereas for $1/\sqrt{3} < \alpha < 1$ it occurs in Schw-type where there is a lower bound on temperature of the black hole. As expected from Figs. 2 and 3, in the absence of dilaton field ($\alpha = 0$), the critical quantities reduce to those of charged AdS black hole [4]. In case of $1 < \alpha < \sqrt{3}$, with increasing α, the values of critical quantities increase and diverge for $\alpha \to \sqrt{3}$. It should also be pointed out that in the range $1 < \alpha < \sqrt{3}$, the critical behavior happens in Schw-type region.

In order to fully obtain phase transition and examine phase structure of charged dilaton AdS black holes, we shall study the behavior of Gibbs free energy in the next section.

IV. GIBBS FREE ENERGY

The general thermodynamic description of charged dilaton AdS black hole is provided by studying the Gibbs free energy which exhibits the global stable state. The Gibbs free energy for a fixed AdS radius regime can be obtained through the Legendre transformation of the mass M. Thus, the Gibbs free energy per unit volume ω is given

$$\begin{aligned} G(T,q) &= M - TS \qquad (21) \\ &= \frac{l\left(\Upsilon\left[3 - 4\Gamma - \alpha^2\right] + 2\left(\alpha^2 - 1\right) + \alpha^2 + 5\right)}{32\sqrt{2}\pi\left(\alpha^2+1\right)\Gamma^{3/2-2/(\alpha^2+1)}\left(\Upsilon - 1\right)^{-1/2}}, \end{aligned}$$

where

$$\Upsilon \equiv \sqrt{1 + \frac{4q^2\left(\alpha^2+1\right)\Gamma^{3-4/(\alpha^2+1)}}{l^2\left(1-\Gamma\right)}},$$

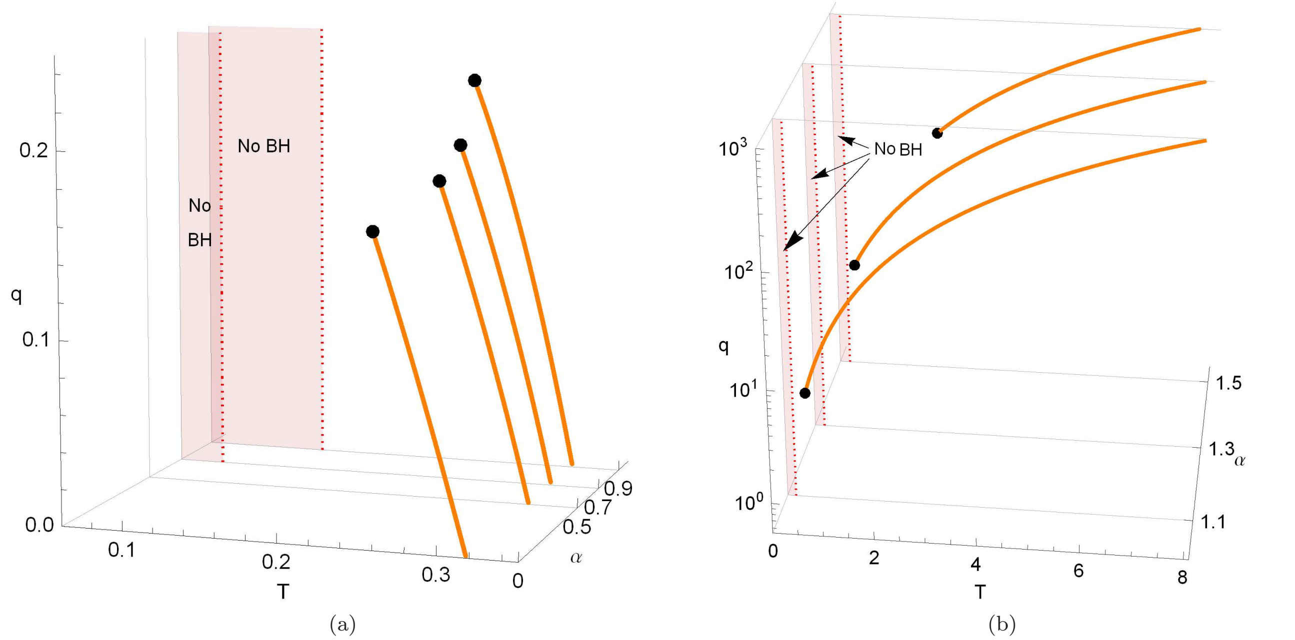


FIG. 6: SBH/LBH phase diagram for $l = 1$ and various values of α. The critical points and first order phase transition curves are highlighted by the black solid circle and solid orange line, respectively. At low temperature, no BH regions correspond to no black hole solution. We use the logarithmic scale on q axis in Fig. (b).

and $\Gamma = 1 - b/\rho_+$, thus we have $\Gamma = \Gamma(T, q)$. Notice that the reality condition of the black hole solutions ($\rho > b$) leads to the constraint $0 < \Gamma < 1$.

The behavior of Gibbs free energy in terms of the temperature T for $\alpha = 0.5, 0.7$ and 1.3 are depicted in Figs. 4 and 5, for different values of charge q. From these figures, it can be seen that the charge dependence of Gibbs free energy is strongly affected by the dilaton coupling parameter α. In case of $\alpha = 0.5 \in [0, 1/\sqrt{3}]$, where charged dilaton AdS black hole is RN-type, the Gibbs free energy is single value in temperature for $q > q_c$ (see Fig. 4(a)). In this case, back hole is locally stable ($C_q > 0$) which is indicated by the solid blue line in Fig. 4(a). On the other hand, when $q < q_c$, black hole becomes thermodynamically unstable where the Gibbs free energy is multi-valued in the certain range of temperature. This corresponds to $C_q < 0$ which is shown by dashed-red line in Fig. 4(a). This unstable behavior of the Gibbs free energy for $q < q_c$ exhibits a *first order* (discontinuous) phase transition between small black hole (SBH) and large black hole (LBH). For $\alpha = 0.7 \in (1/\sqrt{3}, 1)$ case, as illustrated in Fig. 4(b), charged dilaton AdS black hole is Schw-type where the lower (upper) branch of the Gibbs free energy is globally stable (unstable) for $q > q_c$. For $q < q_c$, a first order phase transition occurs between SBH and LBH in the lower branch of the Gibbs free energy which is stable. For $\alpha = 1.3 \in (1, \sqrt{3})$, a novel behavior happens in for Schw-type black hole (see Fig. 5). Indeed, in contrast to what occurs in Fig. 4(b), in this case a first order phase transition takes place between SBH and LBH for $q > q_c$. This behavior has not been observed in previous studies on phase transition of charged AdS black holes [25, 31]. It is notable to mention that we do not find any other phase transition for charged dilaton AdS black hole.

The corresponding SBH/LBH phase diagram of dilaton AdS black hole for different values of the dilaton parameter α is sketched in Fig. 6. It is clear from Fig. 6 that the critical points are denoted by black spots at the end of the first order phase transition curves (orange). In Fig. 6(a), the first order phase transition curves separate the SBH from LBH for $q < q_c$, while in Fig. 6(b), these curves distinguish the SBH from LBH for $q > q_c$. Also, no BH regions implies that no BH solutions exist at the low temperature.

V. SUMMARY

To sum up, we have revisited critical behaviour and phase structure of charged dilaton black holes in the background of AdS spaces. The motivation for study phase behaviour of dilaton black holes in AdS spacetime is mainly inspired by AdS/CFT correspondence and is expected to shed light on the microscopic structure of black holes. We adopted the view point that cosmological constant can be regarded as a fixed parameter, while the charge of the black hole varies.

To understand the impact of the dilaton field on the heat capacity, C_q, which determines the local thermodynamic stability of the system, we have studied the behavior of the temperature T as a function of entropy S for different values of α. By expanding T for small values of S, we have distinguished several black hole systems, with thermal stability/instability, depending on the values of α. In order to obtain the coordinates of the critical point, we numerically solved the system of equations and plotted the quantities at the critical point in terms of α. We observed that there is no critical point in cases with $\alpha = 1$ and $\alpha \geq \sqrt{3}$, while for $0 < \alpha < 1$, the critical quantities are not significantly affected by α, except entropy which abruptly decreases close to 1. In the absence of dilaton field ($\alpha = 0$), the critical quantities reduce to those of charged AdS black hole.

We have also studied the Gibss free energy, which exhibits the global stable state of the system, for different values of α and q. We have realized several cases, depending on α, including whether or not the Gibbs free energy is single/multi-valued and whether or not the system is thermally stable/unstable. Interestingly enough, we realized that unstable behavior of the Gibbs free energy for $q < q_c$ exhibits a *first order* (discontinuous) phase transition between SBH and LBH for $0 \leq \alpha < 1$. For $1 < \alpha < \sqrt{3}$, however, a novel first order phase transition happens between SBH and LBH provided $q > q_c$. The later has not been observed in the previous studies on phase transition of charged AdS black holes and is one of the new result of the present paper.

Acknowledgments

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