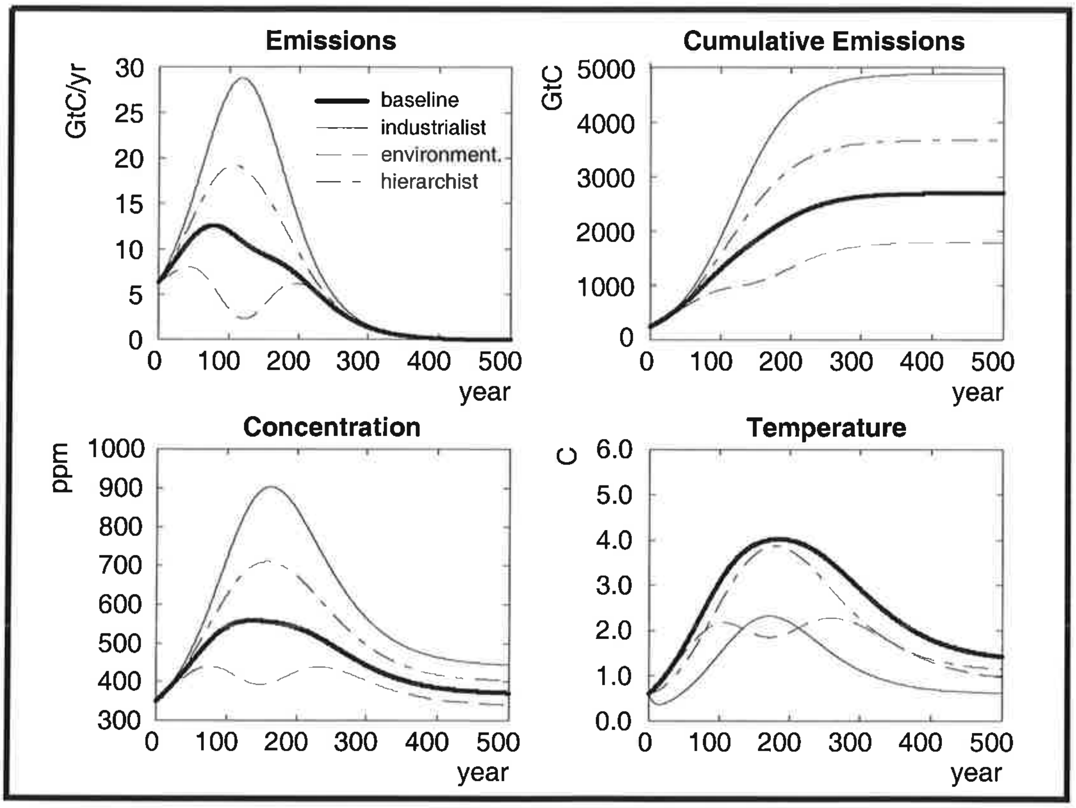
Max-Planck-Institut für Meteorologie

EXAMENSARBEIT Nr. 78



THE POTENTIAL INFLUENCE OF NATURAL CLIMATE VARIABILITY AND INCERTAINTY IN THE DESIGN OF OPTIMAL GREENHOUSE GAS EMISSION POLICIES

by
Victor Ocaña

HAMBURG, September 2000

ISSN 0938-5177

Dissertation zur Erlangung des Doktorgrades

Autor:

Victor Ocaña

Max-Planck-Institut
für Meteorologie

MAX-PLANCK-INSTITUT
FÜR METEOROLOGIE
BUNDESSTRASSE 55
D - 20146 HAMBURG
GERMANY

Tel.: +49-(0)40-4 11 73-0
Telefax: +49-(0)40-4 11 73-298
E-Mail: <name> @ dkrz.de

The potential influence of natural climate variability and uncertainty in the design of optimal greenhouse gas emission policies

Dissertation
zur Erlangung des Doktorgrades
der Naturwissenschaften im Fachbereich
Geowissenschaften
der Universität Hamburg

vorgelegt von

Victor Ocaña
aus Zaragoza (Spanien)

Hamburg
1999

ISSN 0938-5177

Als Dissertation angenommen vom Fachbereich Geowissenschaften der Universität Hamburg

auf Grund der Gutachten von Prof. Dr. Klaus Hasselmann
und Prof. Dr. Hans von Storch.

Hamburg, den 10. Dezember 1999

Prof. Dr. G. Miehlich
Dekan
des Fachbereiches Geowissenschaften

Dedicated to my father

Abstract

The possibility that human activities, through the use of land for agriculture, deforestation and emission of pollutants into the atmosphere, may alter the climate of the earth has been long recognised. Also, in recent times, the possible magnitude of those changes and their relevance for human societies has turned into a major social concern. Consequently great efforts have been made to understand the mechanisms governing climate change, both in its physical and social dimensions, and to design socio-economic actions that would prevent its adverse consequences.

In order to pursue this two-fold effort, the most widely accepted scientific paradigm consists in the use of mathematical models of diverse complexity that describe the coupled system climate - society. At present there are yet many uncertainties and imperfections in our knowledge of the system, but delaying political action until uncertainties are solved could have important consequences. Furthermore the system being considered is stochastic in nature and its natural variability may interact with and mask the causal relation between human activities and climate change.

In this work we aim at a preliminary study of the potential effects of climate's natural variability and imperfect knowledge in the design of policy action directed to reduce greenhouse gas emissions. To this end the robustness of a structural climate - economy coupled model is tested against different assumptions related to the variability of climate and the uncertainties of the system. The model adopts the form of a stochastic optimal control problem, in which an optimal greenhouse gas emission policy is sought that minimizes a stylized cost function that comprises both the damages caused by climate change and the socioeconomic costs of reducing emissions.

Results show that although some of the basic results of deterministic models remain valid, natural variability of climate may play an very important role in the design of climate protection policies, specially if it is coupled to the man-made greenhouse effect. Specially adaptation and flexibility emerge as central issues. Unfortunately, both the nature and magnitude of this coupling are highly uncertain and thus the results should be further tested.

Also uncertainties related to the magnitude and timing of climate change but most importantly to the magnitude and nature of the economic damages generated by it, may play a major role in the design of climate policies. It turns out that values and ranges of the relevant parameters of the model are very poorly known, and strong assumptions are needed to assign values to them. These assumptions are in turn highly dependent on beliefs and political agendas. As a result, the often invoked precautionary principle has to be characterised with more detail, as to where the major uncertainties are perceived and the relative values of different parts of the coupled system.

Much work needs to be done yet, in order to elucidate the nature and characteristics of climate variability and its interaction with climate change. Also the values and uncertainty ranges of parameters relevant to the design of climate policies have to be further narrowed in order to state the nature and magnitude of their influence in the policy process.

Contents

CHAPTER 1 Introduction

1.1 General framework

One of the main issues in the present debate about climate change is the design of 'climate protection' policies. That is, the design of measures to counteract the increase of the atmospheric concentration of greenhouse gases (GHG) and, most importantly, the possible effects of that change on the biosphere and on human societies. To achieve this goal, the most widely used tool has been the construction of models to perform cost-benefit analysis. The cost of reducing or stabilizing GHG emissions is evaluated against the possible damages caused by a global change in climate. One of the essential characteristics of the climate-society-biosphere interaction is the wide range of time scales involved and the presence of feedbacks and delayed responses, so that the models should be, from the beginning, dynamic (as opposed to static equilibrium models). These modelling exercises fall within the generally accepted scientific paradigm for the interaction of the environmental and socioeconomic systems, which can be, in principle, described with Global Environment and Society (GES) models (see Hasselmann 1991, Hasselmann et al. 1997) whose main subsystems and interactions are schematically depicted in figure 1.

It is usually assumed that society has reached an agreement on the definition of global welfare (as well as its distribution among the different participants of the agreement), so that a maximization task can be defined and implemented as a problem that involves a single decision maker. The definition of global welfare is the role of the *decision makers* in figure 1. Ideally, once this definition is set, models of both relevant subsystems, i.e. climate-global change and society-economy, can be coupled and used as constituent elements of the maximization task. Unfortunately there are no realistic models available. The optimization task, in addition, typically requires the iterated integration of the models and is therefore resource consuming.

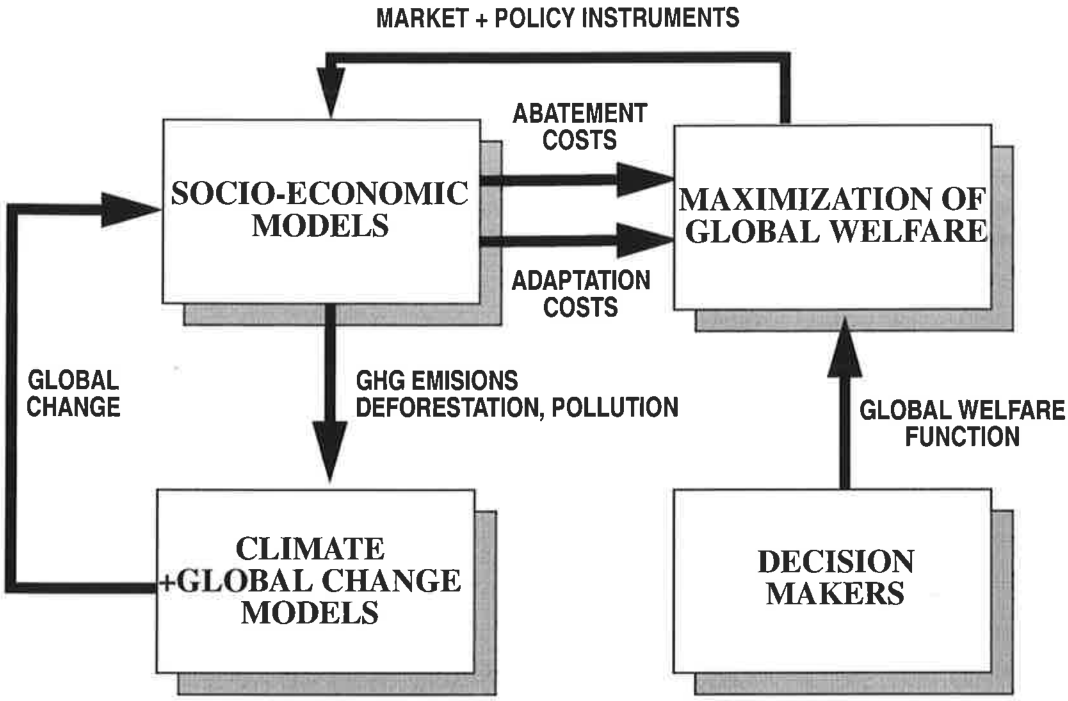


FIGURE 1. Schematic representation of a Global Environment and Society (GES) model (adapted from Hasselmann, 1991).

The usual approach in the cost-benefit analysis consists in the use of a standard economic model (the neoclassical growth model or a general equilibrium model), to which a simplified box-type climate model is added. Both modules (climate and economy) are then coupled through two links: the build-up of GHG in the atmosphere as a result of economic activity and the loss of output and welfare due to a modified climate (see for example Nordhaus (1994), Peck and Teisberg (1992), Nordhaus and Yang (1996), Tol (1997)). The degrees of complexity and aggregation vary with the models but they all share some common assumptions:

- Damage costs originated by a possible climate change are expressed as a function of global surface temperature increase since pre-industrial times (more generally, a function of the changes in whatever variables are used to monitor climate state);
- The only source of change in the climate system is the enhanced greenhouse effect (that caused by anthropogenic GHG emissions, rather than the natural one, corresponding to preindustrial concentrations of GHG). The climate system is assumed to be in an equilibrium state (no natural variability), deviations from which are originated by human activity;
- There is a one-to-one *known* causal relation between different parts of the system (emissions-temperature increase, temperature change-damages, abatement policy-costs, etc.).

Some fundamental problems appear, though, in this approach. Processes and interactions are not well known. Our knowledge of the climate system is far from perfect and predictions of future climates, based on General Circulation Model (GCM) results, uncertain (IPCC (1996a) condenses present knowledge, giving, along with the results and predictions, some uncertainty ranges). On the economic side there is, for example, the ongoing discussion about the costs of emissions-reduction (e.g. top-down vs. bottom-up models). Finally the damages caused by a change in climate conditions are very difficult to evaluate, and estimations, to the present date, are merely speculative. Also, by its nature this problem requires to plan and forecast the future, so from the point of view of modelling (or of deciding) we do not have a world (e.g. an experiment or a laboratory) to compare with. Most importantly, because of the long time scales involved in some climate processes (for example ocean uptake of CO_2 or heat) it is necessary to extend the analysis long into the future. The future is, though, uncertain, and the assumption that the structure of the world (markets, international relations) and societal values will remain as we know them, might prove wrong. Objectives are not always well defined, because of different national interests and social values, i.e. a societal agreement on global welfare has not been reached. As a result, 'climate protection' is not a well defined task. Second, the climate-socioeconomic coupled system is not deterministic, but rather stochastic. It has its own natural variability that will (at least) add to human-made changes. Thus its future evolution, as well as the effects of policies implemented, is not certain.

As a result, the schematic GES modelling approach is plagued with uncertainties, that arise from the non deterministic nature of the system at hand (what we will call *stochasticity* of the system) as well as from the imperfect knowledge of the system (what we will call *uncertainty*). We can schematically categorise uncertainty in three essentially different groups:

- the climate uncertainty, that has its origin in the imperfect knowledge of the state of the system (basic theoretical principles are known but uncertainty appears due to the huge range of space and relevant time scales)
- the valuation of costs, where basic principles are not always well defined or known, nor the interactions between the subsystems involved;
- the perception problem, where different views (from national interest to moral judgements) are intertwined with the process of valuation and policy adoption.

On the climate side, there is an ongoing scientific debate including a number of issues related to global change and the physics of climate, although there is a general agreement that the first principles and laws governing it are well known, and the discrepancies between models would be due to different parametrizations and resolution. In the climate change debate (and more precisely the greenhouse (GH) effect) there is high uncertainty concerning the different feedback mechanisms that would enhance the direct radiative effect of GHG (clouds, water vapour). Also, for a variety of reasons, some subsystems are not easily modelled, contributing to the differences between models (e.g. precipitation, oceanic circulation, criosphere). Finally some more fundamental questions arise as to whether climate models are capable of predicting future climates far away from present conditions. Another interesting and relevant debate is that of the attribution of a possibly observed climate change to a particular cause. The two main difficulties in this attribution are first the detection of a change (there are still some issues concerning the observed record of global temperatures, like the disagreements between land based observations and satellite observations, or the possibility of spurious trends in both time series) and the attribution or the discrimination between possible causes (natural variability, solar forcing, aerosols or a combination thereof). In this regard, the fact that climate has its own internal variability, that is not the result of an external forcing, and which has to be told apart from the anthropogenic climate change, is of critical importance.

A second and different level of uncertainty is found in the definition and modelling of the interaction between the climate and economic subsystems, i.e. the definition of a damage function (relating a certain change in climatic conditions to a loss of economic efficiency, output, welfare or utility) and the definition and characterization of socioeconomic instruments and mechanisms that would prevent such effects. Climate change is either perceived as an externality not accounted for in existing economic models or as a factor that directly reduces the efficiency or output of the economic subsystem under consideration. The precise mechanisms through which these effects take place are though not well characterised, and the scientific community has not agreed upon a consistent description. Nor are the effects of the possible intervention measures (e.g. taxes, carbon permits) completely known or predictable.

Finally, the assumed agreement on a global welfare definition is far from being realised in reality. Several actors involved (i.e. interest groups, nations, economic sectors, etc.) have different views of the climate-society interactions and the possible courses of action to influence it, that are shaped not only by the diverse economic and political interests but also by cultural, religious and moral beliefs. Some issues have taken much attention both in the public and the scientific arena, like intergenerational equity, the regional distribution of the burden of a climate protection policy, the appropriate rate of discount of the future or the economic valuation of human life. All of them share the characteristic that moral or subjective elements are intertwined with the objective valuation, and the descriptive and prescriptive parts of the economic analysis cannot be readily differentiated. A further source of uncertainty originates in the poor definition of the problem itself in many instances. For example, the word 'sustainability' (or the related terms sustainable development, self regenerating system and carrying capacity of the earth) is used without clearly specifying what it refers to, although there are several precise definitions of the concept (not all of them convergent). See Hasselmann (1998).

To come around these difficulties, usually a set of possible future scenarios is tested (sensitivity analysis), but the range of policies obtained for the different possible states of the world is large enough to make models almost useless as policy instruments. Also, when considering a not fully known future, a version of the *precautionary principle* is often invoked (but not quantified): in the presence of uncertainty, policies (or more aggressive policies) should be adopted to protect climate. Otherwise uncertainty and stochasticity are basically ignored or treated as a second order problem. It is known, though, that in a decision process, uncertainty has a two-sided effect: first, uncertainty biases the cost benefit analysis, even for a risk neutral planner, toward policy adoption when uncertainty lies mainly on the effects of not taking action against increasing emissions/concentration (which are always costs, negative income), i.e. the precautionary principle; second, it delays the decision on an irreversible action if the passage of time is likely to bring new information. Thus uncertainty may play a central role in the dynamics and timing of a possible climate protection policy.

1.2 Objectives of the study

The objective of the present work is basically two-fold: first we try a closer scrutiny of some of the basic assumptions fundamental to the present scientific paradigm, specially a) the concept of a *cost function*, that accounts for the economic consequences of climate change, and b) the consideration of uncertainty and stochasticity as second order effects. These issues, among others, are at the heart of the discrepancies between poles of the climate change debate. Second, we attempt to contribute further to the study of the role of uncertainty and stochasticity in the design of climate protection policies.

To this end we will, in turn, cover two main issues: the characterization of the welfare optimization problem as one of sequential optimal decision (i.e. a stochastic dynamic control problem) and the application of that analytical framework with the use of a structural coupled climate-economy model. These objectives will be realized through the detailed description of the modules (climate and climate costs) integrating the model and a collection of experiments to study the effect of stochasticity and natural variability of the system on the policy design and the possibility of including uncertainty in the decision process, beyond the if-then recipes provided by the scenario analysis.

The solution of the model will rely heavily on the well developed theory of Stochastic Optimal Control (SOC), that has found numerous applications in economic theory. Classically, three related problems are identified:

- *Estimation* of the state of the system from measurements
- *Optimization* or optimal control of the system given an appropriate objective functional
- *Identification* of the system, i.e. definition and estimation of the model parameters, functional forms and interrelations that integrate the model.

Mainly the first two tasks will be considered, combined in the form of the optimization of a stochastic system with imperfect knowledge of the state. The problem of identification will not be treated strictly in the framework of stochastic optimal control, but rather the model will be derived from first principles without explicitly including an endogenous learning process in the model (see chapter 2).

The rest of the work is organised as follows: chapter two presents the general framework within which the following sections develop. The design of optimal climate protection policies is formulated as a sequential optimal decision problem in which the relevant subsystems are a stochastic climate and a simplified economy. Also some of the basic assumptions, mainly concerning the cost function, are revised and explained. In chapter three these theoretical principles are realized in a structural simple model. Chapters four and five apply the model to two sets of experiments, to study the roles of stochasticity and uncertainty in the optimal control problem respectively. Both chapters show a selection of representative examples that highlight the major features relevant to the problem, rather than an exhaustive description of all experiments done. Chapter six concludes.

CHAPTER 2

General description of the problem

2.1 Characterisation of climate policy design as a stochastic optimal control problem

This chapter will present some general ideas and the framework within which the rest of the work will develop. The question of how much to curve the predicted future GHG emissions in order to avoid the negative consequences of global climate change, is formulated in the form of a cost-benefit analysis, that studies and balances the costs of reducing GHG emissions, resulting from human economic activity, against those originated by climate change, resulting from the build up of those gases in the atmosphere. The study of the interaction between different natural systems and human societies from an economic point of view has received much attention in recent years, and issues like environmental economics or integrated assessment of climate change have become separate fields of study on their own right. In particular, the economics of the GH effect have been intensively studied and a number of models have been designed to investigate the problem as a cost-benefit analysis (see for example Nordhaus (1991), Nordhaus (1994), Tahvonen et al. (1994), Peck and Teisberg (1992), Manne and Richels (1995), Hasselmann et al. (1991), Wigley et al. (1996), Tol (1997)). Also the applicability and its limits of standard economic techniques like cost-benefit analysis, and other methodological issues have been studied (Munda (1996), Risbey et al. (1996), O'Neill et al. (1997)).

We also introduce the basic fact that the system being modelled is stochastic in nature rather than deterministic, and that the knowledge available is not complete, and sometimes scarce. As a consequence, stochasticity and uncertainty become constituent elements of the problem, and the model includes both elements from the beginning adopting the form of a stochastic optimization. Also many authors have recognised the importance of uncertainty and, to a lesser extent, stochasticity in the

GH problem (Peck and Teisberg (1993), Peck and Teisberg (1995), Baranzini et al. (1995), Yohe (1996)).

As a result of the stochastic nature of the problem, the nature of the decision process also changes, in that flexibility, understood as the ability of the policy maker to react to the continuously incoming information, becomes a relevant issue. Climate policies have to be revised continuously and adapted to the actual value of the stochastic variables in the system. Previous work has been done in this direction: Grubb et al. (1995) analyse adaptability and inertia in the energy sector; Lempert et al. (1996) use a model in which once during the planning period the emission-reduction policy is revised; Fisher and Hanemann (1990) use a simple model to study the problem of information flow in environmental protection.

In this chapter thus, we aim at a consistent characterisation of the climate protection policy design as a Stochastic Optimal Control (SOC) problem, which lets us include uncertainty and stochasticity in our model as constituent parts. We use the mathematical structure of Dynamic Programming (DP) to quantify and specify a basic model and the different policy options available to the decision maker.

2.2 The basic model

As stated in the introduction, the basic question facing us is the design of policies to minimize the impacts of a possible climate change resulting from economic activity (and its associated emission of GHG). The main elements of our system are:

- The climate system (atmosphere, oceans, cryosphere)
- The economic-social system (markets, social preferences)
- Their mutual interaction (CO_2 build-up, damages)
- The decision process: definition of objectives and choice/implementation of policies

We use a highly aggregated model to represent the complex system described above. Due to the inherently global nature of the greenhouse warming problem, we consider an aggregated economy described by a global welfare (utility) function W, thus ignoring the problems of reaching an agreement on the definition of W, and the distribution of costs and benefits associated with climate change. This function follows a Business as Usual (BAU) path disturbed by climate costs C, which come from the damage and adaptation costs due to climate change and from the economic measures needed to abate emissions and avoid that change. That is

$$W = W_{BAU} - C$$

The decision process is summarized in the objective of maximizing welfare and the choice of some protection policy (that in our case will be the choice of a GHG emission policy). This policy is summarized by the vector of *control variables* $\boldsymbol{u}$. Since W_{BAU} is exogenously given, the maximization of W is equivalent to the minimization of C. Since the variables describing the system are not deterministic but random, the objective is properly formulated as the maximization of expected welfare (neither damages nor the costs or effects of our policy options are known with certainty) $max\mathcal{E}\{W\} = min\mathcal{E}\{C\}$. The operator $\mathcal{E}$ is the expected value defined on the set of random variables described below. Notice also that cost-benefit analysis represents to a certain extent an arbitrary choice of the function to be optimized, but can be regarded as an objective comparison tool, that under certain conditions (monetarization or transformation into utility units of all the assets in our system) provides the means to attach relative value to different policy options. In a similar way the choice of the expected value of total utility (or costs) as the argument for the minimization process provides a natural way of comparing outcomes of different options when the system contains stochastic components. One can also attempt to optimize other functions of the costs as defined above as long as certain basic conditions are fulfilled and they have a meaning (can be interpreted in terms of economic or physical quantities). For instance, if we are concerned with undesired-low-probability outcomes, we might minimize the variance of costs (or more properly the variance of deviations from a prescribed path). Alternatively, we can maintain $\mathcal{E}\{.\}$ as the optimization criterion, but use a different function of C as the argument, so as to make it, for instance, more risk averse.

The climate system is monitored through a low-dimensional system of (prognostic) *state variables* $\boldsymbol{x}$ (more precisely their deviations from an equilibrium state), which summarize the relevant features of the high dimensional complete climate. There are basically two sources for internally generated climate variations: non-linear interactions and integration of noise (von Storch and Hasselmann, (1994)). In general a simple climate model is required for our problem, which is unable to reproduce those non-linear interactions, so we will focus our analysis on the variability generated by the integration of noise. In our general framework noise represents both weather (fluctuations with characteristic times much shorter than the system of interest) and external natural forcing like variations of the solar constant or vulcanism.

Following Hasselmann (1976), (part of) the variability of a complex system can be explained as the response of its slow varying components to the stochastic forcing that the fast varying components constitute. We assume that the climate system can be broken down in two subsystems $\boldsymbol{x}$ (oceans, cryosphere) and $\boldsymbol{y}$ (atmosphere) with very different characteristic times $\tau_x >> \tau_y$. We are not interested in short time weather fluctuations, but rather on long term variations, so we centre our attention on the set of variables $\boldsymbol{x}$. Their evolution is described by

$$\begin{aligned} \dot{x}(t) &= f(x, y) \\ \dot{y}(t) &= g(x, y) \end{aligned}$$

where in the second set of equations $\boldsymbol{x}$ is usually regarded as a constant term (as in weather models) and in the first, an averaging operation is carried out, so that $\dot{x}(t) = \langle f(x, y) \rangle$ depends only on the statistical properties of $\boldsymbol{y}$. While that is right in an average sense (would deliver the mean evolution of the slow varying components), is not right for one particular realization of the climate evolution. Rather we have an equation

$$\dot{x}(t) = \langle f(x, y) \rangle + w(x, y)$$

where $\boldsymbol{w}$ is a term with zero mean that for time scales relevant for $\boldsymbol{x}$ can be regarded as ***white*** noise. Furthermore if $\boldsymbol{x}$ represent deviations from an equilibrium state $\boldsymbol{x}_0$, and for integration times short enough compared to τ_x, we can write the equations as

$$\dot{x}(t) = f(x) + w(x_0)$$

where in f the averaging operation is carried out and the statistics of $\boldsymbol{y}$ already expressed as functions of $\boldsymbol{x}$, and $\boldsymbol{w}$ is a stationary random process with statistics depending only on the reference state $\boldsymbol{x}_0$. We can broaden this definition of the random forcing to include other types of external influences not necessarily related to the internal dynamics of the system, but also acting as a stochastic forcing for the climate system (solar cycles, vulcanism, other GHGs), and drop the assumption that $\boldsymbol{w}$ is a white noise, substituting it by a more general red noise forcing.

Additionally, there may be external deterministic forcing, that in our case is the human interaction with the climate system. This anthropogenic influence is represented by the set of control variables $\boldsymbol{u}$, so that the complete climate equation is

$$\dot{x}(t) = f(x, u, t) + w$$

(the dependence on the reference state is not explicitly written any more, but the explicit time dependence indicates that some elements may be non-stationary).

The second problem we shall study is the imperfect knowledge of the modelled system, i.e. the uncertainty on the system, that translates into the fact that the model coefficients are not known with certainty. We assume that the climate evolution equations are known, i.e. the dynamics of the system, but not the particular values of the coefficients. We model this fact by considering those coefficients as random variables, that can be summarized in the random vector π. The equations now are

$$\dot{x}(t) = f(x, u, \pi, t) + w$$

(note that in general π enters the equations in a non-linear fashion). These are the *equations of motion* of the system.

Note that these two types of stochastic forcing are conceptually different: $\boldsymbol{w}$ is a true random variable, whereas π is a set of unknown parameters that let itself be modelled as a random variable.

The mutual interaction between climate and economy is described by the cost function, which is expressed as the sum of time specific costs

$$C = \int_{t_0}^{t_f} c(t)dt = \int_{t_0}^{t_f} (D(t) + G(t))dt$$

where D are damage costs and G the abatement costs.

Damages are a function of the deviations $\boldsymbol{x}$ of climate from the reference state and possibly their rate of change. Abatement costs are a function of the control (usually the rate of reduction of emissions relative to the BAU path) and its rate of change

$$D = D(x, \dot{x}, t)$$
$$G = G(u, \dot{u}, t)$$

We have now defined all the elements necessary to solve our model as a stochastic optimal control problem in which we look for the optimal policy $\boldsymbol{u}^*(t)$ that minimizes C subject to the equations of motion for $\boldsymbol{x}$, namely

$$min \mathcal{E}\{\int c(x, u, t)dt\}$$

subject to the equations

$$\dot{x}(t) = f(x, u, \pi, t) + w$$

so that both problems (intrinsic stochasticity and imperfect parameter knowledge) are part of a general set of problems of finding optimal control policies when stochastic elements are involved, i.e. stochastic optimal control.

Finally, since decisions are met in discrete time intervals, and for calculus purposes, the model will be solved in the discrete form

$$min \mathcal{E}\left\{\sum_i c_i\right\}$$

$$x_{i+1} = f_i(x_i, u_i, \pi_i) + w_i$$

where subindex i indicates time period.

2.3 The damage function

In this section we briefly analyse the main features and caveats of the cost function introduced in the previous section. As we have seen, it consists of two separate contributions: the costs of reducing carbon emissions below the levels that would be reached if the socioeconomic system would follow its BAU path, and the damages caused by climate change. The first contribution can be estimated and explained with the tools of economic theory, and is a well defined quantity. We will thus focus on the second one. The definition and estimation of the damages caused by climate change, and the design of a damage function, need contributions from several fields of knowledge like the physics of climate, biology, sociology or economics. Also, knowledge on impacts and costs of climate change, and their dynamics, is still in a very preliminary stage, and thus many interesting practical an methodological questions arise in the process of defining and quantifying the damage function. As we have seen in the previous section, a functional relation between the state of the climate and the socioeconomic system is a natural requirement for the cost-benefit analysis of the problem of designing optimal climate control policies. It provides an objective means to directly compare the economic effects related to the environment, that result from a given course of action i.e. a given climate protection policy. The choice of an optimal policy is then dictated by the comparison of the (expected) cost of adopting that policy (cost of abatement measures, introduction of new technologies) and the benefits derived from it (damages avoided and, arguably, secondary or indirect benefits).

To put this approach into perspective it is important to mention the on-going debate about *cost-benefit* vs. *cost-effectiveness* (or cost oriented vs. target oriented approaches; see for instance Tahvonen et al. (1994), Nordhaus (1997)). The former approach, as explained above, compares net costs against net benefits derived from a particular policy option. The latter sets exogenously an objective (typically a concentration or emission threshold that should be met or not surpassed) and tries to find the most (cost-) effective way and instruments to reach that objective. Both approaches are in fact very closely related to each other. Advocates of the cost-effectiveness approach argue that a cost-benefit approach might result in solutions (policies) that are not acceptable according to some previously established criteria, like sustainability or equity. Also the functional relation between climate and economy is hard to define and very uncertain, and has a certain degree of arbitrariness, in that monetarization of many assets is not straight forward or unique (for instance leisure or human life, to which cultural values are attached). For the same reasons, the definition of optimality of a particular policy would also be arbitrary. On the other hand, the cost-effectiveness approach sets the task of defining and justifying a particular objective to be met, process in which we also introduce high uncertainty and arbitrariness. Notice that in this choice we are implicitly defining a cost functional in the sense defined above and making a definition of optimality. Also the definition of objectives is often dictated by cost benefit analysis in one way or another.

It must be noted that both approaches serve different purposes and provide answers to different questions. Cost benefit analysis enlightens the basic properties of the system mainly from an economic perspective, and aids in setting typical and extreme values on the relevant parameters of the system. In the cost-benefit approach we hope we can improve our knowledge on the system and accordingly improve our choice possibilities. In the end, though, the action to be taken would respond to a political decision, which will be guided by different criteria than scientific knowledge or economic optimality.

The Damage function

As was already mentioned, a damage function, relating the state of the climate system to the socioeconomic impacts resulting from climate change is a natural requirement of the cost-benefit approach. Unfortunately, only point estimates of damages caused by an equilibrium climate are available at present, rather than a function that expresses damages as a function of a dynamic (not necessarily in equilibrium) climate state. We can schematically describe the process of constructing such a function in the following steps:

- First, point estimates for damages are derived using an equilibrium climate and comparing to a pre-defined reference socioeconomic state.
- Next, we may derive a functional relation between the economic system and different equilibrium climates.
- Since in a dynamic optimization problem we are typically not in equilibrium we may attempt at finding a relation between the economic system and the *instantaneous* climate state.

A large number of impact studies have been published that investigate the relation between climatic variables and economic factors, that vary in the regions, sectors and variables considered and resolution (see for example Cline (1992), Fankhauser (1995), Tol (1996) for valuation of costs associated to the greenhouse effect; also for an extensive review and lists of references, including impact an costs for different socioeconomic sectors and regions, see IPCC (1996b), IPCC (1996c)). All of them share some basic assumptions and methodology: typically a climate scenario derived from a GCM experiment is applied as external forcing to a model representing some sector of the socioeconomic system (agriculture, energy production, health/disease spread, coastal systems, etc.), or impacts are estimated based on *expert* opinion. For the climate scenario definition, some variables are chosen that are relevant to the socioeconomic model, as well as the appropriate resolution both temporal and spatial. The outcome of these modelling exercises is the reaction of the particular socioeconomic subsystem to a different (from present) equilibrium climate or a particular realisation of a transient climate, i.e. the first step in our idealised process.

Further, some authors (see for example Nordhaus(1993), Tahvonen et al. (1994), Tol(1996), Hasselmann et al. (1997)) postulate a simple (polynomial) function that expresses global damages as a function of some proxy climate variable, usually globally averaged surface temperature, based on the equilibrium damage estimates. We will also follow this approach, expressing damages as a function of globally averaged surface temperature and its rate of change, and using the previously mentioned point estimates to give values to the parameters of the function. It is important to keep in mind some fundamental assumptions implicitly made in this definition:

- Using a single variable, in our case globally averaged surface temperature, as proxy to represent climate change, amounts to considering all changes in climate simultaneous. A more complete function should include not only temperature, but also other relevant climate variables like sea level rise, soil moisture, precipitation, etc., which exhibit significant time lags with respect to surface temperature. This de-aggregation of the damage function would allow for a more detailed description of damages, not only in terms of timing, but also of magnitude.
- Also when using point estimates to build the damage function, we assume that equilibrium values are a good proxy for a transient climate change. The differences between transient and equilibrium values may be, though, significant. For instance, at the time of CO_2 doubling, only 50% to 80% of the equilibrium warming is realised in transient climate change experiments (IPCC (1996a)).
- Impacts and damages follow instantaneously and necessarily to a certain climate change. In reality significant time lags may also occur between climate change and climate impacts and damages. For instance forests or population migrations may take some time to react to climatic conditions. Also the damage function does not properly include adaptation of natural and socioeconomic systems to climate change. Numerous examples show, however, that adaptation to climatic conditions may play a decisive role in the climate-economy interaction (see for instance Fischer et al. (1996), Schelling (1991)) being able to offset or cancel the effect of climate change or even reverse the sign of damages.

All this simplifying assumptions suggest ways in which our definition of damages may be improved and fields in which further research is needed. Current contributions to these questions include for instance Fankhauser and Tol(1996), in which several dynamical aspects of the damage function are investigated. Concerning the role of adaptation as a strategy to cope with climate change, it has been argued that it may play a much more important role than thought until now, not only because mitigation efforts may fail according to their own goals, but because adaptation may, in its own right, be an efficient course of action to deal wit the problem at hand. For an excellent analysis of the subject see Pielke (1998).

It must be noted that the climate system has its own natural variability, both internally and externally generated, i.e. the climate state changes over time without the need of an anthropogenic forcing. We will further use the damage function above, thus assuming that natural changes and anthropogenic forced ones have the same characteristics, in terms of time scales and spatial patterns (differing possibly in intensity), and cannot be distinguished from each other. Incidentally, this also means that climate policies can control not only man-made changes, but also, with the same instruments, natural ones. In other words, we are assuming a way of partially *controlling* the climate system, which raises some interesting technical and ethical questions.

A final point arises from the economic side when considering a climate with natural variability. Damages are measured as losses compared to a reference socioeconomic-climatic state, i.e. it is assumed that under pre-industrial climatic conditions the socioeconomic system has fully adapted to its environment, and hence deviations from those conditions originate economic losses. If we consider natural variations of climate, though, we have to state precisely what these conditions are, not only in terms of the mean state but also in terms of its variability in the relevant time scales. Interannual variability, phenomena like El Niño or the Little Ice Age, all affect the socioeconomic system, or parts of it, in a variety of ways and time scales, from variations on agricultural yields to population migrations, and in general force human societies to adapt. In connection to this point it has to be further investigated if natural variations of climate have a comparable or relevant influence on growth or production, and to what extent, as the man-made changes, both in the short time scales (some studies have looked at short time seasonal and daily variability and its effect on plant growth and agricultural production, see for example Abrol and Ingram (1996); Dalton (1997) examines the welfare effects of climatic variability) and longer ones.

Since we will use a very aggregated and simple damage function, that cannot distinguish these different characteristic times nor resolve the different socioeconomic subsystems, we use the *instantaneous* climate state as the argument for the damage function. Since we describe the problem using a mathematical model that is discrete in time, instantaneous means actually averaged over the time step length of the model. This means that the damage function will take into account variations on time scales of the length of the model time step, and ignore variability in shorter time scales. Typical time steps used in economic models are 10 to 20 years, thus not capturing variations over shorter periods. For our integrations we use $\Delta t = 1 yr$.

Also in defining the damage function we arbitrarily choose the *mean* climatic preindustrial conditions as the point for which damages are zero, i.e. as our reference state. Notice though that the actual climate state will seldom be found in that particular state. On the other hand, if natural variations are relatively small, this choice of a reference state will be a good approximation.

2.4 Definition of optimal strategies

The two main elements that will characterize different policy options are information flow and flexibility, the latter understood as the possibility of postponing decisions and adapt or change policy options in the course of time. We will face then a *sequential* decision process. Information will, in turn, appear in two forms: a) knowledge of the state of the system $\boldsymbol{x}_i$ and b) of its statistical properties summarised by the transition probabilities $\mathcal{P}(\boldsymbol{x}_{i+1}|\boldsymbol{x}_i,\ldots,\boldsymbol{x}_0)$. As a starting point we will assume that the state of the system, $\boldsymbol{x}$, can be measured with certainty and is known at current time (although not in the future, since it is a random variable); also, the system $x_{i+1} = f_i(x_i, u_i, \pi_i) + w_i$ has the Markov property (knowledge of the state at time i is equivalent to knowledge at all times before i and i), i.e. all we need to describe the system is the initial probability distribution $\mathcal{P}(\boldsymbol{x}_0)$ and the one step transition probabilities $\mathcal{P}(\boldsymbol{x}_{i+1}|\boldsymbol{x}_i)$. This is a very general formulation, since all random sequences that depend on their finite past can be transformed into Markov sequences by means of state augmentation, adding $\boldsymbol{x}_{i-1},\ldots, \boldsymbol{x}_{i-N}$ as new *dummy* state variables. By augmenting the state we can thus deal with some problems including, for instance, higher order derivatives in the equations of motion or in the cost functional (see also Appendix B for the application to our model). We may also find situations in which the transition probabilities $\mathcal{P}(\boldsymbol{x}_{i+1}|\boldsymbol{x}_i)$ contain unknown elements, that we represent by a set of parameters θ (and $\mathcal{P}$ as $\mathcal{P}(\theta)$) which can also be characterised as random variables. We will assume that any new information concerning θ will come from external processes or actions, i.e. learning is exogenous. Therefore, in our experiments information available either does not change with time (if $\mathcal{P}(\boldsymbol{x}_{i+1}|\boldsymbol{x}_i)$ is known exactly), or comes from external sources, but the information set at time i is not affected by our choice of controls or by the state ($\boldsymbol{x}_i$ and $\boldsymbol{u}_i$ contain no information on θ). A system in which the properties of the random components are perfectly known (thus leaving no room for learning) is called a purely random system. A system in which some of the parameters have unknown statistical moments, and in which learning is endogenous, is called an adaptive system. In these systems (that will not be treated in this work), we can improve our knowledge on $\mathcal{P}(\theta)$ in a bayesian way by measuring the state, past controls and disturbances. In our model, we deal with a purely stochastic system, but the probabilities $\mathcal{P}$ are allowed to change according to factors outside the model, that have to be exogenously specified. It is important to note that information will become available (either in the form of the exact value of the state or the reassessment of the probability distribution $\mathcal{P}$) progressively in time, so that *flexibility* translates basically into the ability of using new information as it comes including it in the optimal policy.

In the following we define a **strategy** as a *rule*, i.e. a function that tells us what to do depending on available information (state of the system, statistics of stochastic components). On the other hand, a **policy** or a decision is a particular realization of the strategy for a determined set of values of their arguments. In the following paragraphs we precisely define the different types of strategies available to the decision maker, depending on the possibility of postponing decisions and the use of information.

Closed Loop or Feedback Strategies

A *closed loop* (CL) or *feedback* strategy is a set of functions $(F_1, \ldots, F_N)$ that specify a policy at each time period as a function of the information available at that time, i.e.

$$u_i = F_i(x_i)$$

The strategy $(F_1{}^*, \ldots, F_N{}^*)$ that minimizes the given cost functional is then the optimal strategy, and $\boldsymbol{u}^*{}_i$ gives the optimal policy corresponding to a particular realization of the random elements.

In the CL strategy, the planner takes full advantage of present information available about the climate-economy system and of the perspective of attaining new information in the future. He also has the possibility of postponing emissions reduction decisions, so that the optimal control for time *i* can be decided at that time. This situation is schematically represented in figure 2.

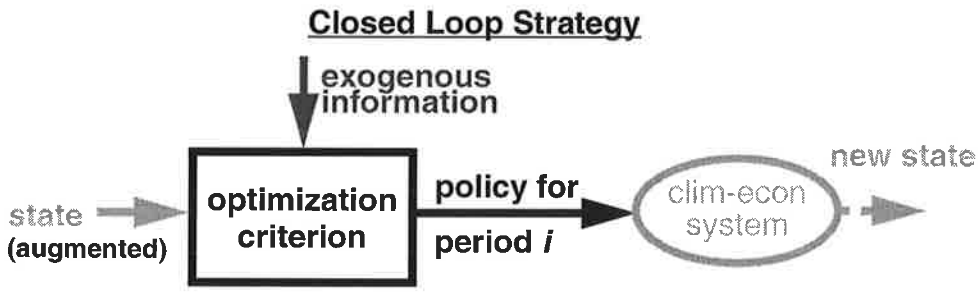


FIGURE 2. Schematic representation of the closed loop (CL) strategy. The structure showed repeats itself at each time step, at which the state of the system is measured, and knowledge revised in view of other external sources of information. An optimal policy is then selected according to the strategy given by *F*, that, when applied, generates the next system's state.

Notice that the optimization criterion includes all costs in the relevant time interval (i.e. does not act myopically considering at time *i* only costs at that time). In this way the optimization criterion box contains a rule that prescribes the proper action in view of present state and exogenous information at time *i*. Notice though that this rule takes into account both future costs and the fact that new information will become available in the future (the rule is typically obtained applying DP; see Appendix A). To obtain the optimal rule *F* we use the recursive expression (given by the DP algorithm; it is the discrete version of Bellman's equation)

$$J_i^{CL}(x_i) = min_{u_i} \mathcal{E}_i\{c_i + J_{i+1}^{CL}[f_i(x_i, u_i)]\}$$

where we define J_i as the *cost to go* function at time i, i.e. the total *optimized* costs from time i to the end of the planning horizon. Also a subindex i in the expected value function indicates that expected value is performed with (conditional to) the information available at time i, and c_i represents the instantaneous costs at time i. The CL optimal strategy adopts the form $u^*_i = F^*_i(x_i)$, i.e. is a function of present (augmented) state and time. In words, at each time i we choose the policy $\boldsymbol{u}^*_i$ that minimizes the sum of present costs and minimized future costs.

Since the state $\boldsymbol{x}_i$ and the exogenous information will only become available at time i, the optimal policy $\boldsymbol{u}^*_i$, $i=1,...,N$, is also a random variable that will be known only at time i, $i=1,...,N$.

The planner can also act myopically, by ignoring the possibility of attaining new information in the future, i.e. uses new information as it arrives at each time step, but assumes no new information will come in the future (not even the knowledge of the present state of the system). We call this course of action, following the usual notation of optimal control theory, an *Open Loop Feedback* strategy (OLF) (notice though that is a closed loop strategy, in that it specifies the optimal policy as a function of present available information). In this case, the planner calculates a whole time-dependent policy $\boldsymbol{u}^*_i$, $i=j,...,N$, from which only $\boldsymbol{u}^*_i$, $i=j$ will be applied. Then, at time $i+1$ the process is again carried out, all decisions are calculated till time N in view of new information but only the $i+1$ decision is applied.

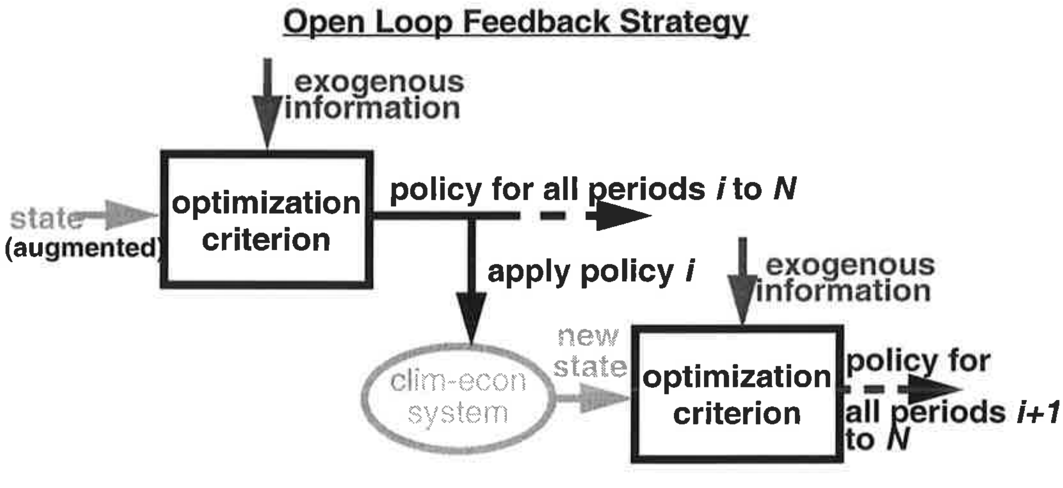


FIGURE 3. Schematic representation of the OLF strategy. As opposed to the CL strategy, the possibility of obtaining information in the future is ignored. Consequently a whole time-dependent policy $\{u_j, j=i,...,N\}$ is calculated, but only u_i is applied. The modular structure depicted repeats itself at each time step.

Once more the planner has a rule that tells him what to do in view of present information. In this case, however, the rule ignores at each time the perspective of attaining new information in the future.

The OLF strategy is obtained with the formula

$$J_i^{OLF}(x_i) = min_{u_i, \ldots, u_N} E_i\left\{ \sum_{j=i}^{N} c_j \right\}$$

that is, replaces J_{i+1} by its estimate using information available at time i. At time i all remaining decisions until time N are calculated, but feedback takes place in that only $\boldsymbol{u}_i$ is applied, and at time $i+1$ all remaining decisions are recalculated with the new information available.

In both the CL and OLF strategies the economic system represented by the control variables is assumed a certain degree of flexibility, i.e. is capable of reacting to present conditions and/or changes in the climatic-economic coupled system, by changing policies, introducing regulations or tuning existing ones (e.g. fuel taxes or agricultural adaptation). On the other hand including the rate of change of the control in the abatement costs represents the fact that the economy is not without memory, and past actions, to a certain degree, influence present ones. In this sense, capital accumulation or public perception play an important role.

Open Loop Strategies/Policies

The planner may also utterly ignore future available information (for instance if he cannot or will not postpone decisions until the corresponding time period) and take all future decisions in view of information available at initial time. He uses the *open loop* policy (OL), that specifies all future actions as a function of initial state and time. Notice that in this case OL strategy and policy are the same, since the information available is all known at initial time. In this case there is no distinction between strategy and policy or, properly, an OL policy is a particular case of the CL case, in which the functions $\boldsymbol{u}=F(\boldsymbol{x})$ are constant functions, i.e. independent of $\boldsymbol{x}$.

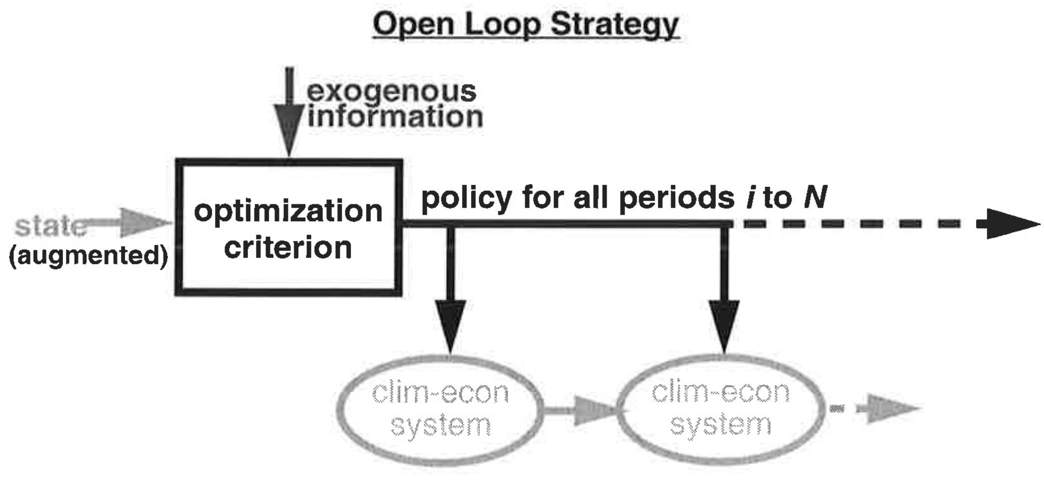


FIGURE 4. Schematic representation of the OL policy. At initial time a time-dependent policy $\{u_j, j=i,...,N\}$ is calculated, and then applied independent of the state of the system or other incoming information.

The decision rule is generated through

$$J_0^{OL}(x_0) = min_{u_1, \ldots, u_N} \mathcal{E}_0 \left\{ \sum_{i=1}^{N} c_i \right\}$$

so that all decisions are taken simultaneously.

Finally, the decision maker can ignore stochasticity and substitute all random variables with a best guess estimate (which does not have to be the expected value, but, for example, a median if he assumes the distribution non symmetric), thus using an *open loop deterministic* solution (OLD). He then uses a deterministic rule

$$J_0^{OD}(x_0) = min_{u_1, \ldots, u_N}\left\{\sum_{i=1}^{N} \bar{c}_i\right\}$$

where $\bar{c}$ designs the costs as a function of the best guess values of the parameters (and similar changes in the equations of motion).

These OL policies resemble the actual decision making process: decisions are based mainly on best guess estimates and are expressed in form of fixed emissions reduction objectives (see for instance the type of policy agreements reached in the Kyoto conference, 1997). This approach also rises important questions about the irreversibility of these kind of measures, as opposed to more flexible/less distortion-inducing instruments.

Relation between Open Loop and Closed Loop Policies

In a deterministic optimization problem CL and OL policies are identical and can be derived from one another. Starting for instance with the OL policy, that specifies the optimal control path $\boldsymbol{u}^*(t)$ as a function of time, we can integrate forward the equations of motion

$$\dot{x} = f(x, u^*, t)$$

and obtain an optimal state path $\boldsymbol{x}^*(t)$. There is though a one to one relationship between $\boldsymbol{u}^*(t)$ and $\boldsymbol{x}^*(t)$ (this is not true for any other control $\boldsymbol{u}(t)$, since a state $\boldsymbol{x}(t)$ can be reached in general with infinitely many feasible controls, only one of which is optimal) so that we can solve for $\boldsymbol{u}^*(t)$ to obtain $\boldsymbol{u}^*(t)=F^*(\boldsymbol{x},t)$. Conversely, if we start with the CL relation $\boldsymbol{u}^*(t)=F^*(\boldsymbol{x},t)$, we can integrate the equations of motion $f(x,\phi)$ to obtain $\boldsymbol{x}^*(t)$ that, when substituted in F^* delivers the control as a function of time only.

The central point here is that in order to calculate the present optimal solution we need to know not only the present, but, most importantly, the future (in order to decide an optimal emission policy now, we need to know the present state of climatic and economic variables, but also how will the climate system react or, for instance, the price of reductions in the future). In a deterministic problem, though, all information ever to be available is contained in the equations of motion and the cost functional, which are known from the start.

In this respect, the OL approach transforms the dynamic problem into a "static" one, in which controls at different times are regarded as independent decisions, and the equations of motion as restrictions on the state variables. On the other hand, in the CL approach, the recursive DP algorithm (or, for that matter, the Maximum Principle) transmits backwards the information contained in f and the cost functional.

In a stochastic problem, the situation changes, in that the future is not known and cannot be known by integrating the equations of motion. Now OL and CL policies are in general different and must be obtained from different methods. The former will specify an optimal control as a function of time and initial state. It is an irreversible policy, fixed in time. The CL policy will specify the optimal control at each time as a function of present information, which is typically the present state of the system and transition probabilities $\mathcal{P}$ (perfectly known in a purely stochastic system). Each of the policies is optimal within the class of policies to which they belong, but CL policies deliver equal or smaller total costs (the option of ignoring information is also available in the CL strategy).

In this case, the DP algorithm can transmit backwards part of the information about the future which is necessary, namely the statistical properties of the random variables and the deterministic elements, but cannot transmit the other part, the particular realization of the random variables. Thus, the form of the function $\phi^*(\boldsymbol{x},t)$ can be calculated based on the information available at initial time, but the argument $\boldsymbol{x}(t)$ will be known only at time t. This separation is possible because the state $\boldsymbol{x}(t)$ contains no information on the statistical properties of the system. On the other hand, the OL policy necessarily ignores all future information.

CHAPTER 3

Design of a simple structural climate-economy model

3.1 Introduction

In this chapter we will apply the formalism described in chapter 2 to derive a coupled structural climate-economy model. In doing so, we will be able to identify where the major uncertainties of the model are, which will be used later (chapter 5) to study their role in the decision making process. In particular we find the best guess values and ranges for some of the parameters that have the greatest influence in the optimal emissions policy. We also describe the stochastic component of the climate model that will be the basis for the experiments of chapter 4. Finally a summary of the sensitivity analysis is presented that highlights the major features of the deterministic optimal solution of the model and defines a baseline run, to which the other experiments are compared.

For the experiments performed in this work, a simple integrated coupled climate-economy model is used that includes climate change and its interaction with human activities, based on the models of Tahvonen et al. (1994) and Hasselmann et al. (1997). The main characteristics on the economic side are an exogenously determined growth rate in the absence of abatement or damages, and an also exogenous b.a.u. emission path. Climate is monitored through atmospheric carbon concentration and globally averaged surface temperature, both measured as deviations from the preindustrial state. Total discounted costs, defined as the sum of time-specific damages (D) and abatement costs (G) are minimized through the whole planning horizon

$$J = min\mathcal{E}\left\{\int_0^{\infty}(D(t) + G(t))e^{-\delta t}dt\right\} \qquad \textbf{(EQ 1)}$$

subject to the equations

$$\begin{aligned} dF(t) &= E dt \\ dC(t) &= (bF + \beta E - \sigma C)dt + dw^C \\ dT(t) &= (\mu C - \alpha T)dt + dw^T \end{aligned} \quad \text{(EQ 2)}$$

where

$$G = \gamma_G(\rho^2 + (a\dot{\rho})^2)U_0 e^{rt} \qquad \rho = 1 - \frac{E}{E^b}$$

$$D = \gamma_D\left[\left(\frac{T}{T_m}\right)^2 + \left(\frac{\dot{T}}{\dot{T}_m}\right)^2\right]U_0 e^{rt} \quad \text{(EQ 3)}$$

F(t) represents the cumulative CO_2 emissions up to time *t*, *C(t)* is the atmospheric carbon concentration relative to preindustrial time (ca. 1860), *T(t)* is the globally averaged surface temperature deviation relative to preindustrial time and E^b is the business as usual emission path. ρ is the percentual reduction in emissions relative to E^b, and we further define $\delta_r = \delta - r$ as the effective discount rate. Notice that δ_r is chosen to be negative (i.e. $\delta < r$), meaning that the future is perceived as less valuable than the present, and ensuring that the integral in equation 1 converges. Both damages and abatement costs are expressed as a percentage loss of the undisturbed (no damages or abatement) output $U_0 e^{rt}$. w^C and w^T are white noise processes with known statistical properties (Gauss-Markov random processes with known mean and variance). In addition to the additive noise forcing, some of the parameters are also considered to be random variables, to account for the poor knowledge on their value. We have included in *D*, for the sake of completeness, the rate of change of surface temperature, but, as we shall see, this term is only of secondary importance relative to the temperature related one.

We face the control of a linear stochastic system with quadratic criteria. An interesting interpretation of the model is that we try to keep the system as close as possible to a certain predetermined state (in our case the state is zero deviation from preindustrial conditions, but the model can be readily extended to minimize $(x-\bar{x})Q(x-\bar{x})$ where $\bar{x}$ is our time dependent desired state) with admissible amounts of control. The quadratic dependence ensures that large deviations from the desired state, i.e. deviations from the preindustrial climatic state, are highly penalized compared to smaller ones. Also high amounts of control, which means high reductions of the carbon emissions with respect to the b.a.u. baseline, are penalized. Such models are usually termed linear-quadratic controllers in the SOC literature.

Note that the economic part of the model has been strongly simplified: there are no dynamics in the economy, but rather an exogenous growth rate *r* is given. That means global output grows at that exponential rate, and costs, which are a percentage of output, too. All other constituent parts of the economy are left out and only costs of reduction and damages are considered.

3.2 The climate model

The climate model will be derived in two separate steps: first the functional forms and parameters are derived as if they were deterministic and then the stochastic component is added. This approach will suffice for our purposes.

The response $\boldsymbol{x}(t)$ of a complex system to a known forcing $\boldsymbol{y}(t)$ (if perturbations are small enough) can be accurately modelled in the following way

$$x(t) = \int_0^t \mathcal{G}(t-\tau)y(\tau)d\tau = \int_0^t \frac{dy}{d\tau}(\tau)\mathcal{R}(t-\tau)d\tau$$

where $\boldsymbol{x}$ is a vector of climate variables, describing the climate state, $\boldsymbol{y}$ is a forcing vector and $\mathcal{G}(t)$ the linearized impulse-response function, i.e. the response of the system to a δ-type forcing, and $\mathcal{G}(t)=d\mathcal{R}/dt$, where $\mathcal{R}(t)$ is the linear transient response to a step function forcing (both $\boldsymbol{y}(0)$ and $\mathcal{R}(0)$ are zero).

This response can be expanded as a linear superposition of individual modes

$$\mathcal{G}(t) = A_0 + \sum_{i=1}^{m} A_i e^{t/\tau_i}$$

$$\mathcal{R}(t) = \sum_{i=1}^{m} \frac{A_i}{\tau_i}(1 - e^{t/\tau_i})$$

We are interested in deriving the equivalent model in differential form, which can be easily done by performing the derivative of $\boldsymbol{x}(t)$ and using the expansion of $\mathcal{G}(t)$. For our purposes, as we shall see below, the first few modes of the expansion will be enough. Notice that both integral and differential versions of the model are totally equivalent: the expansion of $\mathcal{G}$ as a sum of exponential functions is equivalent to the choice of a set of linear differential equations, and the number of individual modes in the expansion of $\mathcal{G}$ to the number of equations.

We apply this formalism to the climate state vector, considering each component of the vector separately, the atmospheric concentration of carbon as a response to the anthropogenic emissions and the surface temperature change as a response to the concentration (it is assumed that with no anthropogenic forcing the system is in equilibrium with $C(t)=0$ and $T(t)=0$)

$$C(t) = \int_0^t \mathcal{G}_C(t-\tau)\beta E(\tau)d\tau$$

$$T(t) = \int_0^t \mathcal{G}_T(t-\tau)\mu C(\tau)d\tau$$

where a constant β has been introduced to transform units (from GtC to ppm) and functions $\mathcal{G}$ are dimensionless. Forcing on temperature is thus assumed to be a linear (rather than logarithmic) function of C. After taking derivatives we obtain

$$\dot{C}(t) = bF(t) + \beta E(t) - \sigma C(t)$$
$$\dot{T}(t) = \mu C(t) - \alpha T(t)$$
$$b = \frac{A_{C0}}{\tau_{C1}}\beta \qquad \sigma = \frac{1}{\tau_{C1}} \qquad \alpha = \frac{1}{\tau_{T1}}$$

since $G_C(0)=1$ (initially all emissions are retained in the atmosphere) and $A_{T0}=0$ (surface temperature returns to its preindustrial equilibrium state if concentration forcing is removed). Notice that after the anthropogenic carbon emissions have been set to zero, a fraction A_{C0} of the emitted CO_2 will remain in the atmosphere (A_{C0} is the asymptotic fraction), i.e. C does not return to its preindustrial equilibrium state after it has been perturbed.

A generalization of this model including a more accurate representation of the climate system, by retaining further terms in the exponential expansion of the Green's functions G_C and G_T is straightforward, resulting in a set of additional variables (and their corresponding linear differential equations). These variables can be physically interpreted as different parts of the carbon concentration and temperature going into different reservoirs with corresponding characteristic times. For CO_2, reservoirs are sinks in which carbon is sequestered at different rates (ocean, biosphere), and for temperature different thermal reservoirs (mixed layer, deep ocean). Also the formalism can be applied to other climate variables relevant to the interaction with human systems, such as sea level rise or precipitation changes.

Numerical values

The coefficient values of the approximation for the Green's functions can be empirically fitted to both observational records and the output of dynamical 3-dimensional carbon cycle models. The value of β (the conversion ppm/GtC ratio) is set to 0.47 after Meier-Reimer and Hasselmann (1987).

In the carbon cycle model we have basically two free parameters to determine: A_{C0}, the fraction of the emitted carbon that stays asymptotically in the atmosphere, and σ, basically the rate at which CO_2 is sequestered from the atmosphere. τ_{C1} is the e-folding time, i.e. the time necessary to reduce atmospheric concentration to $1/e$ of its initial value. Notice that in our simple approximation we have retained only one exponential term, thus reducing the dynamics of the carbon cycle to two sinks with *e*-folding times infinity and τ_{C1}; in the complete model, or in an approximation that retains more exponential terms several sinks with corresponding time scales come into play, so that τ_{C1} represents an *average* sink.

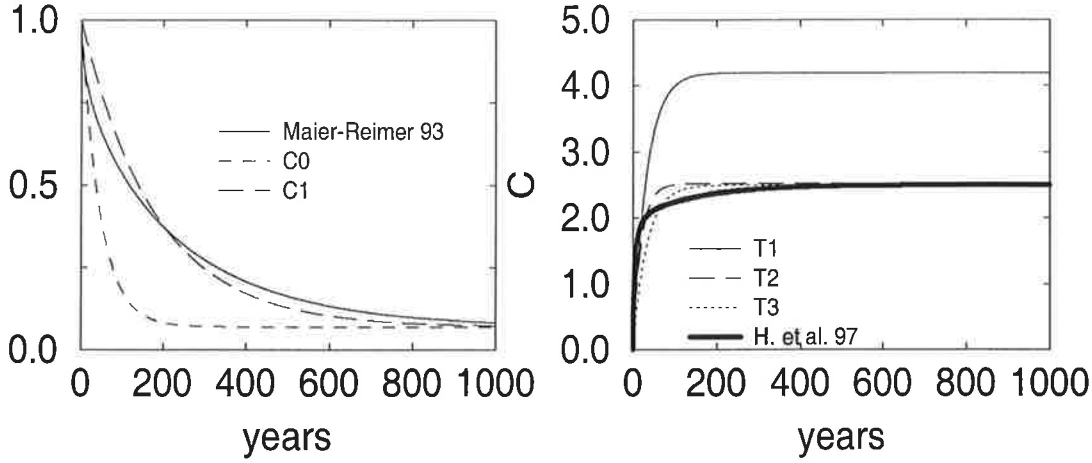


FIGURE 5. a) Response function to an emissions pulse for exponential fit for the model of Maier-Reimer (1993), plus models C0, C1. b) Temperature response to a sudden doubling of CO_2 concentration.

Estimates of A_{C0} vary between 14% in the inorganic ocean carbon cycle model of Maier-Reimer and Hasselmann (1987) and 7% in the more recent organic carbon cycle model of Maier-Reimer (1993). We fix the value of A_{C0} to the latter more recent estimate of 7% and fit σ to the observed record of CO_2 emissions and atmospheric concentration, yielding a value of 0.021 $yr.^{-1}$ (or corresponding e-folding time of τ_{C1} = 47.2 yrs.). The model resulting from the fitting exercise is named C0. Due probably to the fact that the short historical record does not capture the full time dependent dynamics of the system, we obtain too low a value for τ_{C1}, and carbon is sequestered too fast from the atmosphere (see figure 5 *a*). For example, C0 underestimates the remaining atmospheric fraction of CO_2 after 100 yrs. in about 40% compared to the model of Maier-Reimer. In a second exercise, the value of σ for fixed A_{C0} was fitted to the exponential approximation of the full model of Maier-Reimer, yielding a value of *0.0055* $yr.^{-1}$ (or τ_{C1} = *181.6 yrs.)*.The resulting model is C1(figure 5 *a*).

TABLE 1. Parameter values for both carbon cycle models C0 and C1.

Model	A_{C0}(%)	b(*ppm/GtC*)	s($yr.^{-1}$)	τ_{C1}(*yrs.*)
C0	7	$6.9x10^{-4}$	0.021	47.2
C1	7	$1.8x10^{-4}$	0.0055	181.5

Not surprisingly, model C0 performs slightly better when reproducing historical concentrations (figure 6 *c*) and rates of atmospheric storage. For conditions characteristic of the decade 1980-1989, we obtain a mean rate of atmospheric accumulation of 4.75 GtC/yr. (2.23 ppm/yr.) for model C0 and 6.5 GtC/yr. (3.05 ppm/yr.) for model C1, compared to the 3.3 ± 0.2 GtC/yr. (1.55 ± 0.01 ppm/yr.). from IPCC (1995a), or the 4.6 ± 1.7 GtC/yr.(2.16 ± 0.80 ppm/yr.) if the part going into the terrestrial sink is considered. Remember that the models used to fit the coefficients of C1 include only the ocean sink. Note also that the uncertainty concerning the absorption by the land biosphere is very high. On the other hand, C0 offers a worse fit to other more complex models for longer time scales that are relevant in the optimization problem, in which the atmospheric carbon concentration is underestimated (see figure 5 *c*, and compare to Maier-Reimer and Hasselmann (1987), where their model is forced with a logistic emission scenario that injects around 5000 GtC into the atmosphere in roughly 400 yrs.). These differences are the result of reducing the dynamics of the carbon cycle, for which only one average sink is considered, and highlight the different time scales that are important in the problem at hand, showing that for the climate system scales of several hundred years have to be taken into account.

The temperature model has also two free parameters to be fitted: μ, the radiative forcing of atmospheric carbon and α, the relaxation term that measures the time needed by the climate system to return to an equilibrium state after it has been forced out of it. Notice that we have chosen a linear model for simplicity, but a better approximation would be a logarithmic dependence of the radiative forcing on atmospheric carbon concentration, specially for the long time scales and high concentrations considered.

Both μ and α were fitted to the (smoothed) observed temperature and CO_2 atmospheric concentration records, obtaining $\alpha = 0.03$ (for an e-folding time of 33.33 yrs.) and $\mu = 0.00045$. A crucial assumption is being made for this fit that the upward trend in the observed temperature record is due to the increase in atmospheric concentration of CO_2.This amounts to considering the detection-attribution problem solved; the form of the equations, i.e. the assumption that higher concentration of GHG in the atmosphere will result in higher atmospheric temperatures can be justified in terms of physics first principles, although the amount of warming, i.e. the climate sensitivity, is contingent to uncertain issues like feedback mechanisms.

Again, due to the shortness of the observed record, the resulting model, T0, projects a very high climate sensitivity (defined as the equilibrium warming due to a doubling of the carbon concentration) of 4.2 °C, already on the upper range of IPCC projections (figure *5 b*). The equilibrium temperature in our model is given by the equation

$$T_2 = \frac{\mu}{\alpha} C_2$$

where C_2 is the doubled concentration (in our case is 280 ppm over preindustrial value). For sensitivity studies, two other temperature models where constructed normalizing them to give the same equilibrium temperature T_2 and calculating respectively the necessary μ and α when the other is that of T0. The corresponding models are shown in Table 2 for a value T_2=2.5 °C, corresponding to IPCC's best estimate.

TABLE 2. Parameter values for global temperature models T0, T1 and T2.

Model	μ(°C/ppm)	α(yrs.$^{-1}$)	t_a (yrs.)	T_2(°C)
T0	0.00045	0.03	33.3	4.2
T1	0.00045	0.05	20.0	2.5
T2	0.00030	0.03	33.3	2.5

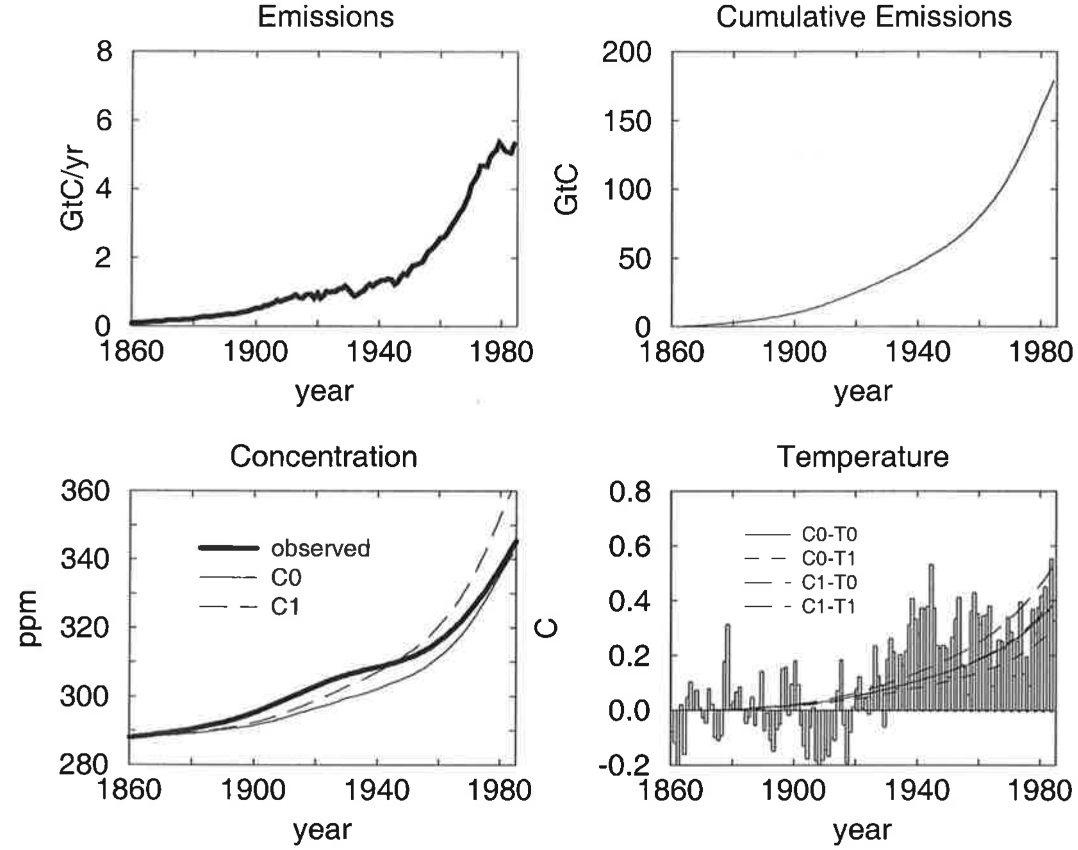


FIGURE 6. Observed emissions (a) and corresponding cumulative emissions (b), concentration (c) and temperatures (d) for all climate models.

Figure 5 *b* shows the corresponding curves for a sudden doubling of carbon concentration. It can be seen that the critical quantity is the climate sensitivity, the individual values of μ and α being not so relevant (see also figure 8).

All combinations of carbon cycle and temperature models where tested for different carbon emission paths. Figure 6 shows that all model combinations are consistent with the observed records of both concentration and temperature. On the other hand, the longer experiment depicted in figure 7, forcing with logistic emissions, shows the marked differences between the models. It must be noted though that the high concentrations and temperature reached in the logistic emissions experiment fall far outside the linearization limit of the system (concentrations reach maximum values between two and four times present day values). Also the linear forcing of concentration on temperature overestimates the climate response to GHG emissions. Nevertheless, this simple climate model captures the main features, and most importantly, the dynamic evolution depicted by more complex models, and clearly shows, as already mentioned above, that time scales of several hundred years (longer than the ones considered in many impact studies) are relevant in the climate system.

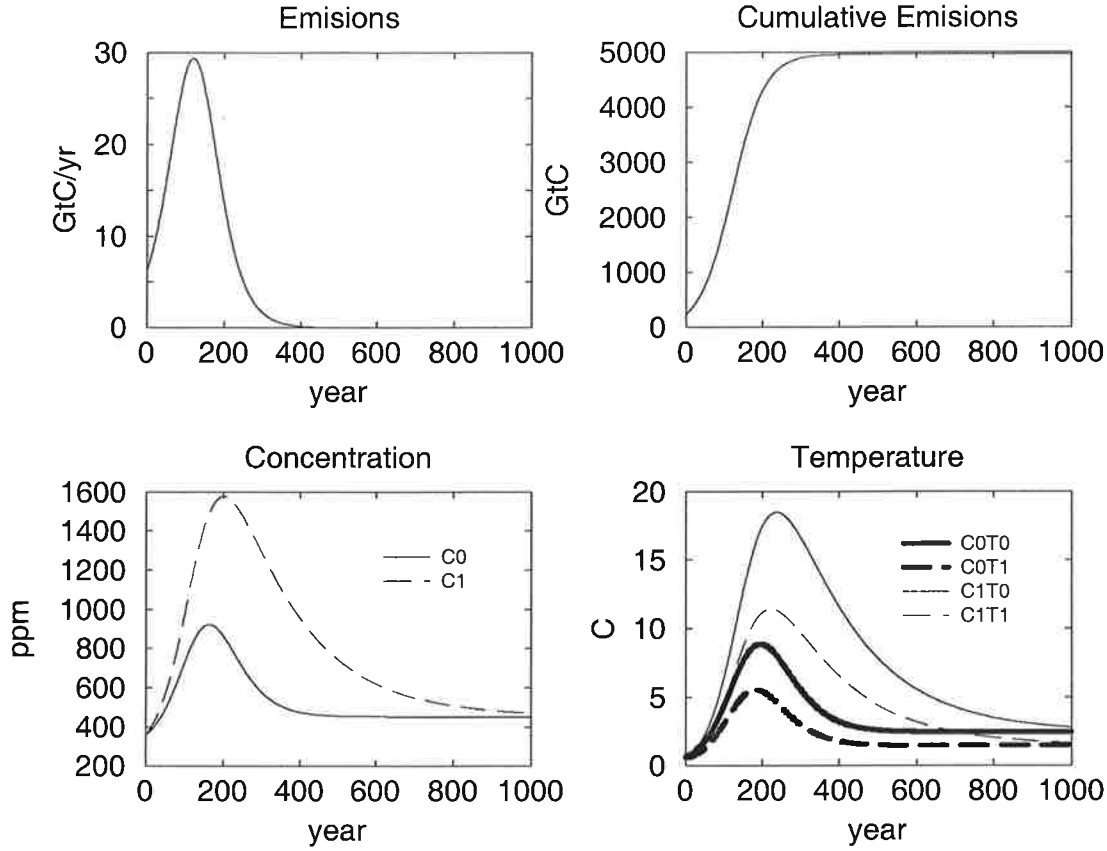


FIGURE 7. Logistic emissions (a) and corresponding cumulative emissions (b), concentration (c) and temperatures (d) for all climate models.

3.3 A stochastic climate model

In this section we will describe only the stochastic forcing related to the natural variability of the climate, i.e. what we described in the introduction as stochasticity of the system. The other form of stochastic forcing, namely the uncertainty associated to the parameters of the model, will be described in chapter 5 and the probability distributions associated will be derived.

A vector of noise processes $[0, w^C, w^T]$ can now be added to the climate model, that will act as stochastic forcing. Note that the simple model adopts the form of a stochastic one (see for example Hasselmann (1976)), in which both w^C and w^T are noise processes independent of the state of the system, with zero average and constant variance. In the absence of external forcing, i.e. if man made emissions are zero, and assuming white noise forcing, the system in equations 2 corresponds to a multivariate first order Gauss-Markov process (an AR(1) process) with the matrix of coefficients given by

$$dx = Axdt + dw \qquad A = \begin{bmatrix} -\sigma & 0 \\ \mu & -\alpha \end{bmatrix}$$

so we have a stable process with preindustrial state as mean state and a constant variance due to the noise terms.

Notice that this simple stochastic model is able to produce its own internal variability *without* external forcing, i.e. periods of higher or lower temperature and carbon concentration than the constant mean. The size and length of these periods depend on the parameters of the model (feedback/relaxation and coupling between both variables) as well as on the variance of the noise term.

Observed Climate Variability

All parameters in these equations have been derived in the previous section except for the variance of the climate variables. The reconstructed records of annual CO_2 concentration show that this quantity has remained remarkably constant for several centuries before preindustrial times, at the level of about 280 ppm. Also since the beginning of industrialization both reconstructed and observed time series show a smooth monotonously increasing curve (except for annual cycle, which gets smoothed out for $\Delta t > 1yr$), so we set $w^C=0$. Variations on longer time scales will not be considered here (for example, there is an estimated difference of around 80 ppm between present conditions and the last glacial period). On the other hand, surface temperature records as reconstructed from paleo-climatological data show significant variability basically in all time scales relevant to our problem (see for instance Harrington (1987), for a review of causes). The observed record of annual mean surface temperature anomalies presents also significant variability, both in the annual and decadal scales.

To estimate χ^T, the covariance of globally averaged surface temperature, we may make use both of modelled and observed data. On a time scale of one year we make use of the 1260 years of modelled data from the coupled run ECHAM1/LSG experiment (see J. von Storch et al. (1997)) and the Jones and Briffa observed surface temperature data set to obtain (both give similar values) a standard deviation of globally averaged surface temperature of 0.2 °C. Note that in both cases it can be argued that the time series is not stationary: the ECHAM1/LSG experiment presents in the first 500 years lower temperatures that indicate that the model may not be in stationary state yet; in the observed record it may be argued that the human influence, through greenhouse warming, is already present in the data (remember that we have used that time series to estimate the values of α and μ, assuming that the anthropogenic GH effect was responsible for the warming in recent years).

On the other hand, models are thought to underestimate natural variability (see for example Barnet et al. (1996)), and the observational record is too short for that purpose. As an alternative we may look at longer reconstructed historical records. Crowley and Kim (1996) propose a tentative 0.5-0.6 °C as a value for trough-to-peak range on the temperature record since the 15th century to preindustrial time, and 0.8-0.9 °C if we also include the period after 1850 (actually those numbers are given as a range of variations rather than a mean of those variations). Temperature reconstructions from ice cores also show radical changes in temperatures (several °C), possibly of hemispheric or global extent, taking place in the time scale of a human life-time or less (IPCC (1996a)). Barnet et al. estimate that model runs (MPI and GFDL) underestimate variability (standard deviation) on the decadal-centennial time scale by a factor of 3-4. Also Hegerl et al. (1996) estimate that on time scales of years to decades variability is underestimated by a factor of 1.5 to 2 (notice also that these control runs made with GCMs only contain variability originating in the internal dynamics of the model, leaving all other sources of change, like changes in solar input, constant). For sensitivity studies we apply the correction factor proposed by Barnett et al. to obtain standard deviation of globally averaged surface temperature of 0.6 °C.

Model Values

First we assume that the globally averaged surface temperature, in the absence of anthropogenic forcing can be modelled by a stationary AR(1) process

$$T(t+1) = \phi T(t) + w^T(t)$$

where w is white noise, with the value of the feedback parameter $\phi = 1 - \alpha\Delta t$ previously adjusted for our model. Then, temperature at each year is the realization of a random variable with constant expected value and variance. Thus we can infer the variance of the noise term, since because of the stationary character we have

$$\chi^T = Var(T) = \frac{1}{1-\phi^2}\Sigma^{TT} \text{ where } \Sigma^{TT} = Var(w^T)$$

With this relation we obtain a standard deviation for w^T of 0.05 °C, for $\alpha = 0.03$ and a time step of one year, corresponding to a standard deviation for temperature of 0.2 °C or 0.14 °C for a standard deviation for temperature of 0.6 °C.

The de-correlation time of our modelled AR(1) process is

$$\tau_D = \frac{1+\phi}{1-\phi}\Delta t$$

For the parameter values above we obtain, for a time step of one year, $\tau_D = 65.7$ yrs. (this long decorrelation time represents the fact that at the surface, atmospheric processes are largely related to the slower ocean (see J. von Storch et al. (1997)). Last, note that these values are derived quantities, that depend on the feedback (memory) parameter ϕ, which in turn is representative of the model's climate sensitivity. For example, the decorrelation time reduces to $\tau_D = 39$ yrs. for $\alpha = 0.05$ $yrs.^{-1}$, showing that for a less sensitive climate deviations from the mean have a shorter duration.

For an AR(1) process x, variability in time scales of one year and longer ones are related through

$$\frac{Var(x)}{Var(\bar{x})} = N'.$$

where $\bar{x}$ is the N year mean of x. If x is white in a period of one year, then $N=N'$; if not, as in our case (where x is the mean surface temperature), $N'<N$. We see that variability (standard deviation) of N-year means and of yearly values are proportional (with a factor $\sqrt{N'}$) so that variability decreases as N, the time scale observed, increases. The climate system though does not show this simple behaviour, and exhibits significant variability in longer time scales. Also, in the analysis above, we have only considered internal natural variability, the one resulting from the integration of *weather* forcing that can be assumed as uncorrelated (white) in time, and not other factors that contribute to the variations in climate like variations of the solar constant and vulcanism.

To include these effects we generalise the stochastic forcing $\boldsymbol{w}$. In order to represent variability in longer time scales and to take into account the fact that the other forms of noise may not be white, we let $\boldsymbol{w}$ have a memory, i.e. we force the climate system with red noise of the form $w(t+1) = \phi_w w(t) + \xi$, where ξ is a white noise process. In this way we have an extra parameter ϕ_w which lets us vary the characteristics of the noise without changing the climate sensitivity of the model, represented by α. Parameter values are chosen again by fixing χ^T and letting the covariance of the white noise forcing adjust for different values of the memory term ϕ_w.

Climate catastrophes and surprises

Another possibility that has been suggested is that of climate catastrophes and surprises, i.e. a possible evolution of the climatic system that would lead to much more drastic changes in climatic conditions than predicted as best guess values. The three major studied catastrophic possibilities (IPCC (1996c)) are the runaway GH effect (in which feedbacks would enhance anthropogenic GH effect), disintegration of the antarctic ice sheet and structural changes in the ocean circulation. In contrast to the stochastic forcing described above, in order to model these effects we have to choose a probability distribution for $\boldsymbol{w}$ that reflects somehow the characteristics of this new one: low probability of occurrence but major changes in the climate system. Unfortunately knowledge in this area is rather scarce. Manabe and Stouffer (1993) predict a shut-down of North Atlantic circulation in their $4xCO_2$ experiments. Other authors (see Jones (1991) for a review) have suggested the possibility of a 'natural salt-oscillator' in which ocean circulation would be naturally shut on and off following small changes in salinity. Evidence is though scarce and controversial, and the mechanisms controlling such changes are not well understood. Also there is a great controversy regarding the nature and sign of the feedback mechanisms that enhance GH effect (mainly water vapour and clouds).

To account for these possibilities we change the nature of the stochastic forcing. Instead of a series of small shocks added continuously, the noise forcing will consist of jumps occurring with very small probability but high amplitude. Also they will have the important characteristic of being influenced by anthropogenic climate change.

3.4 The cost function

Next we couple the climate model above to a highly aggregated cost function, representative of the economic impact of both the climatic change associated to the enhanced GH effect and the economic measures implemented to prevent that change. As stated in section 2, we assume for our model the existence of a globally averaged welfare function W that, in the absence of abatement measures or climate associated damages, follows an undisturbed growth path, the BAU path. Climate change originates deviations from this path through damages, i.e. economic losses related to the new and/or changing climatic conditions. We further assume that climate change, originated by the enhanced GH effect created by the anthropogenic carbon emissions, can be countered by the introduction of emission control policies that will reduce the accumulation of GHG in the atmosphere. These policies, though, imply in general the introduction of technological innovations and structural changes that need initial investments (and possibly initial reductions of present resource/energy consumption levels) and thus originate abatement costs (except for the debated 'no regrets' or 'free lunch' policies). The mission of the planner is thus that of balancing damage and abatement costs in order to minimize total aggregated cost or, equivalently, maximize total global welfare.

Also, according to the general model of section 2, and at a comparable level of aggregation as other models cited in the literature (see Nordhaus (1991) or Tahvonen et al. (1994)), total costs are expressed as a weighted sum (integral) of time-specific costs. Damages are a function of global climate, that in our model is solely represented by the globally averaged surface temperature T, and its rate of change (to account for parts of the earth-biosphere that react sensitively to rapid changes in climatic conditions, like certain ecosystems). Abatement costs are expressed as a function of the emissions reduction rate relative to a pre-specified BAU emissions path, ρ, and its rate of change, the latter accounting for the extra costs incurred by introducing fast distortions of the BAU path. Both damages and abatement costs are expressed as a percentage of global integrated total production. We choose a quadratic form for both components of the cost function, that also has the property of penalizing higher deviations from initial climatic state and higher reduction rates over smaller deviations or amounts of control. The cost function is given by equation 3.

The *critical* values T_m and $\dot{T}_m$ define an elliptical window (corridor) in $(T, \dot{T})$ space characterized by costs equal to γ_D% of global output whenever T_m or $\dot{T}_m$ are reached. Notice that since damage costs are modulated by a factor γ_D, the actual values of T_m and $\dot{T}_m$ are not critical, but only their relative value. Typical values cited in the literature are $T_m \sim 3°C$ and $\dot{T}_m \sim 0.01$ to $0.03°Cyr^{-1}$ but their ratio is highly uncertain. Also, we apply the same factor γ_D to both T_m and $\dot{T}_m$, but actually the contribution of the rate of change of climate is probably much less important than that of the temperature itself (the factor γ_D applied to $\dot{T}_m$ would be correct if we were to use $\dot{T}_m$ alone as a proxy for climate change).

Abatement costs are defined as a quadratic function of the reduction in emissions relative to a prescribed BAU path and its rate of change (thus penalizing fast deviations from the b.a.u. path). Again, the dynamical element is left out, in that a fixed proportion of total output is needed to achieve a percentage reduction. This proportion cannot be improved in our model by means of technological development or otherwise. Also costs are given as *long term costs*, meaning that the economy has full time to find its new equilibrium state (prices in a general equilibrium model) corresponding to the new lower level of emissions. It has been argued that short term costs would be much higher, specially for high reduction rates, since fast distortions of the economic system are more costly (see Nordhaus (1991)). In order to account for the extra costs incurred because these distortions the term $a\dot{\rho}$ is introduced (in a dynamical economy model this economic inertia is originated through the capital stock). The coefficient a is set to a value of 50 years, which means that a change of 2% in the reduction rate from previous year (per unit time) causes costs equal to an instantaneous total shut down of emissions (ρ=1).

Some generally accepted features of abatement costs are represented: the first units of reduction are relatively cheap, and additional units become increasingly expensive. On the other hand, total shut down of carbon emissions amounts for a relatively small percentage of total economic output (precisely γ_G). Another economic question is the assumption that damages grow at the rate of growth of the economy: the rationale behind that would be that the same hurricane will cause more damages in, for example, *civilised* areas (Miami) that in desert (not inhabited) ones, so that damages for equal temperature are higher the richer you are. On the other hand, the same level of physical damages can be covered with a smaller fraction of your total production if you are richer, adaptation possibilities are more and cheaper (insurance, medical care, aid for extreme events, migration possibilities,...). In other words, while damages increase at a rate of r, we could let γ_D decrease at an equivalent rate, since richer countries would be far better prepared, in the event of a catastrophe, than poorer ones, which has been used as an argument for economic development as a climate-related policy (see Murota and Ito (1996)).

Finally, we have to estimate the relative value of γ_G to γ_D, i.e. the relative importance of damage and abatement costs. We assume that if the benchmark values T_m and $\dot{T}_m$ are reached, costs are incurred equivalent to a reduction rate of 50% at an increased rate of 1% per year. A simple calculation delivers a ratio γ_G/γ_D of 1/2. Notice though that this value is calculated relative to the benchmark values of the damage function, so that the actual ratio of costs to damages depends also on T_m and $\dot{T}_m$.

3.5 *Uncertainty in the coupled climate-economy system*

Once an agreement on the global welfare function has been reached, the major obstacles in the design of an optimal climate protection policy are the complexity of the climate-economy system and its uncertainty. The latter is not only a consequence of the former, but results also from the fact that many of the subsystems involved have only recently been studied, and are still poorly known. It is therefore of crucial importance to characterise uncertainties as one of the major forces driving the present debate on climate change. In this section we discuss what are the major unknown factors in the coupled climate economy system. This unknowns will translate into a set of representative values and ranges for the parameters in our model and, eventually (chapter

5), in the estimates of uncertainty that we will use to run the stochastic model. The last section in this chapter and chapter 4 will also make use of these values to perform sensitivity analysis to investigate the characteristics of the modelled system. Finally, in chapter 5 we will turn again to the different definitions of the global welfare function to investigate the implications of assuming a globally agreed optimization criterion.

The Climate System

Probably the most important uncertainty in our present knowledge of climate change is the role of the different feedback processes that take place in the atmosphere. These feedback processes are mainly:

- Albedo (part of the incident solar radiation reflected back to space) resulting from a reduced snow cover in a warmer climate.
- Water vapour, the main GH gas in the atmosphere and hence responsible for much of the natural GH effect, whose atmospheric concentration also could increase if global temperatures do.
- Clouds. Cloud cover is expected to increase in a warmer climate, and although this would also increase planetary albedo, the infrared effect (absorption and re-emission of radiation coming from the earth) is expected to outweigh that effect.

Global warming resulting from direct CO_2 radiative effects for a doubling in atmospheric concentration has been estimated to be 0.5 -1.2 °C (see Lindzen (1994)). The predicted warming of 1.5-4.5 °C results then from the additional effect of these feedback mechanisms. Unfortunately the magnitude and sign (specially for clouds) of the feedbacks are very uncertain.

A further crucial element is the role of the ocean in global warming. It carries and exchanges with the atmosphere heat, moisture and carbon in huge quantities, not only through physical processes (transport) but with a variety of chemical and biological processes. It has been suggested though that the oceans may have a much more complex and variable behaviour than previously thought, and present understanding of the ocean circulation is inadequate. As a result forecasts, specially over the long term (decades or more) relevant for climate change, are not reliable (Wunsch, (1994), Wunsch, (1998)).

As a result, the total climate sensitivity has been estimated by IPPC to be in the range of 1.5-4.5 °C, with a best estimate value of 2.5 °C, although it is difficult to state how likely it is to find a value outside this ranges or the significance of the best guess estimate. For a comparison between GCMs that reviews both parameters relevant to our model (radiative forcing and climate sensitivity) see Cess et al. (1993). Jacoby and Prinn (1994) offer an excellent review of the uncertainties involved in climate modelling and Shackley et al. (1998) for an interesting insight in the issue of flux correction.

There is also an on going debate about the reliability of the data sets of globally averaged surface temperature. Most relevant are the discrepancies between land based measurements and satellite or balloon measurements. The land-based instrumental record of surface temperatures shows a net warming since preindustrial times of about

0.3 to 0.6 °C, that seems to be lacking in the other time series. Christy (1994) concluded that satellite measurements of infrared radiation from the earth do not confirm that warming trend, suggesting that the patterns of global warming may have a more complex structure than previously thought.

The Carbon Cycle

There are also major uncertainties in our understanding of the carbon cycle in the atmosphere-biosphere-ocean system, both concerning sources and sinks. Future anthropogenic emission scenarios need assumptions about population and economic growth, energy use and energy efficiency, alternative energy sources, agriculture and land use. As a result, these scenarios are very uncertain and cover a wide range of different futures. Predictions for carbon emissions in year 2100 range from 5 to 35 GtC/yr. for scenarios IS92c and IS92e (IPCC (1996a)). A representative review of these scenarios can be found in IPCC (1996a) and a critique of them in Gray (1998).

Also the fluxes between atmosphere, ocean and biosphere are uncertain. Further, both natural sources and sinks for carbon are affected by climate conditions (temperature, soil moisture, etc.). Most representative of these uncertainties concerns the terrestrial sink, the one that should explain the present imbalance in the global carbon budget between predicted airborne concentration and the observed one. IPCC estimates already cited, amount to 1.3 ± 1.5 GtC/yr. (to be compared to the estimated 7.1 GtC/yr. anthropogenic emissions). Similarly, emissions due to the changes in tropical land use and deforestation show also high uncertainty, being estimated to be 1.6 ± 1.0 GtC/yr. Using these and more recent figures, Gray (1998) concludes that the 90% confidence limits for the accumulation rate of CO_2 in the atmosphere are 72% of the central figure for the period 1980-1989. This value would be 143% for the 95% confidence limit.

The Climate Change Costs

In chapter 2 of this work we reviewed some of the most relevant issues concerning the estimation of the damages associated to a global climate change. The preliminary stage of our understanding and the complexity of the system make it an extremely difficult task to properly define and quantify both the impacts of climate on the socioeconomic and natural systems and the costs resulting from them. Not surprisingly it is equally difficult to identify and quantify the major uncertainties.

A major contribution to the climate change costs uncertainty comes of course from the lack of knowledge on the process and interactions between climate and the rest of nature and human societies. Also the valuation of costs inherits the uncertainties that affect both the predicted climate change (both global and, most important, regional) and the impacts of climate change. Finally, in the valuation of factors like leisure, human life, intrinsic value of species, etc. often subjective and moral considerations overwhelm the objective analysis. Turning again to the dynamical aspects of evaluating costs, not only estimates are presented as a function of a static climate change, but also for static societal values.

To summarise these uncertainties, table 3 shows the percentual contribution to total damages from different damage categories and for the most popular damage estimates (adapted from IPCC (1996c)). We see that there is barely agreement between the authors, with values different by a whole order of magnitude for the same categories. Also, the inclusion of factors like adaptation or CO_2 fertilization may radically change the damages estimates. See for instance Fischer et al. (1996) in which climate change impact on agriculture depends sensitively not only on the GCM from which climate scenarios are derived, but also on CO_2 fertilization, which may completely offset the effect of climate change on yield, and adaptation level, which also may reverse the sign of the impact. In view of table 1, it is actually rather surprising that these estimates converge to a common value of around 1-2% of global output for a standard warming of 2.5 °C corresponding to carbon concentration doubling (Smith (1996) normalises these estimates for the US to come up with an estimate of less than 1% loss of global output). Different estimates are also mainly based on personal beliefs, which differ widely among scientists (see Nordhaus 1994).

3.6 Baseline run and comparative dynamics

In this section we will describe the most salient features of the optimal solution for the deterministic version of the model. Also we will try to identify its main sensitivities to different parametrizations and assumptions. For a thorough sensitivity analysis of a similar model see Hasselmann et al. (1997). The main differences with that model are:

- The more complete version of the carbon cycle model, which is used in the integral form and approximated by a higher number of exponential terms. Compared to that model, carbon is sequestered too fast from the atmosphere.
- A different BAU emission path.
- The use in Hasselmann et al. of differentiated discount rates for damages and abatement costs, the former not being discounted. We will turn below to this point.

Baseline Run

We define a baseline run characterized by models C0 and T0 in the climate side and the parameters in the cost function depicted in table 4. The value of δ, the rate of time preference, and r, the undisturbed growth rate, are combined to give the effective discount rate δ_r. The value of r is set to 0.02 (notice that the particular values of δ and r irrelevant for the optimization process, but only their difference matters; also the global output at initial time U_0 is a multiplicative constant applied to the cost function as a whole, so it does not affect the optimal emission path either).

TABLE 3. Percentual contribution to total damage from different damage categories

DAMAGE CATEGORY	CLINE (2.5 °C)	FANKH. (2.5 °C)	NORDH. (3 °C)	TITUS (4 °C)	TOL (2.5 °C)
AGRICULTURE	29.0	12.1	1.98	0.86	13.5
FORESTS	5.4	1.1	small	31.3	-
SPECIES LOSS	6.5+	12.1	-	-	6.7
SEA LEVEL RISE	11.5	12.95	21.98	4.1	11.5
ELECTRICITY	18.3	-	1.98	4.0	-
NON-ELECT HEATING	-2.13	-	-	-	-
HUMAN AMENITY	small	-	-	-	16.2
HUMAN LIFE	9.5	16.4	-	6.8	50.4
MIGRATION	0.82	1.0	-	-	1.35
HURRICANES	1.4	0.3	-	-	0.4
LEISURE	2.8	-	-	-	-
WATER SUPPLY AVAILAB	11.5	22.5	-	8.2	-
WATER SUPPLY POLLUT	-	-	-	23.4	-
AIR POLLUTION (O_3)	5.6	10.5	-	19.6	-
MOBILE AIR CONDITION	-	-	-	1.8	-

We have chosen a common discount rate for both elements of the cost function, damages and abatement costs, although it has been argued that damages should not be discounted since they belong to a different type of goods or assets that cannot be readily monetarised. See Hasselmann et al. (1997) and the editorial comments By Nordhaus (1997), Brown (1997) and Heal (1997) for a more in-depth analysis of this point. A related point to be noted is that the model has comparable damages and costs, i.e. in the model abatement costs and damages due to climate have very similar values for our choice of coefficients. That is the reason why there is an interesting trade off between both. If abatement costs were much higher than damages, the solution would be very small reductions; if it were the other way around, the solution would be a large reduction rate (or eventually shut emissions down). Notice though that this equality is arbitrarily forced: we choose coefficients that make damages and abatement comparable, but there is few data to prove or disprove this either way.

All experiments start in 1995 and are integrated for 1000 years. The BAU emission path adopts the form of the logistic function already described in the previous section. This scenario follows a slightly faster emissions profile than, for instance, IPCC 92a, reaching values of about 28 GtC/yr. by the end of the next century (compared to roughly 20GtC/yr for IPCC 92a). BAU emissions grow for a period of around 170 years and a maximum of around 30 GtC/yr., decreasing then progressively for another two centuries, after which they stabilize at zero value. The total amount of carbon emitted is 5000 GtC (see figure 7).

We have chosen a value $\dot{T}_m = \infty$, meaning zero damages related to the rate of change of climate for the baseline run, since experiments done with different ratios of $T_m/\dot{T}_m$ show that the main effect of including the rate of change of temperature in the damage function is that of increasing damages, and hence reduce emissions, not changing though the main characteristics of the optimal solution (choice of values often cited in the literature for $\dot{T}_m$ shows also that the contribution to the damages coming from the rate of change of temperature is typically much smaller than from temperature).

TABLE 4. Cost function parameters for the baseline run

Parameter	Value	Units
δ	0.03	yr^{-1}
γ_G	7.4	%
γ_D	3.0	%
a	50	yr
T_m	3	°C
$\dot{T}_m$	∞	°C/yr.

Sensitivity analysis

The basic features of the deterministic model can be summarized in the following points:

- The relevant time scales in the climate system extend over several hundred years, much longer than the ones usually considered in climate change studies. Both the carbon cycle and the thermal inertia of the atmosphere ocean system need probably centuries to stabilize after the predicted CO_2 emissions have been reduced or stabilised. Accordingly, climate change impacts and costs may also extend over a long future, and thus optimal climate protection policies need to take into account this long term future. This has to be contrasted with the comparatively short time scales (years to decades at most) in which the socioeconomic system can be predicted or modelled.

- The form of the BAU emissions scenario conditions the form and timing of the optimal emissions path specially in the long term. It is assumed that in the future emissions will reduce to zero even in the absence of an agreed climate policy. Consequently, optimal emissions will reduce necessarily to zero and the only relevant feature to be considered is the new equilibrium state reached by the climate system. In our case, BAU emissions start to decay in about 170 years and reduce to zero after approximately 400 years, so that these two dates become the relevant scales also for the rest of the system. Uncertainty is though very high, as to the time evolution and quantity of future emissions.
- In all cases, optimal emissions paths increase for some decades before they start decreasing, rather than prescribing immediate drastic reductions. This suggests (see also Hasselmann et al. (1997)) that the optimal climate policy should proceed progressively with small initial reduction rates, thus taking full advantage of technological development and with a progressive transition to non-carbon intensive energy production. This also confirms results from other authors (Nordhaus(1993)).
- The details of the optimal emissions path are sensitive to the evolution of the climate system. Figure 8 shows the optimal emission path and associated climate response obtained for different combinations of the carbon cycle/temperature models. Changes in both models have similar effects on the optimal solution: smaller emissions for parameter combinations resulting in higher temperature responses, either from high accumulation rates of carbon in the atmosphere (and corresponding radiative forcing) or from high sensitivity of temperature to radiative forcing; since only temperature enters the cost function, it cannot distinguish the origin of the temperature variations, whether they are related to high carbon concentrations or to high sensitivity of surface temperature. Also, in the temperature model the climate sensitivity, i.e. the ratio μ/α, is the important quantity rather than the particular values of μ and α.
- Also the ratio of damages to abatement costs influences greatly the optimized emissions. The value of γ_D (or the ratio of γ_D/γ_G), as expected, controls the amount of optimal reduction by altering the relative cost of emission-reduction to damages. In the baseline we have chosen damages in the high end and abatement costs in the low end of estimates (see also IPCC95). See figure 9.

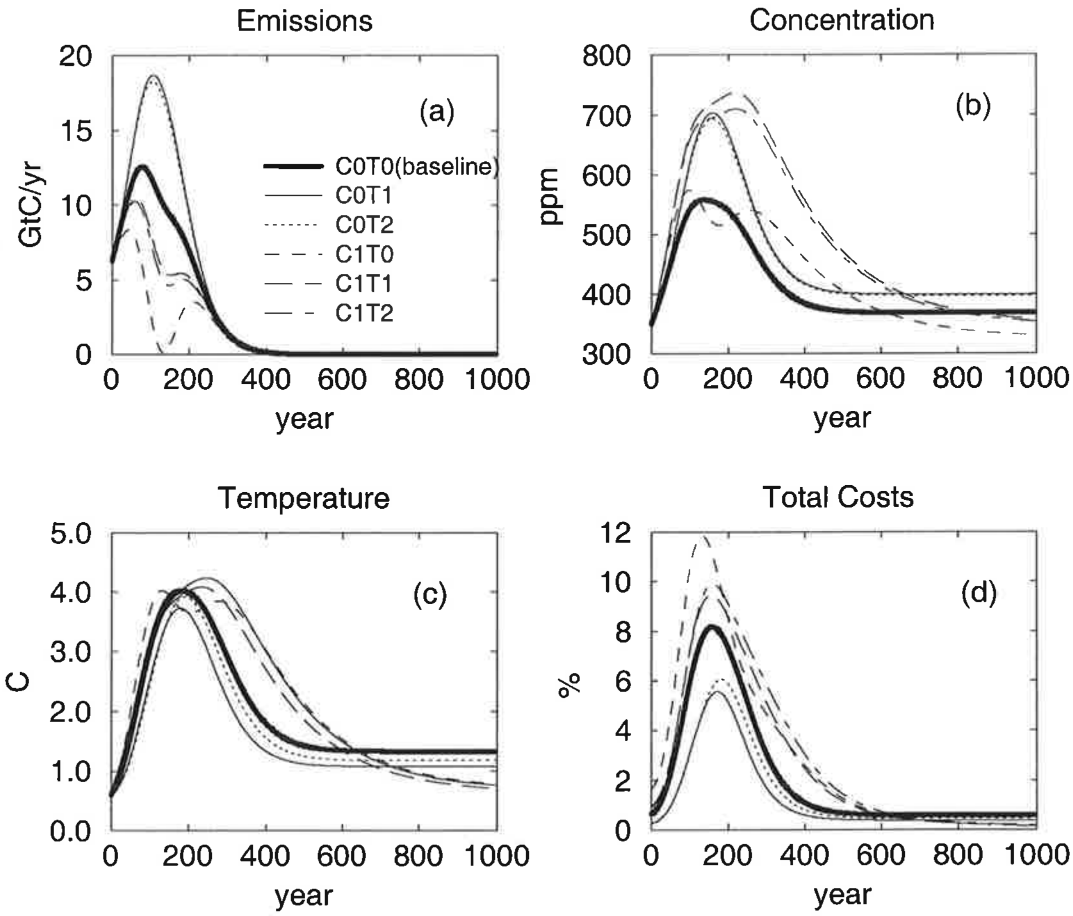


FIGURE 8. Optimized emissions (a), concentration (b), temperature change (c) and total costs (d) for different carbon cycle-temperature model combinations.

The baseline optimal emission path obtained and its associated climate response are depicted in figure 8. Emissions increase initially staying close to the BAU path, but with increasing reduction rates, reaching a maximum of 12 GtC/yr. in roughly 80 years. Thereafter emissions decrease continuously, eventually (after 300 years approx.) following again the BAU path as it approaches zero. The initial fast reduction in emissions after the maximum value is reached is followed by a period of slower reduction, which is determined by the fact that the BAU emissions start themselves going down, so that an equivalent level of emissions can be achieved with a lower reduction rate, i.e. at a lower cost. In terms of the reduction rate relative to BAU, the optimal path is reached with reductions increasing progressively to a level of about 70% in about 130 years (the date at which BAU emissions are maximum). The corresponding climate response is characterised by carbon concentrations and temperatures pretty much following the emissions path, and reaching maximum levels of 550 ppm and 4°C respectively. Asymptotically, a new equilibrium climate is reached at 360 ppm and 1.2 °C. Optimal costs (not discounted) reach a maximum value of about 8% of global integrated output and asymptotically stabilise at 0.3%, associated to the new equilibrium climate state

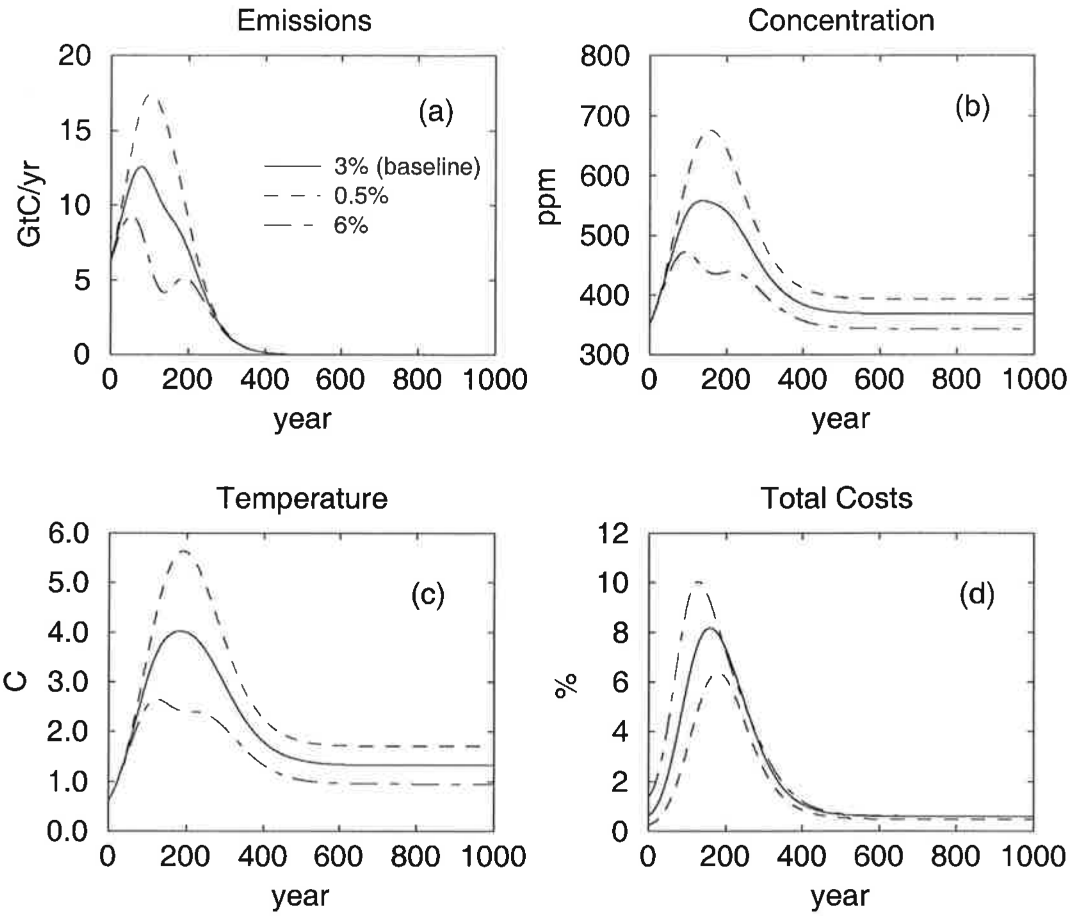


FIGURE 9. Optimized emissions (a), concentration (b), temperature change (c) and total costs (d) for different values of γ_D.

Emission profiles for different parameter assumptions are shown in figures 8, 9 and 10, for different climates and damages/abatement costs ratios. They range from relatively close to the BAU path, reaching annual emissions of around 18 GtC at the peak, and corresponding to medium climate sensitivity (ΔT_2 =2.5 °C) and carbon model C0, to high reduction levels up to 99% of the BAU for high sensitivity (ΔT_2 =4.2 °C) and model C1. Notice that temperature evolution is remarkably similar for the different climate model combinations (as opposed to the carbon concentration). In all cases high reductions are followed by a period of stabilised or even increasing annual emission rates, determined by the fact that BAU emissions start themselves going down, and at lower BAU emissions level higher reduction rates, and thus abatement costs, are needed to obtain an equivalent level of emissions.

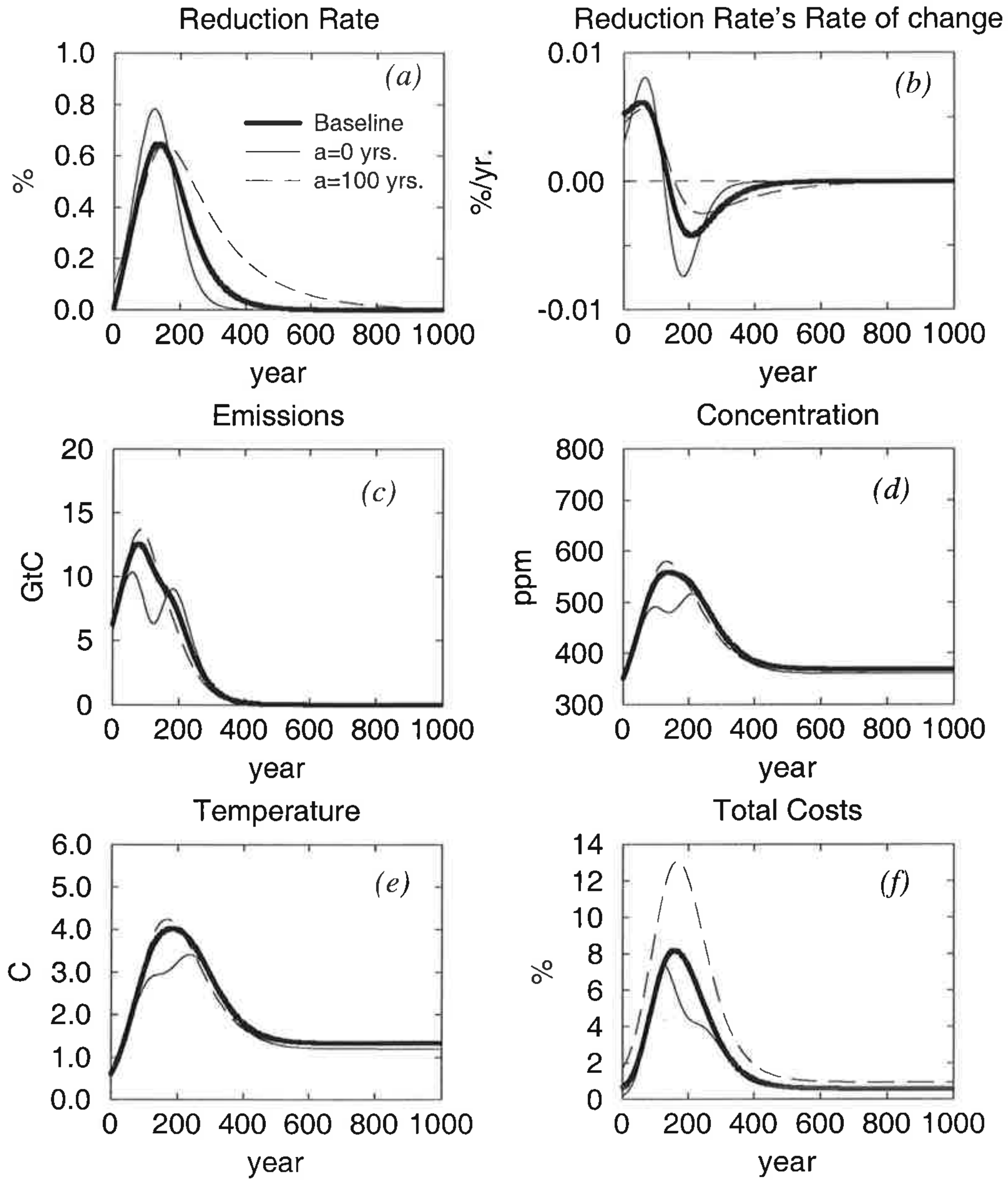


FIGURE 10. **Reduction rate (a), reduction rate's rate of change (d), optimal emissions (c) and corresponding climate response (d-f) for different values of the economic inertia *a*.**

Irreversibility

As we shall see through the reminder of this work, some of the fundamental characteristics of the problem of designing optimal control problems can be described in terms of its irreversibilities. Basically, irreversibility is two-fold, becoming apparent both in the climate system and in the economic system. On the climate side the irreversibility reveals itself in the long residence time of GHGs in the atmosphere and the thermal inertia of the atmosphere-ocean system already described in the previous sections. This phenomena are captured in the climate equations describing atmospheric concentration of CO_2 and globally averaged surface temperature. On the economic side, abatement capital irreversibility has been identified as a major feature (Kolstad (1994), Kolstad (1996), Baranzini et al. (1995)), i.e. abatement measures need initial investments that cannot be readily undone or redirected. This two types of irreversibilities are at the heart of the climate change debate, and different (opposed) views stress, accordingly, one or the other type.

The cost function described in equation 3 takes the latter form of irreversibility explicitly into account by including a term which is a function of the abatement rate's rate of change, $\dot{\rho}$. This term induces extra costs when changes in the reduction policy are introduced, i.e. once engaged in a particular policy there is an incentive to maintain it. Large values of the economic inertia result in reductions taking place at a later date, although since BAU emissions decrease also, high reduction ratés far in the future have a relatively small effect on total emissions.

CHAPTER 4

Optimal emission policies for an stochastic climate system

4.1 Introduction

As already has been explained in previous chapters, one of the salient features of the earth's climate is that it presents natural variations of relevant magnitude in basically all time scales. Briefly, these variations can be attributed to internal dynamics (non-linear interactions, integration of weather noise) or to external forcing (sun-spot cycle, vulcanism, Milankovitch cycle). These natural variations will be added to the man made induced ones, in particular the GH effect caused by anthropogenic emissions of carbon dioxide (and other GH gases).

From the point of view of designing climate protection policies three main characteristics are crucial to describe climate variability:

- Its *smoothness*, i.e. whether variations take place over a long time at a slow rate or if they happen abruptly. Variations between the little ice age and present (or preindustrial) climate conditions have occurred smoothly over the last several centuries; on the other hand ice-core temperature reconstructions seem to show changes of hemispheric extent and of several degrees C, happening in a human life-time.
- A second important characteristic is the magnitude of the variations.
- Finally, we consider the influence of anthropogenic forcing on natural variability. It has been speculated that global warming will change not only the mean state of the climate, but also its variability in several time scales (interannual, decadal), although it is not clear in which direction or the magnitude of this effect (see IPCC (1996a)). We also mentioned before the possibility that global warming could trigger sudden climatic changes like the shut down of the conveyor belt or the melting of the ice sheets.

A number of experiments were designed according to these characteristics of climate variability. Smooth variations of global temperature were simulated by adding small shocks that take place at each time step and are integrated by the climate system, generating low-frequency variations. Cases are considered in which these shocks are independent of the current temperature and in which their statistics are a function of average surface temperature, to simulate the influence of global change on the variability of climate. These smooth variations will be analysed in sections 4.2 and 4.3. Climate catastrophes and surprises are simulated with a forcing term consisting of climate shocks with very low probability of occurrence but very high amplitude. This amplitude is furthermore a function of the already existing global temperature. The effect of climate surprises is analysed in section 4.4. Throughout the chapter all parameters of the model are supposed to be known, except in the sensitivity analysis, for which parameters take different but fixed and known values.

To characterise the effect of natural climate variability on the design of climate protection policies we will study the optimal carbon emission reduction rate. The associated costs will be compared to the expected costs resulting from applying the deterministic best guess policy, i.e. the emission reduction policy that would be optimal if the problem had no stochastic component. We define the quantities

$$\Delta J_1 = J^{OD} - J^{CL}$$

$$\Delta J_2 = J^{OL} - J^{CL}$$

ΔJ_1 is the difference between the expected integrated costs associated to the deterministic policy (J^{OD}) and the optimal costs resulting from the CL policy (J^{CL}). ΔJ_1 represents thus the extra expected costs resulting from ignoring stochasticity of the climate system in the design of climate policies. ΔJ_2 is the extra costs that can be avoided by the choice of the best optimal policy, the cost of irreversibility. For a more intuitive analysis we express them as percentages in the following form

$$\Delta JP_1 = \frac{J^{OD} - J^{CL}}{J^{CL}} \times 100$$

$$\Delta JP_2 = \frac{J^{OL} - J^{CL}}{\Delta J_1} \times 100$$

We further define a third quantity ΔJ_D as the difference between the expected costs associated to a deterministic policy (J^{OD}) and the deterministic estimate of the costs (J^{DET}). Notice that when calculating J^{DET} stochasticity is fully ignored. ΔJ_D represents the estimation error that we make by ignoring stochasticity of climate change in the calculation of climate protection costs. We have then

$$\Delta J_D = J^{OD} - J^{DET} \qquad \text{or} \qquad \Delta JP_D = \frac{J^{OD} - J^{DET}}{J^{DET}} \times 100$$

Notice that these quantities are expected values.

General results

We can summarise the main results of this chapter in the following points:

- The first important consequence of climate's natural variability is that optimal costs are higher than in the deterministic case, i.e. by considering the stochastic problem as a deterministic one we underestimate the real value of the objective function. This is a general result of stochastic optimal control, and other authors have also recognised and studied this fact in regard to climate change; see Dalton (1997). Notice that even in the absence of human intervention that drives the climate system out of its equilibrium state, damages are not zero, since global temperature has in general a value different from its mean, which we have used to characterise that equilibrium state. The error in the estimate of climate change costs made if natural variability if climate is ignored may be very big.
- Part of the extra costs induced by climate variability can be avoided by the choice of policy instruments that allow for flexibility and react to the changing conditions of the climate system. We see then that minimal expected costs are achieved with a CL strategy, one that interactively takes into account the present state of the climate vector and *reacts* to it. The use of OL policies deliver higher optimal costs than that of CL. This is also a general result of SOC (it can be shown that the set of feasible policies available for OL policies is a strict subset of that for CL, so that for the latter case we can find at least as good a minimum, or possibly a better one, as for the former).
- The optimal climate protection strategy is in general a combination of adaptation and prevention. OL policies can prevent some of the potential effects of climate natural variability and its combined effect with climate change. Additionally, the choice of CL policies allows for the updating of emission strategies to the particular realization of the climate vector, i.e. allows for adaptation to present conditions. The relative importance of this two mechanisms depends on the particular nature of the processes studied (feedback between climate change and natural variability, climate surprises).
- Since the climate system is unknown a priori, so too is the optimal CL reduction rate (the optimal policy) that minimizes expected costs, so that flexibility, the ability to adapt our decisions to present conditions, in the choice of a climate protection policy becomes a relevant issue. Also, in general, information 'degrades' with time, so that the further we look into the future, the wider the range of values where we find the optimal reduction rate. This effect is countered in our case by the fact that the BAU emission path becomes zero in the finite future, thus making the optimal reduction rate also zero with certainty.
- The coupling between natural variability and climate change may have a dramatic effect on the optimal GHG emission path. Although the precise nature (or the sign) of this coupling is not yet well understood, this feedback mechanism is identified as potentially very important, since natural variability may a) enhance or counter the GH effect and b) be enhanced through the interaction with global warming.

Further research is necessary to understand the interaction between climate variability in different time scales, specially those relevant to the socioeconomic system, and climate change.

- A second potentially important effect is the risk of climate catastrophes or surprises that would be triggered by global warming, like sudden changes in the ocean circulation or the melting of the ice sheets. A form of the precautionary principle applied to this problem suggests that if the risk of climate catastrophes is significant, GHG emissions should be also significantly lower to prevent them. Similarly to the previous point, the knowledge on the potential catastrophes and their impacts is still scarce and speculative, and further research is needed to investigate the relation of these sudden changes to the climate system and their role within it.
- As a first approximation we consider climate variability is independent of climate change, i.e. the noise forcing is independent of everything else. If noise is independent of the state of the system, the OL policy is the same as the deterministic policy, i.e. certainty equivalence prevails. This fact is due to the linear-quadratic structure of the model, and we can interpret it as a first order approximation, i.e. to first order, if natural variability is not affected by climate change, the OL policy will not take stochasticity into account.

A very important assumption in our model, specially if climate surprises are expected, is that damages occur simultaneously with the changes in the state of the system. That means that adaptation cannot be very efficient (i.e. CL policies do not avoid that much cost), specially when noise has no memory (is white), since by the time we adapt damages already took place, and the new state is not related to the one we already know. In this sense, information gathering is more important the higher the memory of natural variations, since the present state contains more information on future states of the climate system.

Additionally, the variability of the system is probably underestimated in the model, in that the equations of motion represent only the physical climate system. In general, the economic system should be represented also by a suitable set of state variables with their corresponding natural variability (business cycles, technological improvement), probably with a more complicated structure than additive noise. The role of adaptation and the flexibility associated are underestimated, leaving abatement of CO_2 emissions as the only policy option.

The ratio of natural variability to man made changes is very important in our results. In the baseline run described in section 4.2 we assume that natural excursions from the mean are small in magnitude compared to the expected amplitude of anthropogenic climate change. The mean value of these excursions is taken to be 0.6 °C in the baseline scenario, to be compared to the critical value of $T_m = 3$°C in the cost function. As a consequence, damages due to natural fluctuations of the climate system are comparatively small.

4.2 Optimal emission policies in the presence of climate natural variability independent of climate change

In this section we will describe the effect of natural climate variability that is unaffected by climate change, or in general by the particular state in which the climate system is, on the climate protection policy problem. Natural climate variability is simulated by adding in each time step small random temperature shocks. We will attend to two main characteristics: the amplitude of the shocks and their *memory*, i.e. their persistence over several time periods of the model.

We define a baseline set-up characterised by the parameters of the deterministic baseline run (i.e. the one without stochastic forcing described also in chapter 3). The baseline stochastic forcing is a white noise term added to global temperature and characterised by

$$\mathcal{E}\{w_i\} = 0 \qquad \Sigma = 2.1\times10^{-2}{}^\circ C^2$$

which corresponds to a standard deviation of global mean surface temperature of 0.6 $^\circ C$. The random variables w_i are further assumed to be normally distributed and independent from each other at each time i.

As explained in chapter two, this white noise forcing represents the effect of the fast varying components (weather) of the climate system. We also add a more general type of stochastic forcing that represents not only the internal dynamical forcing, but also the effect of other external forcings not related to the emission of CO_2, like the variations of the solar constant, the effect of other GHGs, vulcanism or longer term variability (thermohaline circulation), that may not be fully deterministic in nature or that are unknown to us. This other (possibly external) influences are modelled by forcing the climate model with red noise: an autoregresive process of order 1, i.e. AR(1) process, whose characteristics can be manipulated without changing the sensitivity of climate to carbon concentration in the atmosphere. The forcing adopts thus the form

$$w_{i+1} = \phi_w w_i + \xi_i$$

where now ξ is a white noise Markov process with zero mean and constant variance. The *redness* of the noise can be controlled with the parameter ϕ_w (the white noise forcing case corresponds to the extreme case $\phi_w = 0$), so that we can easily vary both the natural variability of our climate system (changing χ^T) and the characteristics of the stochastic forcing. Note that adding red noise may transform the problem in one in which perfect information does not prevail any more, if we suppose that w is not readily observable.

Table 5 gives a key to the experiment names for this section. Experiment names starting with W indicate white noise is used, whereas those with R are for red noise, for which the central case is characterised by ϕ_w=0.99. Suffix 0 indicates central case, and otherwise parameters are changed one at a time, assuming they are independent from each other, and experiment names are indicated by the first latin letter of the parameter name

TABLE 5. **Key to experiment names**

Experiment name	Parameter Value
F1	$\phi_w = 0.999$
F2	$\phi_w = 0.9$
S1	Std(T)= 1 °C
S2	Std(T)= 2 °C
AL2	$\alpha = 0.02$ yrs. $^{-1}$
M3	$\mu = 0.0003$ °C/ppm
G1	$\gamma_D = 1\%$
G6	$\gamma_D = 6\%$
A0	$a = 0$ yrs.

Optimal Emissions Policy

The inclusion of natural variability in the system to be optimized has the consequence that CL feedback policies and OL irreversible ones are not equal any more. CL policies deliver at least smaller expected costs than OL, since they take advantage of all information available. Furthermore, since the optimal CL emission policy is a function of the system's state, which is a random vector variable, it is itself random, and cannot be known with certainty beforehand, and rather has to be characterised through some of its statistical properties: expected value and variance, both as time dependent functions. In our case, due to the linear-quadratic structure of the model, and the characteristics of the noise forcing (zero mean and additive), the so called *certainty equivalence* prevails, and the OL reduction rate is identical to the optimal policy for the corresponding deterministic problem (although not the optimal costs). Also, for the same reasons, the expected CL reduction rate is identical to the OL or deterministic one, although not the actual reduction rate, that will be chosen in accordance to the particular realisation of the climate system that we find. Notice, though, that certainty equivalence is only a first order approximation, resulting from the linear-quadratic structure of the model and the independence of noise and system's state, and will not prevail if some of the fundamental assumptions named does not hold.

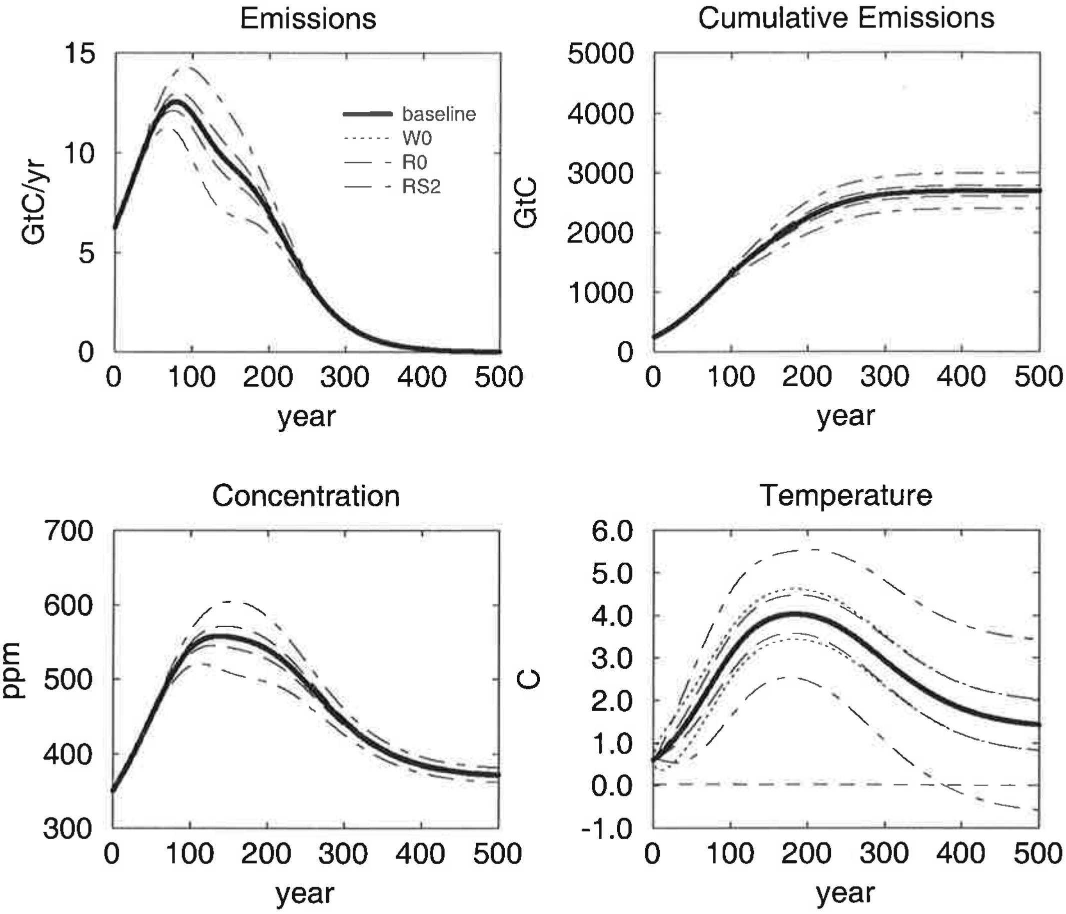


FIGURE 11. Optimal expected emission policy and corresponding expected climate response +- one standard deviation for different stochastic climate experiments.

In order to characterise the differences between the future OL and CL reduction rates we will use the standard deviation of the reduction rate as seen from initial time. Since the noise forcing will introduce small variations at each time step, that will be integrated by the climate system, this quantity will inform us of where, on average, can we expect to find the actual optimal reduction rate in the future. Or, in other words, how good an approximation is the OL policy to the best optimal policy, if irreversibility in the policy adoption process is unavoidable or desired. Notice that a secondary consequence of the carbon emissions being a random variable is that even the climate components that are deterministic in nature (atmospheric concentration) turn into random variables, and their future evolution is not known with certainty.

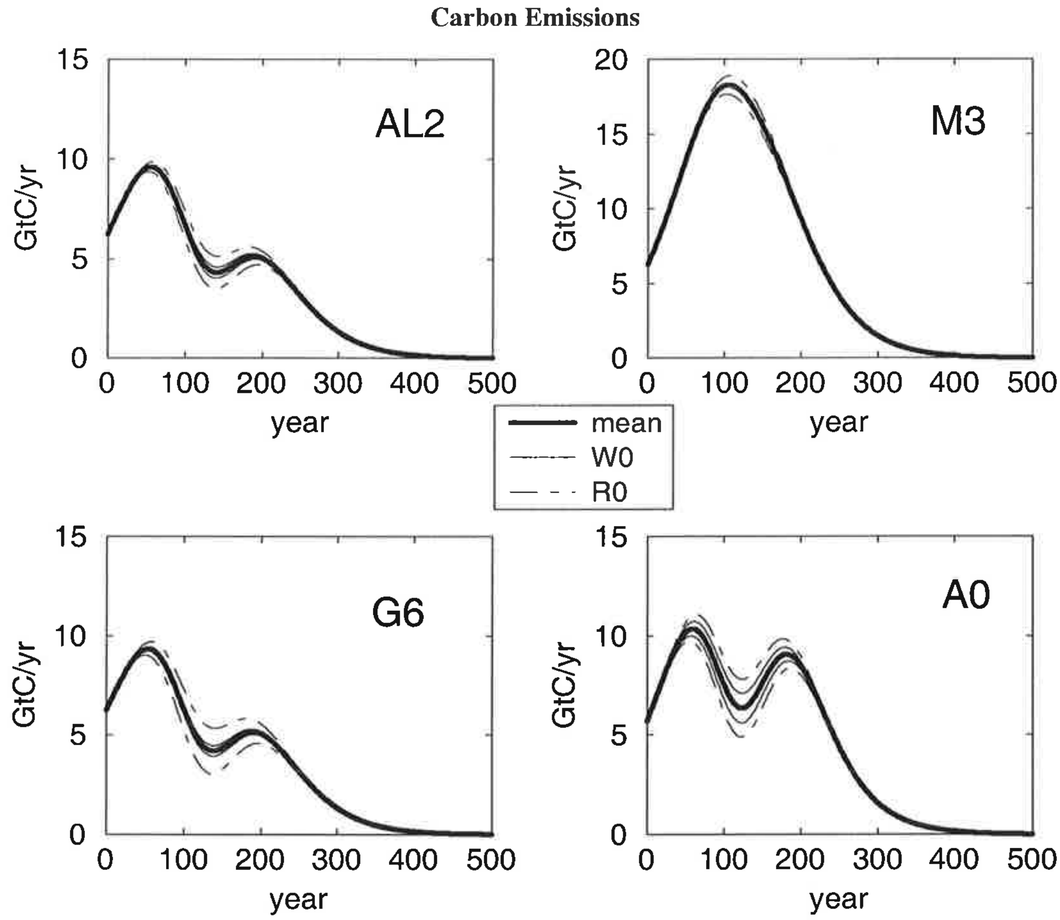


FIGURE 12. Carbon emissions +- one standard deviation for experiments with central additive noise (both white and red) and different values of other parameters in the model.

Figure 11 shows optimal policy and climate response for different values of the amplitude and memory of the stochastic forcing (experiments W0,R0,RS2) and the margins given by one standard deviation of the control. We see that the effect of the noise forcing in the optimal CL policy for both central cases, white and red noise, is rather small (in the case of W0 both the mean and the curve with added standard deviation cannot be told apart), although, as expected, when noise is correlated the effect is greater, since deviations of climate from the mean are longer lived, which allows for the optimal emission path to *follow* those deviations without the extra costs associated to rapid changes of the reduction rate. The range within which we know the future optimal emission path varies between a modest maximum of 0.2 GtC/yr., about 2% of expected emissions, for the standard deviation of the emissions in the case of W0, to a value of around 3 GtC/yr., around 25% of expected optimal emissions, for red noise forcing and a high natural variability of the climate system.

The other factor that influences the CL policy is the economic inertia: for an economic system with no inertia ($a = 0$) the effect of the noise forcing is noticeable even for small climate variability and white noise. The rest of the parameters of the climate model and the cost function do not affect the results explained above: small influence of climate variability on CL climate policy and influence of noise's memory (see figure 12).

Due to the inertia of both the climate and the economic systems, the effect on the CL policy of the introduction of stochastic forcing makes itself noticeable first at the end of the next century. That means that the OL policy will be a good approximation to the CL one for a long time into the future, so that this form of climate variability may be regarded in the short term as a second order effect. Notice finally that since the BAU emissions path is zero in the future, so too are the optimal emissions, with certainty (for very small BAU emissions any deviation from it would require a very high reduction rate and corresponding abatement costs).

We see then that flexibility (CL policy, economic inertia) and information flow (in the form of the system's state) are the main issues that influence the effect of natural variability of climate on the optimal policy. On the other hand, notice that the main features of the optimised emissions are mainly controlled by the deterministic GH effect, because we choose values of the climate natural variability that are small compared to the projected anthropogenic GH effect.

Optimal expected costs

Because certainty equivalence holds, and the open loop and deterministic policies are equal, the values of J^{OD} and J^{OL} are identical, and ΔJP_2 is therefore always 100%, which shows that in this case the choice of CL policies, i.e. flexibility, is important. Also the value of ΔJP_1, the extra costs originated by choosing an OL policy rather than the CL one, is rather small. Its value is mainly influenced by the availability of information, i.e. by the assumption that the noise forcing is observable. If it is not, the extra costs originated by the choice of a deterministic policy increase, though still having a modest value (see figure 13, top panel).

Much of the analysis of ΔJ_D can be simplified by explicitly calculating the value of this quantity, which can be expressed as

$$\Delta J_D = \frac{\gamma_D U_0}{T_M^2} \sum_i \chi_i^T e^{\delta_r (i-1)\Delta t}$$

where i indicates time step and χ_i^T is the covariance of temperature as seen from initial time (i.e. $\chi_{i|0}^T$ using the notation introduced in Appendix B). From this expression it is clear that the two parameters that affect ΔJ_D are the relative cost of damages to abatement and the natural variability of climate. The value of ΔJ_D can be thus seen as the expected damage costs due to the natural variability of climate. As expected, this value is directly proportional to γ_D, the percentual loss of welfare due to a given climate change.

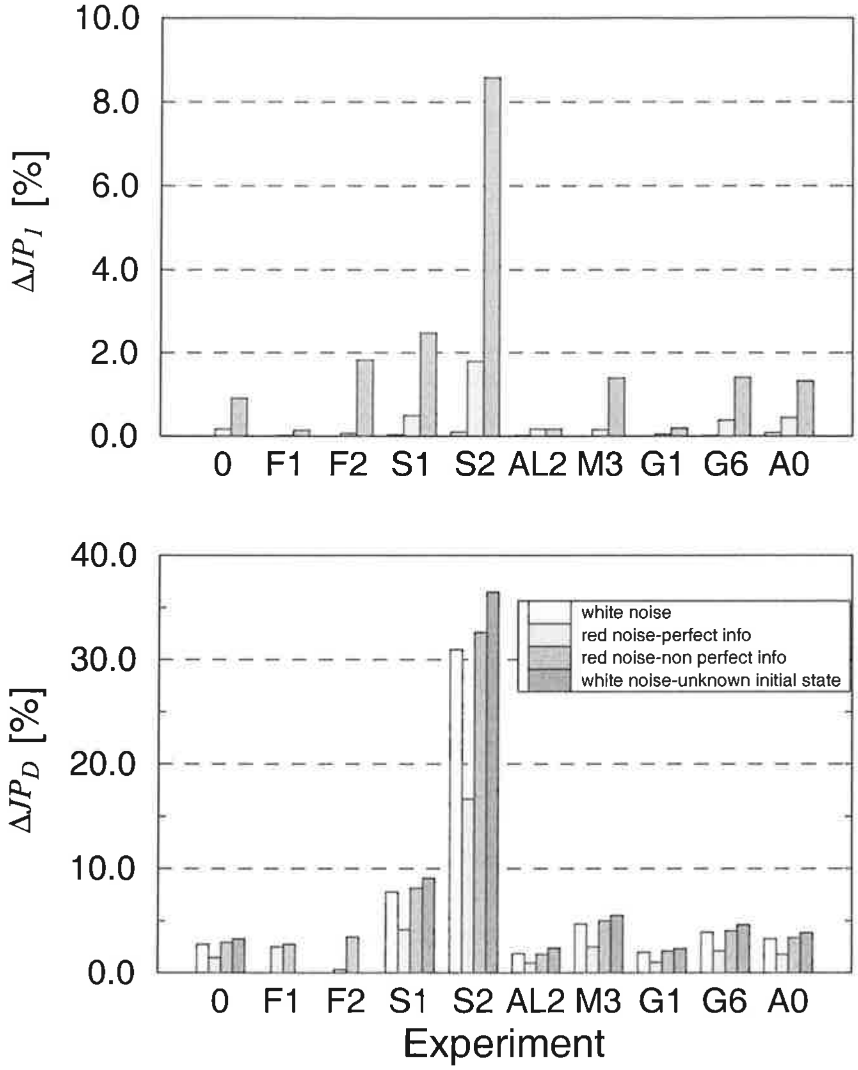


FIGURE 13. Values of ΔJP_I, the extra costs caused by choosing the best guess deterministic policy over the CL policy, for the various experiments in which stochastic forcing is independent of the state of the climate system.

The other parameters in the cost function relate to the abatement costs G and thus do not affect ΔJ_D (remember that the OL policy prescribes a set of actions for the whole planning horizon that are not affected by the presence of noise, so that the abatement costs incurred are not affected).

On the climate side, both the amplitude of $\chi^T{}_{eq}$ and the time evolution of χ^T affect the value of ΔJ_D. The dependence on the amplitude of $\chi^T{}_{eq}$ is pretty straight forward to explain: the bigger natural variations of climate, the higher the costs associated to them. The influence of the time evolution of χ^T (basically the speed at which $\chi^T{}_{eq}$ is reached) is determined by the assumption of perfect information, which implies the climate state at initial time is known with certainty, i.e. $\chi^T_{0|0} = 0$. We can then choose u_0^{OL} optimally (will be the same as the CL policy), but progressively climate moves away from its expected value, eventually reaching its equilibrium variability. In other words, when designing the OL policy we can single out time step 0, when we know the state of the climate system with certainty, from all other times i>0 when that is not the case. In this sense, $\chi_{i|0}$ measures the speed at which the climate system moves away from its expected value given the initial state is known, and the further away, the less optimal u_0^{OL} is, i.e. the higher the costs. To further study this *artificial* dependence on the initial conditions, we may consider the case in which at time zero we know the mean state of the climate system and $\chi^T_{0|0} = \chi^T_{eq}$. In this case, the value of ΔJ_D simplifies, up to a constant, to a product of χ^T_{eq} times γ_D, and the optimal costs increase. Under these conditions, the particular details of the noise are irrelevant (white or red). The time evolution of χ^T (the speed at which reaches χ^T_{eq}) can be influenced either by the value of α or that of ϕ_w. In both cases, small values of α and large values of ϕ_w result in slower excursions from the mean climate and accordingly smaller costs. For μ small, ΔJP_1 increases since the radiative forcing is smaller, so that the deterministic part of costs is also smaller.

Values for ΔJP_D are condensed in figure 13, bottom panel. The value of χ^T_{eq} is the most influential quantity and sets the upper bound for the values of ΔJP_D, that vary between a modest 2.8% for the baseline white noise scenario to a 30% for the rather unrealistic value of Var(T) = 2°C. The addition of red noise translates into smaller expected costs, since climate moves away more slowly from its expected value. The effect is important, reducing the value of ΔJP_D in almost half for the baseline ϕ_w=0.99. The influence of γ_D on ΔJP_D is less than linear, since higher damage costs originate also higher values of J^{DET}, the deterministic part of the costs, resulting in a modest 4% value of ΔJP_D for the high damage estimate(γ_D=6%). This shows again that, in the baseline scenario, the major contribution to the total costs comes from the deterministic anthropogene-induced climate change rather than the natural fluctuations of climate. To put this numbers in perspective, notice that total integrated costs for the baseline white noise case amount to roughly 3.8% of total integrated undisturbed output.

4.3 Optimal emission policies in the presence of climate natural variability coupled to climate change

One possible effect of global warming is that the natural variability of the climate system also changes as a result of GHG forcing, so that if warmer conditions due to global warming should prevail, also a new equilibrium climate variability would be found. On long time scales (centuries to millennia), ice core and other proxy data indicate that glacial periods are associated with higher variability and interglacial, warmer, periods with smaller variability (see McManus et al. (1999) and the references therein). In shorter time scales and recent times, there are no conclusive arguments to prove or disprove the hypothesis that a warmer climate would be a less (more) variable one or with less (more) damaging climate events: observed climate time series do not show significant trends in the statistics analysed and models run with various GHG forcings also show no coherent results. Some experiments seem to show slight decreases in annual-decadal variability associated with ENSO-like phenomena, whereas others indicate an increase in variability of precipitation and temperature specially in the tropics.

Also the feedback between natural variability of climate and climate change would work in both directions, i.e. climate change may also be affected by natural variations. Simulations indicate that decadal and longer time scale variability affect the particular realisation of climate change, both in terms of the rate at which it takes place and the patterns realised, and for some phenomena natural long-term variations are as large as the GH induced mean changes (IPCC (1996a)).

We simulate these effects by including a feedback between the noise forcing term w and the state of the climate system, in our case monitored through surface temperature

$$w = f(T, \xi)$$

where ξ is white (uncorrelated) noise.

Changing the mean of the stochastic forcing

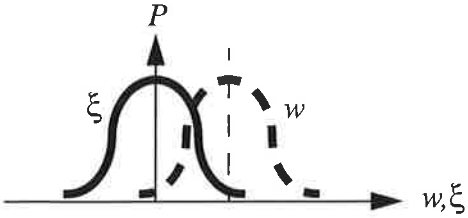
We first assume a linear relationship

$$w = \Lambda_M T + \xi$$

where ξ is white (uncorrelated) noise, with zero mean and variance Σ. We then have

$$\mathcal{E}\{w\} = \Lambda_M \mathcal{E}\{T\} \qquad Var\{w\} = \Lambda_M^2 \chi^T + \Sigma$$

This feedback between the state of the system and the stochastic forcing has the effect of displacing the mean value of the latter by an amount $\Lambda_M T$, rather than change its variance, which is independent of the state of the system. This effect is indicated schematically in the diagram below. Under these conditions the probability that the stochastic forcing w is positive (or negative, depending on the sign of Λ_M) becomes higher, thus further contributing to the increase in global temperature (or counteracting it if $\Lambda_M < 0$).



In more physical terms this situation could be exemplified by an intensification of large scale processes like El Niño, that would become more intense and frequent in a warmer world. The case of a negative Λ_M would mean the opposite effect, i.e. more intense and frequent cold La Niña events as a result of global warming.

We assume that noise is affected instantaneously by the system's state, without any time lag. Then we can reduce the system to a new AR(1) process

$$T_{i+1} = (\phi + \Lambda_M)T_i + \xi_i$$

with parameter $\phi+\Lambda_M$ (parameter ϕ was defined in chapter 3 as $\phi = 1-\alpha\Delta t$).

In the absence of external anthropogenic forcing the system reaches a stationary state characterised by

$$\mathcal{E}\{T\} = 0 \qquad Var\{T\} \equiv \chi^T = \frac{\Sigma}{1-(\phi+\Lambda_M)^2}$$

We use the second relation to derive the value of Λ_S by fixing the other elements in the equality. Being an AR(1) process, if there is an external constant forcing Φ_0, the system is stationary with

$$\mathcal{E}\{T\} = \frac{\Phi_0}{1-(\phi+\Lambda_M)}$$

and the same variance as in the absence of forcing. The mean and variance of the new climate equilibrium state will then be a function of the coupling between stochastic forcing and state Λ_M, but the variance is independent of the state and its mean value. Λ_M represents then the coupling between temperature and the stochastic forcing, and for $\Lambda_M = 0$ we get the system from section 4.2, with noise independent of the climate's state. For the system to be stable, Λ_M has to meet the conditions

$$\phi + \Lambda_M < 1 \quad \text{or equivalently} \quad 0 < \Lambda_M < \alpha$$
$$-\phi < \Lambda_M < 0$$

for a positive and negative feedback respectively. Table 6 indicates the experiment names (**N**oise **T**emperature **C**oupling-**M**ean-**P**ositive/**Negative**) and the vale of Λ_M. Also shown are T_2, the equilibrium temperature at doubled carbon concentration, and the values

$$\Delta_w = \Lambda_M \overline{T}$$

$$\Delta_P = \frac{\Delta_w}{\sqrt{\Sigma}} \times 100$$

which are the amount that we displace the mean value of the noise, in °C (Δ_w) and as percentage of the noise's standard deviation (Δ_P, basically the inverse of the coefficient of variation) respectively. In all cases χ^T has the central value of 0.6 °C.

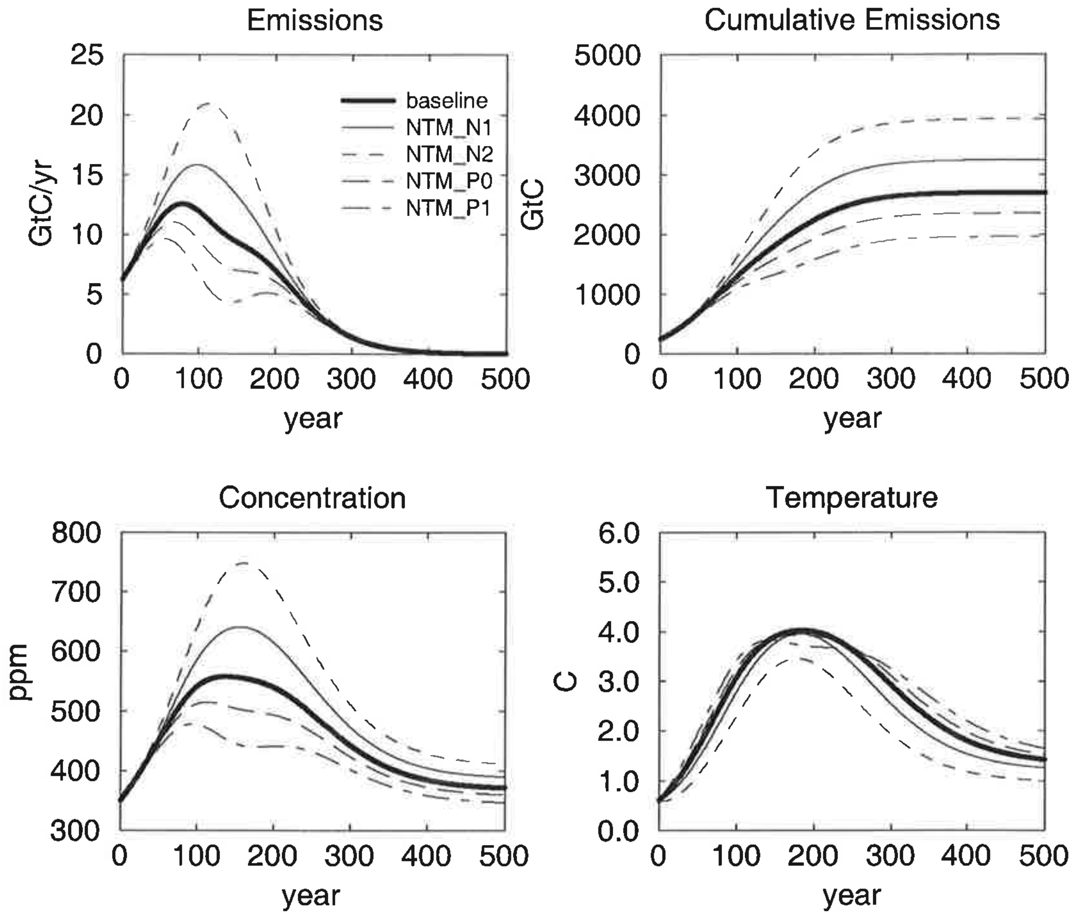


FIGURE 14. Expected carbon emissions and climate system evolution for NTM experiments, in which the mean of the stochastic forcing of climate is coupled to present climate state.

TABLE 6. Key to experiment names

Experiment name	Λ_M	$\mathbf{T_2}$ **[°C]**	Δ_w **[°C]**	Δ_P **[%]**
NTM_P1	0.010	6.3	0.063	50.0
NTM_P0	0.005	5.4	0.027	20.3
Baseline	0	4.2	0	0
NTM_N1	-0.01	3.2	-0.018	-18.8
NTM_N2	-0.02	2.5	-0.050	-26.7

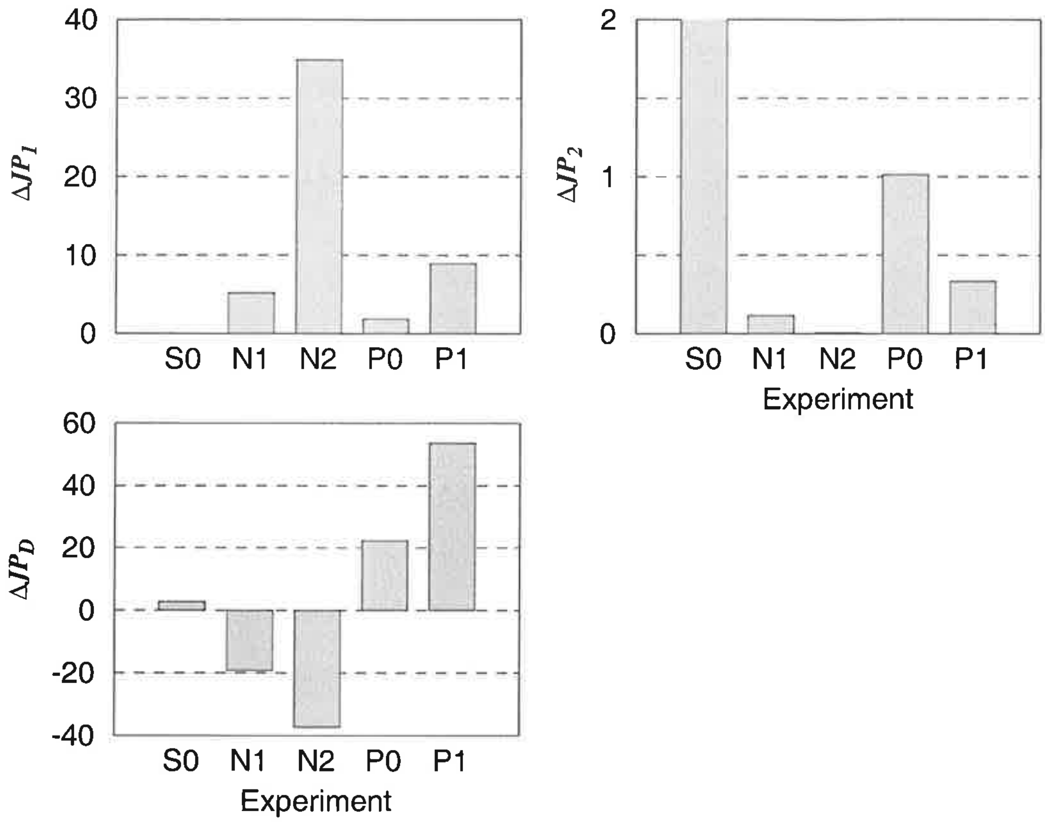


FIGURE 15. Cost analysis for NTM experiments in which the mean of the stochastic forcing of climate is coupled to present climate state.

The main effect of the coupling through Λ_M is effectively enhancing (or counteracting, if $\Lambda_M < 0$) the greenhouse effect with a positive (negative) feedback. As a consequence carbon emissions are curved, compared to the baseline, to prevent the extra warming induced through the feedback process. If $\Lambda_M < 0$ the effect is obviously the opposite, since the coupling acts as a stabilizing process counteracting GH warming more the higher the warming gets, and thus allowing for higher emission rates.

The effect of the noise-state coupling in the optimal emission policy is quite dramatic, and a comparatively small displacement of the noise forcing causes a strong effect on the emissions policy. Shown in Figure 15 are the optimal emissions and expected climate evolution for the experiments summarised in table 6. For example, a value of Λ_M of 0.005 (experiment NTM_P0) means a displacement in the noise forcing's mean of only three hundredths of a degree for equilibrium temperature T_2, but causes this equilibrium temperature to rise from the baseline 4.2 °C to 5.4 °C. Accordingly, carbon emissions are 15-20% less at the end of next century and through the following one. The equivalent negative feedback (Λ_M = -0.01, experiment NTM_N1) displaces the mean of the noise forcing by two hundredths of a degree, and lowers T_2 to 3.2 °C. Carbon emissions are 33% higher than the baseline by year 2100.

Total extra costs originated by ignoring the stochastic component of the climate, ΔJP_1, also reach significant values (up to 32% of optimal CL policy costs for a strong negative coupling of climate and noise). On the other hand the part of those costs that are avoided by an interactive CL policy is only very small. This is also explained through the fact that, in this form of coupling, the main effect is that of enhancing (counteracting) the GH warming, an effect that can be taken into account by an OL policy. Meanwhile, short term temperature variations are still small and with no memory, so that the CL policy cannot do much more to avoid the extra originated costs. The bottom panel on figure 16 shows that the error in the estimated costs made by ignoring stochasticity is also important.

Changing the variance of the stochastic forcing

Assume now w adopts the form

$$w_i = \Lambda_S T_i \xi_i + \eta_i$$

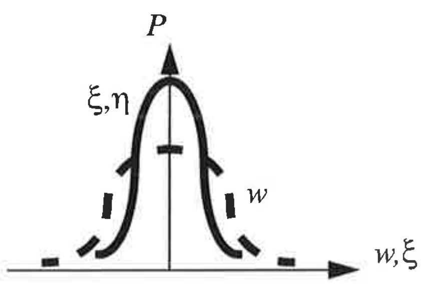
where ξ and η are white noise processes and additionally

$$\mathcal{E}\{\xi_i, \eta_j\} = 0 \qquad Var\{\xi_i\} = Var\{\eta_i\} = \Sigma$$

i.e. they are independent from each other and we have assumed for simplicity that they have equal variance. Under these conditions we have

$$\mathcal{E}\{w\} = 0 \qquad Var\{w\} = (1 + \Lambda_S^2(\mathcal{E}\{T\}^2 + \chi^T))\Sigma$$

This effect is represented in the schematic diagram below. The mean value of the stochastic forcing remains unchanged and equal to zero, but the spread of its distribution increases if global temperature increases.



Going back to our El Niño analogy, this situation corresponds to that in which both warm and cold episodes would be more intense, but their relative frequencies remain unchanged.

The resulting stochastic system differs from the well studied AR(p) processes analysed in the previous section. In the absence of external anthropogenic forcing the system reaches a stationary state characterised by

$$\mathcal{E}\{T\} = 0 \qquad Var\{T\} \equiv \chi^T = \frac{\Sigma}{1 - (\phi^2 + \Lambda_S^2 \Sigma)}$$

We will use the second relation in a similar way to the previous sub-section to derive the value of Λ_S by fixing the other elements in the equality. Similarly to an AR(1) process if the system has an external constant forcing Φ_0, it is stationary with

$$\mathcal{E}\{T\} = \frac{\Phi_0}{1-\phi} \qquad Var\{T\} = \frac{\Sigma(1+[\Lambda_S\mathcal{E}\{T\}]^2)}{1-(\phi^2+\Lambda_S^2\Sigma)}$$

Notice then that the equilibrium variance of the system is a function of its mean state. Thus if a different mean climate equilibrium is reached it will be associated with a higher variability. Also the mean state is actually independent of the coupling between state and stochastic forcing Λ_S.

In order for the system to be stationary both in mean value and variance the conditions to be met are

$$\phi^2 + \Lambda_S^2\Sigma < 1$$

since the condition $\phi<0$ is already met. For simplicity, we consider only the case of a positive coupling, i.e. the case in which a warmer climate is associated with higher natural variability.

Table 7 describes the set of experiments done with this set up. Experiment names indicate **N**oise **T**emperature Coupling-**V**ariance-**L**ow/**H**igh preindustrial climate variability Std(T). Column three shows the value of k_S, the factor by which climate variability, monitored by Std(T), is multiplied for equilibrium T_2 temperature (equilibrium temperature corresponding to double carbon concentration). It can be easily shown that $k_S = (1-\Lambda_S^2 T_2^2)^{1/2}$.

TABLE 7. Key to experiment names

Experiment name	Λ_S	k_S	*Std(T)*
NTV_L0	0.5	2.33	0.6 °C
NTV_L1	1.0	4.32	0.6 °C
NTV_H0	0.5	2.33	1.0 °C
NTV_H1	1.0	4.32	1.0 °C

As opposed to the NTM experiments, the effect of the coupling of climate state and stochastic forcing through Λ_S, is that of increasing the amplitude of χ^T, leaving the mean state unchanged. Thus, similar to section 4.2, the extra costs ΔJP_1 are mostly originated by the short term fluctuations of global temperature, and their value is small compared to the deterministic part: remember that the threshold climate change value in the cost function is T_m= 3°C. On the other hand, and for the same reasons, the CL policy can avoid a greater part of those costs by adapting to the climate fluctuations hence increasing the value of ΔJP_2.

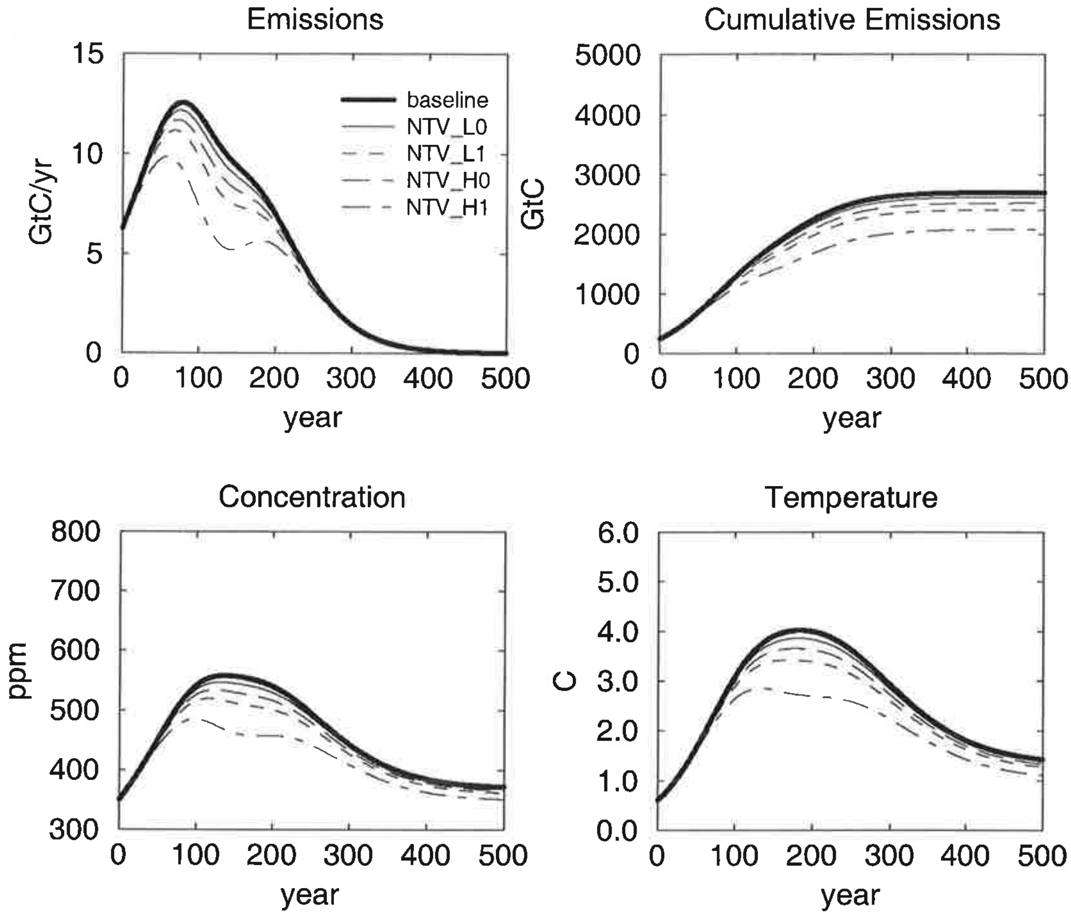


FIGURE 16. Optimal expected emission policies and corresponding expected climate response for NTV experiments, in which the variance of the stochastic forcing of climate is coupled to present climate state.

Nevertheless, since the variability of climate can be influenced by choice of the emission policy, the latter is affected by the coupling Λ_S, and emissions are curved to avoid high values of χ^T (see figures 16 and 17). The optimal policy is again a combination of adaptation and prevention.

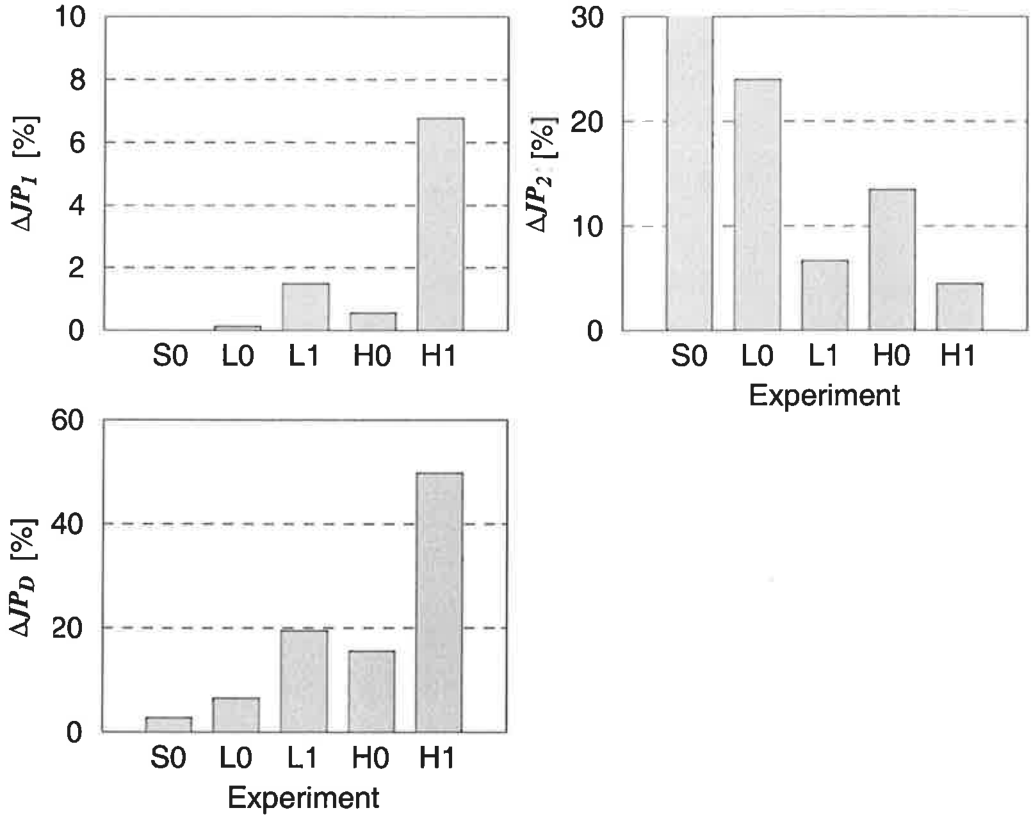


FIGURE 17. Cost analysis for NTV experiments in which the variance of the stochastic forcing of climate is coupled to present climate state.

4.4 Climate catastrophes and surprises

An issue that has raised much interest is the possibility that the climate system reacts to an external forcing in either unlikely but possible ways that would have catastrophic effects, or in unexpected or unpredictable ways. IPCC (1996a) identifies three major types of possible catastrophes resulting from GHG-induced global warming: a) runaway greenhouse effect, i.e. the possibility that positive feedbacks enhance the GH effect producing much larger warming than predicted by *best guess* studies; b) disintegration (melting) of the antarctic ice sheet; c) changes in ocean currents (e.g. shut down of the Gulf Current). All these events have small but unknown probability of occurrence. Also the changes in climate associated with them both in global and regional scales are basically unknown. Finally, the economic effects, i.e. the damages associated with a *catastrophic* climate change are only a matter of speculation, and only few studies have tried to characterise them (for a brief review of present knowledge on the matter see IPCC (1996c)).

We make a preliminary attempt at modelling the effect of such catastrophic possibilities in the design of optimal emission reduction policies by introducing a different stochastic forcing. The basic assumptions of the model are kept, in particular the damage function, i.e. we assume that we can represent such catastrophic economic effects just by increasing the argument of the damage function. For example the (remote) possibility of a much colder climate in northern Europe, as a consequence of the shut down of the Gulf current, is modelled within our assumptions with a high value of global surface temperature, that causes high damages when introduced in the cost functional. We make otherwise no attempt to model the particular nature or effects of each of the catastrophes described above. Also catastrophes occur always in the direction of high damages, i.e. we do not consider feedbacks that would act to counter climate change.

From the point of view of our model, there is a constant probability that at each time step a catastrophe, i.e. a sudden jump in global temperature and its associated damages, occurs, but the magnitude of the catastrophe is proportional to the already existing climate change.

We implement a stochastic forcing of the form

$$w_i = \begin{cases} 0 & \text{w.p. } \pi \\ kT_i & \text{w.p } 1\text{-}\pi \end{cases}$$

so that, in this formulation, we have two free parameters to set, namely the probability of finding a jump at any given time, π, and the amplitude of the jump, *k*. For a given value of the global temperature we have

$$\mathcal{E}\{w_i | T_i\} = (1-\pi)kT_i \qquad Var\{w_i | T_i\} = \pi(1-\pi)k^2T_i^2$$

We already noticed that the fact that damages and abatement are of similar magnitude is very important for the optimal reduction path, and also the fact that abatement is typically only a small percentage of total output. When we allow for these surprises, we let damages be typically much greater than abatement costs with the resultant bias toward policy adoption.

In order to assign values to both parameters in the probability distribution of the random forcing it is useful to consider first the probability π of finding a jump at any given time. Independent from the value of *k*, the set of random forcings $\{w_i | 1 \le i \le N\}$ has a binomial distribution with parameters (1-π) and *N*, where *N* is the number of time steps that we take into consideration. We can then easily derive the expected number of jumps to be found Π

$$\Pi = N(1-\pi)$$

We see then that even for a relatively small probability π of finding a jump at each time step, the expected number of jumps will be high, since it increases rapidly (linearly) with the length of the planning horizon. Or, in more physical terms, the probability of finding a climate catastrophe increases rapidly with the length of the period taken into consideration. We can further pursue this simple analysis for very small probability π of finding a jump in any year. We can then approximate the distribution by a Poisson distribution with parameter Π/*N*, and can be easily shown that the probability of not finding any such jumps in a period of *N'* years is $e^{-N'(\Pi/N)}$. If we find, for instance 1 jump on average every 1000 years, i.e. Π=*1*, the probability of not finding any jumps in a particular 1000 year period is $1/e$, i.e. around 1/3 (and, consequently, the probability of finding at least one jump is about 2/3). Notice also that for our analysis we assume without further consideration that catastrophes (jumps) at different time periods are independent from each other (the probability of finding a jump at any time period is independent from previous jumps that may have taken place already).

The value of k determines the amplitude of the jump that will take place, by multiplying the already realised climate change. Since the noise vector w is added to the global surface temperature, a value $k_C=1$ means a sudden doubling of current temperature (in general, for k_C arbitrary, we have a k_C+1 fold increase of the current temperature). Table 8 gives a key to the experiments done using this model set up. A value $\pi = 0.999$, corresponds to $\Pi = 1$, i.e. the case in which we find, on average, one jump every 1000 years.

TABLE 8. Key to experiment names

Experiment name	k_C	π_C	Π
CCS_P0	1	0.995	5
CCS_P1	1	0.990	10
CCS_K0	1	0.999	1
CCS_K1	3	0.999	1
CCS_K2	5	0.999	1

The first relevant feature of these set of experiments is that the expectation of a future climate catastrophe that can be triggered through climate change, will bias the optimal climate policy towards higher reductions. Thus we have a way of quantifying the *precautionary principle*, stating how much more aggressive our policy has to be as a function of the perceived probability and magnitude of a climate catastrophe. The expected CL equals, though, the OL policy, so that we cannot say beforehand how both will differ. The CL policy will react with more aggressive reductions after the jumps take place, but we do not know when this jumps will occur and consequently we do not know their amplitude either. In both sets of experiments it is to be noted that the finiteness of the BAU emission path that is zero after approximately 400 years greatly conditions the results, since, independent of the probability that a climate jump might take place, the optimal emissions will be zero after that date. In practical terms, the number of such jumps that may have an influence in policy adoption is reduced.

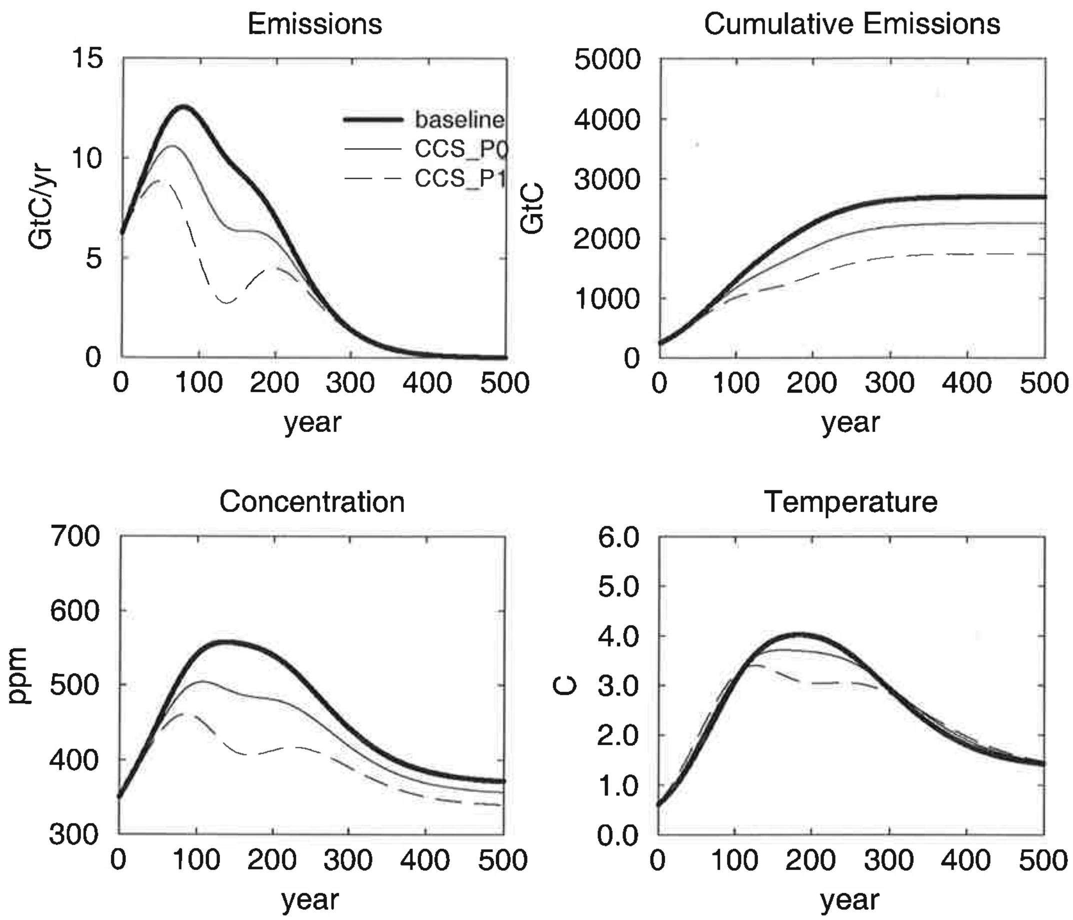


FIGURE 18. Optimal expected emission policies and corresponding expected climate response for climate surprise experiments and different values of parameter π_C.

We can interpret these results in terms of risk, which combines both the probability of a catastrophe and its amplitude. Thus, situations in which either of them is small (small probability of a jump or small amplitude) will be perceived as low risk situations, since even high amplitude jumps, when weighted with the corresponding low probability, deliver a low risk and corresponding relatively low costs.

Figures 18 and 19 show the optimal emissions policy and the associated climate response for the sets of experiments specified in table 8, for different values of the multiplying constant k_C and probability π_C. The risk of a climate catastrophe may have a noticeable effect on the emissions policy. It is important to notice that the response to this risk is conditioned by the asymmetry of the noise forcing, i.e. we can only have positive jumps that originate high costs, and not events that would act to counter anthropogenic climate change, as opposed to sections 4.2 and 4.3 in which natural variations of climate could act to counter anthropogenic GH effect. Results are also very sensitive to both the perceived probability of a catastrophe and its amplitude.

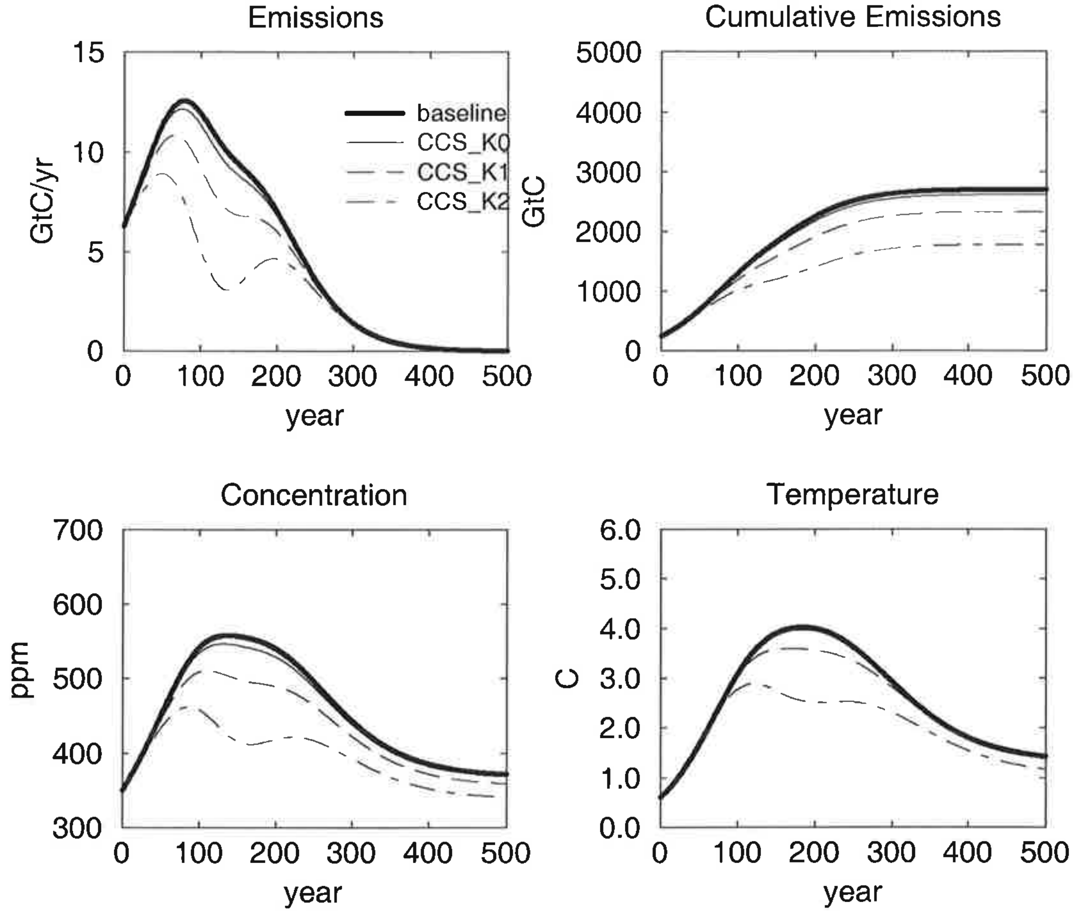


FIGURE 19. Optimal expected emission policies and corresponding expected climate response for climate surprise experiments and different values of parameter *k*.

Figure 20 shows the values of ΔJP_1 and ΔJP_2 for these experiments. Both these values have moderate values even for high risk situations (experiments K2 and P1) in which either the probability of a catastrophe or its amplitude are high. On the other hand, the value of ΔJP_D is rather high, specially for high risk. This is a consequence of the assumption that jumps, an the associated costs incurred take place instantaneously, i.e. unpredicted catastrophes are instantaneous and originate very high costs that cannot be avoided any more, so that adaptation is not very effective. This shows that climate catastrophes can be very expensive, so that the precautionary principle applies, and restrictive emission policies are necessary to prevent high risk outcomes.

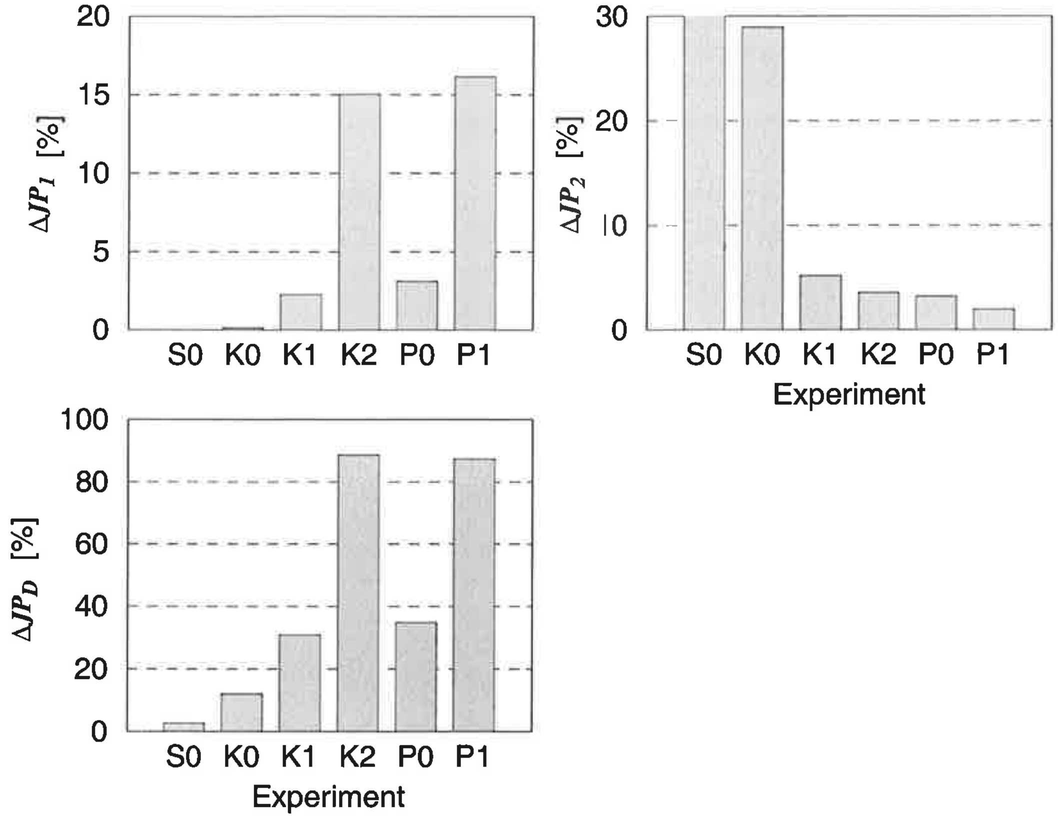


FIGURE 20. Cost analysis for climate catastrophes and surprises experiments.

4.5 The role of information gathering

As explained above, the advantage (which translates in smaller costs) of the CL policy, lies in the possibility of continuously gathering information and updating emission reductions in view of the present climatic conditions. Hence, a relevant change in the structure of the problem takes place in that the timing of the decision process and the possibility of postponing actions become a relevant issue. As a consequence, part of the costs caused by changing climate conditions can be avoided by choice of an interactive feedback policy (i.e. ΔJ_2 is a strictly positive quantity). In this sense we can interpret ΔJ_2 as a measure of the value of information on the climate system, although its particular value is conditioned by several assumptions and model parameters and by the fact that the highest costs are caused by the deterministic GH-induced climate change. We have seen that even for the choice of a OL policy the information available is important. In this case, the only information relevant (or available) is the initial state of the system.

Also we have introduced time correlated (red) stochastic disturbances, but kept the assumption of perfect information, i.e. we have assumed that the noise forcing vector w is observable (although not the value of ξ). We may now relax this assumption and suppose that we do not have access to the value of the random forcing at each decision period: it is easy to imagine this set up, in which for instance the solar constant or the concentrations of other greenhouse gases are not readily measurable. We can still use the mathematical tools of DP, although the linear feedback that relates the optimal reduction rate to the present climatic conditions, has to be substituted by a feedback on our *estimation* of the climate state (for details see appendix B). Under these conditions, both the values of J^{OL} and J^{CL} increase due to the error in the estimation of the climatic state (notice that for J^{OL} only the estimation of the initial state matters, whereas for J^{CL} we have to continuously estimate the present value of the climate state). On the other hand the former value increases more than the latter, and as a result, the value of ΔJP_I increases dramatically, showing that if information is scarce the use of whatever knowledge is available becomes ever more important. Figure 13 illustrates these results.

We can further pursue these considerations and analyse the structure of the optimal policy. We can decompose the optimal feedback policy into a proportional and an integral part (see appendix B)

$$u_i^* = L_i\mathcal{K}_i y_i + L_i \sum_{j=0}^{i-1} \tilde{\mathcal{K}}_{i-j} y_{i-j}$$

where $\boldsymbol{y}$ is the vector of observations and K the gain matrix of the Kalman filter (matrices $\tilde{\mathcal{K}}$ are also derived in appendix B, and are functions of $\mathcal{K}$ and of the coefficients of the problem at hand). This decomposition shows that at any time period, the optimal reduction rate will be a function not only of the present measurements, but also the past information will be used. The relative importance of both terms is a function of the quality of the measurements, i.e. of how well we know the state of the system (in the extreme case of perfect observations, $\mathcal{K}$ is the identity matrix and the integral part reduces to zero).

CHAPTER 5

Optimal emission policies for an uncertain climate economy system

5.1 Introduction. Optimal emission policies under uncertainty

High uncertainty is present basically in all elements of the coupled climate-society system. Even where functional forms are available, that describe the corresponding subsystems, the particular values of the parameters or coefficients of their mathematical description still present high uncertainty. Also in the introduction it was briefly discussed how we find three basic levels of uncertainty: the climate system, whose physical laws are thought to be well known (we may include here, to a certain extent, economic models); the interaction between climate and socioeconomic structures, which are not always properly defined or known; and the *perception* of the global warming problem, as a result of different cultural values and economic and political interests. We can identify these three forms of uncertainty easily in our simple structural model, matching them to its three major elements: the equations of motion, that describe only the climatic evolution; the cost function that includes the economic damages caused by climate change and the costs of avoiding carbon emissions; and the choice of the relevant parameter values and probability distributions relevant to the problem. In the rest of the chapter we will make no methodological distinction between all three forms of uncertainty. We assume all functional forms are known, and only the coefficients are uncertain.

A number of authors have characterised the design of optimal carbon emission policies as highly irreversible, arguing that climate protection policies, once started, cannot be easily revised or abandoned. A representative example is the model of Baranzini et al. (1995), in which carbon emissions reduction policies are studied as an option value problem, and one of the main questions studied is that of the optimal date of intervention, i.e. the date at which the application of a particular given policy option becomes optimal. Similarly other studies (Nordhaus (1993), Peck and Teisberg (1993)) try to solve the dilemma of 'act now' vs. 'wait and see' as a policy option, i.e. the effect of postponing the application of a particular policy action.

Another interesting approach to this problem is the use of montecarlo techniques (Tol (1997), Peck and Teiberg (1993)) that let us test different scenarios, i.e. different realizations of the unknown parameters, and possibly assign them probability distributions. In particular we can estimate the expected value of perfect information, i.e. the difference between the expected total costs incurred when policies have to be defined under uncertainty and the corresponding value if all elements of the model were known with certainty. This value serves as an upper limit to the value of information under different sets of assumptions, that are not so restrictive as that of assuming perfect knowledge of the system. As we shall see this value depends strongly on the probability distributions assigned to the uncertain elements of the model and thus perception of the global warming issue becomes a relevant element of our model.

We aim at studying the effect of uncertainty in the deterministic policies designed with available integrated GES models, similar, for instance, to SIAM (Hasselmann et al. (1997)). We also test the robustness of these policies against uncertainty. We test thus the validity of the best guess approach and its robustness against possible futures.

In order to solve the problem of finding an optimal emission policy under uncertainty, we will transform the problem, through state augmentation, into a deterministic one. In the following we define a *state of the world* as a particular realization of the parameter values that are not known with certainty, i.e. a particular point in our parameter space $\Gamma(\theta) \subset \Re^p$ (where θ is a p-dimensional vector of unknown parameters). We further assume that each component in θ, i.e. each uncertain parameter, can only take a finite number of values with fixed and known probabilities, that are described by the probability function $\mathcal{P}(\theta)$. Under these conditions we define the problem

$$min \mathcal{E}_{\mathcal{P}(\theta)}\{J(x^{\theta}, u)\}$$
$$x_{i+1}^{\theta} = f^{\theta}(x_i^{\theta}, u_i)$$

where $\mathcal{E}_{\mathcal{P}(\theta)}$ is the expected value taken over the probability distribution associated to the different states of the world, and superindex θ indicates the particular state of the world. Notice that instead of the original system of equations of motion, we have a set of systems, one for each state of the world. Additionally we impose the condition that the optimal control be equal across all states of the world.

In this formulation of the problem we restrict ourselves to the study of OL irreversible policies, thus ignoring the effect of incoming information about the system's state (if there is an stochastic component in the climate system) on the optimal emission path. Note that this form of endogenous learning is not the main mechanism, and at any rate not the only one, through which knowledge of the parameters in our GSE models (and the laws governing the coupled system) will be improved. While direct observation of the state of the system is essential in understanding and improving our knowledge, specially in the climate subsystem (detection and attribution of climate change is probably the most relevant case), the description and characterization of the interactions and parameter values needs and uses other sources of knowledge besides direct observation of the climate state variables. Rather, external actions, like basic research or technology development, will bring information about our system exogenously. We do not attempt to model the optimal investment in technology or basic science, which is for itself a different field of study, and restrict ourselves to the OL solution of the model, i.e. we consider information available does not change in the course of time.

Other authors have also approached the problem of finding an optimal climate policy under uncertainty and the irreversibilities involved in global change. Kolstad (1994, 1995) presents a stochastic model with learning in which the effect of uncertainty and learning rate on the climate policy at initial time is analysed. Welsch (1995) and Eismont and Welsch (1996) analyse the problem of finding optimal GHG emissions under ambiguity, defined as the situation in which the probability distributions assigned to the unknown parameters are themselves unknown and subject to revision.

Climate Policy under Uncertainty and Adaptation

Our approach to the design of optimal policy aims at analysing the effect of uncertainty in a set up where learning and adaptation (understood as the revision of climate policy in view of new information available in the future) are not possible. It has been widely recognised, though, that the expectation of acquiring information in the future and the structural changes that the socioeconomic system will undergo in the future may play a major role in the policy decision process, rendering irreversible policies suboptimal. Questions of the form 'act now' vs. 'wait and see' implicitly assume decisions are irreversible. Allowing for adaptation raises a different set of questions, namely that of what to do now, in view of the information available to us, and how to implement policy options and instruments in a way that lets us remain as flexible as possible.

Webster and Reiner (1998) argue the need of flexibility in any response to climate change, since uncertainty is unavoidable. Valverde et al. (1998) present a framework for sequential climate decisions under uncertainty. Lempert et al. (1996) present a model in which a rudimentary form of adaptation is allowed, and the optimal emission policy is revised once during the planning period. They also find that policies in which adaptation is allowed are more robust, i.e. generate smaller expected costs, than those in which adaptation does not exist.

The problem described above can, in principle, also be treated within the mathematical framework of SOC (see also appendices A and B). By means of state augmentation we can treat the set of unknown parameters θ as a new set of state variables with the equation $\theta_{i+1} = \theta_i$, since the parameters are unknown but do not change with time. The problem becomes one of imperfect information, since the parameters, that are now treated as new state variables, cannot be measured. The transition probabilities necessary are now $\mathcal{P}(x_{i+1}, \theta_{i+1} | y^i, u^i)$, where y represents the measurements available to the planner. Similar to the problem presented in chapter 4 in which perfect information was not available (red noise stochastic forcing not measured), these transition probabilities are not known with certainty and have to be estimated from the information available, namely those state variables that can be measured and past controls. In this kind of problem the control vector serves a *dual purpose* (Kumar and Varaiya, (1986): first, the control can alter the evolution of the state variables $\boldsymbol{x}$ in the future; on the other hand can also alter the information available on them. This dual aspect of the control makes problems in general more difficult to solve, since both have to be balanced, i.e. the problems of identification and control have to be solved simultaneously in what is called *adaptive* control. Both bayesian methods (that assume a prior distribution for the parameters, and actualise it in view of new information) and non bayesian methods (that assume no prior knowledge of the probability distribution, and use techniques as maximum likelihood estimators to incorporate new information) can be used. The problem of adaptive control of an stochastic system in the form described above, becomes then one of imperfect information and highly non linear. Though no new basic or theoretical principles have to be invoked to solve such a problem, we loose the ability of finding an analytical form for the optimal policy, and the derivations of the necessary probability distributions have to be carried out numerically and is not trivial, with the subsequent loss of clarity and the imbalance between the conceptual nature of the model and the mathematical structure needed. As a result, this solution approach is of limited value for our present work, and will not be further developed.

5.2 Experiment design and general results

For our analysis in the following sections we will characterise uncertainty through the parameters α, representative of the climate sensitivity; γ_D, the percentual damages due to a climate change; and a, the inertia in the economic system. Furthermore, we will assign probability distributions to these parameters, that will in turn take different values to take into account not only the corresponding sensitivity analysis, but also the problem of perception. The experiments described below show that the inclusion of uncertainty in the model can have a very important effect in the design even of irreversible policies, since they will try to avoid undesirable or dangerous outcomes associated to some possible futures. It turns out though that our particular description (parametrization) of the uncertainties has an overwhelming effect on the system's response for parameter and uncertainty ranges consistent with present knowledge of the system.

The first necessary step is to identify and quantify the probability distributions assigned to the uncertain parameters. It has to be mentioned that just like the estimation of best guess values for these parameters, the estimation of their extreme values or possible ranges and, most importantly, the probabilities of finding each of them, is a task that finds itself in its very beginning, and a great deal of speculation is needed in the absence of experimental evidence or consistent modelling efforts. See Jacoby and Prinn (1994) and Webster and Sokolov (1998) for a description and quantification of uncertainty in climate change predictions. In the following the *states of the world* are particular realizations of the parameter values that are not known with certainty, i.e. a particular point in parameter space $\Gamma(\gamma_D, a, \alpha) \subset \Re^3$.

We will assume that all three parameters are independent from each other so that their corresponding probability distribution can be expressed as

$$P(\gamma_D, a, \alpha) = P_\gamma(\gamma_D)P_a(a)P_\alpha(\alpha)$$

Furthermore we will consider two different specifications for this probability distributions, depending on whether they are symmetric or not. Notice that the use of a non-symmetric probability distribution implies that the most probable and the expected values are not necessarily the same any more. The objective function, though, is defined as the expected value of total costs, and due to the linear-quadratic formulation of the model, we will only find in its solution the expected value and the variance of the stochastic parameters; the different probability distributions will thus manifest themselves through these two quantities.

First we will assume that the individual probability distributions P_γ, P_a and P_α are symmetric. We will follow a rather simple approach in order to quantify mean and variance of the parameters: first we identify a maximum and minimum value which in turn specifies the mean. Next, the standard deviation is derived assuming that the parameter is drawn from either a normal distribution (for a low estimate of the variance) or a uniform distribution (for a high estimate). In the first case, i.e. normal distribution, the extreme values are taken to be the 3σ values of the distribution. Typical values for the uncertain parameters have been already used for the sensitivity analysis in the previous chapters, so that we will use them to characterise the probability distributions.

Non symmetric distributions are assumed to take the form of a Gamma distribution with density

$$\mathcal{P}_\theta(\theta) = \frac{1}{\Gamma(n+1)k_\theta^{(n+1)}}\theta^n e^{-\theta/k_\theta}$$

where $1/\Gamma(n+1)k_\theta^{(n+1)}$ is a normalizing constant to ensure that the integral of the density $\mathcal{P}_\theta$ equals 1, and θ represents one of the three uncertain parameters. Some interesting values are easily calculated for this distribution

$$\begin{aligned} \arg[max\{\mathcal{P}_\theta(\theta)\}] &= nk_\theta \\ \mathcal{E}\{\theta\} &= (n+1)k_\theta \\ Var\{\theta\} &= (n+1)k_\theta^2 \end{aligned} \qquad \textbf{(EQ 1)}$$

where we can fix either the value of maximum probability or the expected value of the parameter and adjust k_θ accordingly (notice that the value of θ corresponding to the a maximum probability density cannot be directly interpreted as the one having maximum probability, since in a continuous distribution the probability of occurrence of any particular value is zero; we take it though as the limiting case of a corresponding discrete distribution in which the maximum frequency of the parameter is found in the neighbourhood of that value). As an example, figure 21 shows the density and distribution functions for α.

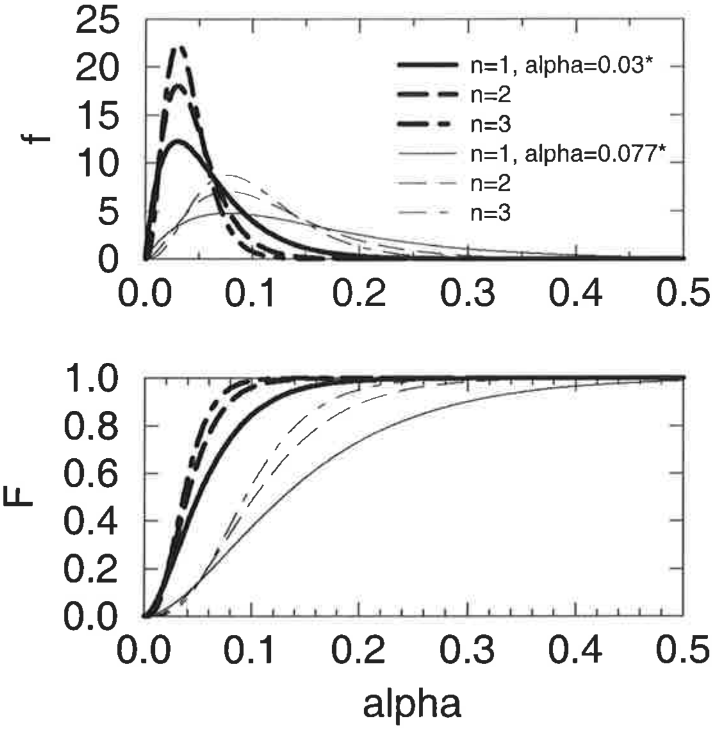


FIGURE 21. Density and distribution functions for Gamma-distributed parameter α and for different values of n and k_{α} (* the value corresponding to arg{max[$\mathcal{P}(\alpha)$]}).

We can interpret the different distributions as representing the degree of certainty we have in our estimates of the parameters. A uniform distribution represents a very poor knowledge of the relative probabilities for different states of the world. On the other hand, a normal distribution assumes a relatively high confidence on the best guess estimate. In the gamma distribution, higher values of n correspond to smaller probabilities of high values of the parameter and higher probability of finding the best guess.

Note that we have two different situations, depending on whether uncertainty is found in the cost functional or the equations of motion. In the former case the expected value operation in the cost functional already takes care of the uncertainty in the parameters, so that what we are doing is redefining a cost function that includes the statistical properties of the perceived probability distributions. On the other hand, if uncertainty is found in the equations of motion, we need to augment the problem to take into account the set of possible states of the world. By chance in our model the whole economic part is contained in the cost functional, and the climate model in the equations of motion, so that these two situations will correspond to economic and climate uncertainty respectively.

Also, since we only consider OL policies, we will only analyse the value of ΔJ_D, the error made in the estimation of costs if uncertainty is ignored.

General results

As the experiments in the next sections will show, uncertainty plays a major role in the design of an optimal policy and the estimation of expected costs associated to climate change. Deterministic policies that ignore uncertainty may then perform not very well, in an average sense, when tested against a range of possible states of the world, i.e. possible realizations of the unknown system. Accordingly, the approximation of treating uncertainty as negligible is inappropriate to design robust climate protection policies that will perform optimally under a wide range of different worlds. In this sense also adaptation an flexibility, both in policy options and institutions, will play a major role. We could try if-then recipes and find an optimal policy for each possible state of the world, but uncertainty is likely to persist, so we have to look for robust policies across all possible states of the world. We can summarise other general results in the following points.

- Experiments show that the particular parametrization of uncertainty is also very important, and results are sensitive to the probability distributions used. Much research is necessary to characterise and quantify uncertainty in climate change not only in the form of best guess and extreme values but also with probability distributions. The first step would be then to reduce 'uncertainty on uncertainty'.
- Although the precautionary principle is frequently invoked to ask for prompt action to reduce GHG emissions to prevent global warming, results show that the outcome of the precautionary principle depends on the particular parametrization of uncertainty used. Depending on where uncertainty is placed (e.g. the damages of global warming or the costs of emissions reduction), and the particular distribution attributed to the uncertain parameters (e.g. high or low probability of high climate sensitivity or of high economic inertia), the optimal policy will react accordingly with higher or lower reduction rates. We should then formulate precisely the precautionary principle in terms of the particular estimate of uncertainty at hand.
- We can characterise the forces dominating the present social debate about global change, with the use of cultural theory, in terms of their attitudes toward nature and uncertainty. We use a simplified description of the *views of nature* to identify three typified attitudes: industrialist (global warming is not a big threat), environmentalist (global warming is a major threat and uncertainty not relevant) and hierarchist (global warming is a potential threat, but uncertainties are important). In the origin of the disagreement there are: a) scientific uncertainty that allows for wide range of beliefs and b) different interests, lengths of planning (political, economic, social), and priorities (e.g. short term development of underdeveloped countries vs. future generations).
- Optimal emission policies and climate costs depend sharply on the particular view of nature adopted, and thus the assumption of an international agreement affects the optimal emission path. Also the economic value of information depends on the attitude towards nature.

5.3 Uncertainty in the costs of climate change

The value of γ_D, the percentage loss on global output due to a benchmark (T_m) climate change, has been estimated to lie between a negligible value and 21%, with a mean of 3.6%, for a global warming of 3 °C taking place by year 2090 (see Nordhaus (1994) and IPCC (1996c)). Cline 1992 estimates losses of around 6% of global output for long term warming (10 °C). We choose the values 0% and 6% as extreme values for the symmetric distribution of γ_D, for a mean value of 3%, already used as baseline value in the previous chapters. Notice, though, that there is a great deal of arbitrariness in this choice, and the wide discrepancies suggest that in this case the uniform distribution for the parameter is not a bad choice. The gamma distribution will take the best guess value of 3% as the most probable one, and adjust k_γ accordingly for different values of n.

The values for a are difficult to estimate, since the term $(a\dot{\rho})^2$ was introduced in a rather artificial way to represent the effect of processes like capital accumulation/depreciation, technological progress or information diffusion. It is thus not easy to relate the simple value of the parameter to the real processes and complicated links taking place in the real world or in more complicated models. We choose tentatively the extreme values of 0, for a very fast reacting and technologically oriented economic system (representative of the bottom-up modelling strategies), and 100 years for a slow reacting one (typical of top-down models). A review of existing literature (see IPCC (1996c) for an extensive one) reveals that also in the economic costs of emission abatement measures, as well as the particular details of the costs functions, there are great uncertainties. The gamma distribution will use the best guess value of 50 yrs. as most probable one.

Table 9 summarises the values of these two parameters and the probability distributions associated with them.

TABLE 9. ***a*) Parameter probability distribution estimates: maximum, minimum, mean, and standard deviation both for normal and uniform distributions. *b*) Parameter probability distribution estimates: expected value and standard deviation for Gamma distribution and different values of n.**

a)	Min.	Max.	Mean	Std($\mathcal{N}$)	Std($\mathcal{U}$)
γ_D (%)	0.0	6.0	3.0	1.0	1.7
a (yrs.)	0.0	100.0	50.0	16.7	28.9

b)		$\mathcal{E}\{\theta\}$	Std$\{\theta\}$	k_θ
γ_D(%)	n=1	6.0	4.2	3.0
	n=2	4.5	2.6	1.5
	n=3	4.0	2.0	1.0

b)		$\mathcal{E}\{\theta\}$	Std$\{\theta\}$	k_θ
a (yrs.)	n=1	100.0	70.7	50.0
	n=2	75.0	43.3	25.0
	n=3	66.8	33.4	16.7

Climate change costs in our model depend in a linear fashion on γ_D. That means that the extra costs, positive or negative, associated to higher or lower values of γ_D than the mean, are equal. On average, then, positive and negative values of the extra costs will cancel each other. As a consequence, after applying the expected value operator, we have only the mean value left, and the particular probability distribution chosen or its variance are irrelevant.

If the probability distribution is symmetric, also expected value and most probable one are equal, so that the optimal policy in this case is identical to the best guess policy, i.e. our baseline emission path. What we have is a situation in which values of the parameter higher than the mean are equally probable as lower ones *and* excess costs associated with higher or lower values are equal (except for the sign), so that outcomes with higher costs cancel out exactly those with lower costs when taking expected value. On the other hand, if there is a significant probability that damages are very high, i.e. if γ_D is gamma distributed, the optimal policy will curve emissions to prevent incurring in the high costs associated to those potentially dangerous states of the world. Notice that in our particular case the long tail in the distribution of γ_D implies that high damages of climate change are more probable than low ones. Figure 22 shows the different optimal policies associated to different uncertainty estimates of γ_D.

For the economic inertia a the situation is different, since costs have a quadratic dependence on it: the expected costs associated to states of the world with a higher than the mean are higher, more than linearly, than the corresponding for smaller values of a (i.e. $\mathcal{E}\{a^2\}= \mathcal{E}\{a\}^2+\text{Var}\{a\} >\mathcal{E}\{a\}^2$). As a consequence, the optimal policy reacts with smaller reduction rates (abatement measures are effectively more expensive). Figure 23 shows the optimal emission policies associated to different estimates of uncertainty for a. Since the effect of the economic inertia on the model's optimal emissions is only small, also uncertainty has a moderate effect on the optimal solution.

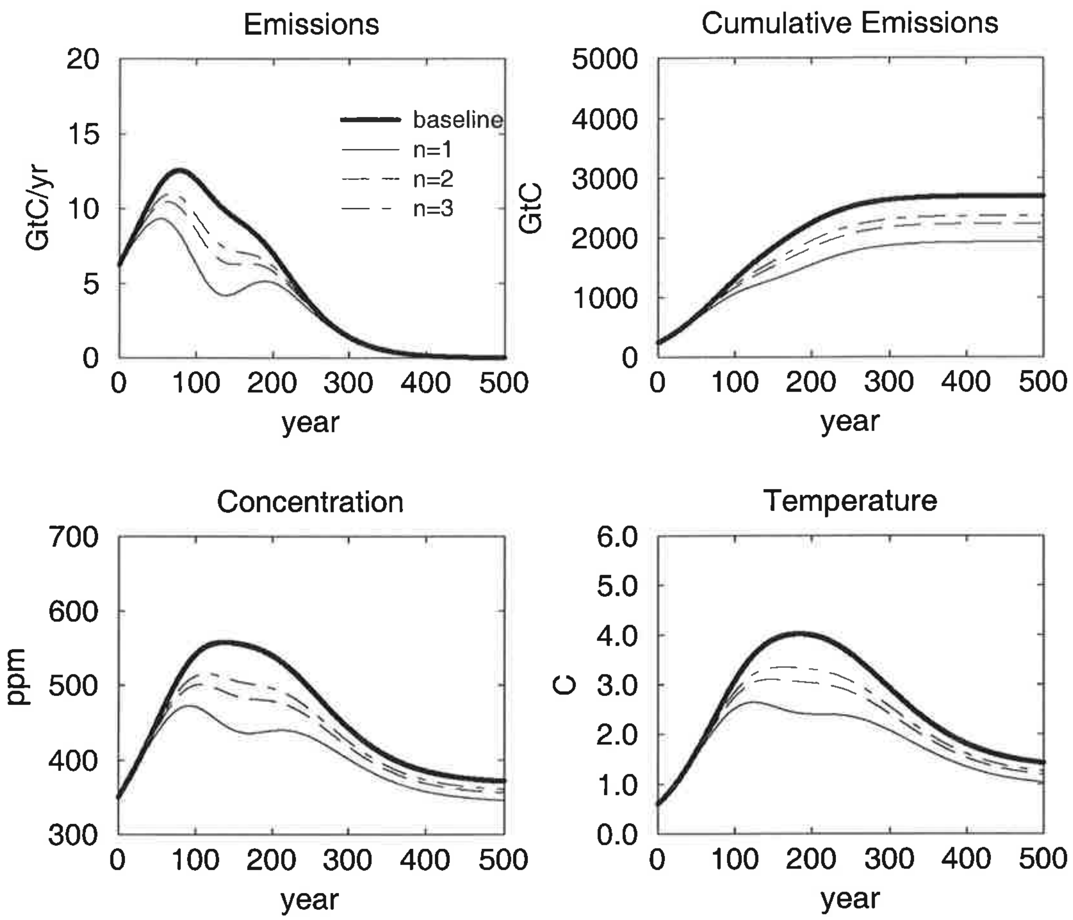


FIGURE 22. Optimal emissions policy and corresponding climate response for parameter γ_D uncertain. Values are shown corresponding to a gamma distribution described in table 9 and for different values of n. Notice that for symmetric distribution of γ_D, we obtain exactly the baseline emission path.

These results illustrate an application of the precautionary principle: the expectation that undesirable states of the world may be realised, calls for prevention. They also illustrate the fact that depending on the sign of the costs associated with those states of the world (high climate damages or high abatement costs) emissions will be curved in one direction or the other. The effect on the optimal policy may be significant, as shown in figures 22 and 23. Ignoring uncertainty leads also to a severe underestimation of the expected costs associated to climate change.

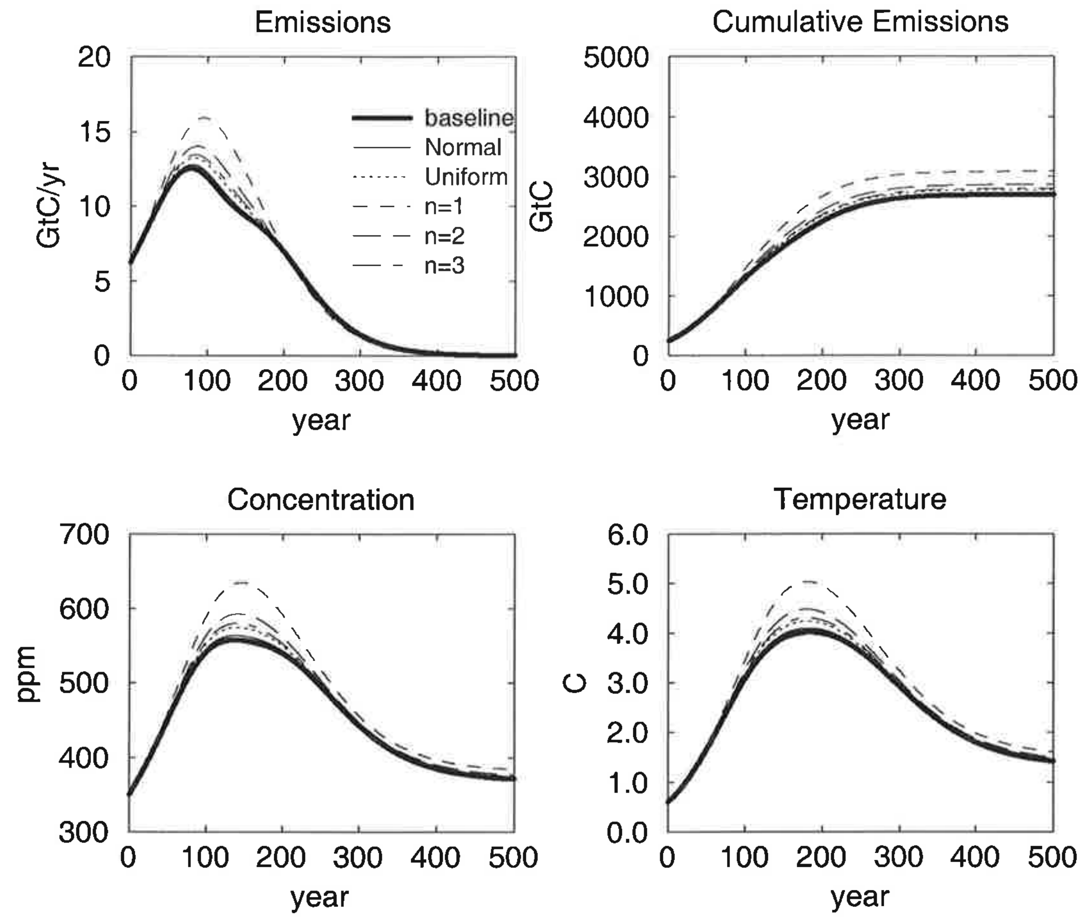


FIGURE 23. Optimal emissions policy and corresponding climate response for parameter *a* uncertain. Both symmetric and gamma distributions for different values of n are shown

Finally, the degree of confidence that we have in the estimate of the most probable value has also an important effect on the optimal policy and costs. If knowledge on the parameter distribution is very poor, the effect of uncertainty in the optimal emissions and the error in the estimation of costs are higher (although this effect disappears in the case of γ_D uncertain, due to the linear dependence of costs). Furthermore, if the probability of finding dangerous states of the world is significant (n low in the gamma distribution) emissions are significantly changed and the error in the estimation of costs is high. This effects are smaller the higher the value of n, i.e. the better we know the value of the parameters.

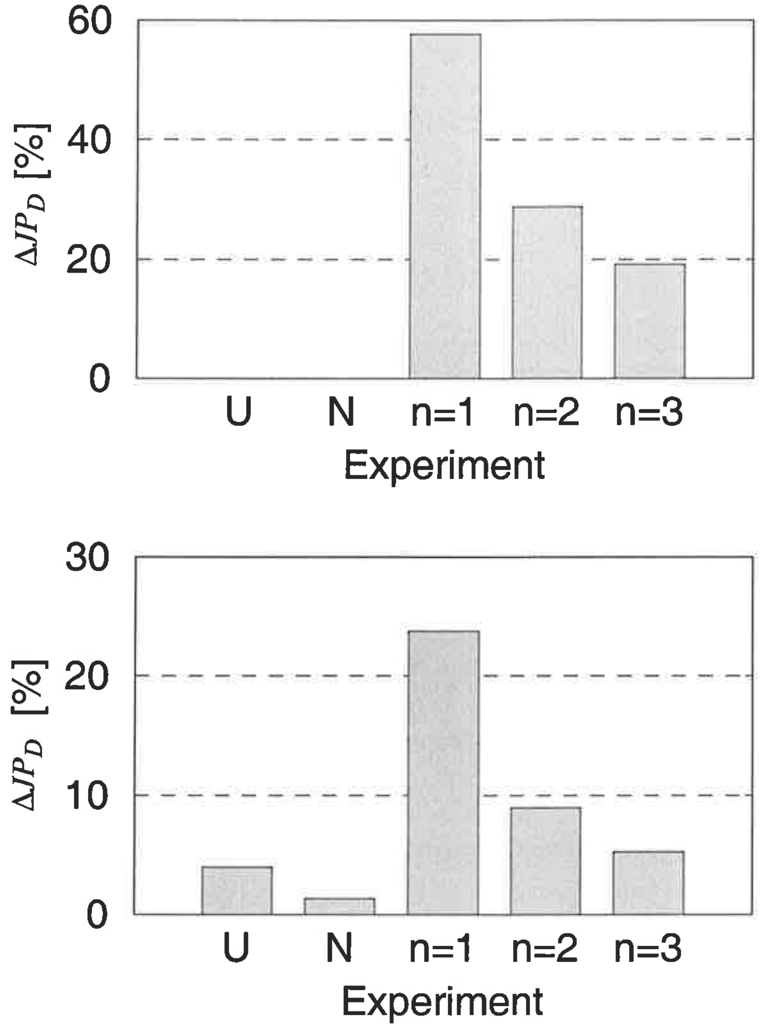


FIGURE 24. Values of ΔJ_D for different assessments of uncertainty for parameters γ_D (upper panel) and a (lower panel).

5.4 Uncertainty in the climate system

IPCC (1996a) estimates that the sensitivity of the earth's climate, i.e. the equilibrium temperature increase due to a doubling of atmospheric CO_2 concentration is in the range of 1.5 °C to 4.5 °C, with a best guess value of 2.5°C (these estimates have been extensively reviewed, discussed and cited in the literature). To account for this range of values in our climate model, we assume that the radiative forcing, represented in our global temperature equation by parameter μ, is constant and known, so that we modulate climate sensitivity of the model through α, the relaxation term in the temperature equation (representative of the time needed by the climate system to return to an equilibrium state once it has been driven out of it). Notice then that our best guess value of $\alpha = 0.03$ °C/yr. represents already the upper bound of these estimates, corresponding to a climate sensitivity of 4.2 °C. We choose for the symmetric distribution extreme values of 0.13 (1°C) and 0.028 (4.5°C). The gamma distribution uses the baseline best guess of 0.03 °C/yr. as the most probable value. An extra experiment is performed, named L5, with a most probable value of 0.077 °C/yr. (1.6 °C) and n=1.

Table 12 summarizes these parameter values

TABLE 10. ***a*) Parameter probability distribution estimates: maximum, minimum, mean, and standard deviation both for normal and uniform distributions. *b*) Parameter probability distribution estimates: expected value and standard deviation for Gamma distribution and different values of n and k_α. Experiment names are shown in brackets.**

a)	Min.	Max.	Mean	Std($\mathcal{N}$)	Std($\mathcal{U}$)
α(°C/yr.)	0.130 (1.0°C)[a]	0.028 (4.5°C)[a]	0.049 (2.6°C)[a]	0.028	0.016

b)		$\mathcal{E}\{\theta\}$	Std{θ}	k_α
α (°C/yr.)	*n*=1 (L1)[b]	0.060 (2.1 °C)[a]	0.042	0.030
	n=3 (L3)[b]	0.040 (3.2 °C)[a]	0.020	0.010
	n=1/2 (L4)[b]	0.090 (1.4. °C)[a]	0.073	0.060
	n=1 (L5)[b]	0.145 (0.9 °C)[a]	0.109	0.077

(*a*)Corresponding value of the climate sensitivity for a fixed value of μ=0.00045
(*b*)Experiment name

Much of the analysis made in the previous section for the uncertainty in the costs of climate change, can be applied here. Both the estimate of the most probable value and the confidence on its knowledge are the decisive elements in the model. The fact that our baseline set up assumed a value of climate sensitivity already on the high end of estimates is important, since the most probable value associated to the symmetric distributions is quite far from the baseline. As a consequence, the optimal policy deviates significantly from the baseline solution. Also the error in the estimation of costs is very high, although we can interpret much of this effect as the error in the estimation of the best guess value, rather than the effect of the uncertainty itself. The latter effect can be better seen in experiments L1, L3 and L4, in which differences between optimal policies and costs are due exclusively to the uncertainty in the confidence on the knowledge of α, i.e. the spread of its distribution.

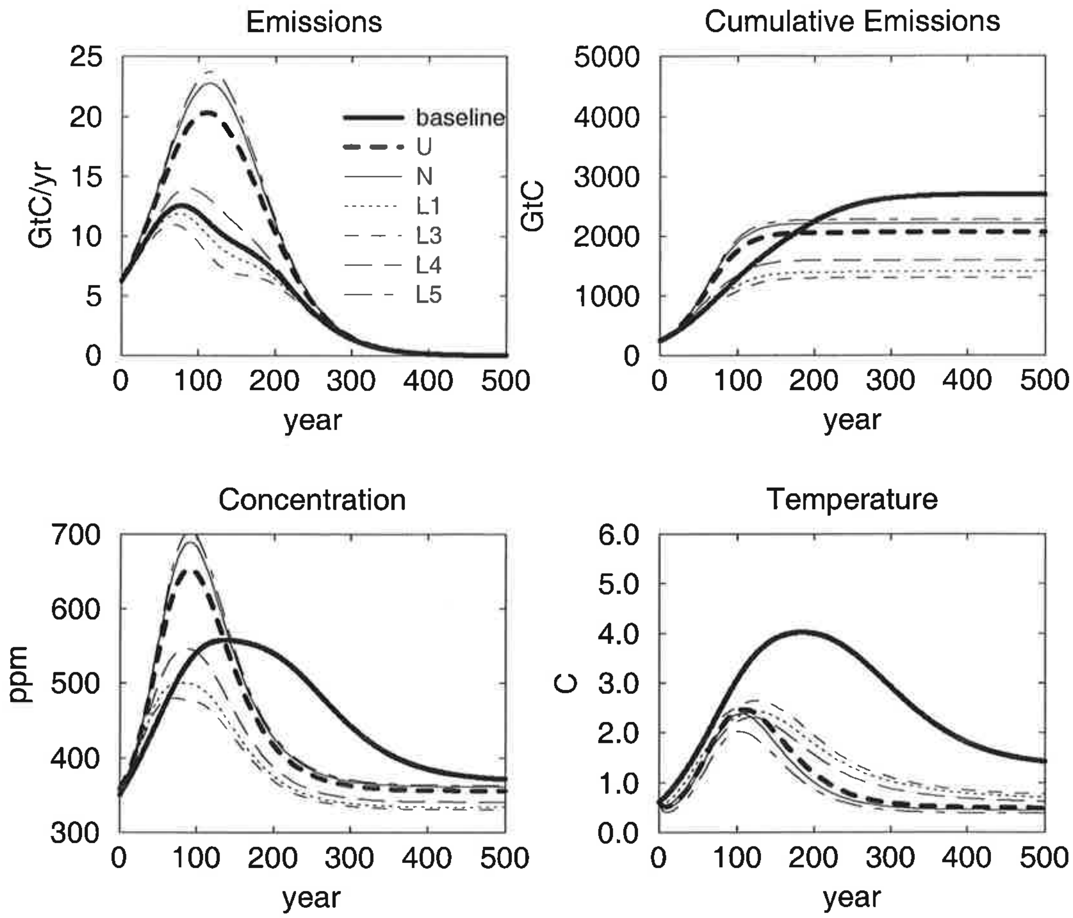


FIGURE 25. Optimal carbon emissions for parameter α under different assumptions of uncertainty: uniform distribution (U), normal distribution (N) and gamma distribution with different values of n and k_α (L1-L5).

Again, we see that we have a way of quantifying the precautionary principle. If climate is not likely to be sensitive, i.e. if high values of α have significant probability, emissions will be higher. Notice though that if confidence in the most probable value is high and the distribution is symmetric, i.e. if the distribution is normal, emissions are higher than in the low confidence (uniform distribution) case, since, in the latter case, states of the world with high sensitivity have a higher relative probability of occurrence. In the gamma distribution experiments L1, L2 and L3, we also see that for higher value of n, states of the world with higher climate sensitivity have a higher relative weight and emissions are further reduced.

The high values of ΔJP_D indicate that deterministic policies ignoring uncertainty might ignore a very big part of the expected costs, i.e. might be very far away from the optimal solution.

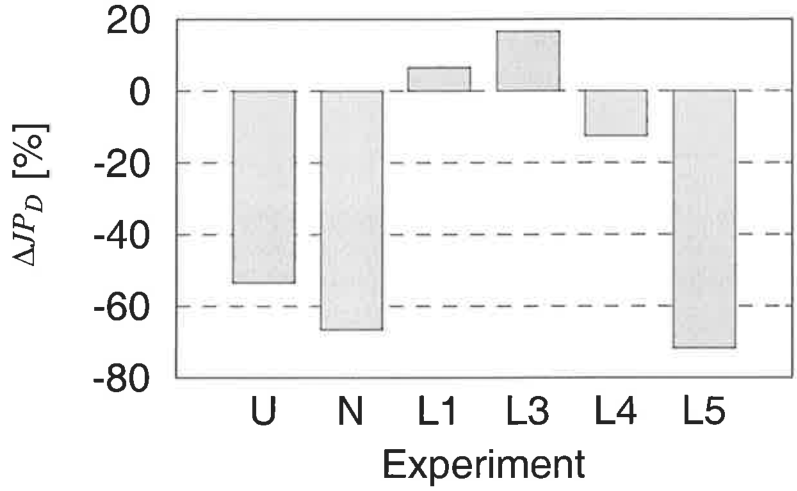


FIGURE 26. Values of ΔJP_D, the error in the estimation of expected costs if uncertainty is ignored, for different assessments of uncertainty for parameters α.

5.5 The perception problem. Expected value of perfect information

One of the assumptions made throughout this work is that an international agreement is reached among all actors (countries, interest groups, economic sectors) to define the welfare function that the central planner maximizes. Dropping this assumption transforms the problem into a multi-actor optimization problem, in which several of the actors try to maximise their own welfare functions, that can be treated in the framework of game theory (for a description of some of these questions with a similar model as the one used here, see Hasselmann (1996)). One has then to characterise the corresponding welfare functions and actors playing the game attending to different criteria: different countries with the corresponding economic parameters, energy producers vs. energy consumers, environmental-ecologist groups vs. industry groups.

In order to approach the perception problem in relation to global warming we will adopt a simplified description of the different attitudes of mankind towards nature, or different views of the natural system, that has become some what standard in the climate policy debate (see van Asselt and Rotmans (1996), Paoli (1994), Pendergraft (1998)).

Figure 17 illustrates these different views that can be grouped in four idealised conceptions of nature. Nature is seen as a ball in a potential well. The industrialist perceives the well as having infinite walls, so that no matter how large the perturbation of the system is, it will eventually return to its original state, i.e. considers that nature, the earth, has the ability and capacity of coping with any human interference. In this sense does not conceive human interaction with nature as a problem. On the other end of the spectrum, the environmentalist sees the well as a positive potential, i.e. a hill rather than a well. Any perturbation, no matter how small, will drive the system out of its unstable equilibrium state, to which it is impossible to return. Nature is perceived as very sensitive to human activities and permanently on the verge of disaster. Third the hierarchist conceives the system as a finite well: nature can take some man-made perturbations, but only up to a limit, after which catastrophic consequences are to be expected. Finally, the fatalist considers the problem as being out of our hands, since nature will follow its course independent of any actions that mankind may take. Since our interest is in the design of climate protection policies, we will centre our experiments in the first three groups (in the fatalist's view, no policy intervention would make sense, since one cannot predict its effects, supposing it has any effect).

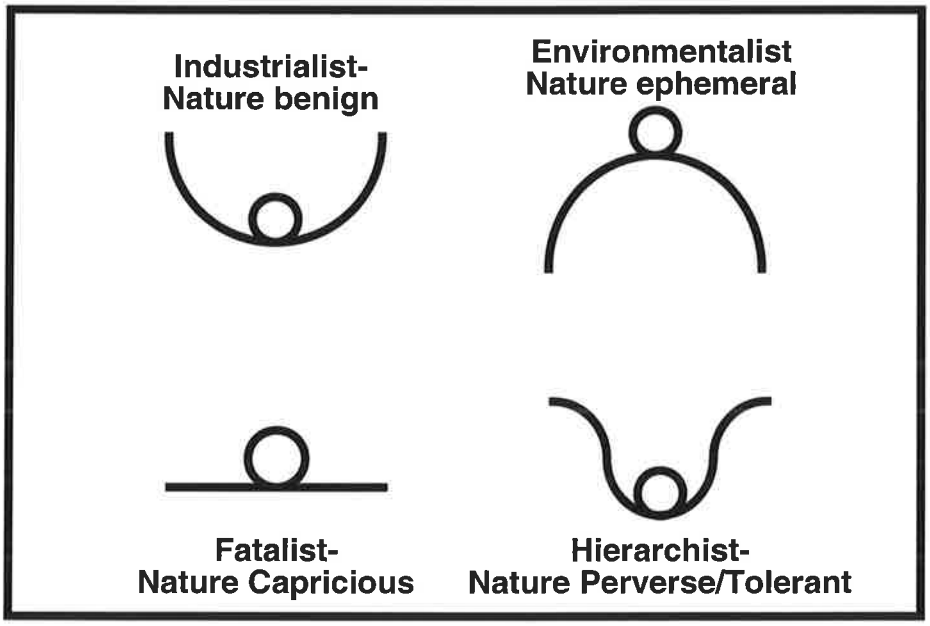


FIGURE 27. **Schematic representation of the different views of nature (adapted from van Asselt and Rotmans (1996)).**

Using the characterisation of uncertainty of the previous section through parameters γ_D, a and α we can also describe these different views of nature. In general, the industrialist will be characterised by small values of γ_D (little effect of human activities on the climate system), and moderate to high values of economic inertia a (costs of perturbing economic growth would be high). On the other extreme, the environmentalist would be characterised through high values of γ_D (very sensitive nature) and rather small values of a. Both these views of nature would assign to the unknown parameters relatively small variance since they both consider their beliefs as almost certainly true. Finally, the hierarchist would model the system with moderate mean values of the parameters with rather high variance, to account for the high uncertainty involved. Table 11 summarises the probability distributions that each of the views of nature assigns to the uncertain parameters. For the sake of simplicity we assume all distributions are Gaussian.

TABLE 11. Characterization of the different views of nature through probability distributions.

	Industrialist		Environmentalist		Hierarchist	
	Mean	Stdev.	Mean	Stdev.	Mean	Stdev.
γ_D (%)	1.0	0.6	6.0	0.6	3.0	1.7
a (yrs.)	100.0	9.6	20.0	9.6	50.0	28.9
α (°C/yr.)	0.12	0.005	0.03	0.005	0.049	0.016

Parameters are chosen somewhat arbitrarily, to illustrate the differences in beliefs and interests that are still consistent with accepted parameter ranges and the effects they may have on the optimal emission path (figure 28). We see then that this views of the world, that though very simplified, represent the main positions dominating the man-nature interaction debate, dominate also the behaviour of the model, specially the optimal emission path. In particular the way in which we quantify uncertainty, the relation between the best guess values and the spread of their distribution, the expectation of low probability-high impact states of the world (that translate into non-symmetric distributions, with long tails) have pivotal importance for the results. Remember once again that the parameter combinations chosen to represent this idealised views of nature fall fully in the presently accepted ranges considered as possible. Consequently, quantitative predictions on future optimal emission policies or more generally climate protection strategies, specially ones looking far into the future, should be interpreted cautiously.

The value of ΔJP_D represents, according to each view of nature, how wrong is the baseline policy when calculating the costs of climate change (figure 29). Also here differences are important, resembling the positions in the social debate about climate change. The industrialist considers almost all costs calculated by the baseline are artificial, i.e. an artifact of wrongly estimating the state of the world. The environmentalist finds costs have been underestimated and the issue of climate change has not been properly prevented. Finally, in the hierarchist point of view the baseline calculations overestimated total climate costs, mainly as a product of choosing too high a value of the climate sensitivity.

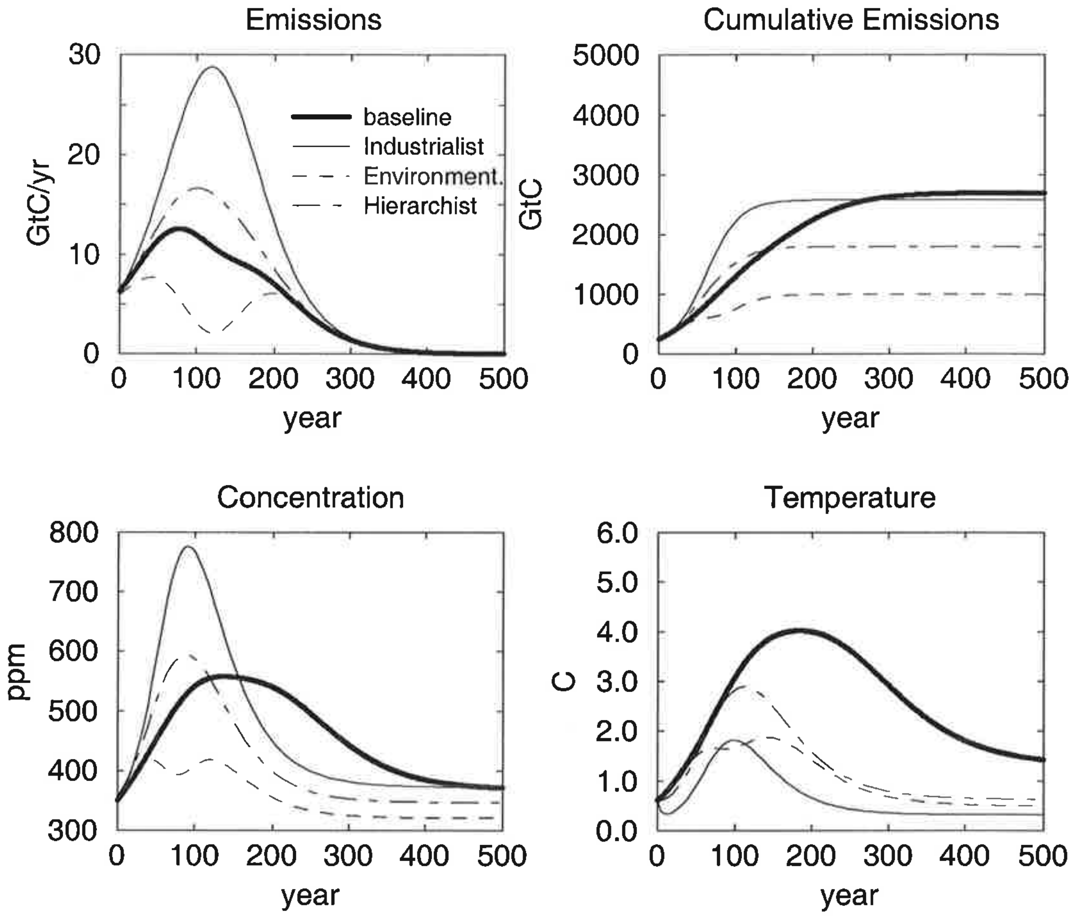


FIGURE 28. Optimal emissions policy and corresponding climate response for the different simplified views of nature.

The expected value of perfect information

As we have seen in this section, the particular believes of the planner as to the precision of his knowledge on the socioeconomic system, translated into its particular realisation of a probability distribution for the uncertain parameters, has a big influence on the design of an optimal climate policy. A related consequence is that information or the resolution of uncertainty have different values depending on the particular view of nature.

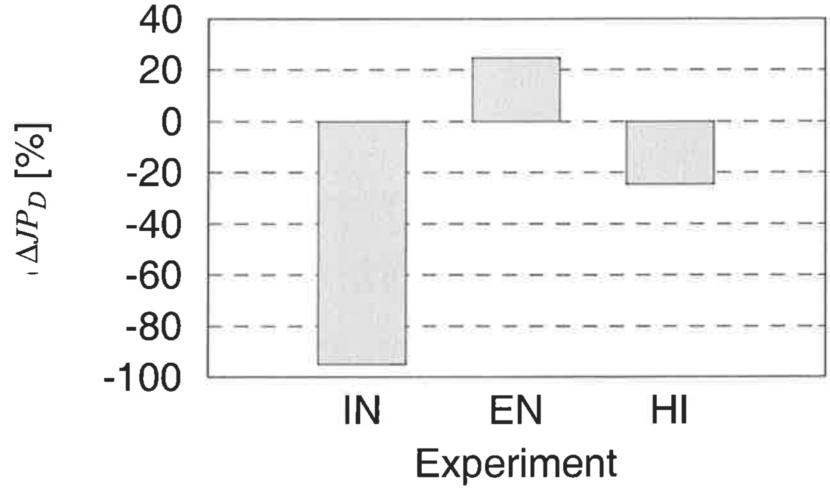


FIGURE 29. **Values of ΔJP_D, the error in the estimation of expected costs if uncertainty is ignored, for different assessments of uncertainty according to the three views of nature: industrialist (IN), environmentalist (EN) and hierarchist (HI).**

Decision analysis provides a general paradigm to calculate the economic value of information. We define the **E**xpected **V**alue of **P**erfect **I**nformation as

$$EVPI = \mathcal{E}_{\mathcal{P}(\theta)}\{J(u)\} - \mathcal{E}_{\mathcal{P}(\theta)}\{J_\theta(u_\theta)\}$$

$\mathcal{E}_{\mathcal{P}(\theta)}$ is the expected value taken over the probability distribution associated to the different states of the world, and subindex θ in the second term indicates the particular state of the world. The EVPI is then the difference between: a) the expected costs resulting if a single optimal policy has to be chosen under uncertainty and b) the expected costs resulting if the state of the world was known beforehand and an optimal policy could be calculated for each state of the world. EVPI is a non-negative quantity, since the previous knowledge of the state of the world allows the decision maker to tailor an optimal policy to the state at hand thus finding a better optimum, i.e. smaller costs.

We express the expected value of perfect information as a percentage of the costs if the state of the world is known.

$$EVPI_P = \frac{EVPI}{\mathcal{E}_{\mathcal{P}(\theta)}\{J_\theta(u_\theta)\}}$$

This quantity is easily calculated for the different views of the world presented in the previous subsection (notice that $\mathcal{E}_{\mathcal{P}(\theta)}\{J(u)\} = J^D$ in our formulation).

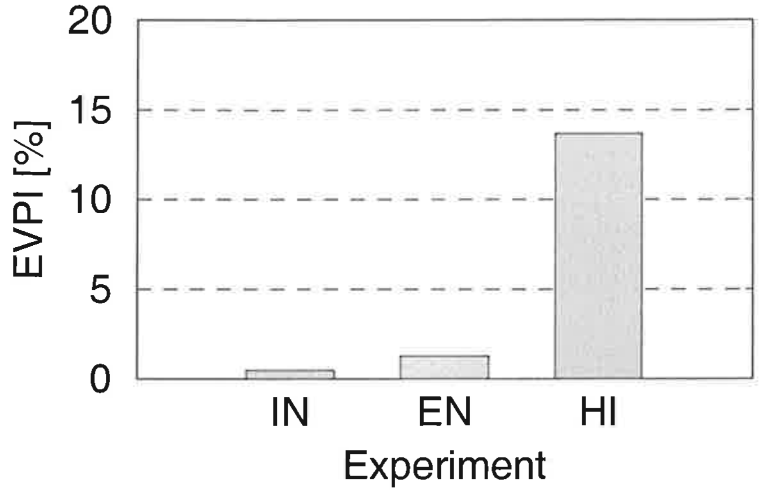


FIGURE 30. Expected value of perfect information (EVPI) according to each of the views of nature: industrialist (IN), environmentalist (EN) and hierarchist (HI).

Figure 30 shows the EVPI for the industrialist, environmentalist and hierarchist views of nature and illustrates an interesting, although expected, result. The economic value of information is much higher for the hierarchist view of nature than for any of the other two, that are characterised by a high confidence on the own beliefs on nature (and thus assign much smaller covariance to their estimates of the state of the world's probabilities).

This results let us further characterise the attitudes of the different views of nature toward uncertainty and information and to explain their optimal policies in terms of these quantities. The industrialist, as we saw before, does not consider global warming a major threat, and information on the possible uncertainties has not much value. As a result, its optimal policy deviates only little from the BAU emission path. On the other extreme of the spectrum, the environmentalist believes that global warming is a major problem resulting in high costs; information is also not very valuable, since the system is thought to be well enough understood. Accordingly, the course of action proposed is to intervene aggressively to counteract and prevent global change, and not pay much attention to solving the uncertainties involved. Finally, in the hierarchist's view, global change might suppose a major threat, but uncertainties are considered high. The resulting optimal policy acts carefully reducing emissions moderately to prevent climate change. Also, the economic value of information is high, so that a part of the efforts and resources devoted to the problem of global change should be directed toward achieving a better understanding of the socioeconomic system, that would allow for a better and less costly design of a climate protection policy.

CHAPTER 6

Conclusions

One of the major tasks facing us is the design of climate protection policies to prevent the welfare losses related to a possible future climate change. To that end, GES models, trying to capture the essential phenomena and dynamics involved in the coupled climate-economy system, are used. One of the essential characteristics of the problem is that policy design has to be done under great uncertainty. Among the many sources of uncertainty, two essentially different origins were identified: first, the inherent stochasticity of the climate system (and of the coupled climate-economy system) that generates from its internal dynamics; second, the uncertainty associated with the imperfect knowledge of the system and in part introduced in the modelling process through the necessary simplification and aggregation. Although the major assumptions and elements of the deterministic GES models can be kept, some structural changes are needed, most importantly the redefinition of the objective function (maximizing of global welfare) in order to have a well formulated problem: in our case the redefinition reduces to maximization of the *expected* welfare or minimization of expected climate protection costs.

In order to further study and quantify these effects, a simple structural coupled climate economy model is constructed. Together with the uncertainty introduced in the modelling process, the model includes a stochastic climate module, capable of generating its own natural variability, and inherits the major uncertainties in the system that still remain unsolved. Many of the mechanisms generating natural climate variations are still unknown or not well understood and the model adopts accordingly several configurations to cover different possibilities:

- Natural variability independent of the climate state: this form of variability can have its origin both in internal dynamics and non linear interactions, and on external sources like variations of the energy input from the sun or vulcanism.

- Natural variability coupled to the climate system, in particular to global warming; the natural and anthropogenic variations of climate influence each other in a feedback process.
- Climate catastrophes and surprises: this is a special form of climate variability in which variations of the climate system take the form of sudden, large amplitude changes, possibly triggered by global warming.

Also, knowledge of the climate system and most importantly, its interactions with the socioeconomic system, in the form of GH effect and the impacts of the associated global warming, is still incomplete and debated in many areas. We identify three different categories in uncertainty:

- Uncertainty in the climate system. The main unknowns are the feedback mechanisms related to albedo, water vapour and clouds, that would enhance (or counter) global warming. Also, the role of the ocean in global warming is a crucial element.
- Uncertainty in the socioeconomic system. Most important to our work are the impacts and costs caused by climate change, summarised in the cost function. Knowledge in this field is still fragmentary and controversial.
- Differences in perception of the nature-society interaction, i.e. the perception problem. Notice that while the other two sources of uncertainty can to a great extent be reduced through research, this latter form may not, since differences in the views of nature are not only based on scientific knowledge, but on cultural and moral values and in different interests.

Since the mechanisms involved in natural climate variability and the sources of uncertainty are far from being completely understood or solved, much research has to be done to improve our knowledge, and only their potential effects can be roughly estimated. In general, we argue that both natural climate variability and uncertainty may not be second order effects, and the design of optimal climate control policies has to take both of them into account in order to come up with realistic solutions to the global change problem. The effect of both phenomena can be important and ignoring them might render policy efforts sub-optimal or even not feasible.

The Precautionary Principle is often invoked to ask for more restrictive GHG emission policies if high impact outcomes of climate change are possible. It turns out that it is necessary to quantify in more detail what high impact means, in terms of whether the major uncertainties are in the impacts of climate change or in the costs of abatement measures. This is in part due to the use of the expected value in the expression of climate costs, that weights different outcomes with their corresponding probabilities, so that low probability events mean in general low risk. This raises questions as to the appropriateness of using the expectation of costs as a decision criterion, since it smooths out risks in the averaging process. Other useful criteria might try to avoid high impact outcomes independent of their probability or may weight risk in different ways.

6.1 Climate policy and climate natural variability

The presence of natural climate variability and its interaction with climate change highlights the importance of adaptation as a climate policy option. Since the future evolution of the climate system is not known with certainty, emission policies do their best when the actual evolution of climate is taken into account. This can be translated into the optimality of choosing flexible policies and institutions, rather than fixed irreversible ones. Also, due to the coupling between natural climate variability and climate change and to the memory of the system, the optimal policy is in general a combination of adaptation and prevention, since part of the effects of climate variations can be known beforehand.

The coupling between natural climate variability and climate change is identified as a potentially very important mechanism. Results vary depending on the sign and nature of the coupling, but they could have similar magnitude to the estimated deterministic GH effect.

The expectation of climate catastrophes and surprises triggered by global warming may, in some cases call for much more restrictive emission policies than if variations of climate conditions are smooth. This is an expression of the Precautionary Principle, advising to avoid high impact climate events even if they have relatively low probability of occurrence. We can quantify this effect as a function of the risk of a catastrophe, that includes both the probability of a climate surprise and its magnitude.

In addition, ignoring climate variability and its interaction with climate change can lead to a severe underestimation of climate costs. The expected climate protection costs are higher for the stochastic system than for the corresponding deterministic one. The extra costs have their origin in the error in estimating the system's future state; part of these costs cannot be avoided, since the particular realization of the stochastic components (climate natural variability or business cycles) cannot be known in advance, but another part can be, by means of observing the system and adapting to it.

6.2 Climate policy and uncertainty

Since uncertainty is high and likely to persist, we need to design robust policies that perform well in a variety of possible states of the world. These might differ significantly from any deterministic policy based on best guess estimates of the uncertain elements and ignoring otherwise uncertainty. Once again, failing to recognise this effect might lead to the application of policies that are suboptimal when their result is averaged over the different possible states of the world. Furthermore, under uncertainty, adaptation will probably play a major role as a policy option. Notice that uncertainty can be solved by means of research and improved scientific knowledge, so that the future is likely to bring progressively new information on the climate-society interactions. In this sense the importance of choosing flexible policies and institutions to cope with global warming has to be stressed again.

The adoption of optimal policies to protect climate has been characterized as highly irreversible. Due to the inertia of the climate system, global warming is also a highly irreversible effect. These two forms of irreversibility, in combination with uncertainty, request a response in opposite directions: the former would lead to a delay in the adoption of a climate policy, specially if the passage of time is likely to bring information, whereas the latter would require aggressive action to avoid the build-up of GHG in the atmosphere and the long term consequences of global warming. Both effects, characterised in our model through the economic inertia of the system and the climate sensitivity, have a relevant effect in the optimal emission policy and costs.

The optimal climate policy will probably be a combination of adaptation and prevention, in order to avoid undesirable high impact outcomes of our policy and at the same time improve the design of policies and institutions in view of the new information available in the future, new scientific knowledge, the actual climate evolution, technical possibilities and different social or political values. Notice that this adaptation will eventually take place anyway, and what becomes important will be the cost of abandoning or modifying already existing policies. Thus, together with the evaluation of the possible future evolution of the coupled system, the flexibility of the actions undertaken, the degree of irreversibility in the system and the flow of information become the relevant issues.

Results show that the way in which we quantify uncertainty, i.e. the choice of particular probability distributions and parameter values for them, plays a very important role in the final outcome. This is a point in which much work has still to be done, since, at present, knowledge on parameter mean values, extremes and probability of occurrence is fragmentary and not always convergent. Quantification of the precautionary principle depends strongly on our beliefs on how the system works and how well we know it. Overall, it is our view of nature, that translates in our estimation of parameter values and functional forms, that dominates the particular outcomes of our modelling efforts.

With the help of Cultural Theory, three relevant simplified types were identified, with which the present dominant positions in the global change debate can be described: the industrialist, for whom climate change is not a relevant issue; the environmentalist, which considers nature fragile, and global warming a major threat; and the hierarchist, for whom nature is able to adapt only to a certain point, and global warming is a potential threat. Different views of nature can be characterised not only through the mean values of the uncertain parameters, but also through their attitudes toward uncertainty and the value of information. Both the industrialist and the environmentalist consider their beliefs to be almost certainly true, and therefore do not consider uncertainty to be relevant or information valuable. On the other hand the hierarchist recognises the uncertainty involved in the problem and considers information to be very valuable, since it would help to appropriately tailor a climate policy to the real world.

6.3 Outlook

As we have seen throughout this work, there are many interesting and open questions concerning the design of optimal climate protection policies. A high level of uncertainty is present in all subsystems and a better understanding of the climate and socioeconomic systems and their interaction is necessary to attempt their control. An assessment of uncertainties, not only in the form of extreme and mean values, but with probability distributions is necessary. Also, structural uncertainties, the ones related to the functional forms and the nature of the interactions between systems (not only parameter values for processes whose basic dynamics are understood) are very high and need to be reduced. In particular, the cost function, relating climate change and its impact on socioeconomic systems needs to be improved. In this sense it would be necessary to enhance the definition of impacts and damages to include both natural variations of climate and a dynamical component, in which adaptation would probably play a major role.

In addition, the interactions of climate change and climate variability have been identified as potentially being important in the global warming problem. The mechanisms generating climate variability and their changes in response to anthropogenic induced change are not yet well understood. Similarly, only little is known about the possibility of sudden catastrophic changes, whether influenced by human actions or of natural origins, and only some speculations are available. A better understanding of these processes is necessary in order to asses their potential effect on the global change process.

On the socioeconomic side too, several lines of research are already active. As we have seen the attitude of the central planner toward nature is decisive in the policy making process. Different views of nature are present in the present social debate and policy making process, and are influenced by both the scientific uncertainty and different beliefs and interests. A better understanding of the reasons behind these discrepancies and the interactions of different attitudes in a game theoretical framework is needed. Also other optimization criteria should be further investigated, which would specify different attitudes toward risk and uncertainty, and study the effect of these differences in the policy making process. Finally, the effects of learning and the acquisition of information are further influential issues in the global warming problem.

Acknowledgements

I thank Professor Dr. Klaus Hasselmann and Professor Dr. Hans von Storch for giving me the opportunity of developing this work at the Max-Planck Institut für Meteorologie, and for showing a great deal of confidence in me. I am also very grateful to Professor Dr. J.M. Savirón and Professor Dr. R. Usón, at the University of Zaragoza (Spain), for their encouragement and help on the early stages of my stay in Germany. I also thank Dr. Eduardo Zorita, who was my first contact at the MPI in Hamburg and Dr. Ulrich Cubasch for whom I worked after my arrival to Hamburg.

This study has benefited from many discussions with Dr. Ulrich Kilian, who had many useful comments and ideas. Dr. Viatcheslav Kharin was also a great help specially in the early stages of the work. During that time the whole of the von Storch Group at the Max-Planck Institut für Meteorologie was a very friendly and stimulating company. Dr. Carlos Ocaña read earlier versions of the work and made many useful comments. Simon Blessing helped me with the german part of my dissertation. I also thank Dr. Jesús Fidel González and Juan Pedro Montávez for their encouragement and help during the last stages of the work.

Patrick Heimbach, Marisa Montoya and Thinka Schlüter made this work possible with their friendship and humour. My brother Jaime Ocaña was a continuous inspiration and a friend.

List of figures

List of acronyms

BAU	Business As Usual
CL	Closed Loop
DP	Dynamic Programming
EVPI	Expected Value of Perfect Information
GCM	General Circulation Model
GES	Global Environment and Society
GH	Green House
GHG	Green House Gas
OL	Open Loop
OLF	Open Loop Feedback
SOC	Stochastic Optimal Control

References

Abrol, Y.P., Ingram, K. T., (1996), 'Effects of Higher Day and Night Temperatures on Growth and Yields of some Crop Plants',in Bazzaz, F., and Sombroek, W., (eds.) *Global Climate Change and Agricultural Production,* John Willey & Sons.

Baranzini, A., Chesney, M., and Morisset, J., (1995), 'Uncertainty and Global Warming. An Option-Pricing Approach to Policy' Policy Reasearch Working Paper 1417. The Workld Bank.

Barnett, T.P., Santer, B.D., Jones, P.D., Bradley, R.S. and Briffa, K.R., (1996) 'Estimates of low frequency natural variability in near-surface air temperature' *The Holocene* **6**, 255-263.

Brown, P.G. (1997), 'Stewardship of Climate. An Editorial Comment', *Clim. Change* **37**, 329-334.

Cess, R.D. et al. (1993), 'Uncertainties in Carbon Dioxide Radiative Forcing in Atmospheric General Circulation Models', *Science* **262**, 1252-1255.

Christy, J. R. and McNider, R. T. (1994), 'Satellite Greenhouse Signal', *Nature* **367**, 325.

Cline, W. R., (1992), *The Economics of Global Warming*, Institute for International Economics, Washington D.C.

Crowley, T. J., and Kim, K., (1996), 'Comparison of Proxi Records of Climate Change and Solar Forcing', *Geophisical Research Letters* **23(4)**, 359-362.

Dalton, M. G., (1997), 'The Welfare Bias from Omitting Climatic Variability in Economic Studies of Global Warming', *Journal of Environmental Economics and Management*, **33**, 221-239.

Eismont, O., Welsch, H., (1996), 'Optimal Greenhouse Gas Emissions under Various Assesments of Climate chage Ambiguity', *Environmental and Resource Economics*, **8**, 129-140.

Fankhauser, S. (1995), *Valuing Climate Change: The Economics of the Greenhouse*, EarthScan, London.

Fankhauser, S., Tol, R.S., (1996), 'Climate Change Costs. Recent advancements in the economic assessment', *Energy Policy*, **24(7)**, 665-673.

Fischer, G., Frohberg, K., and Rosenzweig, C., (1996), 'The Potential Effects of Climate Change on World Food Production and Security',in Bazzaz, F., and Sombroek, W., (eds.) *Global Climate Change and Agricultural Production,* John Willey & Sons.

Fisher, A. C., and Hanemann, M., (1990), 'Information and the Dynamics of Environmental Protection: The Concept of the Critical Period', *Scand. J. of Economics* **92(3)**, 399-414.

Gray, V. (1998), 'The IPCC future projections: are they plausible', *Clim. Res.* **10**, 155-162.

Grubb, M., Chapuis, T., and Duong, M.H., (1995), 'The Economics of Changing Course. Implications of Adaptability and Inertia for Optimal Climate policy', *Energy Policy* **23**, No. 4/5, 417-432.

Harrington, J. (1987), 'Climatic Change: a Review of Causes', *Can. J. For. Res.* **17**, 1313-1339.

Hasselmann, K. (1976), 'Stochastic Climate Models. Part 1: Theory', *Tellus* **28**, 473-485.

Hasselmann, K. (1991),'How well can we predict the climate crisis?', in Siebert, H. (ed.), *Conf. on Environmental Scarcity: The international dimension*, 5-6 July, Kiel, Symposien und Konferenzbände des Instituts für Weltwirtschaft an der Universität Kiel, J.C.B. Mohr (Paul Siebeck), Tübingen, pp. 165-183.

Hasselmann, K. (1998), 'Intertemporal Accounting of Climate Change: Harmonizing Economic Efficiency and Climate Stewardship',Max-Planck-Institut für meteorologie, Report 257.

Hasselmann, K., Hasselmann, S., (1996), 'Multi-actor optimization of greenhouse gas emission paths using coupled integral climate response and economic models', Max-Planck-Institut für meteorologie, Report 209.

Hasselmann, K., Hasselmann, S., Giering, R., Ocaña, V. and von Storch, H. (1997), 'Sensitivity study of optimal CO_2 emission paths using a simplified structural integrated assesment model (SIAM)', *Clim. Change* **37**, 345-386.

Heal, G. (1997), 'Discounting and Climate Change. An Editorial Comment', *Clim. Change* **37**, 335-343.

Heimann, M. (1993), 'The Global Carbon Cycle in the Climate System', in Wllebrand, J. and Anderson D.L.T. (eds.) *Modelling Oceanic Climate Interactoions* NATO ASI Series, Vol. I 11, Springer - Verlag, pp. 299-336.

Hegerl G.C., v. Storch, H., Hasselmann, K., Santer, B. D., Cubasch, U. and Jones, P.D. (1996), 'Detecting greenhouse gas induced Climate Change with an optimal fingerprint method', *J. Climate* **9**, 2281-2306.

IPCC (1996a), 'Climate Change 1995', Houghton, J.T., Meiro Filho, L.G., Callander, B.A., Harris, N., Kattenberg, A., and Maskell, K., (eds.) *The Science of Climate Change. Contribution of Working Group I to the Second Assesment Report of the Intergovernmental Pannel on Climate Change.* Cambridge University Press, Cambridge.

IPCC (1996b), 'Climate Change 1995', Watson, R.T., Zinyowera, M.C., Moss, R.H., Dokken, D. J., (eds.) *Impacts, Mitigation and Adaptation of Climate Change: Scientific-Technical Analyses. Contribution of Working Group II to the Second Assesment Report of the Intergovernmental Pannel on Climate Change.* Cambridge University Press, Cambridge.

IPCC (1996c), 'Climate Change 1995', Bruce, J.P., Lee, H., Haites, E.F., (eds.) *Economic and Social Dimensions of Climate Change. Contribution of Working Group III to the Second Assesment Report of the Intergovernmental Pannel on Climate Change.* Cambridge University Press, Cambridge.

Jacoby, H. D. and Prinn, R. G. (1994), 'Uncertainty in Climate Change Policy Analysis', MIT Joint Program on the Science and Policy of Global Change, Report No. 1.

Jones, G. A., (1991), 'A stop-start ocean conveyer', *Nature* **349**, 364-365.

Kolstad, C. D., (1994), 'George Bush versus Al Gore. Irreversibilities in greenhouse gas accumulation and emission control investment', *Energy Policy* **22(9)**, 771-778.

Kolstad, C. D., (1996), 'Learning and Stock Effects in Environmental Regulation: The case of Greenhouse Gas Emissions', *Journal of Environmental Economics and Management* **31**, 1-18.

Kumar, P.R., Varaiya, P., (1986) 'Stocahstic Systems: Estimation, Identification and Adaptive Control'. Prentice Hall.

Lempert, R. J., Schlesinger, M. E., and Bankes, S.C., (1996), 'When we don't know the costs or the benefits: adaptive strategies for abating climate change', *Clim. Change* **33**, 235-374.

Lindzen, R. S., (1994), 'Climate dynamics and global change', *Annu. Rev. Fluid Mech.* **26**, 353-378.

Maier-Reimer, E., (1993), 'The Biological Pump in the Greenhouse', *Global Planetary Clim. Change* **8**, 13-15.

Maier-Reimer, E., and Hasselmann, K., (1987), 'Transport and Storage of CO2 in the Ocean - an Inorganic Ocean-Circulation Carbon Cycle Model', *Clim. Dynamics* **2**, 63-90.

Manabe, S., and Stouffer, R.J., (1993), 'Century-scale Effects of increased atmospheric CO_2 on the ocean-atmosphere System', *Nature* **364**, 215-218.

Manne, A. S., and Richels, R.G., (1995), 'The Greenhouse Debate: Economic Efficiency, Burden Sharing and Hedging Strategies', *Energy J.* **16**, 4.

McManus, J. F., Oppo, D.W. and Cullen, J.L., (1999), 'A 0.5-Million-Year Record of Millennial Scale Climate Variability in the North Atlantic', *Science,* **283**, 971-975.

Munda, G. (1996), 'Cost-Benefit Analysis in Integrated Environmental Assesment: Some Methodological Issues', *Ecological Economics* **19**, 157-168.

Murota, Y., and Ito, K., (1996), 'Global Warming and Developing Countries. The Possibility of a solution by accelerating developement', *Energy Policy* **24(12)**, 1061-1077.

Nordhaus, W.D. (1991), 'To Slow or Not to Slow: The Economics of the Greenhouse Effect', *Econ. J.* **101**, 920-937.

Nordhaus, W.D. (1993), 'Rolling the "DICE": An Optimal Transition Path for Controlling Greenhouse Gases', *Resource Energy Econ.* **15**, 27-50.

Nordhaus, W.D. (1994), 'Expert Opinion on Climate Change', *Amer. Sci.* **82**, 45-51.

Nordhaus, W.D. (1997), 'Discounting in Economics and Climate Change. An Editorial Comment', *Clim. Change* **37**, 315-328.

Nordhaus, W.D. and Yang, Z. (1996), 'A Regional Dynamic General-Equilibrium Model for Alternative Climate-Change Strategies', *Amer. Econ. Rev.* **86**, 741-765.

O'Neill, B.C. Oppenheimer, M., and Graffin, S. R. (1997), 'Measuring Time in the Greenhouse', *Clim. Change* **37**, 491-503.

Paoli, G., (1994), 'Climate Change: Policy Making or Science', in Climate Change Uncertainty and Decision Making, Greg Paoli, Editor. Institute of Risk Research, University of Waterloo (Ontario, Canada).

Peck, S.C. and Teisberg, C.J. (1992), 'CETA: A Model for Carbon Emissions Trajectory Assesment', *Energy J.* **13**, 55-77.

Peck, S.C. and Teisberg, C.J. (1993), 'Global Warming Uncertainties and the Value of Information: An Analysis using CETA', *Resource and Energy Economics* **15**, 71-97.

Peck, S.C. and Teisberg, C.J. (1995), 'Optimal CO_2 Control Policy with Stochastic Losses from Temperature Rise', *Clim. Change* **31**, 19-34.

Pendergraft, C. A., (1998), 'Human Dimensions of Climate Change: Cultural Theory and Collective Action', *Climatic Change.* **39**, 643-666.

Pielke, R. A., (1998), 'Rethinking the Role of Adaptation in Climate Policy', *Global Environmental Change* **8**, 2, 159-170.

Risbey, J., Kandlikar, M., and Patwardhan, A. (1996), 'Assessing Integrated Assessments', *Clim. Change* **34**, 369-395.

Shackley, S., Risbey, J., Stone, P., and Wynne, B., (1998), 'Adjusting to Policy Expectations in Climate Change Modelling: An Interdisciplinary Study of Flux Adjustments in Coupled Atmosphere-Ocean General Circulation Models'. MIT Joint Program on the Science and Policy of Global Change, Report No. 53.

Schelling, T. C., (1991), 'Benefits of Mitigation and Adaptation',in White, J. C., (ed.) *Global Climate Change: the Economic Costs of Mitigation and Adaptation*, Elsevier Science Publishing Company.

Smith, J.B. (1996), 'Standardized estimates of climate change damages for the United States', *Clim. Change* **32**, 313-326.

Tahvonen, O., von Storch, H., and von Storch, J. (1994), 'Economic Efficiency of CO_2 Abatement Programs', *Environmental Modelling and Assesment* **2**, 151-163.

Tol, R.S. (1996), 'The damage Cost of Climate Change Towards a Dynamic Representation', *Ecological Economics* **19**, 67-90.

Tol, R.S. (1997), 'On the optimal control of carbon dioxide emissions: an application of FUND', *Clim. Res.* **4**, 127-141.

Valverde, A., Jacoby, H. D. and Kaufman, G.M. (1998), 'Sequential Climate Decisions under Uncertainty: An Integrated Framework'. MIT Joint Program on the Science and Policy of Global Change, Report No. 38.

van Asselt, M. and Rotmans, J. (1996), 'Uncertainty in Perspective', *Global Environmental Change,* **6**, 2, 121-157.

von Storch, H., Hasselmann, K., (1994), 'Climate variability and change', Talk presented at *The European Conference on Grand Challenges in Ocean and Polar Sciences* in Bremen, September 12-16,1994. Max-Planck-Institut f ür Meteorologie, Report No. 152.

von Storch, J.-S., Kharin, V., Cubasch U., Hegerl G.C., Schriever D., von Storch, H., Zorita, E., (1997), 'A 1260-year control integration with the coupled ECHAM1/LSG general circulation model', *J. Clim.* **10**, 1525-1543.

Webster, M.D., and Reiner, D.M., (1997), 'Beyond Emissions Paths: Rethinking the Climate Impacts of Emissions Protocols in an Uncertain world'. MIT Joint Program on the Science and Policy of Global Change, Report No. 31.

Webster, M.D., and Sokolov, A., (1998), 'Quantifying Uncertainty in Climate Predictions'. MIT Joint Program on the Science and Policy of Global Change, Report No. 37.

Welsch, H. (1995), 'Greenhouse gas abatement under ambiguity',*Energy Economics* **17**, 2, 91- 100.

Wigley, T. M. L., and Richels, R.G., and Edmonds, J.A. (1996), 'Economic and Environmental Choices in the Stabilization of Atmospheric CO_2 Emissions', *Nature* **379**, 240-243.

Wunsch, C. (1994), 'The Ocean Circulation and Climate', in *The general circulation of the oceans*, Paola Malanote Rizzoli (ed.), Instituto Veneto di Scienze, Lettere ed Arti, pp. 9-53.

Wunsch, C. (1998), 'The Ocean Circulation and Climate', in *Proceedings of the conference on the World Climate Research Programme: Achievements, Benefits and Challenges*, WMO/TD-No. 904, pp. 64-68.

Yohe, G. (1996), 'Exercises in hedging against Extreme consequences of global change and the expected value of information', *Global Environmental Change* **6**, 87-101.

Appendix A: Dynamic Programming

The principle of optimality and the Dynamic Programming Algorithm

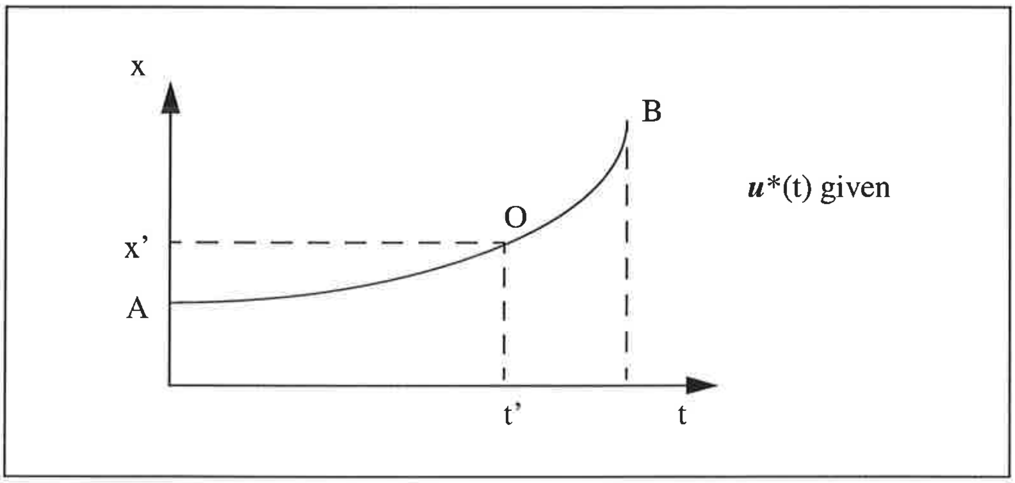
Let the problem to be solved be the maximization of

$$J = \varphi(x_f, \tau) + \int_0^{\tau} I(x, u, t)dt$$

with the equations of motion

$$\dot{x} = F(x, u, t)$$

The DP algorithm makes use of the well known *principle of optimality*, that states that an optimal policy has the property that whatever the initial state and decision are, the remaining decisions constitute an optimal policy with regard to the state resulting from the initial decision.



In the schematic figure above, let AB be the state variable path corresponding to a certain optimal policy $\boldsymbol{u}^*(t)$ for $t_0 \le t \le t_1$. The principle of optimality tells us that the same optimal policy will be applied if we start the problem at any point directly on the trajectory AB, for instance $O \equiv (x', t')$, i.e. that $\boldsymbol{u}^*(t)$, $t' \le t \le t_1$, is the optimal policy for the problem starting at that point.

We may now present our optimal solution in two different ways. The *open loop* solution specifies the optimal control $\boldsymbol{u}^*(t)$ for all times as a function of initial time and state (x_0, t_0), and proceeding forward to a point in the final hypersurface $Q(x_f(t_f), t_f)=0$, which is the locus of points that we can reach with feasible controls. According to the principle of optimality, the problem starting at any point $(x(t), t)$ in the optimal trajectory has the same optimal control, but if we want to know the optimal solution to a problem starting at a point *not* in that trajectory, we have to solve a new optimal control problem starting at that point.

In many cases though, we may want (or need) to know the solution to the problem starting at many different points *(x, t)* and ending at the given final hypersurface, so that we have to calculate a whole family of optimal paths. We may think of the optimal policy $\boldsymbol{u}^*(t)$ as associated to the point *(x, t)* of the corresponding trajectory, and since in general only one optimal path to the final hypersurface will pass through that point, a unique optimal control is associated with it so that we can write $u^* = u^*(x, t)$.
which is the optimal feedback control law or *closed loop* solution, that gives the optimal control as a function of present state and time.
A good example that illustrates these ideas is a thermostat, with which we try to keep the temperature of a room as close as possible to a given value. Given an initial value of the temperature and time (T_0, t_0), we can determine the set of actions $\boldsymbol{u}(t)$ (when to turn on or off the radiator) for the whole interval $t_0 \le t \le t_1$, i.e. the open loop policy. Nevertheless, since we don't know exactly when do we want to start heating or the temperature at that moment, it would be desirable to have an *optimal rule* that tells us what to do for each possible state *(T, t)*. In other words, we want to express the control as a function of the state, i.e. calculate the closed loop policy (we incorporate a thermometer that measures the room temperature and possibly find a rule that minimizes $(T\text{-}T_{desired})$). Another case in which we need the control in a closed form is that in which the evolution of temperature is not fully deterministic (for instance if it depends on the weather outside the room), i.e. it is not certain what is the future effect of present actions.

Notice that in the deterministic case, for a given initial state and time, both the closed and open loop solutions are identical.

The dynamic Programming algorithm.

Let the problem to be solved be

$$J_0(x_0) = min\mathcal{E}_w\left\{g_N(x_N) + \sum_{i=0}^{N-1} g_i(x_i, u_i, w_i)\right\}$$

subject to

$$x_{i+1} = f_i(x_i, u_i, w_i) \qquad x_0 \text{ given}$$

Let $J_i^*(\boldsymbol{x}_i)$ be the optimal integrated costs for the problem starting at time i and state $\boldsymbol{x}_i$ (then the solution of the problem at hand is $J_0^*(\boldsymbol{x}_0)$). Now, according to the principle of optimality the solution to the problem starting at $(i+1, \boldsymbol{x}_{i+1})$ is $J_{i+1}^*(\boldsymbol{x}_{i+1})$. The difference between the two can only come through the intermediate function that adds $g_i(\boldsymbol{x}_i, \boldsymbol{u}_i, \boldsymbol{w}_i)$, so that we have

$$J_i^*(x_i) = g_i(x_i, u^*_i, w_i) + J^*_{i+1} = min_{u_i}\mathcal{E}_w\{g_i(x_i, u_i, w_i) + J^*_{i+1}(f_i(x_i, u_i, w_i))\}$$

which the fundamental recurrence relation.

Another way to see this: he functions $J^*(\boldsymbol{x}_i)$ represent the *cost to go*, the optimal (minimum) costs from time i to the end of the planning horizon once the optimal policy u_i^*, $i = 0, \ldots, N-1$ is chosen. Suppose we are in stage i, and have reached state $\boldsymbol{x}_i$ applying $\boldsymbol{u}_j$, $j = 1,...,i-1$. The cost to go at this stage is, by definition, the sum of the instantaneous costs $\boldsymbol{g}_k$ plus the costs to go at stage $i+1$

$$J_i^*(x_i) = min_{u_i} \mathcal{E}_w \{ g_i(x_i, u_i, w_i) + J^*_{i+1}(f_i(x_i, u_i, w_i)) \}$$

which is again the fundamental recurrence relation.

The boundary condition for this recurrence relation is given by the function $g_N(\boldsymbol{x}_N)$, i.e. we start the relation with

$$J_N(x_N) = g_N(x_N)$$

and proceed backwards applying the recurrence relation.

At each time step, the condition of minimum will let us eliminate $\boldsymbol{u}_i$ from the recurrence relation, solving it as a function of $\boldsymbol{x}_i$, and then solve the corresponding equation for J. We may think of the DP algorithm as a set of problems of increasing length. The boundary condition is nothing else than the problem of length zero starting at the last stage of the problem. Going one step backwards, we apply the recurrence relation at stage N-1, solving the problem of length 1: assuming $\boldsymbol{u}_{N-2}$ known, find $\boldsymbol{u}_{N-1}$ to minimize

$$J_{N-1}(x_{N-1}) = min_{u_{N-1}} \mathcal{E}_w \{ g_{N-1}(x_{N-1}, u_{N-1}, w_{N-1}) + J_N(f_{N-1}(\ldots)) \}$$

The condition of minimum allows us to eliminate $\boldsymbol{u}_{N-1}$ as a function of $\boldsymbol{x}_{N-1}$, and substitute it in this equation. In practice we have to solve the minimization task numerically, so we have to calculate J_{N-1} for all possible values of $\boldsymbol{x}_{N-1}$ (or a finite number of them in a discretized mesh), obtaining a different value of J_{N-1} and $\boldsymbol{u}_{N-1}$ for each of them. Also, the set of possible values of $\boldsymbol{x}$ is at each time (at time 0 the only possible value is $\boldsymbol{x}_0$ and at time N the set of possible values is the terminal hypersurface $Q(x_N, N)=0$). We then repeat the process for each stage of the problem (possibly interpolating J_{i+1}, for the set of feasible values of $\boldsymbol{x}_i$).

The optimal policy is obtained by *integrating* the process forward: we start at $\boldsymbol{x}_0$, apply $\boldsymbol{u}_0$ which is fully determined since the initial state is unique and advance the state one time step with f_0. We measure the resulting $\boldsymbol{x}_1$, choose the corresponding $\boldsymbol{u}_1$ and advance the state. Notice that we find the optimal policy interactively, i.e. we can wait until time i to decide control $\boldsymbol{u}_i$.

Finally note that we are using the Markov property for our system: the whole history of the system, $(\boldsymbol{x}_0,..., \boldsymbol{x}_i, \boldsymbol{u}_0,..., \boldsymbol{u}_{i-1})$ can be described with only $(\boldsymbol{x}_i, \boldsymbol{u}_{i-1})$, i.e. knowing last state and control is equivalent to knowing all previous states and controls. This is actually not a very severe restriction, since, as stated in the main text, any system depending on its finite history can be described as a Markov chain by means of state augmentation.

Appendix B: Numerical solution of the model

Discretization of the model. Linear-Quadratic Controller.

In order to solve the model, a discrete version of it is constructed. The discretization scheme is very simple, with first derivatives approximated with a first order difference equation and the integral replaced by a summation

$$J = min\mathcal{E}\left\{\sum_{i=1}^{\infty}(D_i + G_i)e^{-i\delta\Delta t}\right\}$$

The equations of motion are

$$\begin{aligned} F_{i+1} &= F_i + E_i\Delta t \\ C_{i+1} &= bF_i\Delta t + \beta E_i\Delta t + (1-\sigma\Delta t)C_i + w_i^C \\ T_{i+1} &= \mu C_i\Delta t + (1-\alpha\Delta t)T_i + w_i^T \end{aligned}$$

where initial conditions are given and subindex i stands for time.
The infinite horizon problem will be divided into two separate tasks: a transient period up to time step N (that will be the main objective of our study) and an asymptotic part, from N up to infinity. We assume that at some future date, GHG emissions will stabilize at a finite level (or zero), and that will determine the date N.
Defined are also

$$G_i = \gamma_G(\rho_i^2 + a^2\dot{\rho}_i^2)U_0e^{ir\Delta t} \qquad \rho_i = \left(1 - \frac{E_i}{E_i^b}\right)$$

$$D_i = \gamma_D\left[\left(\frac{T_i}{T_M}\right)^2 + \left(\frac{\dot{T}_i}{\dot{T}_M}\right)^2\right]U_0e^{ir\Delta t}$$

We will now re-write our model in a slightly different form, through state augmentation, in order to present it as an autonomous linear controller with quadratic criteria, whose solution can be found analytically and relatively easy implemented (there is extensive literature concerning the solution of linear quadratic gaussian (LQG) problems, mainly due to the ease of their solution and implementation). To that end we need to express the cost functional in terms of the state variables only and make the problem autonomous. Both transformations can be made augmenting the state (i.e. introducing new (diagnostic) variables or renaming existing ones).
To eliminate the derivative of T from the cost functional, we define damage costs precisely as

$$D_i = \gamma_D\left[\left(\frac{T_i}{T_M}\right)^2 + \left(\frac{\Delta T_i/\Delta t}{\dot{T}_M}\right)^2\right]U_0 e^{ir\Delta t} \qquad \frac{\Delta T_i}{\Delta t} = \frac{T_i - T_{i-1}}{\Delta t}$$

We can now augment the state by introducing a new state variable $Y_i^1 = T_{i-1}$. In terms of the new variable the damage costs are

$$D_i = \gamma_D\left[\left(\frac{T_i}{T_M}\right)^2 + \frac{1}{(\dot{T}_M\Delta t)^2}((T_i)^2 + (Y_i^1)^2 - 2T_iY_i^1)\right]U_0 e^{ir\Delta t}$$

Also the term including the derivative of the control can be transformed using the approximation of the derivative as a finite difference. We obtain

$$\rho_i^2 + a^2\dot{\rho}_i^2 = \rho_i^2\left(1 + \frac{a^2}{(\Delta t)^2}\right) + a^2\rho_{i-1}^2 + \frac{2a^2}{(\Delta t)^2}\rho_i\rho_{i-1}$$

We introduce a new state variable $Y_i^2 = \rho_{i-1}$, and the obvious reformulation of the cost functional.

Second, note that since our control is the reduction rate ρ, but in the equations of motion we have the emissions directly, there is a known vector time-dependent function in the equations of motion (basically the BAU emission path)

$$E_i = (1-\rho_i)E_i^b = E_i^b - \rho_i E_i^b$$

To make the problem autonomous we introduce a new state variable

$$\dot{Y}^3 = 0, Y^3(0) = 1 \text{ or } Y_{i+1}^3 = Y_i^3, Y_1^3 = 1$$

so that the time varying function is part of the time dependent matrix of coefficients.

We can summarize these transformations as follows: define the vectors

$$x_i = \begin{bmatrix} F_i & C_i & T_i & Y_i^1 & Y_i^2 & Y_i^3 \end{bmatrix}' \qquad w_i = \begin{bmatrix} 0 & w_i^C & w_i^T & 0 & 0 & 0 \end{bmatrix}' \qquad u_i = \rho_i$$

(where the apostrophe denotes transposition) and matrices

$$A_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & E_i^b\Delta t \\ b\Delta t & (1-\sigma\Delta t) & 0 & 0 & 0 & \beta E_i^b\Delta t \\ 0 & \mu\Delta t & (1-\alpha\Delta t) & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad B_i = \begin{bmatrix} -E_i^b\Delta t \\ -\beta E_i^b\Delta t \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

The cost functional is characterized through

$$Q_i = \begin{bmatrix} 0\ 0 & 0 & 0 & 0 & 0 \\ 0\ 0 & 0 & 0 & 0 & 0 \\ 0\ 0 & \frac{\gamma_D}{T_M^2} + \frac{\gamma_D}{(\dot{T}_M \Delta t)^2} & \frac{-\gamma_D}{(\dot{T}_M \Delta t)^2} & 0 & 0 \\ 0\ 0 & \frac{-\gamma_D}{(\dot{T}_M \Delta t)^2} & \frac{\gamma_D}{(\dot{T}_M \Delta t)^2} & 0 & 0 \\ 0\ 0 & 0 & 0 & \frac{a^2 \gamma_G}{(\Delta t)^2} & 0 \\ 0\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} U_0 e^{-i\delta_r \Delta t} \qquad R_i = \gamma_G \left[1 + \frac{a^2}{(\Delta t)^2}\right] U_0 e^{-i\delta_r \Delta t} \qquad V_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{-a^2 \gamma_G}{(\Delta t)^2} \\ 0 \end{bmatrix} U_0 e^{-i\delta_r \Delta t}$$

where R_i is a positive definite matrix and Q_i is positive semidefinite. $\boldsymbol{x}_i$ is a vector of state variables and $\boldsymbol{u}_i$ a vector of control variables.
With this notation we can write our model as

$$J = min\mathcal{E}\left\{ x'_N Q_N x_N + \sum_{i=1}^{N-1} \begin{bmatrix} x_i & u_i \end{bmatrix}' \begin{bmatrix} Q_i & V_i \\ V'_i & R_i \end{bmatrix} \begin{bmatrix} x_i \\ u_i \end{bmatrix} \right\}$$

with the equations of motion

$$x_{i+1} = A_i x_i + B_i u_i + w_i \qquad x_0 \text{ given}$$

$\boldsymbol{w}$ is a vector of purely random Gauss-Markov processes characterized by

$$\mathcal{E}\{w_i\} = 0 \qquad \mathcal{E}\{w_i' w_j\} = \Sigma_i \delta_{ij} \qquad \textbf{(EQ 1)}$$

and matrices A and B are random and independent of $\boldsymbol{w}$.
Notice that with the new definition of the state vector, the optimal solution will depend on the state of the climate (*F,C,T*) but also on the past state and controls (Y^1, Y^2) and on external forcing (Y^3).

The term $x'_N Q_N x_N$ represents the value of the world (total discounted integrated costs from time N to infinity) after the last stage in the decision process. Since in all experiments the BAU emission path reduces to zero at a date prior to stage N, implying that the control and the associated abatement costs also vanish (and provided discounting, and considering that we use the model as a tool to compare different policy options, rather than to calculate reliable costs estimates), we choose to ignore the costs originated by the new climatic equilibrium state and its variations (i.e. we let $x'_N Q_N x_N$ represent the damage costs at time N).
The expected value operator in the cost functional includes all sources of uncertainty, both stemming from the additive white noise $\boldsymbol{w}$ and from the uncertain parameters both in the equations of motion and the cost functional itself, but since the parameters in the equations of motion and the cost functional are independent, the expected value

over Q, R and V can explicitly be taken care of beforehand, reducing these matrices to deterministic ones.

We can now calculate the CL and OL policies applying the DP algorithm and a modified version of the Merriam's Parametric expansion idea respectively.

Closed Loop Solution. Dynamic Programming.

The optimal CL solution of the model is obtained applying the DP algorithm outlined in appendix A and given by the following expressions

$$u^*_i = L_i x_i$$

$$J_i(x_i) = x'_i K_i x_i + \sum_{j=i}^{N-1} \mathcal{E}\{w'_j K_{j+1} w_j\}$$

with matrices

$$L_i = -(\overline{B'_i K_{i+1} B_i} + R_i)^{-1} (\overline{B'_i K_{i+1} A_i} + V'_i)$$

$$K_i = \overline{A'_i K_{i+1} A_i} + Q_i - P_i \qquad K_N = Q_N$$

$$P_i = (\overline{A'_i K_{i+1} B_i} + V_i)(\overline{B'_i K_{i+1} B_i} + R_i)^{-1} (\overline{B'_i K_{i+1} A_i} + V'_i)$$

In particular for our problem, and after transforming the noise-related term

$$J_0(x_0) = x'_0 K_0 x_0 + \sum_{j=0}^{N-1} \mathcal{E}\{w'_j K_{j+1} w_j\} = x'_0 K_0 x_0 + \sum_{j=0}^{N-1} Tr(K_{j+1}\Sigma)$$

Open Loop Solution. Merriams Parametric Expansion

In order to find the open loop feedback solution (from which the last stage is the open loop solution) we will use a stochastic version of Merriam's parametric expansion method (see Aoki...).
The cost functional above can be expressed as follows

$$J_0 = \sum_{i=0}^{N} W_i = \sum_{i=0}^{N} \mathcal{E}\{x'_i Q_i x_i + u'_i R_i u_i + 2x'_i V_i u_i\}$$

The cost to go functions are then defined as $J_k \equiv \sum_{i=k}^{N} W_i$.
which can be rewritten as

$$J_k = \mathcal{E}\{x'_k Q_k x_k + u'_k R_k u_k + 2x'_k V_k u_k + J_{k+1}\}$$

This difference equation is satisfied by a quadratic form in $\boldsymbol{x}_k$, $\boldsymbol{u}_k,\ldots,\boldsymbol{u}_{N-1}$, and $\boldsymbol{w}_k,\ldots,$ $\boldsymbol{w}_{N-1}$. So we can write

$$J_k = \mathcal{E}\left\{x'_k H(k) x_k + \sum_{i,j=k}^{N} w'_i Z_{ij}(k) w_j + \sum_{i,j=k}^{N} u'_i S_{ij}(k) u_j + 2x'_k \sum_{i=k}^{N} T_i(k) u_i\right\}$$

with appropriate matrices H, Z, S and T (where already the fact that $\mathcal{E}\{\boldsymbol{w}\}=0$ has ben used to leave out all terms linear on $\boldsymbol{w}$).
Now, for time step k the condition of minimum implies

$$\frac{\partial J_k}{\partial u_i} = \sum_{j=k}^{N} u'_j S_{ij}(k) + x'_k T_i(k) = 0 \qquad i = k, \ldots, N-1$$

which delivers $\boldsymbol{u}_k$ among others.

In order to derive the matrices H, Z, S and T, we substitute the quadratic form above in the difference equation for J_{k+1} to obtain and make use of the fact that $x_{k+1} = A_k x_k + B_k u_k + w_k$ to eliminate $\boldsymbol{x}_{k+1}$ from the equation above and obtain

$$H(k) = Q_k + \overline{A'_k H(k+1) A_K} \qquad H(N) = Q_N$$

$$S_{kk}(k) = R_k + \overline{B'_k H(k+1) B_k} \qquad S_{NN}(N) = R_N$$

$$S_{ki}(k) = 2\overline{B}'_k T_i(k+1) \qquad i > k$$

$$S_{ij}(k) = S_{ij}(k+1) \qquad i, j > k$$

$$T_k(k) = V_k + \overline{A'_k H(k+1) B_k} \qquad T_N(N) = V_N$$

$$Z_{kk}(k) = H(k+1)$$

$$Z_{ij}(k) = \delta_{ij} Z_{ij}(k+1) \qquad i, j > k$$

which recursively gives the matrices of the expansion. They can be calculated for all k off-line, since they only depend on the parameters of the problem (note that subindices indicate the dependence on the different controls, whereas index k in brackets indicate the stage at which we start calculating the problem; for instance the matrix $S_{ij}(k)$ is the form (we have $(N\text{-}1\text{-}k)^2$ from them for each step k) that multiplies controls $\boldsymbol{u}_i$ and $\boldsymbol{u}_j$, i.e. the control vectors for time i and j, when we start calculating at time k (the open loop solution has k=0, i.e. the open loop solution is the solution to the last step for the open loop feedback solution).
We can use this method to find the solution for the deterministic problem in open form, if we choose to ignore uncertainty or randomness.
Finally the boundary conditions for these recurrence relations are the same as for the CL problem.

Expected evolution of the optimally controlled system

It is of great interest to calculate the expected evolution of the controlled system. We will study the evolution of the expected mean and covariance (which in the gaussian additive noise case characterise completely the system). For simplicity we will consider the case in which matrices B_i are deterministic and known. Since we have assumed that the parameter matrices A_i are random and independent of everything else (in particular have no time correlation) we have

$$\bar{x}_{i+1} = \bar{A}_i \bar{x}_i$$

where $\bar{x}_i = E\{x_i | x_0\}$. Also we have

$$\chi_{i+1} = A_i \chi_i A_i^T + \Sigma + N_i$$

where $\chi_i = E\{\chi_i | x_0\}$ and N_i is a diagonal matrix whose elements are

$$N_i^{kk} = \sum_{l=1}^{n} \Psi^{kl}[(\bar{x}_i^l)^2 + \chi_i^l]$$

and Ψ^{kl} is the covariance of element *(k,l)* of matrix A_i. We have assumed that parameters in A_i are independent from each other.

The case of non-perfect information

Suppose now that we relax the perfect information assumption, i.e. instead of measuring the state $\boldsymbol{x}$ exactly we receive at each time step the p-dimensional vector z. For simplicity suppose that both are related through

$$z_i = \mathcal{H} x_i + v_i$$

$\boldsymbol{v}$ is a vector of purely random Gauss-Markov processes characterized by

$$\mathcal{E}\{v_i\} = 0 \qquad \mathcal{E}\{v_i' v_j\} = \mathcal{R}\delta_{ij}$$

and $\boldsymbol{w}$ and ν are independent from each other.
Let us also define the quantities

$$x_{i|k} = \mathcal{E}\{x_i | z^k\} \qquad k < i$$

$$\chi_{i|k} = \mathcal{E}\left\{(x_i - x_{i|k})(x_i - x_{i|k})^T \middle| z^k\right\} \qquad k < i$$

Under this conditions, and for the cost functional given above the optimal control is

$$u_i = L_i \mathcal{E}\{x_i | z^i\} = L_i \mu_i$$

where $z^{\cdot} \equiv (z_i, \ldots, z_0)$, and L_i the same as given above.
The estimate of $\boldsymbol{x}_i$ can be easily implemented using the Kalman filter

$$\mathcal{E}\{x_i | z^i\} = x_{i|i-1} + \mathcal{K}_i(z_i - \mathcal{H}x_{i|i-1})$$

where the matrix $\mathcal{K}_i$ is given by $\mathcal{K}_i = \chi_{i|i}\mathcal{H}^T\mathcal{R}^{-1}$.
Notice that the experiments described in section 4.1 in which a red noise stochastic forcing which cannot be measured is added can be solved using these equations, once it is reduced to the proper form using state augmentation.
Also the decomposition of the control into a proportional plus an integral part can be carried out using the expression of the estimate of $\boldsymbol{x}$ given by the Kalman filter as

$$\mu_{i+1} = (I - \mathcal{K}_i\mathcal{H})(A_i + B_iL_i)\mu_i + \mathcal{K}_iz_{i+1} = S_i\mu_i + \mathcal{K}_iz_{i+1}$$

and applying the expression recursively

$$\mu_{i+1} = \mathcal{K}_iz_{i+1} + \sum_{j=0}^{i} S_i \ldots S_j\mathcal{K}_jz_j$$

Notice that in this case the optimal control does not have the markov property any more.