Alternative approach to the critical behavior and microscopic structure of the power-Maxwell black holes

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Employing a new approach toward thermodynamic phase space, we investigate the phase transition, critical behavior and microscopic structure of higher dimensional black holes in an Anti-de Sitter (AdS) background and in the presence of Power-Maxwell field. In contrast to the usual extended P-V phase space where the cosmological constant (pressure) is treated as a thermodynamic variable, we fix the cosmological constant and treat the charge of the black hole (or more precisely Q^{s}) as a thermodynamic variable. Based on this new standpoint, we develop the resemblance between higher dimensional nonlinear black hole and Van der Waals liquid-gas system. We write down the equation of state as $Q^s = Q^s(T, \psi)$, where ψ is the conjugate of Q^s , and construct a Smarr relation based on this new phase space as $M = M(S, P, Q^s)$, while s = 2p/(2p-1) and p is the power parameter of the Power-Maxwell Lagrangian. We obtain the Gibbs free energy of the system and find a swallowtail behaviour in Gibbs diagrams, which is a characteristic of first-order phase transition and express the analogy between our system and van der Waals fluid-gas system. Moreover, we calculate the critical exponents and show that they are independent of the model parameters and are the same as those of Van der Waals system which is predicted by the mean field theory. Finally, we successfully explain the microscopic behavior of the black hole by using thermodynamic geometry. We observe a gap in the scalar curvature R occurs between small and large black hole. The maximum amount of the gap increases as the number of dimensions increases. We finally find that character of the interaction among the internal constituents of the black hole thermodynamic system is intrinsically a strong repulsive interaction.

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I. INTRODUCTION

The study of black holes thermodynamics is one of the most important subject in gravitational physics, which was anticipated by Bekenstein in 1973 [1]. In complete analogy with known non-gravitational thermodynamic systems, black hole spacetime obeys a version of first law of thermodynamics. [1, 2]. It can be specified by an entropy S proportional to the horizon area and temperature T proportional to the surface gravity.

Furthermore, after the advent of AdS/CFT correspondence, phase transition has gained more attention as a thermodynamical property of black holes in asymptotically AdS spaces.

A seminal investigation in this relevance was reported in Hawking and Page's paper [3], where it was demonstrated that in the phase space of AdS-Schwarzchild black hole, phase transition certainly exists. Through the AdS/CFT (gage/gravity) duality the Hawking-Page phase transition can correspond to the confinement/deconfinement phase transition in the dual quark gluon plasma [4]. In view of this duality, the thermodynamic phase space of charged AdS black holes exhibits first order SBH/LBH (small black hole/large black hole) phase transition suggestive of a Van der Waals liquid/gas phase transition (see e.g [5–8]).

In most treatments of phase transition in black hole thermodynamics, in an extended phase space, negative cosmological constant Λ is treated as a thermodynamic variable. In fact, Λ is physically thought of as a pressure and its conjugate variable is considered as a thermodynamic variable proportional to a volume V [9–11]. In the past few years the various class of black hole phase transition such as Multiple reentrant phase transition [12], superfluid-like phase transition [13], zero order phase transition [14] and so on have been studied in an extended phase space (see e.g.[15–22]).

The authors of [23], by using of the Smarr formula, concluded that the mass M of AdS black hole should be identified as enthalpy H rather than internal energy of the spacetime. Therefor, regarding the cosmological constant

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as a pressure P, for a non rotating charged black hole, the first law of thermodynamics should read,

$$\mathrm{d}M \equiv \mathrm{d}H = T\,\mathrm{d}S + \Phi\,\mathrm{d}Q + V\,\mathrm{d}P,$$

where Q and Φ are charge and electrical potential respectively.

Although, there are some reasons to suppose the cosmological constant as a variable, but it is more reasonable to hold it as a constant parameter. In fact, from the physical standpoint it is difficult to consider the cosmological constant as a thermodynamic variable which can take an arbitrary value. Also, in general relativity the cosmological constant is understood as a constant related to the zero point energy of the background spacetime. By these motivations, the authors of [24] proposed the cosmological constant as a fixed parameter and consider the charge of the black hole as a thermodynamic variable. They indicated a phase transition similar to Van der Waals liquid-gas in the black hole system in this manner. In this alternative view, the SBH/LBH phase transition of black hole in $Q^2 - \psi$ plane exactly correspond to the Van der Walls fluid phase transition, whereas ψ is thermodynamic variable conjugate to the Q^2 . Recently, the universality class and critical properties of any AdS black hole in this alternative approach toward the phase space have been addressed in [25].

Besides, the study of the nonlinear electrodynamic field on the thermodynamic and phase transition of the black holes has been attracted a lot of attentions in the literature in the past decade [26-35]. The first one of the nonlinear electromagnetic field models is Born-Infeld electrodynamics introduced in 1930's. Nonlinear behavior in strong electromagnetic field such as the field in region near a point-like charge, is suggested by Dirac in 1964 [37]. Moreover, nonlinear electromagnetics can be due to the nonlinear effects of quantum electrodynamics [38-40].

Power-Maxwell invariant field as an important class of the nonlinear electrodynamics, was introduced in [41–45]. It is worth mentioning that the Lagrangian of the power Maxwell invariant fields are invariant under the conformal transformation $g_{\mu\nu} \longrightarrow \Omega^2 g_{\mu\nu}$, where $g_{\mu\nu}$ is the metric tensor. In the special case linear electromagnetic can be generated by reducing of the power Maxwell invariant field. the authors of [43] have investigated the effect of power Maxwell field on the P - V criticality of black holes and phase transitions in the extended phase space.

In the present work, we would like to study the critical behavior of the AdS black hole in the presence of power-Maxwell field via an alternative viewpoint. It means we keep the cosmological constant as a fixed parameter and instead treat the charge of the black hole (or more precisely Q_p) as a thermodynamic variable which can vary. The advantages of this approach is that we do not need to extend the thermodynamical phase space to see the critical behavior of the system. On the other side, absorbing or emitting charged particles may cause the change in the charge of the black hole in reasonable manner which is more logical than varying the cosmological constant. Phase structure and critical behavior of AdS balck holes with linear [14], and nonlinear [22] electrodynamics, Lifshitz dilaton black holes [46], where the charge of the system can vary and the cosmological constant (pressure) is fixed have been investigated. Recently, it was argued that this method also indeed works for investigating the phase transition of Gauss-Bonnet black holes [47], which further supports the viability of this alternative approach.

The outline of this paper is as follows. In the next section we investigate thermodynamics of the higher dimensional AdS black hole with considering of the effects of the nonlinear power- Maxwell field and obtain Smarr relation by the well-known usual thermodynamic quantities. Moreover we introduce a new phase space and thermodynamic variables ψ , Q_p and find the appropriate Smarr relation in terms of this new variables. In Sec.III, we write down the equation of state according to the offered variables and calculate the critical points and critical exponent. Also, the analogy of the system with Van der Waals liquid gas system is showed in this section. The Gibbs free energy diagrams are investigated in section IV. The swallowtail behavior of the Gibbs free energy represents a first-order phase transition in the system. Finally, in section V, we explore microscopic structure of thermodynamic system and calculate the Ruppeiner scalar curvature in 4,5 and 6 dimensional spacetime.

II. THERMODYNAMIC OF HIGHER DIMENSIONAL ADS BLACK HOLE WITH A POWER MAXWELL FIELD

The action of Einstein gravity in (n + 1)-dimensional spacetime coupled to a power Maxwell field can be written as [42]

$$I = -\frac{1}{16\pi} \int d^{n+1}x \sqrt{-g} \left[R - 2\Lambda + (-F_{\mu\nu}F^{\mu\nu})^p \right], \tag{1}$$

where R is the Ricci scalar, p is a constant determining the nonlinearity of the electromagnetic field, $F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$ is the electromagnetic field tensor and A_{μ} is the electromagnetic potential. Here, Λ is the cosmological constant related to the AdS radius as $l^2 = -n(n-1)/(2\Lambda)$. We apply a spherically symmetric and static metric of (n + 1)-dimensional spacetime

$$ds^{2} = -f(r) dt^{2} + \frac{dr^{2}}{f(r)} + r^{2} d\Omega_{n-1}^{2}, \qquad (2)$$

where $d\Omega_{n-1}^2$ denotes the metric of spherical hypersurface with volume ω_{n-1} and f(r) is given by [42]

$$f(r) = 1 - \frac{m}{r^{n-2}} + \frac{r^2}{l^2} + \frac{2^p \left(2 p - 1\right)^2 q^{2p}}{(n-1) \left(n-2p\right)} r^{\left[(6-2n)p-2\right]/\left[2p-1\right]}.$$
(3)

The quantity q is an integration constant related to the charge Q of the black hole per unite volume ω_{n-1} and one can find it by applying the Gauss law

$$Q = \frac{1}{4\pi} \int r^{n-1} (-F_{\mu\nu} F^{\mu\nu})^{p-1} F_{\mu\nu} n^{\mu} u^{\nu} \,\mathrm{d}r,\tag{4}$$

where $n^{\mu} = f(r)^{-1/2} dt$ and $u^{\nu} = f(r)^{1/2} dr$ are the unit space like and time like normal to the surface, respectively, and Maxwell invariant is $F_{\mu\nu}F^{\mu\nu} = -2F_{tr}^2$. Directly from the generalized Maxwell equation, nonzero electromagnetic field is $F_{tr} = q r^{(1-n)/(2p-1)}$ [42]. Thus, Gauss law relation Eq. (4) reads

$$Q = \frac{2^{p-1}q^{2p-1}}{4\pi}.$$
(5)

The parameter m is known as the geometrical mass parameter which it can be expressed in term of the largest horizon radius r_+ (the largest root of $f(r_+) = 0$)

$$m = r_{+}^{n-2} + \frac{r_{+}^{n}}{l^{2}} + \frac{2^{p} \left(2 p - 1\right)^{2} q^{2 p}}{\left(n - 1\right) \left(n - 2 p\right)} r_{+}^{\left(n - 2p\right)/\left(1 - 2p\right)}.$$
(6)

Using Brown-York method [48], the total mass of the black hole per unit volume ω_{n-1} can be read as follows

$$M = \frac{(n-1)m}{16\pi}.$$
 (7)

The electric potential Φ , measured at infinity with respect to the horizon radius r_+ , is obtained as (details is referred to [49])

$$\Phi = \frac{p \left(2 p - 1\right) \left(\pi Q\right)^{1/(2p-1)}}{(n-2p) 2^{(p-3)/(2p-1)}} r_{+}^{(2p-n)/(2p-1)}.$$
(8)

In the case p = 1, the metric function Eq. (3) and the electric potential Eq. (8) reduce to n + 1-dimensional Reissner-Nordstrom (RN)-AdS black holes [7].

According to the so-called area law of the entropy, the entropy of the black hole is a quarter of the event horizon erea. Using this, one can obtain the entropy of the black hole per unit volume ω_{n-1} as

$$S = \frac{r_+^{n-1}}{4}.$$
 (9)

The Hawking temperature can be calculated as

$$T = \frac{f'(r_{+})}{4\pi} = \frac{(n-2)}{4\pi r_{+}} + \frac{nr_{+}}{4\pi l^{2}} - \frac{2^{p-2}(2p-1)q^{2p}}{(n-1)\pi} r_{+}^{-[2(n-2)p+1]/[2p-1]}.$$
 (10)

One may obtain the generalized Smarr relation for the black hole in the extended phase space by using the definition of total mass M (7), chrge of the black hole Q (5) and the entropy S (9). It is a matter of calculation to show the Smarr formula is

$$M(S,Q,P) = \frac{(n-1)S^{(n-2)/(n-1)}}{2^{2n/(n-1)}\pi} + \frac{PS^{n/(n-1)}}{n2^{2n/(1-n)}} + \frac{(2p-1)^2Q^{2p/(2p-1)}S^{(2p-n)/[(n-1)(2p-1)]}}{(n-2p)\pi^{1/(1-2p)}2^{[n(3p-2)-7p+4]/[(n-1)(2p-1)]}},$$
(11)

where $P = -\Lambda/(8\pi)$.

One can then define the variables conjugate to Q, S and P. As mentioned before, the cosmological constant parameter is treated as pressure P. So, its conjugate variable from the thermodynamic viewpoint should be volume V

$$V = \frac{\partial M}{\partial P}\Big|_{S,Q}.$$
(12)

Likewise, the corresponding conjugate quantity of S and Q are interpreted as a temperature T and electric potential Φ respectively

$$T = \frac{\partial M}{\partial S}\Big|_{P,Q}, \qquad \Phi = \frac{\partial M}{\partial Q}\Big|_{S,P}.$$
(13)

It is easy to show that the usual Smarr mass formula can be written as

$$M = \frac{n-1}{n-2}TS + \frac{(n-3)p+1}{(n-2)p}Q\Phi - \frac{2}{n-2}VP.$$
(14)

Obviously if we set n = 3 and p = 1, Eq.(14) reduces to the well-known Smarr relation for the 4-dimensional Einstein-Maxwell black holes [7]

$$M = 2(T S - V P) + \Phi Q.$$
 (15)

As expected, the obtained thermodynamic quantities satisfy the usual first law of thermodynamics

$$dM = T \, dS + \Phi \, dQ + V \, dP. \tag{16}$$

It is notable that the electric potential Φ must have a finite value at infinity. This leads to the following restriction on the parameter p

$$\frac{1}{2}$$

which obtained it from (2p-n)/(2p-1) < 0 [42].

A. Alternative phase space

The Van der Waals like critical behavior of various types of AdS black holes has been studied by considering the cosmological constant as a thermodynamical pressure in the extended phase space [7, 9]. Also, Q- Φ plane-phase transitions of charged AdS black hole are investigated by [5, 6]. Although the authors claimed that the phase transition is similar to the Van der Waals fluid system, but the phase transition behavior exhibits unusual Van der Waals isotherms. In this approach, a thermodynamic response function $(\partial Q/\partial \Phi)_T$ does not lead to physically relevant quantity. For more details, we refer to Ref. [24].

The most recent work that indicates a complete similarity between the charged AdS black hole and Van der Waals fluid system is important to highlight [24]. In this work, the cosmological constant has been thought as a fixed parameter and instead, the square of the charge of black hole Q^2 has been considered as a thermodynamic independent variable, where $\Psi = 1/(2r_+)$ is the conjugate of Q^2 [24]. It allows the definition of new response function $(\partial Q^2/\partial \Psi)_T$ which is clearly characterized stable-unstable region by its sign [24].

According to this viewpoint, we would like to offer the thermodynamic variables allowing us to complete the analogy of higher dimensional power Maxwell-AdS black hole with Van der Waals fluid system. Hence, we consider the mass of the black hole as a function of Q_p where $Q_p \equiv Q^{2p/(2p-1)}$ instead of the standard Q. Our motivation is that the charge of the black hole appears as Q_p in the expressions of M Eq.(11) and T 10. Another advantage of this choice is to achieve a physically meaningful response function $(\partial Q_p / \partial \Psi)_T$. Thus, with the use of (11) the conjugate of Q_p is directly written as

$$\Psi = \frac{\partial M}{\partial Q_p} \Big|_{S,P}$$

$$= \frac{(2p-1)^2 2^{(4-3p)/(2p-1)}}{(n-2p)\pi^{1/(1-2p)}} r_+^{(n-2p)/(1-2p)}.$$
(18)



FIG. 1: The behavior of isothermal $Q_p - \Psi$ diagrams of power-Maxwell black holes for the case l = 1. The critical points are indicated by block spot.

Now we can rewrite the corresponding Smarr mass formula in this new phase space

$$M = \frac{n-1}{n-2}TS + \frac{2\left((n-3)p+1\right)}{\left(2p-1\right)\left(n-2\right)}\boldsymbol{Q}_{p}\Psi - \frac{2}{n-2}VP.$$
(19)

A simple calculation shows that for p = 1 and n = 3, relation (19) is reduced to the Smarr formula obtained by the authors of [24]

$$M = 2 (T S + Q^2 \Psi - V P).$$
⁽²⁰⁾

Finally, the first law of thermodynamics in this new picture becomes

$$dM = T \, dS + \Psi \, dQ_p + V \, dP. \tag{21}$$

In what follows, we examine the critical behavior of power Maxwell black hole in alternative phase space where the AdS radius (or Λ) is fixed and the electric charge of black hole can vary.

III. EQUATION OF STATS AND CRITICAL POINT

In order to determine the critical point, we need to have the equation of state $Q_p(T, r_+)$. Using (5) and (10) one may write the equation of state as a function of the temperature and horizon radius

$$\boldsymbol{Q}_{p} = \frac{(n-1) \, 2^{5p/(1-2p)}}{(2p-1) \, \pi^{2p/(2p-1)}} \left(n - 2 - 4\pi T + \frac{nr_{+}^{2}}{l^{2}} \right) r_{+}^{n-3+(n-1)/(2p-1)}. \tag{22}$$

The behavior of the black hole electric charge Q_p versus Ψ are plotted for fixed l = 1 and different sets of parameter values in Fig. 1. In Fig. 1, isothermal diagrams show that, for $T = T_c$, a second-order phase transition (critical point) occurs in the point with the following conditions (inflection point):

$$\frac{\partial \boldsymbol{Q}_p}{\partial \Psi}\Big|_{T_c} = 0, \quad \frac{\partial^2 \boldsymbol{Q}_p}{\partial \Psi^2}\Big|_{T_c} = 0.$$
(23)

Solving the above equations yields the coordinates of the critical point as

$$Q_{p_{c}} = \frac{n (2 p - 1) (n - 1) 2^{5p/(1-2p)} l^{n-3+(n-1)/(2p-1)}}{(2p (n - 2) + 1) (p (n - 3) + 1) \pi^{2p/(2p-1)}} \left[\frac{(n - 2) (p (n - 3) + 1)}{np(n - 1)} \right]^{p(n-1)/(2p-1)},$$

$$\Psi_{c} = \frac{(2p - 1)^{2} 2^{(3p-4)/(2p-1)}}{(n - 2p) \pi^{1/(2p-1)}} \left[\frac{(n - 2) (p (n - 3) + 1) l^{2}}{(n - 1) np} \right]^{-[n-2p]/[2(p-1)]},$$

$$T_{c} = \frac{\sqrt{np (n - 1) (n - 2) (p (n - 3) + 1)}}{(2p (n - 2) + 1) \pi l}.$$
(24)



FIG. 2: The relation Q_{p_c} and T_c and p for different values of dimensions. In (b), as p increases the black hole follows direction of arrows.

Following the new definition $\rho_c = \Psi_c Q_{p_c} T_c$ in [24, 46], the universal number for black hole at the critical point is

$$\rho_c = \Psi_c \mathbf{Q}_{p_c} T_c$$

$$= \frac{n(n-1)(n-2)(2p-1)^3 l^{-3+n}}{16(n-2p)(2p(n-2)+1)^2 \pi^2} \left[\frac{(n-2)(p(n-3)+1)}{np(n-1)} \right]^{(n-1)/2}, \qquad (25)$$

according to constrain condition Eq.(17), ρ_c is a positive quantity. Also, it is independent of the AdS radius l only when n = 3. In the conformally invariant case p = (n + 1)/4, the critical quantities of the black hole are

$$\mathbf{Q}_{c}^{c} = \frac{(n-1)^{(n-1)/2} l^{n-1}}{2^{[n+9]/[2(n-1)]} \pi^{(n+1)/(n-1)}} \left[\frac{(n-2)}{n(n+1)} \right]^{(n+1)/2}, \quad \Psi_{c}^{c} = \frac{2^{5/(n-1)-5/2}}{l\pi^{2/(n-1)}} \sqrt{\frac{n(n^{2}+1)}{n-2}}.$$

$$T_{c}^{c} = \frac{1}{2\pi l} \sqrt{\frac{(n+1)(n-1)(n-2)}{n}}, \quad \rho_{c}^{c} = \frac{(n+2/n-3)^{(n+1)/2} l^{n-3}}{16\pi^{2} (n+1)^{(n-1)/2}}.$$
(26)

It is remarkable to note that in 3-dimensional space for a linear Maxwell (p = 1), these critical quantities reduce to those of RN-AdS black holes [24]. To see the effect of p in the range of 1/2 on the critical value of $black hole, we show the behavior of <math>Q_{p_c}$ and T_c for different dimension n in Fig. 2. Figure 2(a) shows, Q_{p_c} vanishes near p = 1/2 in the different dimension. Also, increasing p makes Q_{p_c} higher. According to Fig. 2(b), the critical temperature is not almost influenced by the change of p.

The existence of oscillating isotherms in Fig. 1 are a consequence of physically unstable feature which are remedied by the Maxwell equal area construction [24]

$$\oint \Psi dQ^2 = 0. \tag{27}$$

A. Critical exponent

In the investigation of phase transition phenomena, it is important to study the scaling behavior of thermodynamic system near the critical point and find the corresponding universality class. In particular, the behavior of physical quantities in the vicinity of the critical point can be characterized by the critical exponents. Hence, we would like to calculate these exponents for the new approach in this subsection.

In order to calculate the critical exponents, it is convenient to define the reduced thermodynamic variables

$$T_r = \frac{T}{T_c}, \qquad \Psi_r = \frac{\Psi}{\Psi_c}, \qquad \boldsymbol{Q}_{p_r} = \frac{\boldsymbol{Q}_p}{\boldsymbol{Q}_{p_c}}.$$

Also, since the critical exponents should be studied near the critical point, we rewrite the reduced variables in the form of

$$t = T_r - 1, \qquad \psi = \Psi_r - 1, \qquad \phi = Q_p - 1,$$
 (28)

where t, ψ and ϕ point out to the deviation from critical point. Now, we approximate the equation of state (22) around the critical point as

$$\phi = At + Bt\psi + Ct\psi^2 + D\psi^3 + O(t\psi^3, \psi^4),$$
(29)

where A, B and C are constant quantities depend on n and p, as follows

$$A = -\frac{4p(n-1)(p(n-3)+1)}{(2p-1)^2}, \quad B = \frac{4p(n-1)(p(n-3)+1)(2p(n-2)+1)}{(2p-1)^2(n-2p)},$$

$$C = -\frac{2p(n-1)(p(n-3)+1)(2p(n-2)+1)(2p(n-3)+n+1)}{(2p-1)^2(n-2p)^2}, \quad D = -\frac{2p(n-1)(p(n-3)+1)(2p(n-2)+1)}{3(n-2p)^3}$$
(30)

Due to the fact that during phase transition the charge (\mathbf{Q}_p) remains constant, we have from Eq.(29)

$$Bt\psi_s + Ct\psi_s^2 + D\psi_s^3 = Bt\psi_l + Ct\psi_l^2 + D\psi_l^3,$$
(31)

where ψ_s , ψ_l stand for the small and gas black hole, respectively. On the other hand, by applying the Maxwell construction Eq.27, one obtains

$$\int_{\psi_s}^{\psi_l} \psi(Bt + 2Ct\psi + 3D\psi^2) \,\mathrm{d}\psi = 0.$$
(32)

Equation (31) and (32) have a non-trivial solution given by

$$\psi_{l,s} = \frac{-Ct \pm \sqrt{3t(C^2t - 3BD)}}{3D}.$$
(33)

So, the corresponding expression for the order parameter near the critical point becomes

$$|\psi_s - \psi_l| \sim t^{1/2} \Rightarrow \beta = \frac{1}{2}.$$

This equations yields to the order parameter near the critical point

$$|\psi_s - \psi_l| \sim t^{1/2} \Rightarrow \beta = \frac{1}{2}.$$

The critical exponent γ determines the behavior of the parameter X_T as

$$\chi_{T} = \frac{\partial \Psi}{\partial \boldsymbol{Q}_{p}} \Big|_{T} \sim |t|^{-\gamma}$$

thus from (29),

$$\chi_T \sim \frac{\Psi_c}{B \boldsymbol{Q}_{p_c} t} \Longrightarrow \gamma = 1.$$
(34)

The behavior of charge on the critical isotherm t = 0 is also explained by exponent δ . Hence using (29) one can write $\phi = D\psi^3$ and so $\delta = 3$.

To find the specific heat behavior, one needs to find the critical exponent α such that,

$$C_{\Psi} = T \frac{\partial S}{\partial T} \Big|_{\Psi} \sim |t|^{\alpha}.$$

Since the entropy (9) is independent of t, $C_{\Psi} = 0$ and we can conclude $\alpha = 0$. The values we have found for the set of critical exponents coincide with those obtained for Van der Waals fluid [7].



FIG. 3: The behavior of G versus Q_p for power-Maxwell black holes corresponding to Fig. 1 with l = 1. Note curves are shifted for clarity.



FIG. 4: The transition line of the phase transition between small and large black holes for various values of n, p and fixed l = 1. The critical points are indicated by block spot.

IV. GIBBS FREE ENERGY

Now, we investigate the black hole thermodynamics by studying the thermodynamic potential. In particular, the Gibbs free energy as a thermodynamic potential characterizes the globally stable state at equilibrium process. Hence, to find the phase transition and classification of its type, we explore the Gibbs free energy of power Maxwell black holes. In fixed the AdS radius l (Λ) regime, the Gibbs free energy is calculated by Legendre transformation G = M - TS [24]. Using (5)-(7) and (9) the Gibbs free energy per unit volume ω_{n-1} is obtained as

$$G = G(\boldsymbol{Q}_p, T) = \frac{r_+^{n-2}}{16\pi} - \frac{r_+^{n-2}}{16\pi l^2} + \frac{(2p-1)\left(2p(n-2)+1\right)\pi^{1/(2p-1)}\boldsymbol{Q}_p}{(n-1)\left(n-2p\right)2^{4+5p/(1-2p)}},\tag{35}$$

where $r_{+} = r_{+}(\boldsymbol{Q}_{p}, T)$. The behavior of the Gibbs free energy G is depicted in Fig. 3. As it is clear from Fig. 3, for $T < T_{c}$, the Gibbs free energy is single value and monotonically increasing function of \boldsymbol{Q}_{p} . While for $T > T_{c}$, it becomes multivalued which means that a first-order phase transition occurs between the small and large black holes. The corresponding phase diagrams represented as \boldsymbol{Q}_{p} versus T are shown in Fig. 4. Here, the small and large black holes are distinguished by transition line (blue line). As one can see from Fig. 4(a), for conformal case p = (n+1)/4, the slope of transition line increases with increasing dimension n. In Figs. 4(b) and 4(c), when we increase p, slope of transition line increases too.

V. THERMODYNAMIC GEOMETRY AND MICROSCOPIC STRUCTURE

In this section, we turn to study phase transition structure of power-Maxwell black holes in AdS space from point of view of thermodynamic state space geometry. The Ruppeiner geometry has been proposed as new approach to insight into underlying structure of thermodynamic system from the thermodynamic fluctuation theory [50]. Indeed, Ricci scalar which is obtained from Ruppeiner metric, indicates the dominant interaction between possible molecules



FIG. 5: (a) The behavior of reduced Ruppeiner scalar curvature (R/R_c) along the transition line for large and small black holes for p = (n+1)/4. (b) The dependence of the critical Ruppeiner scalar curvature R_c on the parameter p is depicted for different values of dimension.

by its sign [51, 52]. In fact, the Ruppeiner (Ricci) curvature vanishes for the ideal gas, while for attractive (repulsive) dominant interaction is negative (positive) [53–55]. Recently, various studies on Ruppeiner geometry have been carried out in [56–58].

The components of the Ruppeiner metric in the energy representation are defined as [50]

$$g_{\mu\nu} = \frac{1}{T} \frac{\partial^2 M}{\partial X^{\mu} \partial X^{\nu}},\tag{36}$$

where $X^{\mu} = (S, Q_p)$. With Eqs. (10), (11) and (36) at hand, one can calculate the Ruppeiner scalar curvature

$$R = \frac{8p^{(n+1)/2} \left[\frac{(n-2)(p(n-3)+1)}{n(n-1)}\right]^{(1-n)/2} \left[1 + (\Psi/\Psi_c)^{2(2p-1)/(n-2p)}\right] (\Psi/\Psi_c)^{(n+1)(2p-1)/(n-2p)}}{(2p-1) \left[1 + \frac{(n-1)}{p(n-3)+1} (\Psi/\Psi_c)^{2(2p-1)/(n-2p)} - \frac{(2p-1)^2 (\mathbf{Q}_p/\mathbf{Q}_{pc})}{(p(n-3)+1)(2p(n-2)+1)} (\Psi/\Psi_c)^{2p(n-1)/(n-2p)}\right]},$$
(37)

here l = 1. As can be seen in Table I, the positive sign of R is allowed due to positive temperature, i.e. there is always repulsive interaction. Figure 5(a) shows Ruppeiner curvature R/R_c , for conformal case p = (n + 1)/4, along the transition line in both the small and large black holes. According to Fig. 5(a), the value of Ruppeiner curvature in both small and large black holes is the same at the critical point. Also, there is a gap in Ruppeiner curvature between small and large black holes that is increased in higher dimension. The dependence of the critical Ruppeiner curvature on the allowed range of p is illustrated in Fig. 5(b). In Fig. 5(b) for arbitrary values of dimension, the critical value of Ruppeiner curvature diverges close to p = 1/2.

TABLE I: The allowed ranges	of	R
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	R	R
T	Positive	Negative
validity	allowed	not allowed

VI. SUMMARY AND CONCLUSION

In this paper, we have investigated the critical behavior of higher dimensional AdS black holes with power-Maxwell nonlinear electrodynamics via an alternative approach toward the phase space. We have kept the cosmological constant as a fixed quantity and treated the charge of black hole as thermodynamic variable. To show the complete analogy between the liquid/gas phase transition of the Van der Waals fluid and small/large black hole phase transition, we have investigated the phase space and critical behaviour in $Q_p - \psi$ plane.

We have suggested an algorithmic method to find the charge-independent thermodynamic variable ψ as a conjugate quantity to $f(Q) = \mathbf{Q}_p$ where s = 2p/(2p-1). We have also rewritten the Smarr mass formula in according to the new phase space and emphasized on its correspondence with standard Smarr relation. Furthermore, we have shown the behavior of coexistence curve of SBH and LBH in 4,5 and 6 dimensional spacetime. We have calculated the

Finally, we have studied the microscopic properties of higher dimensional AdS black holes by considering the effects of the conformal invariant power Maxwell field. From the viewpoint of the thermodynamic geometry we have figured out that the interaction between two micromolecules of black hole is a strong repulsive interaction. Actually transition from small to large (n+1)-dimensional black hole is due to this repulsive force. Similar to zero temperature thermodynamic in Fermi gas, we have introduced a temperature T_0 . We have seen that at $T > T_0$, when R = 0, large black hole can be stable and its size is a function of temperature only. Finally, as a result, the maximum amount of the scalar curvature gap Fig. 5(a) microscopic structur has been increased by increasing the number of dimensions.

The advantage of the approach presented in this paper is that we do not need to extend the thermodynamical phase space to see the Van der Wasls behaviour for black hole systems. The results obtained here further support the argument given in [24, 46]. Our study indicates that the approach here is helpful to investigate the critical behaviour of other gravity theories such as Gauss-Bonnet black hole [47] and other different electromagnetic fields without needing to consider the cosmological constant (pressure) as a thermodynamic variable.

Our investigation on thermodynamical properties and microscopic structure of the asymptotically AdS black holes with power Maxwell field further supports the viability of this new approach and confirms that this approach is enough powerful to explore the phase transition of higher dimensional black holes.

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