Quasistatic Evolution of Ideal MHD Equilibria in INTOR and ASDEX-UG

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1. Introduction. For studies of plasma equilibrium and stability in some present-day tokamak devices we consider processes of plasma dynamics which evolve fast on the resistive, but slowly on the Alfven time scale. Under such conditions the time evolution of the plasma configuration can be described in terms of a continuous sequence of ideal MHD equilibria which are controlled by appropriate time-dependent external magnetic fields. Such sequences are considered solving the partial differential equation for the poloidal magnetic flux together with the dynamical equations describing the isentropic motion of an ideally conducting plasma. In doing so we self-consistently take account of the instantaneous action on the plasma equilibrium of externally placed passive conductor elements and, by a corresponding adjustment of the loop voltage, provide for a skin-current free formation of the plasma.

2. Theory. In the plasma region we will use the equations for equilibrium

$$\mathrm{rot}\underline{B}\times\underline{B} = \mu_0 \underline{\nabla} p \tag{1}$$

with $\mathrm{div}\underline{B} = 0$, the conservation equations for magnetic flux, for mass and for entropy:

$$\partial\underline{B}/\partial t = \mathrm{rot}(\underline{v}\times\underline{B}) \qquad \partial\rho/\partial t + \mathrm{div}\rho\underline{v} = 0 \qquad \partial S/\partial t + \underline{v}\cdot\underline{\nabla}S = 0 \tag{2}$$

and an equation of state (more precisely a thermodynamic potential). In these equations all quantities have their usual meaning; we use MKSA-units. With R as the radial distance and with φ the angle about the axis of symmetry in z-direction we will refer to right-handed co-ordinates (R,φ,z). The fluxes of magnetic field and of current density through a surface which is bounded by a field line of the toroidal magnetic field we call Ψ and J, respectively. For the corresponding fluxes through poloidal cross-sectional areas of magnetic surfaces we will use the notations Φ and I. We make the assumption that the temperature is constant on magnetic surfaces. Then the fluxes Ψ and Φ, the currents J and I and the thermodynamic variables p, ρ and T are all surface quantities. The equilibria we are calculating are, besides through the equations (1) and (2), defined by the specification of

(I) a free-boundary equilibrium at an initial time;
(II) the externally produced toroidal and poloidal magnetic vacuum fields controlling and confining the plasma at any time.

By (I) the distribution of the above-mentioned seven quantities over the different magnetic surfaces is given at an initial time, their values at any time are determined by (II), the three conservation laws, by two equations relating fluxes and currents, the equation of state and by the normal component flux surface average of equation (1). A straightforward elimination procedure leads to a second-order (generalized /4/) ordinary differential equation of the form $F(\Psi'',\Psi',\Psi,V,t) = 0$ for the poloidal flux as function of the volume V (of the magnetic surfaces). This equation is to be solved with the boundary conditions corresponding to a constant flux difference between magnetic axis and plasma boundary and with the side condition that the poloidal current is continuous across the plasma-vacuum interface. This side condition excludes poloidal skin currents; the absence of toroidal ones must be externally controlled keeping constant the poloidal flux at the plasma boundary by an adjustment of the loop voltage. The rest of the problem reveals in (a) (not flux-surface averaged) local equations for $V(R,z,t)$ and $\underline{v}(R,z,t)$ and (b) circuit equations describing the inductive interaction between plasma and external active and passive conductor elements. More details on theory will be given elsewhere /1/.

3. Computational Features. The calculations profit by the fact that for given geometry of the magnetic surfaces the problem amounts to only the solution of ordinary differential equations. Solving it iteratively we start with the geometry of the initial free-boundary equilibrium and then proceed in time changing the currents in the external conductors. In order to find the new self-consistent plasma equilibrium we apply the following iteration scheme: (A) We solve the circuit equations for the external currents with the present plasma current density; (B) Use a fast Buneman solver for the determination of the plasma poloidal flux and superpose the vacuum flux of the external currents; (C) With that flux function calculate the new geometry and on the basis of this geometry a new plasma current density; (D) Repeat steps (A) to (C) until convergence is reached. In some cases acceleration of the procedure is expedient. For this case Newton-like iterations are applied shifting the plasma in about two steps to almost the right new position. Afterwards the cross-section and the exact plasma position is evaluated by iterations of the above scheme. In strongly stable configurations a new equilibrium is reached in 10-15 iterations, weakly stable situations need more iterations (about 30).

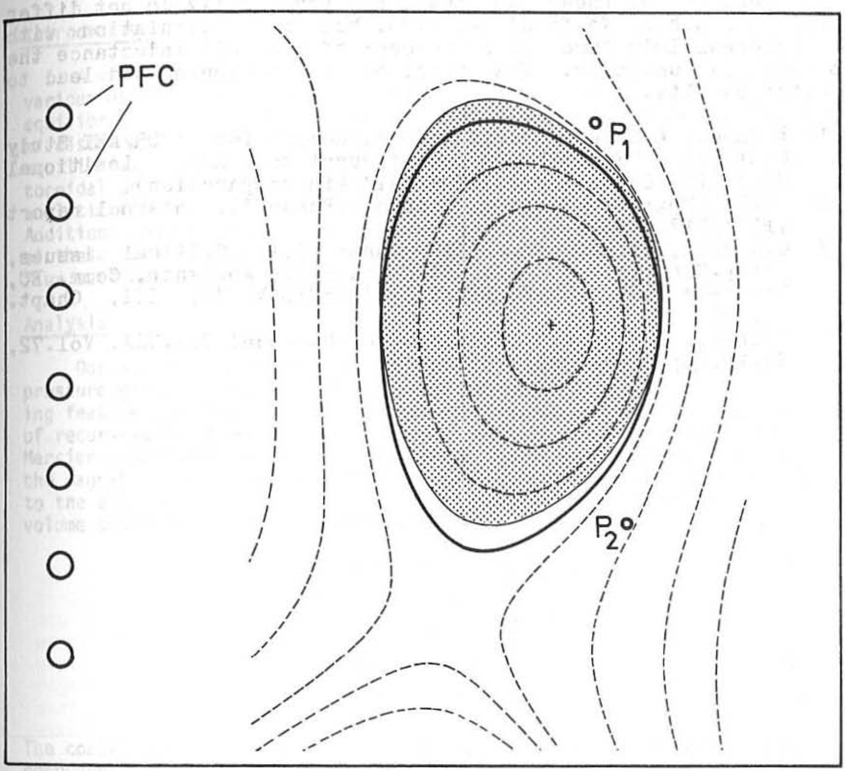
4. Application and Results. We considered the vertical stability of two experiments recently in the planning phase: NET/INTOR and ASDEX- UG. As shown in the Fig. INTOR (and similarly ASDEX-UG) have an elongated cross-section and therefore an unstable vertical position unless passive conductors would stabilize it. One easily can prove that in the case of stabilization the plasma displacement is of the following form:

$$\xi_z(t) = \xi_z(0)\exp(t/\tau), \quad \xi_z(0) = F_P/(f_D(\alpha-1)) \tag{3}$$

where

$$\tau = (\alpha-1)L/R \quad \text{and} \quad \alpha = f_R/f_D \tag{4}$$

ξ_z is a small deviation from the equilibrium position, F_P is a perturbation force (here produced by a radial magnetic field), $F_D = f_D\xi_z$ is the driving force due to an unstable vertical field index, $F_R = f_R\xi_z$ is the restoring force due to the induced currents in the passive coils and L/R is the time constant of the passive coil system in the absence of a plasma. For stability it is necessary that $\alpha > 1$, and the larger α the smaller are the energy requirements on a feed-back system. As $\alpha \sim 1/L$, the stabilizing effect depends on the cross section and the inductance of the connection between the upper and the lower passive coils which have to be connected antidirectional in series because otherwise they would shortcircuit the toroidal electric field.



The Fig. shows the flux pattern of an INTOR configuration for an initial equilibrium state and the equilibrium position of the plasma after application of a radial magnetic field. P_1, P_2 - passive conductor elements; PFC - poloidal field coils

For an INTOR configuration described in /3/ we took two passive coils (P_1, P_2 in the Fig.) with a cross section of 0.09 m^2. Their selfinductance is $L = 51\mu H$. With this value we found α = 1.5 which means that the additional inductances have to be significantly smaller than 25µH. The induced current is $I_{P_1} = -I_{P_2} = \gamma I_P \xi_z$, with $\gamma = 0.15\ m^{-1}$ and I_p the total toroidal plasma current. For larger displacements $\xi_z = 0.1$ m some nonlinear effects give a small enhancement to $\alpha = 1.6$. There was no significant difference between parabolic and flat plasma current distributions.
The ASDEX-UG configuration /2/ has a system of 2x 3 passive coils, the upper and the lower ones in parallel, respectively, with a total cross-section of 2x 0.024 m^2. At present we are not able to treat parallel connected coils, so we connected them in series or separated them into three pairs of two coils. The results of these two treatments practically do not differ from each other. We found $\alpha = 1.25$, but in a calculation with an external inductance of 20 percent of the coil inductance the plasma is unstable. The parallel connection should lead to better results.

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