

## SUPPLEMENTAL MATERIAL

*Magnus expansion.* Solving the time-dependent Schrödinger equation for a time-independent system described by the Hamiltonian  $H$  yields the well-known time-evolution operator  $U(0, t) = e^{-iHt}$ . If the system is time-dependent, the time-evolution operator can be expressed as

$$U(0, t) = e^{\Omega(t)} = \exp\left\{\sum_k^{\infty} \Omega_k(t)\right\}, \quad (8)$$

where  $t \mapsto \Omega(t)$  is a series known as the Magnus expansion. The first three terms in the Magnus expansion are given by

$$\begin{aligned} \Omega_1(t) &= \int_0^t dt_1 A(t_1), \\ \Omega_2(t) &= \frac{1}{2} \int_0^t dt_1 \int_0^{t_1} dt_2 [A(t_1), A(t_2)], \\ \Omega_3(t) &= \frac{1}{6} \int_0^t dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \\ &\quad [A(t_1), [A(t_2), A(t_3)]] + [A(t_3), [A(t_2), A(t_1)]], \end{aligned} \quad (9)$$

with  $A(t) = -iH(t)$ . If the Hamiltonian is time-independent, all Magnus terms but the first vanish.

The time-periodic Hamiltonian of Eqs. (1) and (2) is now plugged into this expansion, and one focuses on times  $t$  which are an integer multiple of the driving period  $T = 2\pi/\omega$ . The first-order term involving  $H_1$  vanishes (since the time-average is zero), and the one involving  $H_2$  yields the second and third term of Eq. (3). The second-order contribution associated with  $[H_1(t_1), H_1(t_2)]$  yields the first term of Eq. (3). The second-order contribution involving  $[H_1(t_1), H_2(t_2)]$  as well as all higher-order terms scale away with  $1/\sqrt{\omega}$  or faster. A benchmark of how well the time-evolution computed using the effective Hamiltonian  $H^{\text{eff}}$  agrees with the one governed by the full  $t \mapsto H(t)$  is shown in Fig. 3, Fig. 6, and Fig. 7.

*Single preparations.* An important question relates to the behavior of individual systems in contrast to disorder averaged ones such as those directly feasible in experiments. In Fig. 8 we show typical individual calculations that enter the disorder average focused upon in the main text. We show that those qualitatively behave like the averaged results.

*Numerical details.* In order to tackle the time-evolution governed by  $t \mapsto H(t)$ , we first compute the time-evolution operator  $U(0, t)$  for  $t = T = 2\pi/\omega$  numerically using a discrete time step of  $\Delta t = T/100$ . The periodicity of the Hamiltonian implies that  $U(t_1, t_2) = U(t_1 + T, t_2 + T)$  and thus  $U(0, 2T) = U(0, T)^2$ ,  $U(0, 4T) = U(0, 2T)^2$ , which allows one to efficiently propagate the system to large times  $t/T \sim 10^7$ .

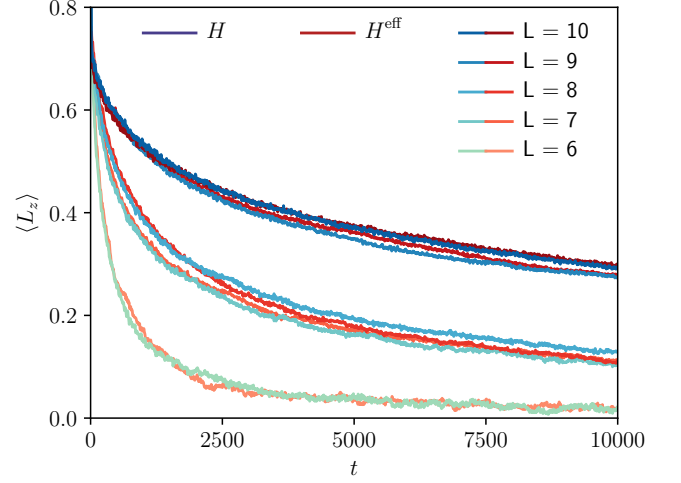


FIG. 6. Same as Fig. 3 of the main text but for more values of the system size  $L$ . The driving frequency is  $\omega = 1000$ .

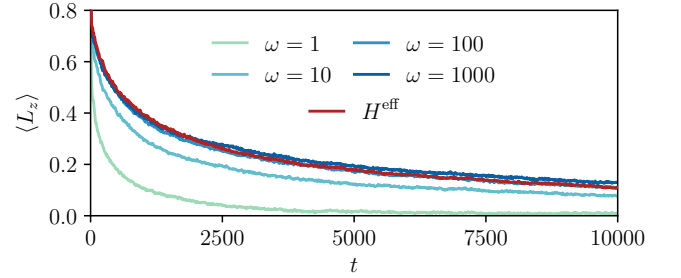


FIG. 7. Same as Fig. 3 of the main text but for various driving frequencies  $\omega$  at a fixed system size  $L = 8$ .

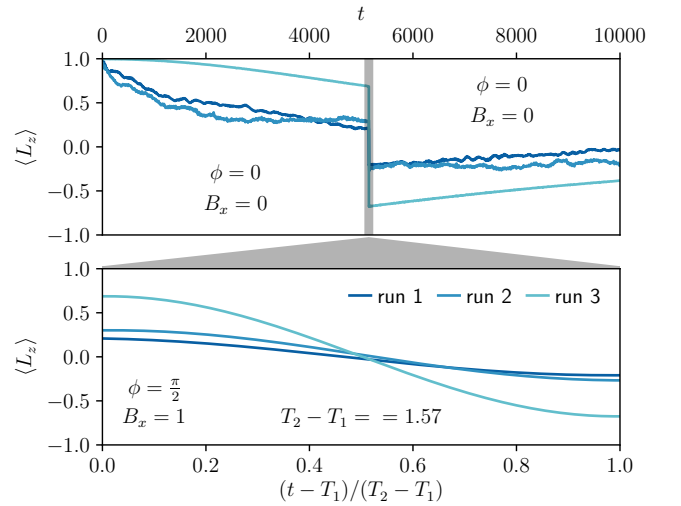


FIG. 8. Same as Fig. 5 of the main text but for three individual disorder configurations for a fixed value of  $T_2 - T_1 = 1.57$ .