

Microstructure of charged AdS black hole via $P - V$ criticality

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We suggest a new thermodynamic curvature, constructed via adiabatic compressibility, for examining the internal microstructure of charged black holes in an anti-de Sitter (AdS) background. We analyze the microscopic properties of small-large phase transition of black holes with pressure and volume as the fluctuation variables. We observe that strong repulsive interactions dominate among the micro-structures of near extremal small black holes, and the thermodynamic curvature diverges to positive infinity for the extremal black holes. At the critical point, however, thermodynamic curvature diverges to negative infinity.

I. INTRODUCTION

Phase transition is a fascinating phenomenon in black holes thermodynamics which has received considerable attentions in recent years. This is mainly motivated by AdS/CFT duality, which states that there exists a correspondence black holes in asymptotically anti-de Sitter (AdS) spacetime and the conformal field theory living on its boundary [1, 2]. A significant interest has been arisen for study phase transition of AdS black hole in an extended phase space in which the cosmological constant can be regarded as thermodynamic pressure which can vary [3, 4]. In this viewpoint, the mass of black hole is identified as the enthalpy [5]. It was shown [4] that the four dimensional charged AdS black hole demonstrates the first order (discontinuous) and second order (continuous) phase transitions between the small and large black holes in an extended phase space. This phase transition is analogous to the Van der Waals gas/liquid phase transition, thus, their critical exponents are the same as well. The investigation on the critical behavior of black holes in this context is often referred to as “ $P-V$ criticality” and has widely explored in the literatures [6–15] and references therein. Some interesting phenomena have been observed in the extended phase space of black holes such as *zeroth order* phase transition [16] and reentrant phase transition [17] as well as triple critical point [18] as well as superfluid like phase transition [19]. More recently, a universality class of the critical behavior of AdS black holes in an extended phase space has been studied by a general approach without specifying the functional form of the spacetime metric [20].

An alternative approach to investigate critical behavior of black holes is to consider the electric charge (Q) of the black hole as a thermodynamical variable while keeping the cosmological constant as a fixed parameter. From the physical point of view, the electric charge of

black hole is a natural variable which can take on arbitrary values and it affects the thermodynamic properties of AdS black hole. In this case, it was argued [21] that there exists a small-large black hole phase transition for the charged black hole in a fixed AdS background. It has been demonstrated [22] that this phase transition is physically conventional in an alternative phase space where the square of the electric charge (Q^2) is viewed as an independent thermodynamic variable of the black hole system. In this perspective, the new thermodynamic response function correctly signifies stable and unstable regimes and the critical behavior of the black hole resembles with Van der Waals fluid, belonging to the same universality class [23]. Phase transition of black holes in an alternative phase space have been explored in different setups [24–26]. More recently, the authors of Ref. [27] investigated thermodynamic phase structure of Born-Infeld and charged dilaton [28] black holes in a fixed AdS spacetime by studying the behavior of specific heat.

The theory of covariant thermodynamic fluctuations provides a powerful geometric framework to study properties of underlying thermal system, completely from the thermodynamic viewpoint [29, 30]. In this context, Ruppeiner defined the Riemannian metric on the equilibrium thermodynamic state space as the second derivatives of entropy. In his series of works [31–33], it has been confirmed that thermodynamic curvature (Ricci scalar) arising out of a such metric is related to the microscopic interactions, where the thermodynamic curvature is positive (negative) for the repulsive (attractive) interaction. In addition, thermodynamic curvature diverges at the critical point for pure fluid systems. With regard to this approach, the microscopic behavior and phase transition of various kinds of black holes have been explored [34–37]. In all these works, thermodynamic curvature has a finite value at the critical point. Recently, a new normalized thermodynamic curvature was proposed to understand the microscopic behavior of charged AdS black hole in an extended phase space where the temperature and volume are treated as fluctuating variables [38–40]. In this formalism, thermodynamic curvature is normalized with respect to the heat capacity at constant volume.

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It was shown that the microstructure of small black hole has a weak repulsive interaction and the thermodynamic curvature goes to infinity at the critical point of phase transition.

In this paper, we offer a new thermodynamic curvature, which is constructed via the adiabatic compressibility, for examining the internal microstructure of charged AdS black holes in an extended phase space with fixed charge. In particular, we analyze the microscopic properties of small-large phase transition of black holes with pressure and volume as the fluctuation variables. Our work differs from [38, 39] in that we allow the pressure and volume to fluctuate and normalize the thermodynamic curvature by the adiabatic compressibility, while the authors of [38, 39] considered the temperature and volume as the fluctuating quantities and normalized the thermodynamic curvature by the heat capacity at constant volume. We observe that strong repulsive interactions dominate among the micro-structures of small black holes where the thermodynamic curvature diverges to positive infinity. It is shown that the thermodynamic curvature diverges to negative infinity at the critical point.

The structure of the paper is laid out as follows. We begin in Sec. II by giving a brief review of the thermodynamics and critical behavior of the four dimensional charged AdS black holes in the extended phase space. In Sec. III, we first introduce the Ruppeiner geometry and obtain the corresponding line element for a thermodynamic system in terms of the entropy and pressure. Then, we use the thermodynamic curvature to investigate in detail the microstructure of charged AdS black hole. Finally, we present some remarks in Sec. IV.

II. THERMODYNAMICS AND PHASE TRANSITION OF CHARGED ADS BLACK HOLES

We start with a brief review on the thermodynamics properties and $P - V$ criticality of Reissner-Nordstrom (RN)-AdS black hole in an extended phase space. The action of Einstein-Maxwell theory in four-dimensional spacetime with a cosmological constant (Λ) is

$$I = \frac{1}{16\pi} \int d^4x \sqrt{-g} (\mathcal{R} - 2\Lambda - F_{\mu\nu}F^{\mu\nu}), \quad (1)$$

where \mathcal{R} is the scalar Riemann curvature, $F_{\mu\nu}$ is the electromagnetic field strength that is defined as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ with the gauge field A_μ . The negative cosmological constant Λ is related to the AdS radius L by the relation, $\Lambda = -3/L^2$. In four dimensions, the line element of the spherically symmetric RN-AdS metric is given by [4]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \quad (2)$$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{L^2}, \quad (3)$$

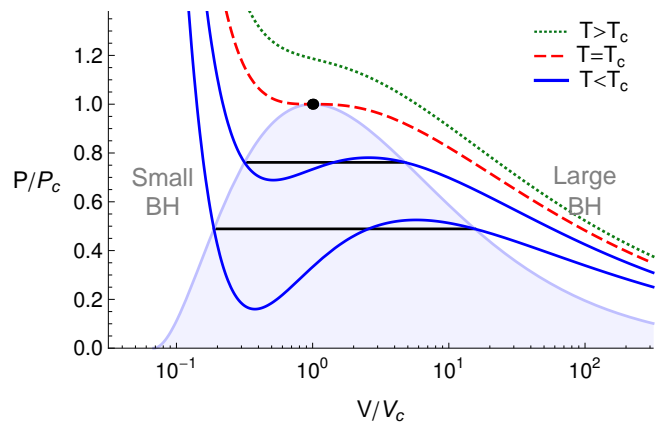


FIG. 1: P - V diagram of RN-AdS black holes. The region of the first order phase transition is identified where the isobars (black horizontal lines) remedy the unstable regime by the Maxwell equal area law. The areas above and below the black isobar are equal one another which is not seen because of logarithmic scale on the horizontal axis. The critical point is marked by a black spot. Note the logarithmic scale on the horizontal axis.

where $d\Omega^2$ is the metric of the unit two sphere. Herein, the parameters M and Q are, respectively, the mass and charge of black hole where the position of the black hole event horizon (r_+) is determined as a largest positive real root of $f(r_+) = 0$. The only nonvanishing component of the electromagnetic field tensor is given by $F_{tr} = Q/r^2$.

The Hawking temperature of the RN-AdS black hole on an event horizon is obtained as [4]

$$T = \frac{f'(r_+)}{4\pi} = \frac{1}{4\pi r_+} \left(1 + \frac{3r_+^2}{L^2} - \frac{Q^2}{r_+^2} \right), \quad (4)$$

and the entropy is

$$S = \pi r_+^2. \quad (5)$$

By interpreting the cosmological constant as a thermodynamic pressure, $P = -\Lambda/(8\pi)$, and its conjugate quantity as a black hole thermodynamic volume, $V = 4\pi r_+^3/3$, the first law of black hole thermodynamics and the corresponding Smarr formula take the form, respectively,

$$dM = TdS + VdP + \Phi dQ, \quad (6)$$

$$M = 2TS + \Phi Q - 2VP, \quad (7)$$

where $\Phi = Q/r_+$ is the electric potential measured with respect to the event horizon. In this consideration, the mass (M) of the black hole is identified as the enthalpy. Also, the thermodynamic process is carried out in the extended phase space. It is worthwhile to mention that according to Eq. (5) and black hole thermodynamic volume formula, the entropy is only a function of area/volume, i.e. $S = S(V)$. This feature of the black hole will be used in the next section.

For the four-dimensional charged AdS black hole, the equation of state, $P = P(T, V)$, is obtained by using

Eq.(4) as

$$P = \frac{T}{2r_+} - \frac{1}{8\pi r_+^2} + \frac{Q^2}{8\pi r_+^4}, \quad (8)$$

where $r_+ = (3V/4\pi)^{1/3}$. The behavior of isotherms in the P - V diagram is shown in Fig. 1. We see that the critical point is an inflection point on the isotherm which is characterized by

$$\left. \frac{\partial P}{\partial V} \right|_{T_c} = 0, \quad \left. \frac{\partial^2 P}{\partial V^2} \right|_{T_c} = 0. \quad (9)$$

One obtains the critical quantities as

$$T_c = \frac{\sqrt{6}}{18\pi Q}, \quad P_c = \frac{1}{96\pi Q^2}, \quad V_c = 8\sqrt{6}\pi Q^3.$$

For $T < T_c$, an oscillating part of the isotherm denotes unstable region where the isothermal compressibility is negative, i.e.

$$\kappa_T = -\frac{1}{V} \left. \frac{\partial V}{\partial P} \right|_T < 0. \quad (10)$$

This instability is replaced by an isobar (the horizontal line) via the Maxwell equal area construction, $\oint V dP = 0$, which means that there exists a first order phase transition between the small black hole and large black hole. The small-large black hole transition region, determined by Maxwell construction, has the following forms [41]

$$\begin{aligned} \tilde{T}^2 &= \tilde{P}(3 - \sqrt{\tilde{P}})/2, \\ \tilde{P} &= \frac{7 + 6\tilde{V}^{2/3} - 4\sqrt{3 + 6\tilde{V}^{2/3}}}{\tilde{V}^{4/3}}, \end{aligned} \quad (11)$$

where the reduced thermodynamic variables are defined as

$$\tilde{T} = \frac{T}{T_c}, \quad \tilde{P} = \frac{P}{P_c}, \quad \tilde{V} = \frac{V}{V_c}.$$

It is worthwhile to note that the two phase of small and large black holes cannot be distinguished above the critical point. In the next section, we examine the behavior of charged black hole in the Ruppeiner geometry.

III. RUPPEINER GEOMETRY

In this section, we apply the concept of the Ruppeiner thermodynamic geometry as a useful tool to study the microscopic structure of charged AdS black holes. The Ruppeiner geometry arises from the Gaussian thermodynamic fluctuation theory which is constructed on the thermodynamic state space [30]. In two dimensions, the Riemannian curvature scalar, R , (thermodynamic curvature) gives complete information about the Ruppeiner

geometry which is connected with the inter-particle interaction in a thermodynamic system. Specially, the positive (negative) sign of the thermodynamic curvature indicates the repulsive (attractive) interaction, while $R = 0$ corresponds to no interaction [31–33]. In the following, we first derive the thermodynamic fluctuation metric in the (S, P) coordinates, where a thermodynamic potential is the enthalpy. Then, using the fact that the entropy of the charged AdS black hole only depends on the volume, we investigate the thermodynamic curvature of black hole through the (P, V) plane.

A. Ruppeiner metric

Consider a thermodynamic system characterized by the entropy (S), internal energy (U) and volume (V) such that the line element between two thermodynamic states is [30]

$$\Delta l^2 = g_{\mu\nu} \Delta x^\mu \Delta x^\nu, \quad (12)$$

where $x^\mu = (U, V)$ and the metric element $g_{\mu\nu}$ is given by

$$g_{\mu\nu} = -\frac{\partial^2 S}{\partial x^\mu \partial x^\nu}.$$

In the entropy representation, the first law of thermodynamics for this system is expressed as follows

$$dS = \frac{1}{T} dU + \frac{P}{T} dV, \quad (13)$$

where T and P are temperature and pressure, respectively. Using the first law of thermodynamics Eq.(13), the line element Eq.(12) can be written as

$$\Delta l^2 = \frac{1}{T} \Delta T \Delta S - \frac{1}{T} \Delta P \Delta V. \quad (14)$$

To express the above line element in (S, P) coordinates, we have

$$\begin{aligned} \Delta T &= \left. \frac{\partial T}{\partial S} \right|_P \Delta S + \left. \frac{\partial T}{\partial P} \right|_S \Delta P, \\ \Delta V &= \left. \frac{\partial V}{\partial S} \right|_P \Delta S + \left. \frac{\partial V}{\partial P} \right|_S \Delta P. \end{aligned} \quad (15)$$

Substituting Eqs.(15) into Eq.(14) and using the Maxwell relation

$$\left. \frac{\partial T}{\partial P} \right|_S = \left. \frac{\partial V}{\partial S} \right|_P,$$

one obtains the thermodynamic line element

$$\Delta l^2 = \frac{1}{C_P} \Delta S^2 + \frac{V}{T} \kappa_S \Delta P^2, \quad (16)$$

where $C_P = T (\partial S / \partial T)_P$ is the heat capacity at constant pressure and $\kappa_S = -1/V (\partial V / \partial P)_S$ is the adiabatic compressibility. Here, the thermodynamic potential is the enthalpy where the independent variables are entropy and pressure.

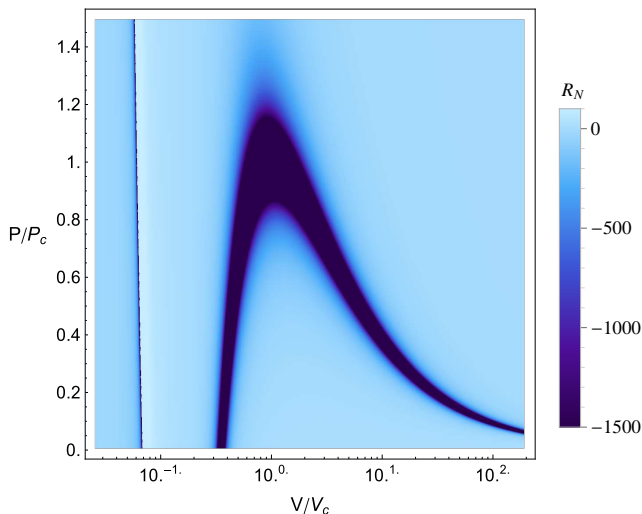


FIG. 2: The normalized thermodynamic curvature as a function of the pressure P and volume V . Note the logarithmic scale on the horizontal axis.

B. Thermodynamic curvature in P - V diagram

Now, we use Eq. (16) to investigate microstructure of RN-AdS black hole in an extended phase space. Due to the fact that the entropy of black hole only depends on the volume, i.e. $S = (3^2\pi/2^4)^{1/3}V^{2/3}$, the line element of the Ruppeiner geometry can be written as

$$\Delta l^2 = \frac{1}{C_P} \left(\frac{\pi}{6V}\right)^{2/3} \Delta V^2 + \frac{V}{T} \kappa_S \Delta P^2, \quad (17)$$

where the pressure and volume are taken as the fluctuation variables. For the black hole, the adiabatic compressibility (κ_S) vanishes similar to the heat capacity at constant volume, i.e. $C_V = T(\partial S/\partial T)_V = 0$ ¹. Hence, following [38, 39], we define a normalized thermodynamic curvature, R_N , based on the adiabatic compressibility

$$R_N = R\kappa_S. \quad (18)$$

In what follows, we analyze in detail the behavior of the normalized thermodynamic curvature as function of the pressure and volume. By performing simple calculations, we obtain the normalized thermodynamic curvature

$$R_N = \frac{16\tilde{V}^{2/3}(3\tilde{V}^{2/3} - 1)(5 - 6\tilde{V}^{2/3} + 9\tilde{P}\tilde{V}^{4/3})}{(1 - 2\tilde{V}^{2/3} + \tilde{P}\tilde{V}^{4/3})^2(1 - 6\tilde{V}^{2/3} - 3\tilde{P}\tilde{V}^{4/3})}, \quad (19)$$

which is expressed in terms of the reduced thermodynamic variables. Remarkably, the R_N is independent

¹ The entropy of the Van der Waals fluid system is a function of the temperature and volume i.e. $S = S(T, V)$ [39, 42]. This would imply that the adiabatic compressibility is non-zero ($\kappa_S \neq 0$) and it has a finite value at the critical point.

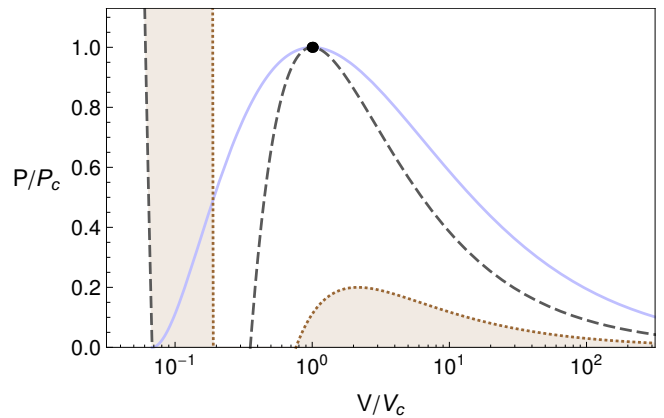


FIG. 3: Transition curve (light blue solid line), vanishing curve (brown dotted line) and diverging curve (gray dashed line) of R_N . The region identified corresponds to positive R_N , otherwise R_N is negative. Both the transition and diverging curves start at $\tilde{V} = 1/6\sqrt{6}$. The region to the left of the shaded region on the left side of the figure is excluded because temperature is negative. The critical point is marked by a black spot. Note the logarithmic scale on the horizontal axis.

of the charge of a black hole in Eq.(19). It should be noted that if one uses Eq.(16) instead of Eq.(17) for the Ruppeiner line element, the normalized thermodynamic curvature (R_N), Eq.(19), does not change. The overall behavior of the normalized thermodynamic curvature as a function of P/P_c and V/V_c is illustrated in Fig.2. As can be ascertained from Fig.2, the R_N goes to negative infinity in certain regions of the plane. From Eq.(19), R_N diverges along the curves

$$\tilde{P}_{div} = \frac{2\tilde{V}^{2/3} - 1}{\tilde{V}^{4/3}}, \quad (20)$$

$$\tilde{P}_{div} = \frac{1 - 6\tilde{V}^{2/3}}{3\tilde{V}^{4/3}}, \quad (21)$$

The divergent curve in Eq. (21) corresponds to the extremal black holes which are at zero temperature. On the other hand, R_N obviously vanishes at the following curves

$$\begin{aligned} \tilde{P}_0 &= \frac{6\tilde{V}^{2/3} - 5}{9\tilde{V}^{4/3}}, \\ \tilde{V}_0 &= \frac{1}{3\sqrt{3}}, \end{aligned} \quad (22)$$

where the dominant interaction between the microstructure of charged black hole changes from attractive to repulsive and vice versa.

To better understand the behavior of the normalized thermodynamic curvature, we show the diverging (gray dashed line) and vanishing (brown dotted line) curves corresponding to Eqs.(20), (21) and (22), respectively, as well as the small-large black hole phase transition (light blue solid line) curve in Fig.3. In Fig.3, the critical point

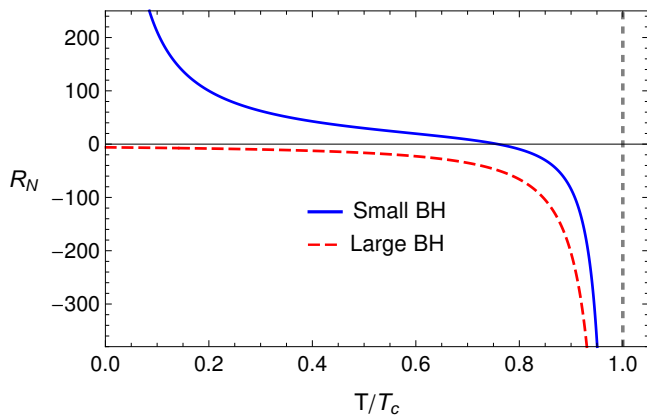


FIG. 4: The normalized thermodynamic curvature R_N for charged AdS black hole along the transition curve in the small and large black holes phases. R_N of the small black hole changes the sign to positive at $\tilde{T} = 3\sqrt{3}(7 - 3\sqrt{5})/2 \approx 0.7581$.

is highlighted by a black solid circle and the shaded regions have positive values for R_N which imply the domination of repulsive interaction. In the other region, R_N is negative which means the microstructure interactions are attractive. As evident from Fig.3, R_N is negative for the large black hole, while there is a certain range of volume in the small black hole phase ($\tilde{V} < 1$) that has positive R_N . In this positive region, R_N diverges to positive infinity when the gray dashed line is approached from large values of volume. i.e, the microstructure interaction of the small black hole is strongly repulsive. A strongly repulsive interaction also exists in the higher pressure regime (above the critical point) at low volume \tilde{V} . The white region to the left of the gray dashed curve on the left side of the Fig. 3, where black holes are sufficiently small, is excluded due to the fact that temperature is negative. Since the equation of state (8) may not hold in the transition region (below the light blue solid curve), R_N does not give any information about the black hole microstructure. Furthermore, as also seen in Fig.3, light blue solid and gray dashed curves coincide at the critical point where the thermodynamic functions of charged black hole are characterized by a set of critical exponents [4]. Hence, the normalized thermodynamic curvature diverges to negative infinity ($R_N \rightarrow -\infty$) at the critical point. This situation is analogous to fluid in the critical point regime, such as Van der Waals system [30, 38, 39], where thermodynamic curvature goes to negative infinity at the critical point.

To obtain an explicit expression of R_N near the critical point, we expand R_N , Eq.(19), around the critical point using Eq.(8)

$$R_N = -\frac{9}{2}t^{-2}, \quad (23)$$

where $t = 1 - \tilde{T}$ is the deviation from the critical temperature. Therefore, R_N has the universal critical exponent 2 and critical amplitude $-9/2$. Further, it is interesting to investigate the behavior of R_N on the transition curve. In this respect, we plotted in Fig.4, R_N along the transition curve in both the small and the large black holes phases from the critical temperature to zero. One observes from Fig.4 that R_N in both phases diverges to $-\infty$ at the critical temperature. In the large black hole phase, R_N uniformly negative and $|R_N|$ decreases as the temperature decreases from the critical temperature, which it is small at $\tilde{T} = 0$. While, in the small black hole phase, R_N changes sign and becomes positive below $\tilde{T} = 3\sqrt{3}(7 - 3\sqrt{5})/2 \approx 0.7581$. Remarkably, R_N diverges to positive infinity as \tilde{T} tends to zero where strong repulsive interactions dominate.

IV. FINAL REMARKS

In this paper, we proposed a new thermodynamic curvature, by using the adiabatic compressibility, for examining the internal microstructure of charged AdS black holes in an extended phase space. We explored the microscopic properties of small-large black holes phase transition by considering the pressure and volume as the fluctuation variables. We defined a normalized thermodynamic curvature, $R_N = \kappa_S R$, where κ_S is the adiabatic compressibility, and studied the behavior of R_N as a function of the pressure and volume. The sign of R_N determines the repulsive or attractive feature of black holes microstructure. When $R_N > 0$, the repulsive interaction dominates, while $R_N < 0$ indicates that the microstructure interactions are attractive. We also observed that a strongly repulsive interaction exists in the higher pressure regime (above the critical point) at low volume. At the critical point however, we have $R_N \rightarrow -\infty$, which is analogous to the Van der Waals fluid in its critical point regime.

Acknowledgments

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