Design and characteristics of a low-frequency magnetic probe for magnetic profile measurements at Wendelstein 7-X

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Equilibrium analysis in fusion-devices usually relies on plasma pressure profiles and magnetic measurements outside the plasma. The kinetic profiles can give indirect information about the equilibrium magnetic field, while the stationary magnetic diagnostics can not resolve current distributions on a smaller scale. This work presents a reciprocating magnetic probe, designed to provide direct plasma response measurements of the magnetic field in the scrape-off layer of Wendelstein 7-X. Hardware design and frequency characteristics are discussed, and a post-processing technique for extending the lower frequency cutoff of the integration scheme is presented.

I. INTRODUCTION

Low-frequency noise is a common hurdle in magnetic diagnostics based on integrated coil signals, due to the large relative am-

plification of low-frequency components showing up as low-frequency drifts. The long discharge durations of the Wendelstein 7-X (W7-X) stellarator^{1,2} further emphasize this issue. Discharges frequently exceed 10 s in length and have reached up to 100 s

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by zeroing or inversion of the input voltage) can be used to capture low-frequency drift in processing stages behind the chopper^{3,4}, but the chopper itself must be set up carefully to avoid channel cross talk and biases introduced by the chopping circuit. Thechopped signal must then be interpreted in software to compensate both chopping the operational amplifier can then be integrated model inputs. of the design. This paper discusses a magnetic probe using such an analog integrator designed to measure the change of the edge magnetic field of W7-X during plasma operation.

particle exhaust⁵⁻⁷. For high-performance change inside the magnetic islands. long-pulse operation, the modification of sensors make in principle an excellent can-

already, with 30 min discharges planned for these islands both due to a change in the a future campaign. Signal chopping (either magneto-hydrodynamic (MHD) equilibrium and bootstrap currents needs to be modeled to ensure operational safety and proper screening of the plasma core from edge impurities. Equilibrium calculation codes, such as VMEC⁸ and HINT⁹ rely on pressure profiles as an input. Usually, these codes also have additional free parameters, such as (usually) the boundary shape in the case of determined drift. However, recently, a new VMEC and the pressure averaging length for implementation strategy for signal chopping HINT. In the past, the presence of these free Multiple vendors now offer parameters has shown to result in differing operational amplifiers with an integrated results between the codes, even outside stabilization circuit. The signal is chopped the plasma¹⁰. For validation of numerical in the amplifier and used to determine an models, it is preferred to rely on diagnostic offset correction for the amplifier. Such an measurements which are unrelated to the One good benchmark for into a conventional analog integrator circuit, equilibrium models is the change in the offering the advantages of chopper-based magnetic field, both in topology (which can stabilization while retaining the simplicity be observed indirectly with edge profiling diagnostics) and magnetic signals.

Magnetic diagnostics at W7-X include flux loops¹¹, Rogowski coils^{12,13} and Mirnov coils localised at the vessel¹⁴. Usually, the observations made by these diagnostics are At the plasma edge, W7-X relies on a either cross-section averaged or localized chain of magnetic islands - intersected by outside of the plasma. A key observation, its divertor target plates - for heat- and however, would be the local magnetic field didate for such low-frequency observations of the magnetic field, but sensor designs that can operate under the high heat fluxes experienced by reciprocating probes are still in development¹⁵. Magnetic coils are a common feature in reciprocating probes 16,17 but are mainly used for fluctuation studies¹⁸. Equilibrium profile reconstruction from coil signals is challenging since the magnetic field created by the plasma (which is in the order of 1 mT to 10 mT) is dominated by the spatial variation of the vacuum magnetic field (which is about 0.1 T in the outboard mid-plane of W7-X), which requires a broad dynamic range while retaining high precision.

In contrast to Hall sensors, the signal of a magnetic probe must be integrated to obtain the plasma response. Methods for integration of magnetic coil signals can be mostly divided into two branches, analog and digital. Analog methods mostly use operational amplifierbased circuitry to obtain an integrated signal before sampling. Digital integration methods sample the un-integrated input signal and inhigh dynamic range and ease of implementasignal acquisition chain. This challenge can ing into the confined plasma core. Magnetic

be addressed by introducing a signal chopper early into the measurement chain for drift calibration. Additionally, digital integration requires continuous data acquisition. Analog integrators can by placed early in the measurement chain, but require a trade-off between dynamic range, stability against parasitic decay and input impedance.

SETUP OF THE DIAGNOSTIC II.

The combined probe Α.

The combined probe is a diagnostic system for the integrated measurement of a multitude of plasma parameter profiles. Its front surface features a triple probe for electron temperature and density measurements, two protruding floating potential pins, a pair of up- and downstream facing Mach probes for parallel flow measurements, a gas inlet, a tungsten sample for exposition and an experimental ion-sensitive probe¹⁹. The probe interior - including the magnetic coils - is shielded by a Boron Nitride cover. The probe tegrate the time trace numerically. Digital is designed for the Multi-Purpose Manipuintegration methods have the advantage of lator (MPM)^{20,21}, which is located in the $\phi = 200.8^{\circ}, z = -17 \,\mathrm{cm}$ line and can theotion, as well as the potential to modify the in- retically plunge up to 35 cm inwards from its tegration scheme later on, but can show sus- parking position. Large heat-fluxes on the ceptibility to low-frequency drift errors in the probe however usually prevent it from plungplasma response measurements are restricted differential coil pair (2198 windings each, opto the outboard side of the magnetic islands, due to interference between the pickup coil system and currents drawn via the Langmuir probes. This interference would manifest itself as a jump in the integrated voltage occuring simultaneously with short transient current bursts appearing on the Langmuir probe channels. To ensure that smaller variations of this interference - potentially not visible to the naked eye on the raw signals - do not accumulate in the integrated signal, the error analysis was designed to be sensitive to differences in the magnetic field measurement during the inward and outward motion.

В. The magnetic sensor

The magnetic probe is a 3D pickup probe consisting of three concentric mutually orthogonal coils. Each coil consists of 500 windings of a 0.1 mm copper wire with a Polyimide film (Kapton) insulation. The coils have a (calculated) effective area per winding of $1.342\,\mathrm{cm}^2$, $1.464\,\mathrm{cm}^2$ and $1.83\,\mathrm{cm}^2$ in radial, toroidal and vertical direction respectively. The common center point is offset 39.2 mm from the front-most pin, 111 mm ure 2). In an ideal scenario, such an intefrom the base of the probe along the probe grator has an amplification factor of $\alpha =$ path, and 6 mm below the MPM center axis $-10 \,\mathrm{s}^{-1}$. The chosen operational amplifier (see figure 1). For measured total effective AD8628 suppresses common-mode drift usareas, see table I. The design also features a ing a built-in signal chopper and feedback

positely wound, 1cm apart) for gradient fluctuation measurements, but these coils were out of operation during the 2018 campaign. The pickup probe is connected via 14 m long twisted cable pairs to the integration circuit.

Integration circuitry

For this probe, an analog integrator was chosen over a chopper-stabilized digital integration for two reasons:

- Robustness of the integration scheme at limited sample rate and piece-wise signal acquisition
- Availability of high-performance stock components and ease of implementation (limited time-frame between the 2017 and 2018 experimental campaigns)

To minimize distortion of the signal prior to integration (mainly by $\frac{1}{f}$ noise), the integrator was implemented as an integrating pre-amplifier. It uses a standard inverting amplifier circuit with the parameters $R_{\rm amp} = 10 \,\mathrm{k}\Omega$ and $C_{\rm amp} = 10 \,\mathrm{\mu F}$ (see fig-

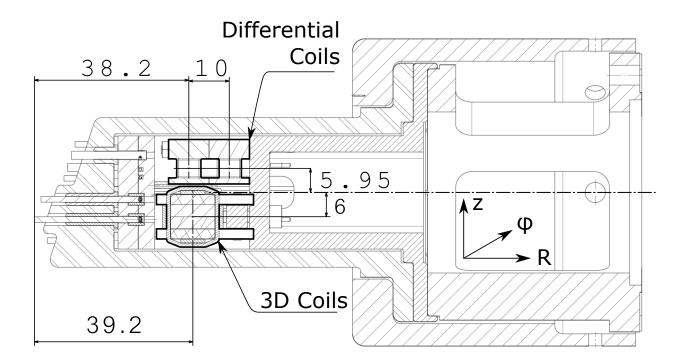


Figure 1. Location of the 3D and the differential coil sensors inside the combined probe

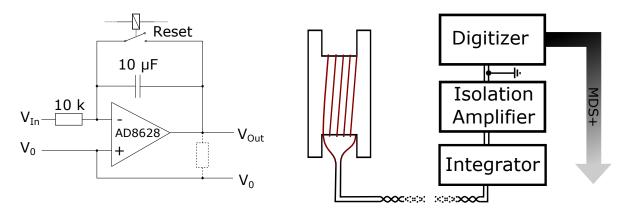


Figure 2. Simplified schematic of the integrating pre-amplifier (unintentional parasitic resistance displayed in dashed lines)

Figure 3. Setup of the signal processing stack for the pickup coil

This configuration provides a strong noise reduction at low frequencies (< 10 Hz) at the cost of additional output noise at the from the high voltages present inside W7chopping frequency. This drawback is mitigated by pseudo-random chopping, spreading of the integrator, are passed through a set the noise up to 15 kHz wide.

\mathbf{D} . Signal acquisition

To protect the data acquisition system X, all probe signals, including the output of DEWETRON® HSI-LV variable isolation

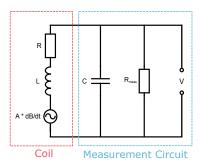


Figure 4. Circuitry model for the coil characterization. A, R and L are the parameters to be determined, while C is externally set and R_{meas} is a function of the voltage measurement device

amplifier before being recorded by a DTAQ® ACQ132CPCI and stored in an MDSplus^{22,23} 3). Data can be acquired tree (see fig. in a single block or over multiple segments. The isolation amplifiers have a bandwidth of 2 MHz, while the sampling rate of the digitizers could be selected from a range between 250 kHz (for up to 16 s) and 2 MHz (for up to 2s) during the last experimental campaign.

III. COMPONENT **CHARACTERISTICS**

Characteristics of the pickup coils Α.

The response curves of the pickup coil (phase and sensitivity) were determined in a pair of Helmholtz coils. The probe was set up as it would be inside the manipulator, so the measurement also includes shieldponents. Adding capacitors parallel to the voltage measurement allowed measurement of the coil's self-inductance and internal resistance. The equivalent circuitry for this measurement is shown in figure 4 and can be used to obtain the frequency response (equation 1) to an external magnetic field (substituting $X = \widetilde{X}e^{i\omega t}$), the derivation of which can be found in appendix A.

$$\frac{\tilde{V}_{\text{coil}}}{\tilde{B}} = \frac{i\omega A_{\text{coil}}}{h(\omega)}$$

$$h(\omega) = 1 + \frac{R_{\text{coil}}}{R_{\text{meas}}} - \omega^2 L_{\text{coil}} C_{\text{meas}}$$

$$+ i\omega \left(R_{\text{coil}} C_{\text{meas}} + \frac{L_{\text{coil}}}{R_{\text{meas}}} \right)$$
(1)

The parameter C_{meas} was varied to better characterize the self-inductances, while the input impedance R_{meas} of the measurement digitizer is fixed at $1 M\Omega$. The parameters in table I were estimated by fitting equation 1 to the measurements in figure 5 in a range from 1 Hz to 10 kHz. In this range, the characteristics of the signal cables do not yet play a significant role. If one were doing a characterization into the MHz regime, one would have to take into account transmission delays as well as impedance matching effects. The nonmonotonic relation between the effective area A and the inductance L shows that the exact winding geometry of the coil plays an important role in determining the self-inductance. Given that the dynamic ranges have approxing effects from the cover and metal comimately a 2:1:1 ratio for toroidal, vertical

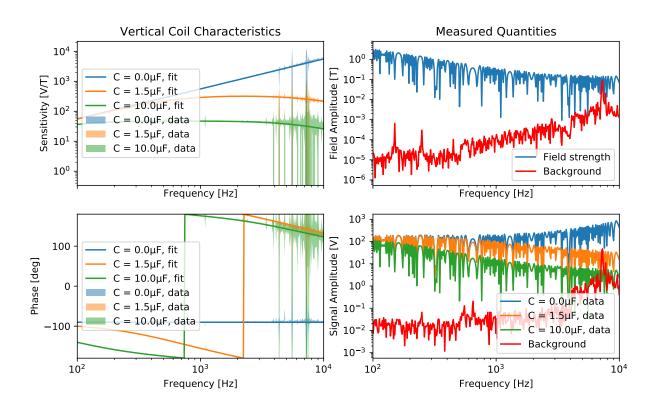


Figure 5. Input and output measurements - as well as background levels - for the radial sensitivity measurement

Coil	$A_{ m radial} \left[{ m cm}^2 ight]$	$A_{ m toroidal} \left[{ m cm}^2 ight]$	$A_{ m vertical} \left[{ m cm}^2 ight]$	$L[\mathrm{mH}]$	$R\left[\Omega\right]$
Radial	675 ± 10	$3.9^{+5}_{-3.9}$	6.9 ± 5	3.0 ± 0.6	167 ± 10
Toroidal	$1.9^{\ +5}_{-1.9}$	707 ± 4	5.4 ± 5	2.7 ± 0.1	167 ± 10
Vertical	10.7 ± 5	3.0^{+5}_{-3}	888 ± 5	3.4 ± 0.1	187 ± 2

Table I. Empirically obtained coil parameters

Coil 1	Coil 2	Mutual inductance [μH]
Toroidal	Radial	34
Vertical	Radial	18
Toroidal	Vertical	16

Table II. Upper bounds for mutual inductances of the coils

and radial signals respectively, the additional error from cross-direction coupling should be bounded at about 2%, which is far below the intrinsic error level of the measurement itself (see section V for a discussion of the magnetic profile error). This limit of course only holds as long as the integration circuitry operates

linearly, as this implies proper error cancellation when combining field measurements.

The uncertainties of the measurement in the upper-frequency region are related to two effects, which are visible in figure 5. Firstly, the current amplitude from the KEPCO power supply powering the Helmholtz coils drops (as the current supply is driven near its specified frequency limit of 10 kHz), reducing both the magnetic field and the coil signal. Secondly, there is an increase in background noise near 8 kHz, which causes the signal level to drop below the uncertainty. The noise peak obtained by the background characterization (where the power supply was set to a zero control voltage) exceeds the measured signal in magnitude at a few frequencies in the coil voltage measurement. This indicates a noise reduction when the power supply is actively driven to a specific frequency, which is possible if the power supply contains a nonlinear feedback loop. This means that the background measurement is likely an overestimation of the actual measurement error, but obtaining a more accurate estimate is experimentally challenging.

By replacing the external Helmholtz coil pair with the pick-up coils themselves as the magnetic field sources, the mutual inducsmall source currents could be applied. Up- addressed in the next iteration of the cir-

per bound measurements obtained as such are shown in table II. Due to the mutual inductances being two orders or magnitude below the self-inductances, and the similar dynamic range of the signals, we assume inductive effects to be dominated by the selfinductances.

В. Characteristics of the analog integrator

In a simple linear model the integrator can be described using equation 2 with the (integrating) input amplification α and the output signal decay time τ .

$$\frac{dV_{\text{out}}}{dt} = \alpha V_{\text{coil}} - \tau^{-1} V_{\text{out}} \tag{2}$$

This model implies the transfer function $\widetilde{V}_{
m out} = rac{lpha}{ au^{-1} + i\omega} \widetilde{V}_{
m coil}$ which has a $1/\sqrt{2}$ cutoff at $f_{1/\sqrt{2}} = \frac{1}{2\pi\tau}$. Table III shows empirical parameters for model 2, obtained by fitting it onto the response to a single sine-shaped pulse of 500 ms duration mimicking a typical magnetic signal during a manipulator plunge. The high decay rate τ^{-1} was found to coincide with a $\approx 3 \,\mathrm{M}\Omega$ scale parasitic resistance, which was only present when the integrators were mounted on their auxiliary power supply board. This parasitic decay stresses the tances of the sensors can be determined. Due importance of proper isolation between the to the high ohmic resistivity of the coil, only signal line and external circuitry and will be

Integrator	$\alpha \left[s^{-1} \right]$	$\tau^{-1} \left[s^{-1} \right]$	$f_{1/\sqrt{2}}[\mathrm{Hz}]$
Radial	-11.68	0.22	0.035
Toroidal	-11.42	0.31	0.049
Vertical	-11.34	0.35	0.055
Ideal	-10	0	0

Table III. Parameters for the linear integrator model described in equation 2

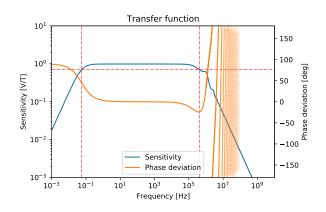


Figure 6. Total system characteristics of the different pickup coil systems (calculated using equation 3). Red lines mark the $1/\sqrt{2}$ cutoff. To improve readability the phase axis does not include the 180° from the inversion during integration.

cuitry.

IV. SYSTEM CHARACTERISTICS AND POST-PROCESSING

Combining the transfer function of the coil with the transfer function of the integrator leads to the total system transfer function de-

rived in appendix A:

$$\frac{\widetilde{V}_{\text{out}}}{\widetilde{B}} = \frac{2i\omega A \exp(ikl)}{(1-\lambda)(1-\kappa) + (1+\kappa)(\lambda + \exp(2ikl))}$$

$$\cdot \frac{\alpha}{1+\frac{1}{i\omega\tau}} \qquad (3)$$

$$\lambda = Z^{-1} (R_{\text{coil}} + i\omega L_{\text{coil}})$$

$$\kappa = Z (R_{\text{meas}}^{-1} + i\omega C_{\text{meas}})$$

$$Z = \sqrt{\frac{L_{\text{cable}}}{C_{\text{cable}}} - \frac{i}{\omega} \frac{R_{\text{cable}}}{C_{\text{cable}}}}{l^2}}$$

$$k = \sqrt{\omega^2 \frac{L_{\text{cable}} C_{\text{cable}}}{l^2} - i\omega \frac{R_{\text{cable}} C_{\text{cable}}}{l^2}}{l^2}}$$

with the manipulator cable resistance $R_{\rm cable} = (2.3 \pm 0.1) \,\Omega$, the cable capacitance $C_{\rm cable} = (11 \pm 1) \, {\rm nF}$, its inductance $L_{\rm cable} =$ $(9.4 \pm 0.1) \,\mu H$ and the integrator's input impedance $R_{\text{meas}} = 10 \,\text{k}\Omega$. The length of the cable is implicitly captured in R, C and Land does not explicitly appear in either kl, κ or λ . As can be seen in figure 6, the system has good sensitivity up into the low kHz regime, which is more than sufficient for accurate integration of the magnetic field in the absence of external disturbance. The lowfrequency behavior is, however, slightly problematic since the average measurement duration lies around 8-10 seconds. While the system is still sensitive at these frequencies, the deviations from the ideal case do already distort the magnetic field measurements in the form of a hysteresis opposite to the measured magnetic field change. Figure 7 b shows such a hysteresis in form of a negative offset after the plunge.

To recover the correct field, two corrections are applied to the integrated signal:

- A linear compensation is added so that $V\left(t_{\text{start}}\right) = V\left(t_{\text{end}}\right) = 0$
- A corrected integrated voltage is defined as $V_{\mathrm{out,corr}} = \int_{t_{\mathrm{start}}}^{t} V_{\mathrm{coil}}$, estimating V_{coil} using equation 2. The backwards application of equation 2 results in a correction shown in equation 4.

$$V_{\text{out,corr}} = \int_{t_{\text{start}}}^{t} \alpha^{-1} \left(\frac{dV_{\text{out}}}{dt} - \tau^{-1} V_{\text{out}} \right)$$
 (4)

It may seem unintuitive to first put in the effort to introduce an analog integrator, only to differentiate the signal and then again apply numerical integration. However, this scheme retains the low susceptibility against intermediate drifts in the signal processing chain. Additionally, V_{out} only varies slowly between manipulator plunges (which does not apply for $V_{\rm in}$), and therefore can be accurately interpolated if the signal acquisition is disabled in-between. A further advantage is that the V_{out} component can be natively sampled at a low rate, while the unintegrated signal must either be sampled at a high rate or preprocessed using an analog low-pass filter. On the downside, while this correction scheme can be used to re-adjust frequency components of the order of τ^{-1} , at even lower frequencies it exceedingly amplifies small noise

integrator hardware's τ value for long-pulse operation.

An example of the signal correction is shown in figure 7 b. Besides a 10% reduction in peak height from 0.21 T to 0.19 T due to the correction of α from -10 to about -11, the correction also significantly reduces the short-term offset of the magnetic field after the plunge, and brings the time-trace more in line with the position measurement (compare figures 7 a and b).

MAGNETIC PROFILES

As can be seen in figure 7 b - d, the postcorrected integrator output (also referred to as "hardware integrated" in figure 7 c and d) shows a significantly improved stability against low-frequency drifts, compared to both a simple software integration with a linear drift compensation (referred to as "software integrated" in figure 7 c and d) and the uncorrected integrator signal shown in figure 7 b. An unusual feature in the signal is a post-plunge oscillation at about 6 Hz that decays over about 1s. This feature is caused by a small vertical oscillation of the manipulator arm after the plunge, which is also visible in the arm's acceleration sensor (figure 7 e, see $also^{20}$) after the plunge. The oscillation is not visible in all field components, requiring an improvement of the components (compare figure 7 c and d). For

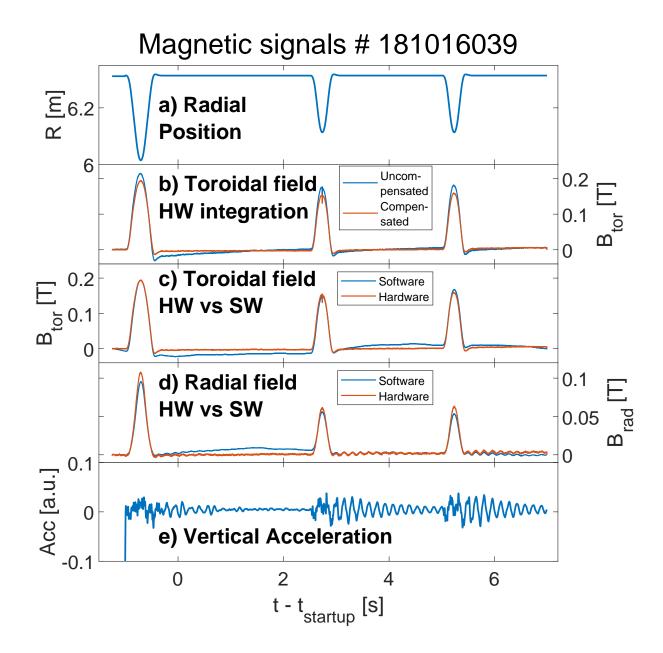


Figure 7. Time-traces of signals related to the magnetic probe measurement - a): Radial position of the magnetic probe, b): Comparison of hardware-integrated toroidal coil signal with and without post-compensation, c) Post-compensated hardware-integrated and software-integrated toroidal coil signal, d) Post-compensated hardware-integrated and software-integrated toroidal coil signal, e) Vertical acceleration measurement in the probe interface on the manipulator arm

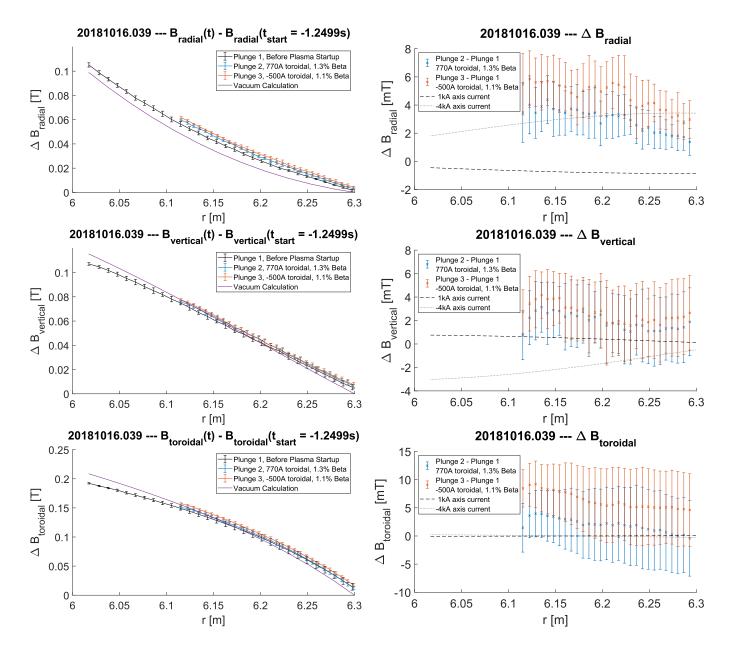


Figure 8. Measurements of magnetic field profiles (left) and plasma response (right)

profile derivation, the radial probe position the parking position. This overshoot is also is obtained using a laser-based distance mea-visible in the magnetic field measurement, surement system. As visible in figure 7 a, especially for the toroidal field profile (figure the manipulator arm does not immediately 7 b and c), which has a steeper gradient at retract to the starting position, but instead the edge. overshoots a little bit on the way back (likely due to inertia of the arm) before returning to

Figure 8 shows example radial profiles of

ideal CAD coil geometry. The radial profiles were obtained by binning the magnetic field time trace radially over the position signal, and using mean and standard deviation of the bin ensembles as value and error estimate respectively. This ensures that any differences between the inward and outward movement of the manipulator are appropriately captured in the error bars. To remove the influence of high-frequency MHD events, the magnetic signal was low-pass filtered before bin analysis. The cutoff frequency was varied between 50 Hz and 500 Hz without any significant variation of the profile or its errors. Toroidal current measurements were obtained from the W7-X Rogowski coil system, and peak plasma beta was estimated from Thomson scattering measurements for electron-density and -temperature measurements assuming $n_I = n_e$ for the ion density and $T_i = \min(T_e, 1.5 \text{ keV})$ for the ion temperature. Because the profiles are obtained by integration, they are only defined up to a constant. This constant is chosen so that the integrated signal is 0 T at the start of the measurement (which is before the the t = 0 s time of plasma startup).

The measured profiles do not agree perfectly with the expected profiles from a Biot-

all three magnetic field components, as well as theoretical field profile calculations obtained by applying the Biot-Savart law to the ideal CAD coil geometry. The radial profiles outer-most radius (which is the start- and were obtained by binning the magnetic field end-point of the plunge) in all components. This is probably a small low-frequency and using mean and standard deviation deviation that could not be fully corrected of the bin ensembles as value and error during the post-processing. The additional estimate respectively. This ensures that any slope disagreements in vertical and toroidal differences between the inward and outward direction could be error fields related to the movement of the manipulator are approprificately captured in the error bars. To remove the influence of high-frequency MHD events, not limited to this discharge, but can be the magnetic signal was low-pass filtered observed systematically during the whole before bin analysis. The cutoff frequency was

Since the first probe plunge was always performed before plasma startup, it can be used as an experimental vacuum reference for plasma response calculation. Error analysis estimates that the magnetic probe has a field resolution of about 4 mT in vertical and radial, and about 10 mT in toroidal direction (the difference is likely due to the stronger overall magnetic field change in toroidal direction). The radial measurement in particular is accurate enough to detect a significant (given the measurement uncertainties) deviation between the vacuum magnetic field and the magnetic field during the discharge. When comparing the measured magnetic field change to the magnetic field generated by a hypothetical current filament of VII.

1 kA on the magnetic axis, it can be seen,
that an on-axis current of about -4 kA would
be required to even poorly match the change
in edge magnetic field. Given that the total toroidal current (as measured by the Rogowski coil¹³) is far lower than that, this magnetic field change is most likely caused by
pressure-gradient-driven edge currents.

VI. SUMMARY

A coil system for magnetic profile measurements using the Multi-Purpose-Manipulator was successfully employed at Wendelstein 7-X. Characterization of the probe and the integrator showed an upperfrequency $1/\sqrt{2}$ cutoff of about 100 kHz and a lower cutoff of 50 mHz. The lower cutoff could be extended using a post-processing technique. The resulting profiles are accurate enough to detect the plasma response in a 10s discharge. While the recovery of the profile using post-processing (which amplifies any present low-frequency noise) is not ideal, we expect to reduce parasitic decay by multiple orders of magnitude in the next iteration of the design. Since the integrated signal good drift shows resilience under these aggravated conditions, we expect the stability to carry over into the next revision.

VII. ACKNOWLEDGEMENTS

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VIII. DATA AVAILABILITY

The data that support the findings of this study related to probe characterization are available from the corresponding author upon reasonable request. Raw data for the magnetic profile measurements were generated at the Wendelstein 7-X large scale facility. Derived data supporting the findings of this study are available from the corresponding author upon reasonable request.

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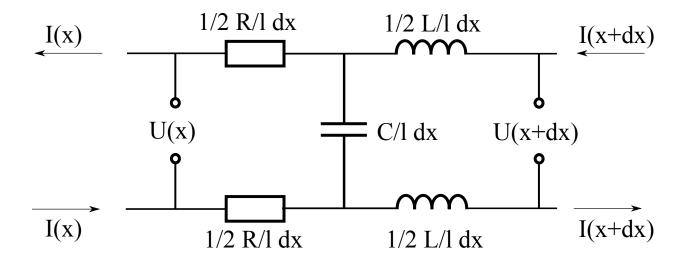


Figure 9. Schematic of the continuous cable model

Appendix A: Derivation of the full transfer model

Assuming the twisted wire pair to follow a standard differential LCR model, the differential signal propagation equations can be extracted from figure 9 as:

$$\frac{dU}{dx} = -\frac{R_{\text{cable}}}{l}I(x) - \frac{L_{\text{cable}}}{l}\frac{d}{dt}I(x)$$

$$\frac{dI}{dx} = -\frac{C_{\text{cable}}}{l}\frac{d}{dt}U(x)$$
(A1)

Using a propagating wave Ansatz $U, I(x,t) = U, I_{\pm} \exp(i(\omega t \pm kx))$ yields the wave dispersion relation

$$\pm ikU_{\pm} = -\frac{R_{\text{cable}}}{l}I_{\pm} - i\omega \frac{L_{\text{cable}}}{l}I_{\pm}$$

$$\pm ikI_{\pm} = -i\omega \frac{C_{\text{cable}}}{l}U_{\pm}$$

$$\Rightarrow$$

$$-k^{2} = i\omega \frac{R_{\text{cable}}}{l} \frac{C_{\text{cable}}}{l} - \omega^{2} \frac{L_{\text{cable}}}{l} \frac{C_{\text{cable}}}{l}$$

$$\Rightarrow$$

$$k(\omega) = \sqrt{\omega^{2} \frac{L_{\text{cable}}C_{\text{cable}}}{l^{2}} - i\omega \frac{R_{\text{cable}}C_{\text{cable}}}{l^{2}}}$$
(A2)

Assuming that the coil is connected at x = 0 and the integrator input at x = l, one gets:

$$\begin{split} \tilde{U}_{\text{coil}} &= U_{+} + U_{-} \\ \tilde{U}_{\text{input}} &= U_{+} \exp\left(ikl\right) + U_{-} \exp\left(-ikl\right) \\ \tilde{I}_{\text{coil}} &= \frac{\omega}{k} \frac{C_{\text{cable}}}{l} \left(U_{-} - U_{+}\right) \\ &= Z\left(\omega\right)^{-1} \left(U_{-} - U_{+}\right) \\ \tilde{I}_{\text{input}} &= Z\left(\omega\right)^{-1} \left(\exp\left(-ikl\right) U_{-} - \exp\left(ikl\right) U_{+}\right) \end{split}$$

with the frequency-dependent cable impedance

$$Z(\omega) = \left(\frac{\omega}{k} \frac{C_{\text{cable}}}{l}\right)^{-1}$$
$$= \sqrt{\frac{L_{\text{cable}}}{C_{\text{cable}}}} - \frac{i}{\omega} \frac{R_{\text{cable}}}{C_{\text{cable}}}$$

The coil follows a linear differential equation and can be subjected to the same rotating wave Ansatz

$$V_{\text{coil}} = \frac{d}{dt} \left[A_{\text{coil}} B - L_{\text{coil}} I_{\text{coil}} \right] - R_{\text{coil}} I_{\text{coil}}$$
$$\tilde{V}_{\text{coil}} = i\omega \left(A_{\text{coil}} \tilde{B} - L_{\text{coil}} \tilde{I}_{\text{coil}} \right) - R_{\text{coil}} \tilde{I}_{\text{coil}}$$
$$= i\omega A_{\text{coil}} \tilde{B} - \left(R_{\text{coil}} + i\omega L_{\text{coil}} \right) \tilde{I}_{\text{coil}}$$

while the integrator has a fixed input impedance of $R_{\text{meas}} = 10 \,\text{k}\Omega$, along with an optional capacitor C_{meas} used in characterization to determine the coil inductance.

$$\tilde{I}_{\mathrm{input}} = \left(R_{\mathrm{meas}}^{-1} + i\omega C_{\mathrm{meas}}\right)\tilde{U}_{\mathrm{input}}$$

Expanding for U_{\pm} yields:

$$U_{+} + U_{-} = i\omega A_{\text{coil}}\tilde{B} - \left(R_{\text{coil}} + i\omega L_{\text{coil}}\right)\frac{U_{-} - U_{+}}{Z}$$

$$\frac{\exp\left(-ikl\right)U_{-} - \exp\left(ikl\right)U_{+}}{Z} = \left(R_{\text{meas}}^{-1} + i\omega C_{\text{meas}}\right)\left[U_{+} \exp\left(ikl\right) + U_{-} \exp\left(-ikl\right)\right]$$

$$U_{-} = \frac{1 + Z \left(R_{\text{meas}}^{-1} + i\omega C_{\text{meas}}\right)}{1 - Z \left(R_{\text{meas}}^{-1} + i\omega C_{\text{meas}}\right)} \exp\left(2ikl\right) U_{+}$$

Let $\kappa = Z \left(R_{\text{meas}}^{-1} + i\omega C_{\text{meas}} \right)$ and $\lambda = Z^{-1} \left(R_{\text{coil}} + i\omega L_{\text{coil}} \right)$ be the normalized dimensionless output- and input-impedances. Then

$$\label{eq:energy_equation} \begin{split} \left[1 + \frac{1 + \kappa}{1 - \kappa} \exp{(2ikl)}\right] U_{+} &= i\omega A_{\mathrm{coil}} \tilde{B} + \lambda \left[1 - \frac{1 + \kappa}{1 - \kappa}\right] U_{+} \\ \Rightarrow \\ \left[1 - \lambda + \frac{1 + \kappa}{1 - \kappa} \left(\lambda + \exp{(2ikl)}\right)\right] U_{+} &= i\omega A_{\mathrm{coil}} \tilde{B} \\ U_{+} &= \frac{1}{1 - \lambda + \frac{1 + \kappa}{1 - \kappa} \left(\lambda + \exp{(2ikl)}\right)} i\omega A \tilde{B} \\ U_{-} &= \frac{\frac{1 + \kappa}{1 - \kappa} \exp{(2ikl)}}{1 - \lambda + \frac{1 + \kappa}{1 - \kappa} \left(\lambda + \exp{(2ikl)}\right)} i\omega A \tilde{B} \end{split}$$

This then gives an input voltage on the integrator of

$$\tilde{U}_{\text{input}} = \frac{\left(1 + \frac{1+\kappa}{1-\kappa}\right) \exp\left(ikl\right)}{1 - \lambda + \frac{1+\kappa}{1-\kappa}\left(\lambda + \exp\left(2ikl\right)\right)} i\omega A_{\text{coil}} \tilde{B}$$

$$= \frac{2 \exp\left(ikl\right)}{\left(1 - \lambda\right)\left(1 - \kappa\right) + \left(1 + \kappa\right)\left(\lambda + \exp\left(2ikl\right)\right)} i\omega A_{\text{coil}} \tilde{B} \tag{A3}$$

In the case of $kl \approx 0$ (neglecting forward and backward phase delays due to the cable), one can simplify this expression to:

$$\tilde{U}_{\text{input}} \approx \frac{2}{(1-\lambda)(1-\kappa) + (1+\kappa)(\lambda+1)} i\omega A_{\text{coil}} \tilde{B}$$

$$= \frac{i\omega A_{\text{coil}} \tilde{B}}{1+\lambda\kappa}$$

$$= \frac{i\omega A_{\text{coil}} \tilde{B}}{1+\frac{R_{\text{coil}}}{R_{\text{meas}}} + i\omega \left(R_{\text{coil}} C_{\text{meas}} + \frac{L_{\text{coil}}}{R_{\text{meas}}}\right) - \omega^2 L_{\text{coil}} C_{\text{meas}}}$$
(A4)