

# Supermassive gravitinos and giant primordial black holes

Krzysztof A. Meissner<sup>1</sup> and Hermann Nicolai<sup>2</sup>

<sup>1</sup>*Faculty of Physics, University of Warsaw  
Pasteura 5, 02-093 Warsaw, Poland*

<sup>2</sup>*Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut)  
Mühlenberg 1, D-14476 Potsdam, Germany*

We argue that the stable (color singlet) supermassive gravitinos proposed in our previous work can serve as seeds for giant primordial black holes. These seeds are hypothesized to start out as tightly bound states of fractionally charged gravitinos in the radiation dominated era, whose formation is supported by the universally attractive combination of gravitational and electric forces between the gravitinos and anti-gravitinos (reflecting their ‘almost BPS-like’ nature). When lumps of such bound states coalesce and undergo gravitational collapse, the resulting mini-black holes can escape Hawking evaporation if the radiation temperature exceeds the Hawking temperature. Subsequently the black holes evolve according to an exact solution of Einstein’s equations, to emerge as macroscopic black holes in the transition to the matter dominated era, with masses on the order of the solar mass or larger. The presence of these seeds at such an early time provides ample time for further accretion of matter and radiation, and would imply the existence of black holes of almost any size in the universe, up to the observed maximum.

## I. INTRODUCTION.

The origin of large (galactic) black holes, present already in the early Universe has been a long standing puzzle, see *e.g.* [1] for information on the most recently discovered behemoth black hole, [2] for a generally accessible update and overview, and [3–5] and references therein for more recent work. It seems generally agreed that such large black holes cannot form by the usual stellar processes (*i.e.* gravitational collapse of stars and subsequent accretion of mass), but must have originated from some other source. One possible explanation is that black holes were already present from the very beginning of the matter dominated period, and in sufficient numbers and with sufficiently large masses to be able to grow further by accretion to very large sizes already a few hundred million years after the Big Bang. Various mechanisms along these and other lines have been proposed towards solving this problem, see *e.g.* [3–5] and references therein. In addition the crucial question must be answered whether an explanation can be found in terms of known physics, or whether such an explanation necessarily involves essentially new physics.

In this paper we present a new proposal towards addressing this problem which relies on the conjectured existence of certain supermassive particles that allow for the formation of black holes already during the early radiation phase, well before decoupling. There are two necessary prerequisites for such a mechanism to work, namely

1. the supermassive particles must be absolutely stable against decay into Standard Model matter; and
2. they must be subject to sufficiently strong attractive forces to enable them to rapidly cluster in sufficient amounts to undergo gravitational collapse.

Although ansätze towards fundamental physics, in particular Kaluza-Klein theory and string theory, abound in massive excitations that might serve as candidates for such a scenario, such excitations usually fail to meet the first requirement (with decay lifetimes on the order of the Planck time  $t_{\text{Pl}}$ ), which is why they are often assumed to play no prominent role in the cosmology of the very early universe. Here we will argue that, by contrast, the superheavy gravitinos proposed in our previous work [6, 8] can meet both requirements. That the requisite particles should be gravitinos, rather than some other particle species, is perhaps unusual, so let us first explain the reasons for this claim.

Our proposal has its origin in our earlier attempt to understand the observed spin- $\frac{1}{2}$  fermion content of the Standard Model, with three generations of quarks and leptons (including three right-chiral neutrinos). It relies on a unification scenario based on a still hypothetical extension of maximally extended  $N = 8$  supergravity involving the infinite-dimensional duality symmetries  $E_{10}$  and  $K(E_{10})$  [6–8] (this proposal itself has its origins in much earlier work [9, 10]). The enlargement of the known duality symmetries of supergravity and M theory to the infinite-dimensional symmetries  $E_{10}$  and  $K(E_{10})$  is absolutely essential here, because without this extension neither the charge assignments of the quarks and leptons, nor those of the gravitinos in (1) below could possibly work, and stability of the gravitinos against decay could not be achieved. A key feature of our proposal, and one that sets it apart from all other unification schemes,

is that besides the 48 spin- $\frac{1}{2}$  fermions of the Standard Model, the *only* other fermions are the eight supermassive gravitinos corresponding to the spin- $\frac{3}{2}$  states of the  $N = 8$  supermultiplet. It is thus a *prediction* that the spin- $\frac{1}{2}$  fermion content of the Standard Model will remain unaltered up to the Planck scale – a prediction that is (at least so far) supported by the absence of any signs of new physics from LHC, and by the fact that the currently known Standard Model couplings can be consistently evolved all the way to the Planck scale. Indeed, the detection of any new fundamental spin- $\frac{1}{2}$  degree of freedom (such as a sterile fourth neutrino, or a fourth generation of quarks and leptons, or any of the ‘*ino*’ fermions predicted by low energy supersymmetry) would immediately falsify the present scheme.

Evidence for infinite-dimensional duality symmetries of Kac-Moody type comes from an earlier BKL-type analysis of cosmological singularities in general relativity [11, 12]. This has led to the conjecture that M theory in the ‘near singularity limit’ is governed by the dynamics of an  $E_{10}/K(E_{10})$  non-linear  $\sigma$ -model [13]. In this scenario space-time, and with it space-time based quantum field theory and space-time symmetries would have to be emergent, in the sense that all the relevant information about space-time physics gets encoded in and ‘spread over’ a hugely infinite-dimensional hyperbolic Kac-Moody algebra. In particular, this scheme goes *beyond* supergravity in that the infinite-dimensional  $E_{10}$  duality symmetry replaces, and quite possibly disposes of, supersymmetry as a guiding principle towards unification. The fermionic sector of the theory is then governed by the ‘maximal compact’ (or more correctly, ‘involutory’) subgroup  $K(E_{10}) \subset E_{10}$ , which can be regarded as an infinite-dimensional generalization of the usual R-symmetries of extended supergravity theories. While an analysis of the bosonic sector of the  $E_{10}/K(E_{10})$  model and its dynamics beyond the very first few levels is severely hampered by the fact that a full understanding of  $E_{10}$  remains out of reach, a remarkable property of its involutory subgroup  $K(E_{10})$  is the existence of *finite*-dimensional (unfaithful) spinorial representations [14–16]. The combined spin- $\frac{1}{2}$  and spin- $\frac{3}{2}$  fermionic degrees of freedom at any given spatial point are then no longer viewed as fermionic members of the  $N = 8$  supermultiplet, but rather as belonging to an (unfaithful) irreducible representation of the generalized R-symmetry  $K(E_{10})$  [14–16]. The link with the physical fermion states is then made by identifying the known  $K(E_{10})$  representation with the Standard Model fermions at a given spatial point, in the spirit of a BKL-type expansion in spatial gradients, as explained for the bosonic sector in [13].

A crucial feature is now that the gravitinos are predicted to participate in strong and electromagnetic interactions (unlike the sterile gravitinos of MSSM-like models with low energy supersymmetry), and that they carry fractional charges. More precisely, as a consequence of the group theoretic analysis in [6–8], the eight massive gravitinos are assigned to the following representations of the residual unbroken  $SU(3)_c \times U(1)_{em}$  symmetry

$$\left(\mathbf{3}_c, \frac{1}{3}\right) \oplus \left(\bar{\mathbf{3}}_c, -\frac{1}{3}\right) \oplus \left(\mathbf{1}_c, \frac{2}{3}\right) \oplus \left(\mathbf{1}_c, -\frac{2}{3}\right) \quad (1)$$

These assignments follow from an  $SU(3) \times U(1) \subset SO(8)$  decomposition of the  $N = 8$  supergravity gravitinos, *except* for the ‘spurion’ shift of the  $U(1)$  charges by  $\pm\frac{1}{6}$  that was originally introduced in [9] for the spin- $\frac{1}{2}$  members of the  $N = 8$  supermultiplet, in order to make their electric charge assignments agree with those of three generations of quarks and leptons (including right-chiral neutrinos). As shown in [6–8], it is this latter shift which requires enlarging the R-symmetry to  $K(E_{10})$ , and which takes the construction *beyond*  $N = 8$  supergravity and *beyond* the confines of space-time based field theory. All gravitinos are assumed to be superheavy, with masses just below the Planck mass. This assumption is plausible because in any scheme avoiding low energy supersymmetry and in the absence of grand unification the Planck scale is the natural scale for symmetry breaking. Despite their large mass *all gravitinos are stable against decays into Standard Model matter*, as a consequence of their peculiar quantum numbers: there is simply no final state in the Standard Model into which they could possibly decay in compliance with (1) and the residual unbroken  $SU(3)_c \times U(1)_{em}$  symmetry. This feature is essentially tied to the replacement of the usual R-symmetry by  $K(E_{10})$ , because in a standard supergravity context a supermassive gravitino would not be protected against decay into other particles.

In the present paper we take a more pragmatic approach by simply proceeding with the assignments (1) as the starting point, but keeping in mind that this scheme is strongly motivated by unification and a possible explanation of the observed pattern of quark and lepton charge quantum numbers, and thus not based on *ad hoc* choices. In [17, 18] we have already begun to explore the possible astrophysical implications of supermassive gravitinos with the above assignments. More specifically, in [17] we have proposed the color singlet gravitinos as novel dark matter candidates, and discussed possible avenues to search for them. In subsequent work [18] we showed that the color triplet states in (1) can potentially explain the observed ultra-high energy cosmic ray events with energies of up to  $10^{21}$  eV via gravitino anti-gravitino annihilation in the crust of neutron stars. In this paper we now turn our attention again to the *color singlet* gravitinos of charge  $\pm\frac{2}{3}$ , to argue that they can in addition play a key role in shedding light on the origin of giant black holes in the early universe.

The structure of this paper, then is as follows: in section II we show that quantum mechanically the wave function of a multi-gravitino bound state is highly unstable against gravitational collapse. In the following two sections we

study the formation and evolution of mini-black holes during the radiation era, also deriving numerical estimates. For the evolution we employ a generalization of the McVittie solution (on which there is already an ample literature, see *e.g.* [26–32] and references therein). In the last section we analyze the energy-momentum tensor for this solution, and show that it has the right form expected for a radiation dominated universe. We also argue that the ‘blanket’ surrounding the primordial black hole can further enhance the growth of massive black holes. These last two sections may be of interest in their own right, independently of the main line of development of this paper.

## II. FORMATION OF MULTI-GRAVITINO BOUND STATES

The main new feature of our proposal is that, as a result of the assumed large mass of the gravitinos, the combined gravitational and electric forces between any arrangement of gravitinos and anti-gravitinos is *universally attractive*. In natural units we define the BPS-mass  $M_{\text{BPS}}$  to be the one for which the electrostatic force between two elementary charges equals their gravitational attraction (modulo sign)

$$e^2 = GM_{\text{BPS}}^2; \quad (2)$$

we refer to  $M_{\text{BPS}}$  as the ‘BPS-mass’ because it is the one relevant for extremal Reissner-Nordström or Kerr-Newman solutions. This equality is written in units where  $4\pi\epsilon_0 = \mu_0/(4\pi) = c = 1$  (here it is worthwhile to recall that these units, with the addition of  $e = M_{\text{BPS}} = 1$ , were introduced already in 1881 by George Stoney, probably the first physicist who seriously contemplated quantization of charge [19]; the electron was discovered only 16 years later, while Planck units were introduced 18 years later). As is well known,  $M_{\text{BPS}}$  is *not* the same as the Planck mass  $M_{\text{Pl}}$ , but differs from it by a factor of the fine structure constant  $\alpha_{em}$  (always with  $c = 1$  from now on):

$$M_{\text{BPS}}^2 = \frac{e^2}{G} = \frac{e^2}{\hbar} \cdot \frac{\hbar}{G} \equiv \alpha_{em} M_{\text{Pl}}^2. \quad (3)$$

We will assume that the gravitino mass lies between these two values, *i.e.*

$$M_{\text{BPS}} < M_g < M_{\text{Pl}} \quad (4)$$

Then we can write for the gravitino charges

$$Q_g = \pm \frac{2}{3} e = \pm \beta G^{\frac{1}{2}} M_g \quad (5)$$

with the ‘BPS-parameter’  $\beta$  obeying  $0 < \beta < \frac{2}{3}$ ; we will denote the (fixed) gravitino mass by  $M_g$  throughout this paper, whereas generic black hole masses will be designated by the letter  $m$ , where  $m$  can also vary with time. The total force between two (anti-)gravitinos is thus determined by the combined electric and gravitational charges  $(1 \pm \beta^2)GM_g^2 > 0$ , so that even for like charges the force remains attractive because the gravitational attraction overwhelms the electrostatic repulsion (reflecting the ‘almost BPS-like’ nature of the gravitinos). In this paper we hypothesize that it is this universal attraction that leads to the formation of multi-gravitino bound states inside the plasma of the radiation dominated phase, starting from small inhomogeneities in analogy with cluster formation of galaxies. The main difference with the latter is that, prior to gravitational collapse, we are here initially dealing with a *quantum mechanical* bound state, not one that can be understood in terms of Newtonian physics. For two gravitinos the bound state would be somewhat analogous to positronium, however with the crucial difference that ‘gravitinium’ can be a longer lived state because the annihilation cross section between two oppositely charged gravitinos is of the order  $\sim \ell_{\text{Pl}}^2$  (but note that in principle positronium can also be long lived, provided the bound state is formed in a state of very large radial quantum number [21], see below).

We wish to study the formation of bound states of gravitinos during the radiation era in the very early universe. For a proper analysis, and as a first step, we would now have to go through a first quantized analysis of the massive Rarita-Schwinger equation in such a homogeneously and isotropically expanding background. This task is substantially simplified by our main assumption (4) which allows us to resort to the non-relativistic limit, and by the fact that this inequality also implies

$$M_g > H(t) \quad (6)$$

for the Hubble parameter during the radiation era, whence we can also drop the usual friction term  $\propto H(t) = \dot{a}(t)/a(t)$  that would normally have to be included in the equation of motion. It is therefore enough to consider the free Rarita-Schwinger equation for a massive spin- $\frac{3}{2}$  complex vector spinor, which reads

$$i\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho + M_g\gamma^{\mu\nu}\psi_\nu = 0 \quad (7)$$

From this one immediately deduces the Dirac and constraint equations

$$(i\gamma^\lambda \partial_\lambda - M_g)\psi_\mu = 0 \quad \text{and} \quad \gamma^\mu \psi_\mu = \partial^\mu \psi_\mu = 0 \quad (8)$$

(see *e.g.* [20] for a more complete account). The latter two equations imply a halving of the available degrees of freedom, and tell us that the vector spinor  $\psi_\mu$  carries altogether four helicity degrees of freedom, with labels  $\sigma, \tau \in \{\pm\frac{1}{2}, \pm\frac{3}{2}\}$  for both gravitino and anti-gravitino. The relevant expansion reads

$$\begin{aligned} \psi_\mu(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E(\mathbf{p})}} & \left[ e^{ipx} f_\mu^+(p) u_+(p) + e^{ipx} f_\mu^-(p) u_-(p) + \right. \\ & \left. + e^{-ipx} g_\mu^+(p) v_+(p) + e^{-ipx} g_\mu^-(p) v_-(p) \right] \end{aligned} \quad (9)$$

where, of course,  $p^2 + M_g^2 = 0$ , and  $u_\pm(p)$  and  $v_\pm(p)$  are the two positive and negative energy solutions of the Dirac equation. The last constraint equations is solved by

$$f_\mu^\pm(p) = \sum_i b_i^\pm(p) \varepsilon_\mu^i(p) \quad , \quad g_\mu^\pm(p) = \sum_i d_i^\pm(p) \varepsilon_\mu^i(p) \quad (10)$$

with the three linearly independent polarization vectors  $\varepsilon_\mu^i(p)$  satisfying  $p^\mu \varepsilon_\mu^i(p) = 0$ . For the other constraint equation we need to impose

$$\begin{aligned} \sum_i \gamma^\mu \varepsilon_\mu^i(p) \left[ b_i^+(p) u_+(p) + b_i^-(p) u_-(p) \right] & \stackrel{!}{=} 0 \\ \sum_i \gamma^\mu \varepsilon_\mu^i(p) \left[ d_i^+(p) v_+(p) + d_i^-(p) v_-(p) \right] & \stackrel{!}{=} 0 \end{aligned} \quad (11)$$

thus eliminating four out of the 12 free coefficients  $b_i^\pm(p)$  and  $d_i^\pm(p)$ , respectively, leaving us with four helicity wave functions for gravitino and anti-gravitino each. As the spin interactions are not relevant for our approximation there is no need here to be any more specific about the parametrization of the helicity wave functions. However, each gravitino degree of freedom is exposed to the gravitational and electric background generated by the other gravitinos (as well as the surrounding plasma which we can neglect). In order to incorporate these interactions in lowest order, one performs the standard Foldy-Wouthuysen transformation on each component of  $\psi_\mu$ , which yields a non-relativistic one-particle Hamiltonian for each gravitino component.

The corresponding multi-particle Schrödinger Hamiltonian therefore reads

$$H = -\frac{\hbar^2}{2M_g} \sum_i (\Delta_{\mathbf{x}_i} + \Delta_{\mathbf{y}_i}) + V(\mathbf{x}, \mathbf{y}) \quad (12)$$

with the universally attractive potential

$$V(\mathbf{x}, \mathbf{y}) = -(1 - \beta^2) \left( \sum_{i \neq j} \frac{GM_g^2}{|\mathbf{x}_i - \mathbf{x}_j|} + \sum_{i \neq j} \frac{GM_g^2}{|\mathbf{y}_i - \mathbf{y}_j|} \right) - (1 + \beta^2) \sum_{i,j} \frac{GM_g^2}{|\mathbf{x}_i - \mathbf{y}_j|} \quad (13)$$

where the positions of the gravitinos and anti-gravitinos are designated by  $\mathbf{x}_i$  and  $\mathbf{y}_j$ , respectively. This Hamiltonian acts on a fermionic wave function  $\Psi(\mathbf{x}_1, \sigma_1, \dots, \mathbf{x}_n, \sigma_p; \mathbf{y}_1, \tau_1, \dots, \mathbf{y}_q, \tau_q)$  which is antisymmetric under simultaneous interchange of the position and spin labels of the gravitinos and anti-gravitinos, respectively. In writing this Hamiltonian we have also neglected the fluctuating external electric and magnetic fields in the radiation plasma. Likewise, as we already explained, we ignore subleading spin-orbit and spin-spin interactions that would follow from the Rarita-Schwinger equation in a fully relativistic treatment (and which would be very complicated). Finally, we can neglect the effect of the protons and electrons from the surrounding plasma (as well as all other Standard Model particles): for them, the gravitational interactions are governed by the factors  $GM_g m_e, GM_g m_p \dots \ll GM_g^2$ , whence their interactions are completely dominated by the purely electromagnetic forces. The latter are, however, screened out because of the overall electric neutrality of the plasma, and can thus be ignored.

Evidently the above considerations only apply to superheavy particles obeying (4) and (6), and would not make any sense at all for ordinary (Standard Model) particles. For the latter all masses and binding energies are far below the temperature of the surrounding plasma, that is  $m_e, m_p, \dots \ll T_{rad}$ , and also below the Hubble parameter,

$m_e, m_p, \dots \ll H$ . In that case, the stationary Schrödinger equation would have to be replaced by a relativistic equation in a time-dependent background, and the friction term involving the Hubble parameter  $H$  would lead to immediate decay of the wave function (as unitarity in the naive sense is violated in a time-dependent background).

We will not attempt here to investigate in any detail the multi-particle Schrödinger equation based on (12), which would amount to a quantum analog of the computations performed in connection with galaxy structure formation. Nevertheless, we can still make some rigorous statements relying on well known estimates (see *e.g.* [22]). Namely, it is a rigorous result [23] that for a system of fermions (that is, particles obeying the Pauli principle with a fully antisymmetric wave function) the lowest energy eigenvalue of the  $N$ -particle Hamiltonian

$$E_0(N) := \inf_{\|\Psi\|=1} \langle \Psi | H | \Psi \rangle \quad (14)$$

(where  $N$  is the combined number of gravitinos and anti-gravitinos) is subject to the upper and lower bounds

$$-AN(N-1)^{\frac{4}{3}}G^2M_g^5\hbar^{-2} \leq E_0(N) \leq -BN^{\frac{1}{3}}(N-1)^2G^2M_g^5\hbar^{-2} \quad (15)$$

with strictly positive constants  $A > B > 0$ . Consequently the lowest energy per particle  $E_0(N)/N$  decreases as  $\propto -N^{4/3}$  with  $N$ , signaling an instability. For a bosonic wave function the fall-off would be even faster with  $E_0(N)/N \propto -N^2$  [23]. Therefore the inclusion of spin degrees of freedom (where one combines a partially symmetric wave function in the space coordinates with an anti-symmetric wave function in spin space) cannot improve the situation. The estimate (15) tells us that the system is unstable, and for sufficiently large  $N$  will thus undergo gravitational collapse, as the fermionic degeneracy pressure is not enough to sustain the system in a stable equilibrium. Because of (6) the basic instability estimate (15) is not affected by the cosmological expansion either.

Now if we consider a bound state of just two gravitinos (a hydrogen-like system) the associated ‘Bohr radius’ is only a few orders of magnitude away from the Planck length, to wit

$$a_B \sim \frac{\hbar^2}{GM_g^3} \sim \mathcal{O}(10^3) \ell_{\text{Pl}} \quad (16)$$

which is not too far from the Schwarzschild radius. If the formation of such bound states took place in vacuum, and the relaxation to the ground state proceeded too fast, the resulting mini-black holes would immediately evaporate by Hawking radiation according to the well known formula (see *e.g.* [24])

$$t_{\text{evap}} \sim t_{\text{Pl}} \left( \frac{m}{M_{\text{Pl}}} \right)^3 \sim 10^{-40} \text{ s} \left( \frac{m}{10^{-8} \text{ kg}} \right)^3 \quad (17)$$

In order to prevent this from happening, and in order to create bigger black holes that can survive for longer and start growing, it is therefore necessary for the bound states to persist long enough to accrete a sufficiently large number of gravitinos *before* gravitational collapse. Meta-stability can be ensured if the initial energy of the bound state is much larger than  $E_0(N)$ , and consequently its overall extension stays well above its Schwarzschild radius for sufficiently long time. Of course, the bound state will eventually relax to lower lying bound states by the spontaneous emission of photons and gravitons, but this process will take some time. For instance, for positronium the average relaxation time for a bound state to settle into the ground state is given as a function of the initial quantum numbers  $n$  and  $l$  by the formula [25]

$$t_{\text{relax}} \sim 2m_e^{-1}\alpha_{em}^{-5}n^3l^5(n+l)^{-2} \sim m_e^{-1}\alpha_{em}^{-5}n^6 \quad (18)$$

(after which the positronium state of course disappears quickly by annihilation). Extrapolating this formula to the present case suggests that, with sufficiently large  $n$  at the time of formation, we can get lifetimes long enough to bind a large number of gravitinos into a meta-stable configuration before the collapse can occur. We also note that at this stage (that is, prior to the formation of a black hole) the absorption of protons and electrons from the ambient plasma plays no role, as these particles, unlike the gravitinos, will be only very weakly bound.

### III. COLLAPSE OF GRAVITINO LUMPS AND MINI-BLACK HOLES

At this point we have lumps, each corresponding to a quantum mechanical multi-gravitino bound state, which are scattered throughout the radiation plasma. Because of the density fluctuations and inhomogeneities in the plasma, and as a result of their strong gravitational attraction these lumps will eventually coalesce before collapsing into small black holes, a microscopic analogue of the clumping of dust into galaxies and stellar matter. In a first approximation

the ensemble of massive lumps can be treated classically (*i.e.* need not be considered as a single coherent wave function). In order to arrive at a rough estimate of the initial mass of the resulting black holes we first estimate the total number of gravitinos contained in a coalesced lump of gravitino matter. Treating them classically with an average kinetic energy per particle equal to the temperature of the plasma we have

$$\langle E_{kin}(t) \rangle \sim NT_{rad}(t) = NT_{eq} \left( \frac{t_{eq}}{t} \right)^{1/2} \quad (19)$$

where  $T_{eq} \sim 1 \text{ eV}$  and  $t_{eq} \sim 40\,000 \text{ yr} \sim 10^{12} \text{ s}$  (we find it convenient to refer all quantities to equilibrium time  $t_{eq}$  rather than Planck units). The potential energy of  $N$  gravitinos and anti-gravitinos is given by

$$\langle E_{pot}(t) \rangle \sim -N^2 \frac{GM_g^2}{\langle d(t) \rangle} \quad (20)$$

where for numerical estimates we take  $M \sim M_{\text{BPS}}$ . The average separation  $\langle d(t) \rangle$  between two gravitinos or anti-gravitinos at time  $t$  is given by

$$\langle d(t) \rangle \sim \left( \frac{M_g}{\rho(t)} \right)^{1/3} \sim (10^2 \text{ m}) \left( \frac{t}{t_{eq}} \right)^{1/2} \quad (21)$$

where we estimate the density  $\rho(t)$  at time  $t$  by scaling back the known density at the equilibrium time (with  $8\pi G\rho_{rad} = 8\pi G\rho_{mat} \sim 4 \cdot 10^{-25} \text{ s}^{-2}$ ), keeping in mind that matter density scales as  $a(t)^{-3}$  also during the radiation era (while the radiation density scales as  $a(t)^{-4}$ ). Importantly we here assume that, at the time of decoupling, most of the matter consists of supermassive gravitinos, in line with our previous dark matter proposal [17].

Gravitational collapse is expected to occur if the total energy is negative:

$$\langle E_{kin}(t) \rangle + \langle E_{pot}(t) \rangle < 0 \quad \Rightarrow \quad N > \frac{T_{eq} \cdot 10^2 \text{ m}}{GM_g^2} \sim 10^{12} \quad (22)$$

(note that the time  $t$  drops out of this relation). Let us stress that this is only a very rough estimate: if the bound state is meta-stable, the collapse can be delayed in such a way that a larger number of (anti-)gravitinos can be accrued. With (22) the mass of the resulting mini-black hole comes out to be

$$m_{initial} \sim 10^{12} M_{\text{BPS}} \sim 10^3 \text{ kg} \quad (23)$$

By formula (17) the Hawking evaporation time for a black hole of this mass would be

$$t_{evap}(m_{initial}) \sim 10^{-7} \text{ s} \quad (24)$$

However, it is important now that Hawking evaporation is not the only process that must be taken into account. There is a competing process which can in fact stabilize the mini-black holes and their further evolution: it is the presence of the dense and hot plasma surrounding the black hole that can feed the growth of small black holes. More precisely, Hawking evaporation competes with accretion according to the following equation:

$$\frac{dm(t)}{dt} = C_0 G^2 \rho(t) \cdot m^2(t) - C_1 \frac{M_{\text{Pl}}^3}{t_{\text{Pl}}} \cdot \frac{1}{m^2(t)} \quad (25)$$

where  $C_0$  and  $C_1$  are constants of  $\mathcal{O}(1)$ . The first term on the r.h.s. originates from the flux of the infalling radiation from the surrounding plasma, which is  $\propto 4\pi R^2(t)\rho(t)c$  (with  $c = 1$ ) for a (time dependent) black hole of radius  $R(t) = 2Gm(t)$  (a ‘fudge factor’  $C_0 = \mathcal{O}(4\pi)$  can be included to account for the fact that not all the surrounding radiation falls in radially, but this is not essential). The second term in (25) governs Hawking evaporation, and follows from the Stefan-Boltzmann law upon substitution of the Hawking temperature

$$T_{\text{Hawking}} = \frac{\hbar}{8\pi Gm} \quad (26)$$

For Hawking evaporation taking place in empty space we can ignore the first term on the r.h.s. of (25), and formula (17) follows directly. In that case any microscopic black hole would disappear, and not be able to grow into a macroscopic black hole. The crucial difference with this standard scenario is embodied in the first term on the r.h.s. of (25) (which is usually disregarded in discussions of Hawking evaporation). This term takes into account the fact

that the decay takes place in an extremely hot surrounding plasma whose density varies with time as  $\rho(t) = 3/4t^2$ . At the initially extremely high temperatures of the radiation era the accretion can thus out-compete Hawking evaporation *even for very small black holes*. More precisely, with the breakeven point at  $T_{rad} = T_{Hawking}$  the simple criterion for black hole accretion to overcome the rate for Hawking radiation reads

$$T_{rad} > T_{Hawking} = \frac{\hbar}{8\pi Gm} \quad (27)$$

where the radiation temperature  $T_{rad}(t)$  at time  $t$  can be read off from (19): for instance, the temperature corresponding to (24) would be  $T_{rad} \sim 3 \times 10^3$  GeV. This inequality is easy to achieve in the initially very dense and hot plasma where  $T_{rad} \sim 10^{17}$  GeV; later, with progressing time and decreasing temperature, the initial mass has to be correspondingly larger. Indeed, there is a delicate balance because from (25) it follows that  $m(t)$  can run away in either direction. In practice, this means that the earlier the mini-black hole is formed, the smaller  $N$ , and thus  $m_{initial}$  can be than the values given in (22) and (23); by contrast, if it is formed later,  $N$  has to be correspondingly larger than  $10^{12}$ . From these considerations we can also derive an upper bound on the formation time, after which the radiation temperature is too low to stabilize mini-black holes of mass (23) against Hawking evaporation; the latest time is

$$t < t_{max} = 10^{-20} \text{ s} \quad (28)$$

Mini-black holes formed after this time will decay by Hawking radiation because  $T_{rad}(t) < T_{Hawking}$  for  $t > t_{max}$ . In summary, the usual argument that small black holes would quickly decay via (17) no longer applies as long as the inequality (27) is obeyed.

Note that we invoke the ‘empirical’ formula (25) mainly to argue that mini-black holes can form in such a way as to remain stable against Hawking evaporation at early times. In fact, this reasoning can be made more precise by substituting  $\rho = \frac{3}{32\pi G}t^{-2}$  into (25) which turns this equation into a simple differential equation that can be studied numerically. However, because this formula is only approximate, and once the stability of the mini-black hole is ensured, we can switch to a classical description by means of an exact solution of Einstein’s equations describing a Schwarzschild black hole in a radiatively expanding universe, to describe its further evolution. This will be explained in the next section.

#### IV. GROWTH OF BLACK HOLES IN RADIATION DOMINATED UNIVERSE

Having motivated the assumption that small black holes stable against Hawking evaporation have formed in sufficient numbers early in the radiation dominated era we can proceed to study their evolution in this background. For this purpose we employ an *exact* solution of the Einstein equations, rather than the ‘phenomenological’ formula (25). This solution can be regarded as a variant of the so-called McVittie solution [26]; for more recent literature, see *e.g.* [27–32] and references therein. The solution that we require here is conveniently presented in terms of conformal coordinates, by starting from the general ansatz

$$ds^2 = a(\eta)^2 \left[ -C(\eta, r)d\eta^2 + \frac{dr^2}{C(\eta, r)} + r^2 d\Omega^2 \right] \quad (29)$$

where  $\eta$  is conformal time, which we use from now on as the time coordinate.  $a(\eta)$  is the scale factor and  $C = C(\eta, r)$  some function to be specified. We will discuss the equations for the general ansatz elsewhere, but for the present purposes it is enough to restrict to the special case, where  $C$  depends only on the radial coordinate, *i.e.*  $C(\eta, r) \equiv C(r)$ . Furthermore, since we are here mainly interested in perfect fluids, for which  $a(t) \sim t^{2/3(w+1)} \sim \eta^{2/(3w+1)}$ , and more specifically, a radiation dominated universe, we right away specialize the scale factor to be

$$a(\eta) = A\eta \quad \iff \quad t = \frac{1}{2}A\eta^2. \quad (30)$$

where in our Universe  $A \sim 4 \cdot 10^{-5} \text{ s}^{-1}$  (while  $a(\eta)$  is dimensionless). With these assumptions it is straightforward to compute the non-vanishing components of the Einstein tensor, and hence the components of the energy-momentum tensor, with the result

$$\begin{aligned} 8\pi G T_{tt}(\eta, r) &= -\frac{1}{\eta^2 r^2} \left( C(r)C'(r)r\eta^2 + C^2(r)\eta^2 - C(r)\eta^2 - 3r^2 \right) \\ 8\pi G T_{tr}(\eta, r) &= \frac{C'(r)}{\eta C(r)} \end{aligned}$$

$$\begin{aligned}
8\pi G T_{rr}(\eta, r) &= \frac{1}{C(r)^2 r^2 \eta^2} \left( C(r) C'(r) r \eta^2 + C(r)^2 \eta^2 - C(r) \eta^2 + r^2 \right) \\
8\pi G T_{\theta\theta}(\eta, r) &= \frac{1}{2C(r)\eta^2} \left( C(r) C''(r) r^2 \eta^2 + 2C(r) C'(r) r \eta^2 + 2r^2 \right) \\
8\pi G T_{\varphi\varphi}(\eta, r) &= 8\pi G \sin^2\theta T_{\theta\theta}(\eta, r)
\end{aligned} \tag{31}$$

where, of course,  $C'(r) \equiv dC(r)/dr$ , *etc.* At this point, this is just an identity (the so-called ‘Synge trick’ [33]); in fact, such solutions trivially exist for *any* profile of the scale factor  $a(\eta)$ . The non-trivial part of the exercise is therefore in ascertaining that the energy-momentum tensor resulting from this calculation does make sense *physically*. The requisite condition for a radiation dominated universe, stated in the most general and coordinate independent way, is the vanishing of the trace of the energy-momentum tensor, *viz.*

$$T^\mu{}_\mu(\eta, r) = \frac{1}{A^2 \eta^2 r^2} \left[ \frac{d^2}{dr^2} (r^2 C(r)) - 2 \right] \stackrel{!}{=} 0 \tag{32}$$

This condition is solved by

$$C(r) = 1 - \frac{2Gm}{r} + \frac{GQ^2}{r^2} \tag{33}$$

with two integration constants  $m$  (mass) and  $Q$  (charge). Remarkably, the metric (29) comes out to be conformal to the Reissner-Nordström metric not as a result of imposing the Einstein equations with an electromagnetic point charge source on the r.h.s., but with the weaker and more general conformality constraint (32)! Taking  $Q = 0$  for simplicity (and also because we do not expect these black holes to carry significant amounts of electrical charge), the resulting solution describes the exterior region ( $r > 2Gm$ ) of a Schwarzschild black hole in a radiation dominated universe. We emphasize that there is absolutely no issue with the causal structure of this solution, because the conformal equivalence ensures that (for  $\eta > 0$ ) the global structure of the space-time outside the would-be horizon  $r = 2Gm$  is the same as for the Schwarzschild solution, and the tracelessness of the energy momentum tensor holds right up to the would-be horizon (the black hole interpretation is also supported by the arguments in [30, 31]). However, there are some subtleties (apart from issues related to de Sitter space and cosmological horizons discussed in [27–31], which are of no concern here) which have to do with the structure of the energy-momentum tensor. Namely, as we show in the following section, closer inspection reveals the existence of an apparent ‘superluminal barrier’ surrounding the surface  $r = 2Gm$ , and shielding the would-be horizon from the outside observer.

For the physical mass of the black hole we take the formula

$$\frac{1}{2\pi} \int d\theta a(\eta) r \Big|_{r=2Gm} = 2Gm a(\eta) \quad \Rightarrow \quad m(\eta) = ma(\eta) \tag{34}$$

keeping in mind that the observer at infinity will in addition measure the integrated matter density outside the apparent horizon, so the above formula is really a lower bound on the total mass accretion. The total mass therefore grows (at least) linearly with the scale factor, and this is also consistent with the fact that  $T_{tr} \neq 0$ . The formula (34) gives

$$m = \frac{m_{initial}}{a(\eta_{initial})} \tag{35}$$

where  $m_{initial}$  is given by (23) and  $a_{initial}$  is the scale factor at the time when the black hole forms. The mass accretion described by (34) is also evident from the non-vanishing mixed component  $T_{tr}$  in (31) which states that there is energy flow into the black hole from the surrounding radiative medium. During the radiation era there is, in fact, an unlimited supply of ‘food’ for the black hole to swallow. This supply will dry up only when inhomogeneities are formed, after which the accretion works in the more standard form.

Evolving the initial mass (4) with the formula (34) we calculate the final mass at the equilibrium time (assuming that  $\eta_{initial} \sim \eta_{PI}$ )

$$m_{final} \sim m_{initial} \left( \frac{\eta_{eq}}{\eta_{PI}} \right) \sim 10^{30} \text{ kg} \sim M_\odot \tag{36}$$

with  $\eta_{eq} \sim 2 \cdot 10^8$  s and  $\eta_{PI} \sim 10^{-19}$  s. This estimate applies to mini-black holes formed very early in the radiation era (for  $\eta \ll \eta_{max}$ ). The same calculation for a mini-black hole at the latest possible time  $\eta_{max}$  given by (28) also yields



a lower bound for the final mass of the primordial black hole upon exit from the radiation era,

$$\sqrt{m_{\text{initial}} M_{\text{Pl}}} < m_{\text{final}} < M_{\text{Pl}} \left( \frac{\eta_{\text{eq}}}{\eta_{\text{Pl}}} \right) \quad (37)$$

or

$$10^{11} \text{ kg} < m_{\text{final}} < M_{\odot} \quad (38)$$

This inequality restricts the possible mass range for primordial black holes at the equilibrium time.

The above analysis can be repeated for matter dominated and exponentially expanding universes, respectively. In this case we need the angular Killing vectors  $k_{\theta}^{\mu} \partial_{\mu}$  and  $k_{\phi}^{\mu} \partial_{\mu}$  to state the pertinent conditions in a generally covariant way. In the matter dominated era we have

$$a(\eta) = B^2 \eta^2 \quad , \quad T_{\mu\nu} k_{\theta}^{\mu} k_{\theta}^{\nu} = T_{\mu\nu} k_{\phi}^{\mu} k_{\phi}^{\nu} = 0 \quad \Rightarrow \quad C(r) = 1 - \frac{2Gm}{r} \quad (39)$$

where we utilize the Killing vectors to state the condition of vanishing pressure. Because this solution does not allow for a non-vanishing charge, this provides another reason for setting  $\mathcal{Q} = 0$  in (33), in order to allow for a smooth transition from the radiation dominated to the matter dominated phase.

Finally, for an exponentially expanding universe we have

$$a(\eta) = \frac{1}{H(\eta_{\infty} - \eta)} \quad , \quad T^{\mu}{}_{\mu} = 2(T_{\mu\nu} k_{\theta}^{\mu} k_{\theta}^{\nu} + T_{\mu\nu} k_{\phi}^{\mu} k_{\phi}^{\nu}) \quad \Rightarrow \quad C(r) = 1 - \frac{2Gm}{r} - C_H r^2 \quad (40)$$

Please note that for  $C_H \neq 0$  this is *not* the well known Kottler solution (that is, de Sitter space in static coordinates). We stress again that for  $C(\eta, r) = C(r)$  and with  $\mathcal{Q} = 0$  and  $C_H = 0$  the causal structure of the space-time is the same as for an ordinary black hole space-time, and only in this case we can have a smooth transition between all phases.

## V. ENERGY-MOMENTUM TENSOR

To gain further insight into the physical properties of our solution let us examine the energy-momentum tensor (31) a bit more closely. Following [34] we parametrize the latter as

$$\begin{aligned} T_{\mu\nu} = & \quad p g_{\mu\nu} + (p + \rho) u_{\mu} u_{\nu} - \Pi_{\mu\rho} Q^{\rho} u_{\nu} - \Pi_{\nu\rho} Q^{\rho} u_{\mu} \\ & - \zeta_1 \Pi_{\mu}{}^{\rho} \Pi_{\nu}{}^{\sigma} \left( \nabla_{\rho} u_{\sigma} + \nabla_{\sigma} u_{\rho} - \frac{2}{3} g_{\rho\sigma} \nabla^{\lambda} u_{\lambda} \right) - \zeta_2 \Pi_{\mu\nu} \nabla^{\lambda} u_{\lambda} \end{aligned} \quad (41)$$

where  $u^{\mu} u_{\mu} = -1$ ,  $Q^{\mu}$  is the heat flow, and  $\zeta_1$  and  $\zeta_2$  are the shear and bulk viscosity, respectively. All variables are assumed to depend on  $\eta$  and  $r$  only. The projector is defined by

$$\Pi_{\mu\nu} = g_{\mu\nu} + u_{\mu} u_{\nu} \quad (42)$$

We will now match the energy-momentum tensor (31) to this formula. For simplicity we assume

$$\zeta_1 = \zeta_2 = 0 \quad (43)$$

We also write  $q_{\mu} \equiv \Pi_{\mu\nu} Q^{\nu}$  (so that  $u^{\mu} q_{\mu} = 0$ ), so that the energy momentum tensor simplifies to

$$T_{\mu\nu} = p g_{\mu\nu} + (p + \rho) u_{\mu} u_{\nu} - q_{\mu} u_{\nu} - q_{\nu} u_{\mu} \quad (44)$$

The assumption of vanishing viscosity coefficients (43) is certainly justified after baryogenesis (that is  $t > 10^{-12}$  s), when the number of photons by far exceeds the number of other particles in the plasma (for instance,  $n_{\gamma} \sim 10^{10} n_b$ ). While the condition (32) leaves  $\zeta_1$  undetermined, we could in principle also admit a non-vanishing  $\zeta_2 \neq 0$ , that is, self-interacting conformal matter (*e.g.* self-interacting massless scalar fields). In that case the relation  $\rho = 3p$  derived below would no longer hold even with vanishing  $T^{\mu}{}_{\mu}$ .

For the comparison we write out (31) explicitly for the solution (33) (with  $Q = 0$ ), which gives

$$\begin{aligned}
8\pi G T_{tt} &= \frac{3\dot{a}^2}{a^2} = \frac{3}{\eta^2} \\
8\pi G T_{rr} &= \frac{r^2(-2a\ddot{a} + \dot{a}^2)}{a^2(r-2Gm)^2} = \frac{r^2}{\eta^2(r-2Gm)^2} \\
8\pi G T_{rt} &= \frac{2Gm\dot{a}}{ar(r-2Gm)} = \frac{2Gm}{r\eta(r-2Gm)} \\
T_{\theta\theta} &= r^2 T_{rr} \quad , \quad T_{\varphi\varphi} = r^2 \sin^2\theta T_{rr}
\end{aligned} \tag{45}$$

Comparing (45) and (44) we read off the unknown quantities on the r.h.s. of (44); we find

$$\begin{aligned}
u_\mu(\eta, r) &= A\eta \left( \sqrt{\frac{r-2Gm}{r}} \cosh \xi, \sqrt{\frac{r}{r-2Gm}} \sinh \xi, 0, 0 \right) \\
q_\mu(\eta, r) &= A\eta q(\eta, r) \left( \sqrt{\frac{r-2Gm}{r}} \sinh \xi, \sqrt{\frac{r}{r-2Gm}} \cosh \xi, 0, 0 \right)
\end{aligned} \tag{46}$$

where

$$\tanh \xi = \frac{Gm\eta}{r^2} \quad (\Rightarrow \xi > 0) \tag{47}$$

and

$$q(\eta, r) = 2p(\eta, r) \tanh \xi \tag{48}$$

(with  $m \equiv m_{initial}$ ). The density and pressure are given by

$$\rho(\eta, r) = 3p(\eta, r) \quad \text{with} \quad p(\eta, r) = \frac{r}{A^2\eta^2(r-2Gm)} \tag{49}$$

as expected for a radiation dominated universe. We stress that there are no pathologies here of the kind encountered in some of the previous literature on McVittie-type solutions. In particular, the energy density  $\rho(\eta, r)$  is strictly positive for  $r > 2Gm$  and at all times  $\eta > 0$ . Moreover, because  $q$  is positive from (48), the radial component of  $q^\mu$  in (46) is also positive, which means that the radial heat flow is *inward directed*, explaining why the mass of the black hole *grows* with time.

To keep  $\xi$  real we must demand

$$\tanh \xi = \frac{Gm\eta}{r^2} < 1 \quad \Rightarrow \quad r > \sqrt{Gm\eta} \quad (> 2Gm) \tag{50}$$

For  $r^2 \rightarrow Gm\eta$  the average velocity of the infalling matter reaches the speed of light, and the expansion (41) in powers of  $u_\mu$  and its derivatives breaks down. Consequently, while the solution (29) remains valid down to  $r = 2Gm$ , the expressions (46), (48) and (49) become meaningless in the region  $2Gm < r < \sqrt{Gm\eta}$  because of apparently superluminal propagation (similar conclusions regarding superluminality were already reached in [27]). Likewise the components of the heat flow  $q^\mu$  diverge for  $\tanh \xi \rightarrow 1$ , indicating an apparent divergence of the temperature in this limit. This is also an unphysical feature in view of the breakdown of the expansion (31). Physically it is tempting to interpret this result as implying that the would-be horizon is shielded from the outside observer by a ‘blanket’ at  $r = \sqrt{Gm\eta}$ , whose extension grows with cosmic time  $\eta$ . However, in recent work [35] it is argued that the gradient expansion (31) must be replaced by a different expansion; adapting these arguments to the present case we conclude that the solution can, in fact, remain meaningful all the way down to  $r = 2Gm$ . Because of the breakdown of the expansion (41), also the apparent ‘firewall’ ( $\equiv$  divergent energy density  $\rho$ ) on the would-be horizon  $r = 2Gm$  is an unphysical feature. This is just as well, because otherwise the total mass at infinity (which includes the integrated energy density for  $r > 2Gm$ ) would diverge, as  $\rho(\eta, r)$  has a non-integrable singularity at  $r = 2Gm$ . At any rate these arguments show that the actual mass value for the black hole will exceed the estimated value (36) if the matter contributions outside the horizon are taken into account, thus further enhancing the growth of primordial black holes.

**Acknowledgments:** We would like to thank B.F. Schutz for correspondence and helpful comments. K.A. Meissner thanks AEI for hospitality and support; he was partially supported by the Polish National Science Center grant DEC-2017/25/B/ST2/00165. The work of H. Nicolai has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (grant agreement No 740209).

- 
- [1] E. Banados et al., Nature **553** (2018) 473  
[2] <https://www.scientificamerican.com/article/zeroing-in-on-how-supermassive-black-holes-formed1/>  
[3] K. Inayoshi, E. Visbal and Z. Haiman, [arXiv:1911.05792\[astro-ph\]](https://arxiv.org/abs/1911.05792)  
[4] P.J. Alister Segal, D.R.G. Schleicher, T.C.N. Boekhold, M. Fellhauer and R.S. Klessen, [arXiv:1912.01737\[astro-ph\]](https://arxiv.org/abs/1912.01737)  
[5] B. Trakhtenbrot, [arXiv:2002.00972\[astro-ph\]](https://arxiv.org/abs/2002.00972)  
[6] K.A. Meissner and H. Nicolai, Phys. Rev. **D91** (2015) 065029  
[7] A. Kleinschmidt and H. Nicolai, Phys. Lett. **B747** (2015) 251  
[8] K.A. Meissner and H. Nicolai, Phys. Rev. Lett. **121** (2018) 091601  
[9] M. Gell-Mann, in Proceedings of the 1983 Shelter Island Conference on Quantum Field Theory and the Fundamental Problems of Physics, eds. R. Jackiw, N.N. Khuri, S. Weinberg and E. Witten, Dover Publications, Mineola, New York (1985)  
[10] H. Nicolai and N.P. Warner, Nucl. Phys. **B259** (1985) 412  
[11] T. Damour and M. Henneaux, Phys. Rev. Lett. **86** (2001) 127  
[12] T. Damour, M. Henneaux and H. Nicolai, Class. Quant. Grav. **20** (2003) R145  
[13] T. Damour, M. Henneaux and H. Nicolai, Phys. Rev. Lett. **89** (2002) 221601  
[14] T. Damour, A. Kleinschmidt, and H. Nicolai, JHEP **08** (2006) 046  
[15] S. de Buyl, M. Henneaux and L. Paulot, JHEP **0602** (2006) 056  
[16] A. Kleinschmidt, H. Nicolai and A. Viganò, *On spinorial representations of involutory subalgebras of Kac-Moody algebras*, [arXiv:1811.11659](https://arxiv.org/abs/1811.11659)  
[17] K.A. Meissner and H. Nicolai, Phys. Rev. **D100** (2019) 035001  
[18] K.A. Meissner and H. Nicolai, JCAP **1909** (2019) 041  
[19] See e.g. [https://en.wikipedia.org/wiki/George\\_Johnstone\\_Stoney](https://en.wikipedia.org/wiki/George_Johnstone_Stoney)  
[20] B. de Wit and D.Z. Freedman, *Supergravity – the basics and beyond*, in *Supersymmetry*, eds. K. Dietz et al. NATO ASI series, Physics Vol.125, Plenum Press (1984)  
[21] D.N. Page and R. McKee, Phys.Rev. **D24** (1981) 1458  
[22] E.H. Lieb and R. Seiringer, *The Stability of Matter in Quantum Mechanics*, Cambridge University Press (2010)  
[23] J.M. Levy-Leblond, Journ. Math. Phys. **10** (1969) 806  
[24] T. Damour, [hep-th/0401160](https://arxiv.org/abs/hep-th/0401160)  
[25] L.D. Landau and E.M. Lifschitz, *The Classical Theory of Fields*, Pergamon, Oxford (1971)  
[26] G.C. McVittie, Mon. Not. R. Astron. Soc. **93** (1933) 325  
[27] B.C. Nolan, Class. Quant. Grav. **16** (1999) 1227, [arXiv:1707.07612\[gr-qc\]](https://arxiv.org/abs/1707.07612)  
[28] V. Faraoni and A. Jacques, Phys. Rev. **D76** (2007) 063510  
[29] C. Gao, X. Chen, V. Faraoni and Y.G. Shen, [arXiv:0802.1298 \[gr-qc\]](https://arxiv.org/abs/0802.1298)  
[30] N. Kaloper, M. Kleban and D. Martin, Phys. Rev. **D81** (2010) 104044  
[31] K. Lake and M. Abdelqader, Phys. Rev. **D 84** (2011) 044045  
[32] Sung-Won Kim and Y. Kang, Int. Journ. Mod. Phys. **12** (2012) 320  
[33] G.F.R. Ellis, private communication  
[34] S. Weinberg, *Gravitation and Cosmology*, John Wiley and Sons (1972)  
[35] M.P. Heller and M. Spaliński, Phys. Rev. Lett. **115** (2015) 072501