Recent Developments in Computer Simulation of Plasmas

D. Biskamp and R. Chodura

Max- Planck-Institut für Plasmaphysik, Garching b.München

In recent years computer simulation has become a valuable tool of plasma physics research. This was essentially due to the development of powerful computers such as the CDC 6600, 7600 or the IBM 360-91, which combine sufficient storage capacity (internal or external) with high-speed execution. To begin with, let us briefly describe the general concept of plasma simulation. The basic idea is to follow numerically a large number of electrons and ions under the influence of their self-consistent fields, in contrast to solving the only approximately valid fluid equations. Now it is easy to see that if one intends to include the details of the microfields around each particle the number of particles, one can handle even on the largest machines, is ridiculously small. However, because of the long range of the Coulomb field, many particles interact with each other simultaneously, the effect of small angle scattering dominates that of close encounters. Thus the main contribution to the interaction is due to certain average fields produced by average charge and current densities.

The simplest model based on this concept of average fields, is the PIC (particle-in-cell) method[1)2)]. Here the fields are computed on a Eulerian grid whose meshsize Δx should be small compared with the wavelengths of the effects under consideration. For collective modes with $\lambda >> \lambda_D, \Delta x \sim \lambda_D$ is sufficiently fine. This method has the double advantage of reducing collisional effects, which allows simulation of collisionless processes with relatively small numbers of particles per Debye cell, and of being numerically very simple, which allows to use a large number of particles i.e. to treat (relatively) large systems. Combined with certain interpolation procedures (area weighting=finite particle size, and field interpolation) this method is the one now generally used in simulations of microscopic plasma processes (another name of this method is CIC=clouds-in-cell[1)]). Fig. 1 shows the Coulomb cross-section in 2D and 3D demonstrating the smoothing properties of finite particle size or cell size Δx. Note the particularly strong dependence in 3D[3].

Let us briefly indicate what kinds of systems can presently be treated by computer simulation. 1D systems do not cause any troubles. Usually $N = 10^4$ particles are sufficient. Only if very large system size is required, or if very weak collective effects are investigated requiring a large number of simulation particles per cell, N must be larger. 2D computations are possible for a number of problems. Typical N's are several times 10^4 to several times 10^5. These numbers can usually only be handled by use of external storage, and 2D runs are in general rather expensive (a number of hours per run). 3D runs are just marginally possible in very special cases and this situation is likely not to change in the near future. Even if there are computers 10 times as powerful as the present ones, in 3D this means only an improvement of a factor of 2 in the linear dimensions. Hence real life simulation is practically not possible, and for a rather long time plasma simulation will have to use appropriate 1D and 2D models.

To characterize developments in plasma simulation of the last one or two years we would like to distinguish three main tendencies:

a) Inclusion of multiple length and time scales. Special topics are turbulent electrical resistance and collisionless magnetic shock waves.
b) Development of fully electromagnetic codes including radiation. First application: simulation of the Weibel instability.
   Main interest: Investigation of plasma interaction with laser light or relativistic electron beams.
c) Investigation of collisional effects, to study collisional transport phenomena such as cross-field diffusion.

This list is by far not complete, the different items given being just examples. There are of course multiple (time and length) scales in any two-species code, and in cases where electromagnetic and electrostatic effects are treated simultaneously. The emphasis in category a) is on computations where either these scales are largely different, for instance if a large mass ratio is required, or where a whole variety of different scales appear explicitely.

If the meshsize Δx of the grid is made smaller than λ_D, discreteness effects appear in the particle model, and in 3D, PIC reliably describes physical binary collisions. This can be used to study collision dominated transport processes, which corresponds to solving the Boltzmann equation in a stable plasma. The most interesting results on cross-field plasma diffusion[4)] were reported in the preceding talk by Dr.Taylor. So we will not touch the last topic c) here. We primarily discuss some results on turbulent heating and on magnetic collisionless shock waves, and then briefly survey progress in simulation of electromagnetic and radiation phenomena.

Electrostatic instabilities excited by an electric current in a plasma are well suitable for investigation by particle simulation. Since typical wavelenths are much shorter than any macroscopic plasma scale, these processes may be regarded as quasilocal and the assumption of a homogeneous system is a good approximation. First simulation results on turbulent heating were presented at the Madison conference a year ago. We think that the situation is somewhat clearer now. The present status of the simulation work on turbulent resistance is as follows:

1) No external magnetic field, $B_o = 0$:

   In 1D no substantial resistance exists in the ion-sound regime $v_d < v_{the}$ [5)6)7)].

   In 2D appreciable electrical resistance is found, which is caused by a broad cone of unstable ion-sound waves. The numerical value of the effective collision frequency ν_{eff}, however, is still too small to explain the resistance in turbulent heating experiments[7)].

   In 3D computations are just marginally possible. It has been found here, that ν_{eff} is about 2-3 times larger than in 2 D, because of the more rapid thermalization of electrons in a 3-dimensional turbulent field[8)]. This value comes closer to experimental observations.

   In all these cases the formation of an electron tail in the distribution fuction gradually reduces the resistance.

2) External field parallel to the current $B \parallel j$:

   This is the usual situation in turbulent heating experiments.

   Simulations in 2D show a resistance comparable to the one in unmagnetized plasmas for $\Omega_{ce}/\omega_{pe} < 1$, and a much lower value when Ω_{ce} becomes greater than ω_{pe}. No appreciable level of electron cyclotron oscillations is observed that could isotropize the electron distribution and thus enhance resistance. Thus if strong anomalous resistance is found in experiments in the regime $\Omega_{ce}/\omega_{pe} > 1$, it cannot be simply interpreted in terms of ion-sound instability.

3) Magnetic field perpendicular to the current, $B \perp j$:

Electron runaway is prevented, when the current is flowing perpendicularly to B, which is the case in high -β configurations such as shock waves and magnetic sheaths. We want to treat this case somewhat more in detail. The discussion about the type of instability producing anomalous resistivity in collisionless shock waves was initiated by the experimental observations of Keilhacker and coworkers[9)]. These experiments seem to indicate that the presence of anomalous resistance is not depending on a high temperature ratio T_e/T_i. Thus the usual ion-sound instability appeared to be a somewhat doubtful candidate. The electron cyclotron drift instability at first sight seems to solve the problem, since it predicts unstable wave growth quite independently of T_e/T_i. However, in a high density plasma, $\Omega_{ce} \ll \omega_{pe}$, this instability will saturate at very low fluctuation levels (Lampe et al.[10)]) and hence cannot explain the anomalous resistivity observed.

On the other hand computer simulations show strong instability and efficient electron heating in a parameter range where the usual two-stream instability does not exist[11)]. An intense discussion on this point came up. We believe that these points are clarified by now and we want to describe briefly the essential results of the $j \perp B$ instability for amplitudes greater than the saturation level of the electron cyclotron drift instability. The following features have been found in the 1D case:

i) The instability threshold is not much different from that of the usual two-stream instability, see Fig. 2, in contrast to the electron cyclotron drift instability. The numerical simulations of Forslund, Morse and Nielson[11)] reported at the Madison Conference had parameters just between the two curves of Fig.2.

ii) Nevertheless, the basic nonlinear mechanism suggested by Forslund, Morse and Nielson is correct. In 1D electron heating occurs by a rather coherent process of trapped electron acceleration, and not by some stochastic (quasilinear) electron scattering . This is seen directly in Fig.3. While the quantity $W/nT \simeq$ const, $W = \langle E^2\rangle/8\pi$, in the turbulent phase, the thermal velocity and hence the effective collision frequency increase linearily with time $v_{the} \propto \nu_{eff} \propto \Omega_{ce} t$ in contrast to the quasilinear prediction $\nu_{eff} \simeq W/nT \simeq$ const.

iii) The electron temperature finally saturates (Fig.2c), when the drift approaches a certain value v_{ds}. The theory of ion-sound instability predicts $v_{ds} \sim c_s$, while the electron cyclotron instability predicts $v_{ds} \sim v_{the}\, \Omega_e/\omega_{pe}$ (although the linear electron cyclotron instability does not play a role in these cases, the strong dependence of the collision frequency on Ω_e may suggest a dependence of the switch-off drift v_{ds} on the magnetic field in the sense of the electron cyclotron drift instability threshold). Running a number of computer experiments up to saturation of T_e, we find that $v_{ds} \simeq 2\, c_s$ and that there is only very weak dependence on B. If one assumes the thickness Δ of a magnetic sheath to be determined by anomalous resistivity in the way that resistive magnetic field penetration occurs until the current density approaches env_{ds}, we would obtain the usual $\Delta \sim c/\omega_{pi}$ (for $\beta_e \sim 1$), and not $\Delta \sim (c/\omega_{pe})(c/v_{the}) \sim c/\Omega_{ce}$ (for $\beta_e \sim 1$) as predicted previously on account the electron cyclotron drift instability[10].

In 2D the general behaviour is rather different from the 1D case. The main result is that the strong coherent electron heating observed in the 1D runs does not exist here, since a broad cone of modes leading to a short correlation length effectively prevents longtime electron trapping. Fig.3 shows some results of a run with $T_{eo}/T_{io}=50$ and $m_i/m_e = 1600$. One finds, that ν_{eff} reaches a maximum at about the same time as W/nT, and that afterwards $\nu_{eff}/(W/nT)$ becomes constant, indicating stochastic heating. The decay of ν_{eff} after the maximum is a common feature of all 2D runs. It is roughly consistent with the following scaling:

$$\nu_{eff}/\omega_{pe} \propto \frac{v_d - \alpha c_s}{v_{the}}, \quad \alpha \sim 2 \ .$$

Thus we find that the gross features of the development of the instability in 2D may be described as an ion-sound instability with electron runaway prevented by gyration. The strong trapped electron heating seems to be an artefact of the 1D system. Nevertheless 1D simulations are useful, since they may give upper or lower bounds of certain quantities which cannot be studied in 2D because of computer limitations. Thus, for example, the fact that in 1D where magnetic effects are strongest, v_{ds} is found independent of B, suggest that this will also be the case in higher dimensions.

It seems that computer simulations have clarified the somewhat confused theoretical situation. However, they can not fully explain Keilhacker's experiments. It appears that to understand these high-β shock waves, additional measurements of the local drift velocity especially at the upstream edge, of possible magnetic oscillations, and of the full ion distribution are required.

After describing the simulation results on turbulent resistivity especially in collisionless shock waves, we briefly discuss some results about the global structure of magnetic shock waves in the high Mach number range. Here the mechanism of ion dissipation has been a major theoretical problem. The question is: are there strong (electrostatic) beam instabilities between the reflected ions and the upstream plasma or is ion thermalization due only to gyration effects. To investigate this point one first omits the magnetic force on the ions. In the case of shock waves propagating perpendicularly to the magnetic field one finds that there is no beam interaction[13], Fig.5a. In the case of a wave propagating obliquely with respect to B, the whistler precursor can trigger a nonlinear ion beam instability which leads to rapid ion thermalization Fig.5b. In addition electrostatic subshocks are formed giving rise to microturbulence which strongly enhances dissipation. These computations have been performed with a two-species code including all electrostatic and electromagnetic length and time scales[14].

Finally we briefly touch recent developments in simulation of electromagnetic properties of plasmas including radiation. An electromagnetic code has to deal with a specific noise problem which does not arise in electrostatic codes. Since in most cases of interest the radiation energy is much smaller than the thermal energy, the plasma is in strong non-equilibrium with the radiation field and collisions will tend to gradually build up radiation by bremsstrahlung. This means that the thermal noise excited by collisions is not constant but increases linearily with time. To reduce this noise production is an important problem. Examples of good electromagnetic codes were given by Morse and Nielson[15] and Boris[16]. Interesting results have been obtained for instance by Morse and Nielson in the 2D simulation of the Weibel instability[15] and in describing the heating of the edge of a plasma pellet by laser radiation. In the latter case strong collisionless absorption was observed, leading to very high energy tails in the electron distribution, a somewhat undesirable feature in the present laser fusion concept.

In conclusion we would like to say that computer simulations have provided considerable insight into basic collective and collisional processes in plasmas. They have enriched theoretical understanding by proceeding into strongly nonlinear regimes where standard analytical methods are mostly inadequate. Topics not mentioned here, where computer simulations are and will be very useful are problems of r.f.heating of plasmas and microinstabilities in toroidal plasmas.

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Figure Captions

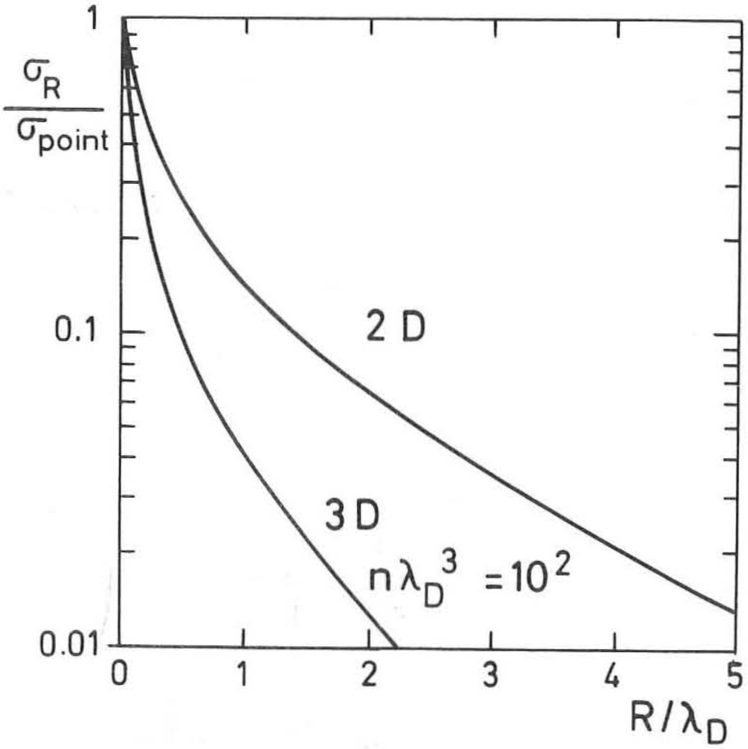
Fig. 1 Coulomb cross section for finite size particles in 2D and 3D, taken from Ref.3

Fig. 2 Critical drift velocity for $j \perp B$ instability in 1D, as obtained from simulations. The critical velocity for two-stream instability is given for comparison.

Fig. 3 Computer run of the $j \perp B$ instability in 1D: $m_i/m_e=1600$, $v_d/v_{theo}=1$, $T_{eo}/T_{io} = 2$. The drift velocity v_d is kept constant during the computatuion. $W \equiv \langle \tilde{E}^2 \rangle / 8\pi$. $\Omega_{ce}/\omega_{pe} = 0.04$.

Fig. 4 Computer run of the $j \perp B$ instability in 2D: $m_i/m_e=1600$, $v_d/v_{theo}=1$, $T_{eo}/T_{io}=50$, $\Omega_{ce}/\omega_{pe} = 0.04$.

Fig. 5 Magnetic collisionless shock waves, propagating from right to left. The ions are not magnetized. Plots of ion phase space, total magnetic field and electric potential. a) Perpendicular propagation; b) oblique propagation $\theta = 45^{\circ}$.



COULOMB CROSS SECTION FOR 2- AND 3-DIM. FINITE SIZE PARTICLES.

Fig.1

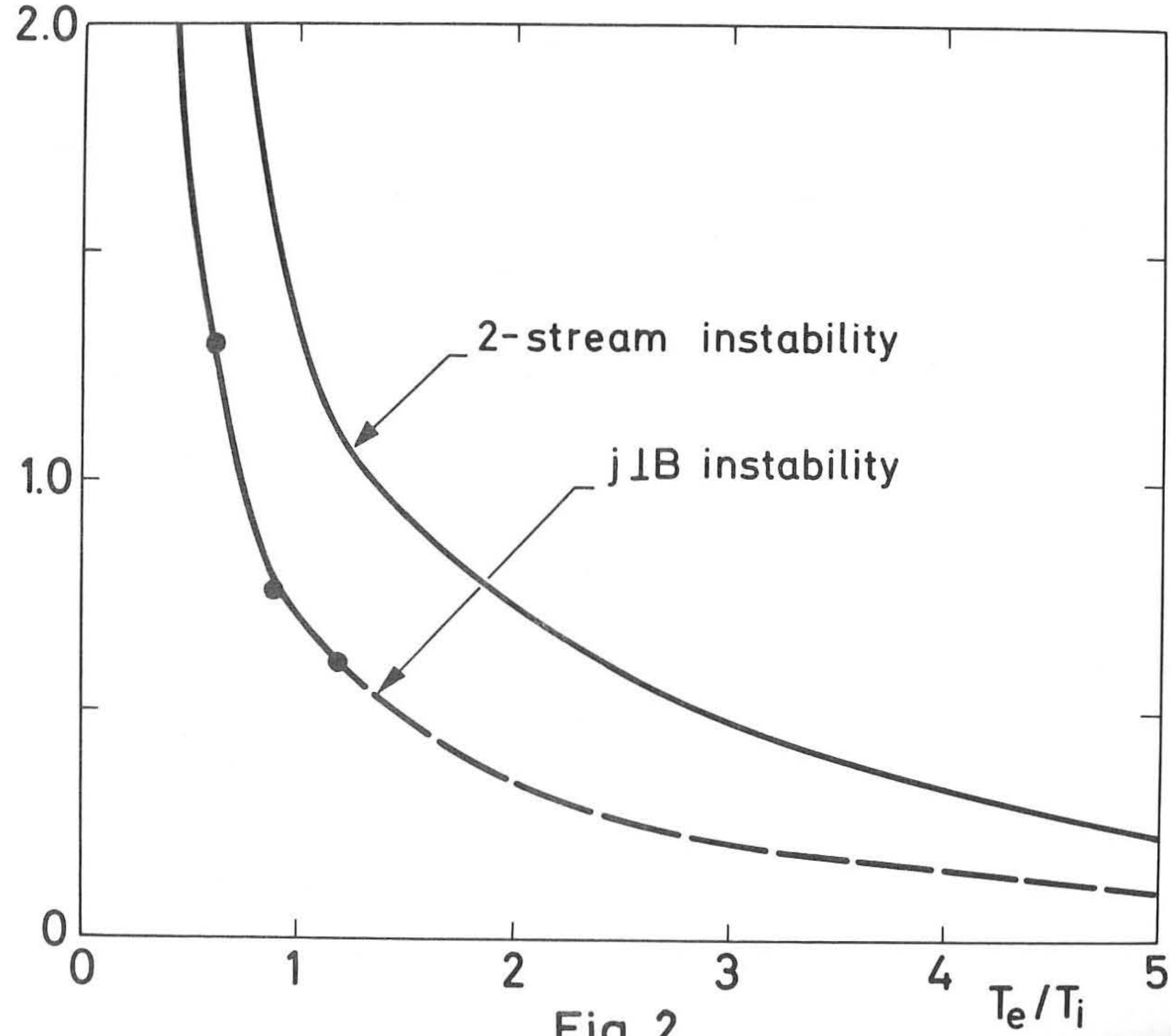


Fig. 2

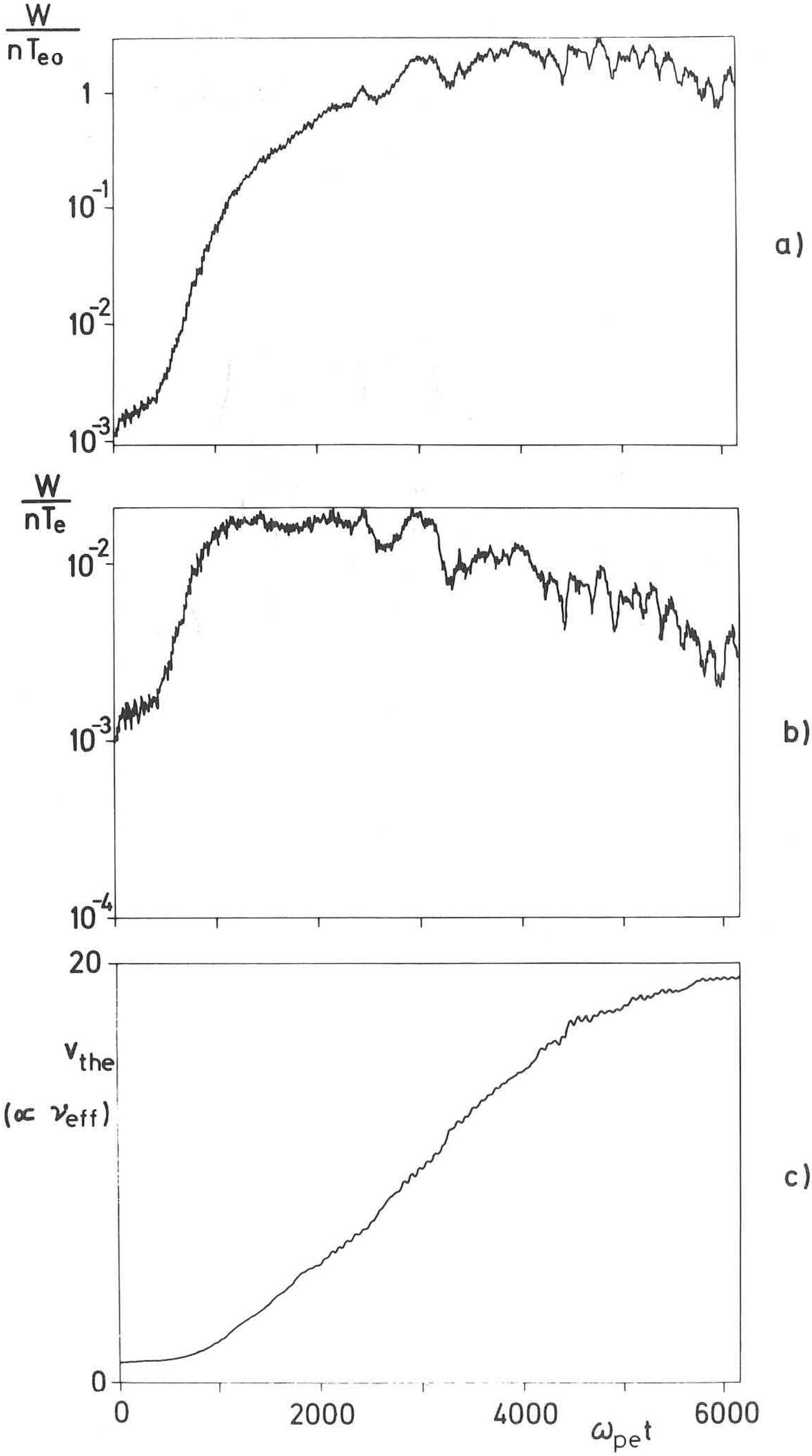

$\frac{W}{nT_{eo}}$
1
10^{-1}
10^{-2}
10^{-3}
a)
$\frac{W}{nT_e}$
10^{-2}
10^{-3}
10^{-4}
b)
20
v_{the}
($\propto \nu_{eff}$)
0
c)
0
2000
4000
6000
$\omega_{pe}t$

Fig. 3

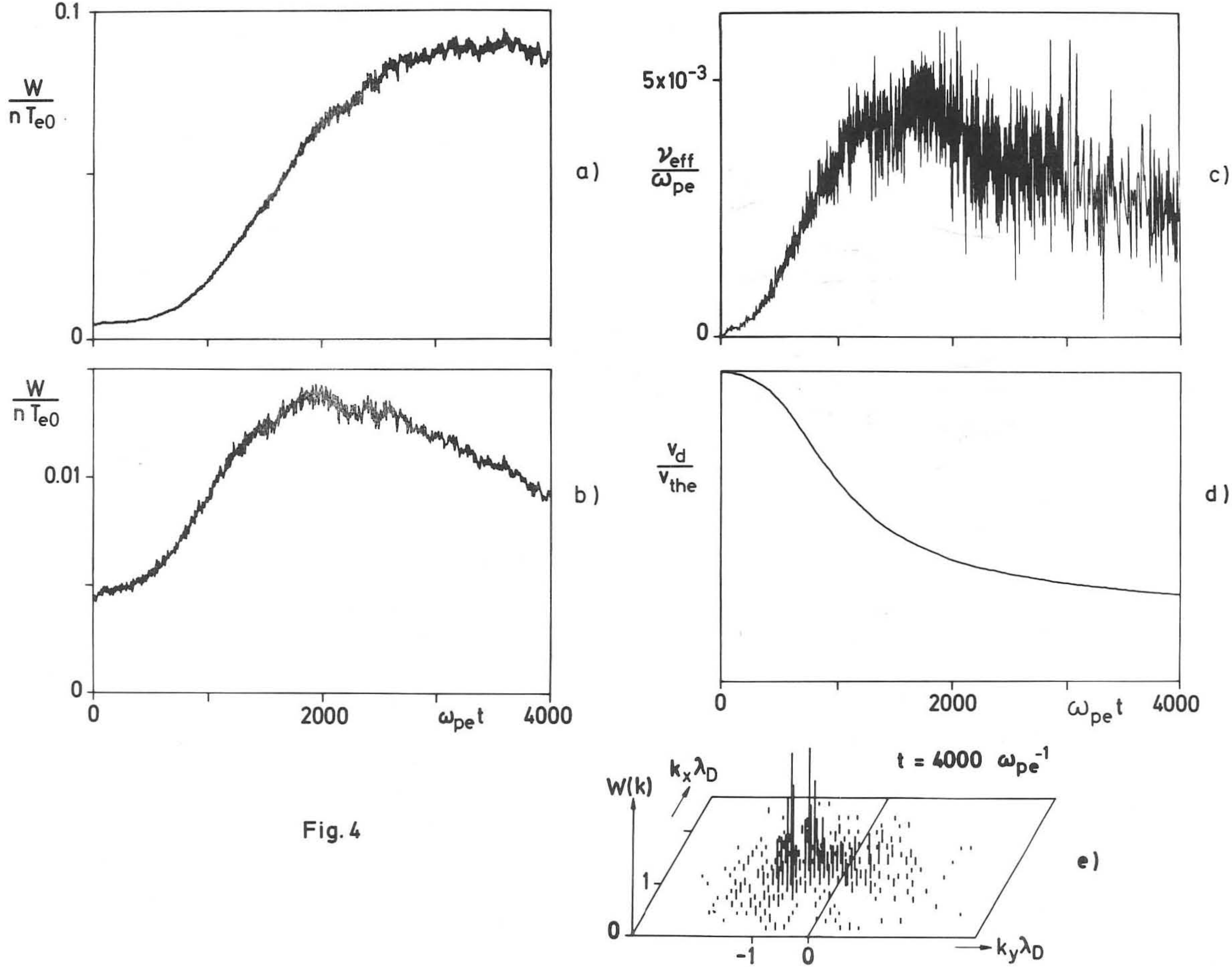

0.1
W / n T_e0
0
a)
0.01
b)
0
2000
ω_pe t
4000
5x10^-3
ν_eff / ω_pe
c)
v_d / v_the
d)
t = 4000 ω_pe^-1
W(k)
k_x λ_D
1
-1
k_y λ_D
e)

Fig. 4

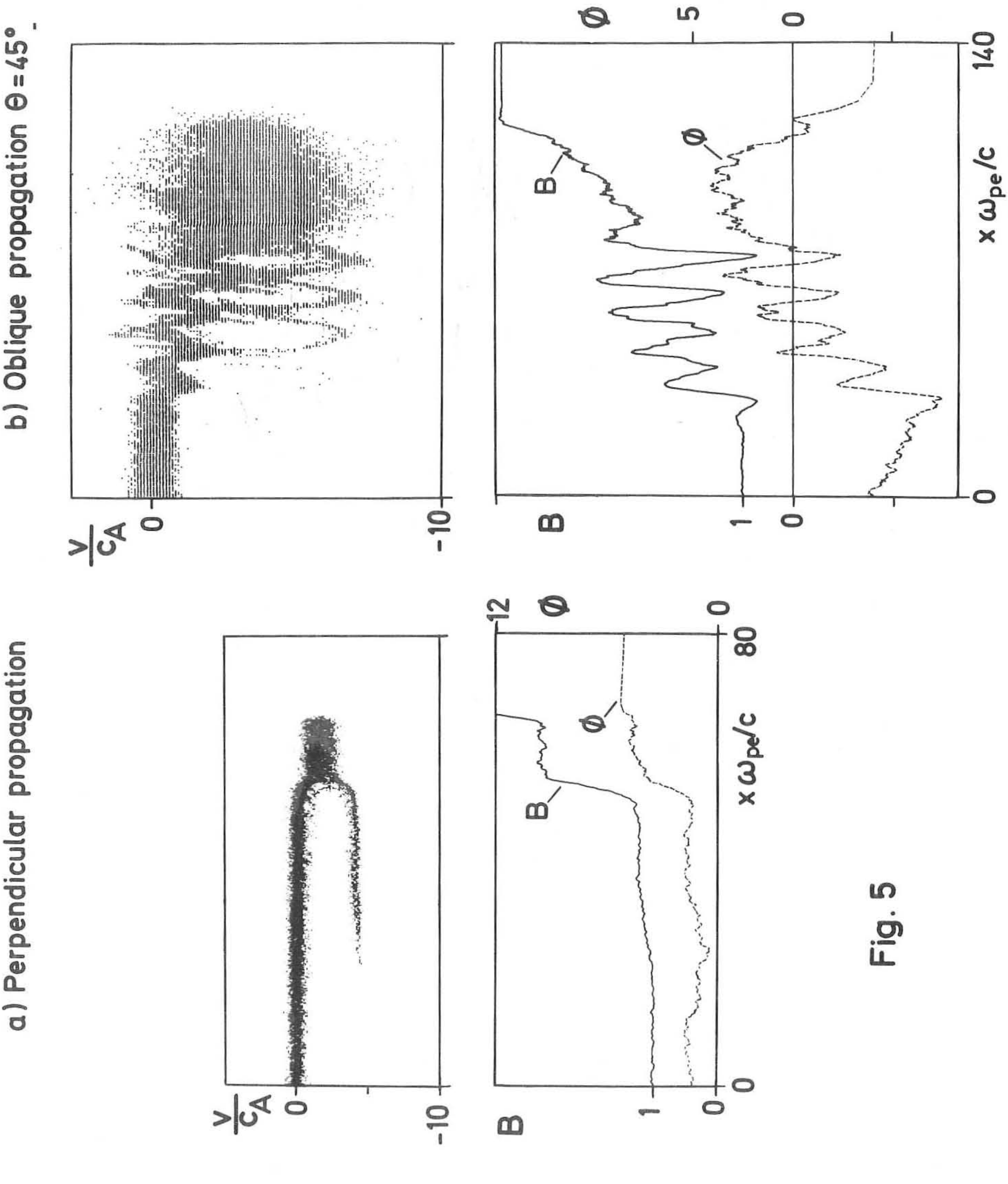

a) Perpendicular propagation
b) Oblique propagation Θ=45°
$\frac{v}{c_A}$
0
-10
B
1
0
Ø
12
5
B
Ø
x ω_{pe}/c
80
140

Fig. 5