Published for SISSA by Springer

Received: October 1, 2020
Revised: November 5, 2020
Accepted: November 27, 2020
Published: January 11, 2021

JHEP01(2021)043

Mimetic gravity in $(2+1)$-dimensions

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Abstract: One of the most important achievements in general relativity has been discovery of the $(2+1)$-dimensional black hole solutions of Einstein gravity in anti-de Sitter (AdS) spacetime [7]. In this paper, we construct, for the first time, the $(2+1)$-dimensional solutions of mimetic theory of gravity. These solutions may provide a powerful background to investigate the physical properties of mimetic gravity and examine its viability in lower spacetime dimensions. In particular, some physical properties of stationary black hole solutions of this theory in the presence of charge or angular momentum are investigated.

Keywords: Black Holes, Classical Theories of Gravity

ArXiv ePrint: 2009.12826

https://doi.org/10.1007/JHEP01(2021)043

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1 Introduction

Black holes are perhaps the most fascinating and amazing astrophysical objects that have ever emerged as the solutions of gravitational theories. These wonderful and enrich structures provide a powerful background for exploring different branches of physics including quantum gravity, thermodynamics, superconducting phase transition, paramagnetism-ferromagnetism phase transition, superfluids, condensed matter physics, spectroscopy, information theory, holographic hypothesis, etc. This powerfully is mainly due to the discovery of the well established correspondence between gravity in d-dimensional anti-de Sitter (AdS) spacetime and the conformal field theory (CFT) living on the boundary of $(d-1)$-dimensional spacetime, known as $\mathrm{AdS}_d/\mathrm{CFT}_{d-1}$ correspondence or gauge/gravity duality. Recently, black holes have also received a renewed attention after some significant steps toward understanding the puzzle of information paradox [1] as well as the shadow of the supermassive black holes as the first results of $M87$ Event Horizon Telescope [2]. It was shown that there exist soft hairs, including soft gravitons and/or soft photons, on the black hole horizon and the complete information about their quantum state is stored on a holographic plate at the future boundary of the horizon [1]. Black hole entropy and microscopic structure near the horizon can be understood through these soft hairs [3–6].

One of the most significant achievements in black hole physics has been the discovery of three dimensional solutions of general relativity in AdS spacetime known as BTZ (Banados-Teitelboim-Zanelli) black holes [7]. In fact, the $(2+1)$-dimensional solution of Einstein gravity provides a simplified model to investigate and find some conceptual issues such as black hole thermodynamics, quantum gravity, string and gauge/gravity duality, holographic superconductors in the context of the $\mathrm{AdS}_3/\mathrm{CFT}_2$ [8–12]. It has been shown that the quasinormal modes in this spacetime coincide with the poles of the correlation function in the dual CFT. This gives quantitative evidence for $\mathrm{AdS}_3/\mathrm{CFT}_2$ [13]. Furthermore, BTZ black holes play a crucial role for improving our perception of gravitational interaction in low dimensional spacetimes [14]. In particular, it might shed some light on

the quantum gravity in three dimensions. Geometry of the spinning $(2+1)$-dimensional black holes has been explored in [15]. It was shown that the surface $r = 0$ is not a curvature singularity but, rather, a singularity in the causal structure which is everywhere constant and continuing beyond it would produce closed timelike curves [15]. The extension to include an electric charge in addition to the mass and angular momentum have been performed [16, 17]. The $(2+1)$-dimensional black holes also provide a powerful background to explore one-dimensional holographic superconductors [18–22]. The studies on $(2+1)$-dimensional solutions of gravitational field equations have been extensively carried out in the literatures (see e.g. [8, 23–33]).

In this paper, we introduce the $(2+1)$-dimensional black hole solutions in the context of mimetic gravity. The theory of mimetic gravity was proposed a few years ago, as an alternative description for the dark matter puzzle [34]. It was argued that the mimetic field can encodes an extra longitudinal degree of freedom to the gravitational field. Thus, the gravitational field achieves, in addition to two transverse degrees of freedom, a dynamical longitudinal degree of freedom which can play the role of mimetic dark matter. Latter, it was shown that a modified version of mimetic gravity can resolve the cosmological singularities [35] as well as the singularity in the center of a black hole [36]. Besides, it has been confirmed that the original setting of the mimetic theory predicts that gravitational wave (GW) propagates at the speed of light, ensuring agreement with the results of the event GW170817 and its optical counterpart [37, 38]. It has also been shown that this theory can explain the flat rotation curves of spiral galaxies [39, 40]. Mimetic theory of gravity has arisen a lot of enthusiasm in the past few years both from the cosmological viewpoint [41–56] as well as black holes physics [57–72].

Till now, black hole solutions of mimetic gravity in $(2+1)$-dimensions have not been explored. Our purpose here is to construct static and stationary analytical black hole solutions of mimetic gravity in three dimensional spacetime and investigate their properties. These new solutions provide a set up, for future investigations, to examine mimetic theory of gravity and its physical consequences in lower spacetime dimensions and examine the viability of this theory. In particular, it may be useful for investigating one-dimensional holographic superconductors in the context of mimetic gravity. We shall consider several cases including whether or not there is a coupling to the Maxwell field or whether or not there is an angular momentum associated with the spacetime. We study the effects of the mimetic field on the casual structure and physical properties of the solutions and disclose that, in contrast to the three dimensional solution of general relativity, in mimetic gravity a curvature singularity emerges at $r = 0$ even in the absence of Maxwell field. This essential singularity might be due to the extra longitudinal degree of freedom of the gravitational field encoded by the mimetic field. Surprisingly, the curvature singularity disappears by adding an angular momentum to the spacetime.

This paper is organized as follows. Section 2 is devoted to introducing the basic field equations of mimetic gravity in $(2+1)$-dimensions. For simplicity we first ignore the coupling to the Maxwell field in this section and construct three dimensional black hole solutions. In section 3, we take into account the Maxwell field and explore charged mimetic black holes in three dimensions. In section 4, we add an angular momentum to the back

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hole and investigate rotating (2+1)-dimensional solution of mimetic gravity. We summarize our results in section 5.

2 Field equations and solutions

We start with the following action

$$S=\int d^3x\sqrt{-g}\left(\mathcal{R}+\lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\epsilon)+\frac{2}{l^2}-F_{\mu\nu}F^{\mu\nu}\right), \tag{2.1}$$

where $\mathcal{R}$ is the Ricci scalar, λ is the Lagrange multiplier, and l is related to the cosmological constant by $-\Lambda=l^{-2}$. $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ is the electromagnetic field tensor and A_μ is the gauge potential. In the above action, $\epsilon=\pm1$, in which the positive and negative sign refer to, respectively, spacelike and timelike nature of vector $\partial_\mu\phi$. We adopt $(-,+,+)$ as our signature and set $8\pi G_N=1$ throughout this work. The equations of motion can be derived from varying action (2.1), yielding

$$G_{\mu\nu}=\lambda\partial_\mu\phi\partial_\nu\phi+\frac{g_{\mu\nu}}{l^2}+T_{\mu\nu} \tag{2.2}$$

$$\frac{1}{\sqrt{-g}}\partial_\kappa(\lambda\sqrt{-g}\partial^\kappa\phi)=\nabla_\kappa(\lambda\partial^\kappa\phi)=0, \tag{2.3}$$

$$\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right)=0. \tag{2.4}$$

$$g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi=\epsilon. \tag{2.5}$$

where

$$T_{\mu\nu}=2F_{\mu\gamma}F_\nu{}^\gamma-\frac{1}{2}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}, \tag{2.6}$$

being the Maxwell energy-momentum tensor. Equation (2.5) restricts the evolution of the mimetic field ϕ and indicates that the scalar field is not dynamical by itself, nevertheless it makes the longitudinal degree of freedom of the gravitational field dynamical [34]. It was argued [34] that if one assume $g_{\mu\nu}=g_{\mu\nu}(\phi,\tilde{g}_{\mu\nu})$, in such a way that $g_{\mu\nu}=\epsilon(\tilde{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi)\tilde{g}_{\mu\nu}$, then one recovers (2.5) immediately (see also [54, 57]). Tracing eq. (2.2), combining with eq. (2.5), yields $\lambda=\epsilon\left(G-T-3/l^2\right)$, where G and T are, the trace of the Einstein tensor and energy momentum tensor, respectively. Substituting λ in the field equations (2.2) and (2.3), they transform to

$$G_{\mu\nu}=\epsilon\left(G-T-\frac{3}{l^2}\right)\partial_\mu\phi\partial_\nu\phi+\frac{g_{\mu\nu}}{l^2}+T_{\mu\nu}, \tag{2.7}$$

$$\partial_\kappa\left[\sqrt{-g}\left(G-T-\frac{3}{l^2}\right)\partial^\kappa\phi\right]=0, \tag{2.8}$$

Our aim here is to derive static $(2+1)$-dimensional black hole solutions of the above field equations. We assume the metric of spacetime as

$$ds^2=-f(r)g^2(r)dt^2+\frac{dr^2}{f(r)}+r^2d\varphi^2, \tag{2.9}$$

where an additional degree of freedom is incorporated in the line element through the metric function $g(r)$ which is expected to reflect an extra degree of freedom of gravitation

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encoded by the mimetic field ϕ. With metric (2.9), the constraint eq. (2.5) transforms to $f(r)\phi'^2 = \epsilon$, which has solution of the form

$$\phi(r) = \int \frac{dr}{\sqrt{|\epsilon f(r)|}}, \tag{2.10}$$

where we have chosen the positive sign and set the integration constant equal to zero, without loss of generality. Equation (2.10) shows the explicit dependence of the mimetic field on the metric function and reveals that it is not an independent dynamical variable. Let us first consider the uncharged solution. In this case the field equations (2.7) and (2.8), have the following solutions

$$f(r) = -M + \frac{r^2}{l^2}, \tag{2.11}$$

$$g(r) = 1 + \frac{b\ r}{\sqrt{|r^2 - Ml^2|}}, \tag{2.12}$$

where b is the constant of integration which incorporates the impact of the mimetic field into the solutions. In general b could be either positive or negative. For $b = 0$, our solutions reduce to the $(2+1)$-dimensional BTZ black holes of Einstein gravity [7]. The horizon is located at $r_+ = l\sqrt{M}$ where $f(r_+) = 0$, while $g(r_+)$ diverges. However, $r = r_+$ is a coordinate singularity and both Kretschmann and Ricci scalars have finite values at r_+. The sign of $|r^2 - Ml^2|$ depends on whether one considers interior solution ($r < r_+$) or exterior solution ($r > r_+$). Expanding $g(r)$ for large r leads to

$$g(r) \approx 1 + b + \frac{bMl^2}{2r^2} + O\left(\frac{1}{r^4}\right). \tag{2.13}$$

Therefore as $r \to \infty$, we have $g(r) \approx 1 + b$, which implies that the remnant of the mimetic field ϕ contributes to the metric function $g(r)$ through constant b. Nevertheless the asymptotic behavior of the solutions is still AdS since the constant $1 + b$ can be absorbed by redefinition of the time at the asymptotic region.

The (tt) component of the metric is given by

$$-\mathbf{g}_{tt}(r) = f(r)g^2(r) = \frac{1}{l^2}\left[\sqrt{|r^2 - Ml^2|} + b\ r\right]^2, \tag{2.14}$$

where its expansion for large r is given by

$$-\mathbf{g}_{tt}(r) \approx -(1+b)M + (1+b)^2\frac{r^2}{l^2} - \frac{bM^2l^2}{4r^2} + O\left(\frac{1}{r^4}\right). \tag{2.15}$$

This confirms that in large r limit, we have $-\mathbf{g}_{tt} \neq \mathbf{g}^{rr}$. On the other hand when $r \to 0$, we have $g(r) = 1$ and $-\mathbf{g}_{tt} = \mathbf{g}^{rr} = f(r)$. The infinite redshift surface can be obtained by setting $\mathbf{g}_{tt}(r) = 0$, which yields

$$r_{s_i} = \frac{r_+}{\sqrt{1 \pm b^2}}, \tag{2.16}$$

where $i = 1, 2$ and $-$ for $r_{s_1} > r_+$ and $+$ for $r_{s_2} < r_+$. This means that we have 2 infinite redshift surfaces and the black hole horizon is located between them, $r_{s_2} < r_+ < r_{s_1}$. Besides, r_{s_1} exists provided $b^2 < 1$.

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We now calculate the scalar curvatures of the spacetime. It is a matter of calculations to show that the Ricci scalar and the Kretschmann invariant are given by

$$R=\frac{2\left[b(Ml^2-3r^2)-3r\sqrt{|r^2-Ml^2|}\right]}{rl^2\left(\sqrt{|r^2-Ml^2|}+br\right)}, \tag{2.17}$$

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}=\frac{4\left\{3r^2(r^2-Ml^2)+2br(3r^2-Ml^2)\sqrt{|r^2-Ml^2|}+b^2(3r^4-2Ml^2r^2+M^2l^4)\right\}}{r^2l^4\left(\sqrt{|r^2-Ml^2|}+br\right)^2}, \tag{2.18}$$

Thus, as $r\to 0$, both Ricci and Kretschmann invariants diverge, they are finite at $r\neq 0$ and go to $R=-6/l^2$ and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}=12/l^4$ as $r\to\infty$. Therefore, the spacetime has an essential singularity at $r=0$. This is in contrast to the $(2+1)$-dimensional black holes of Einstein gravity where $r=0$ is not a curvature singularity but, rather, a singularity in the causal structure [15]. Indeed, for Einstein gravity where $b=0$, the curvature invariants have constant values anywhere, namely $R=-6/l^2$ and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}=12/l^4$, and the curvature singularity disappears.

In the next section, we shall consider the $(2+1)$-dimensional charged black hole of mimetic gravity.

3 Charged mimetic black holes in 3D

In the presence of coupling to the Maxwell field, the gauge potential and the non-vanishing component of the electromagnetic tensor become

$$A_\mu=h(r)\delta^0_\mu,\quad F_{tr}=h'(r), \tag{3.1}$$

where prime indicates derivative with respect to r. Then it is easy to show that the Maxwell equation (2.4) transforms to

$$rh''(r)g(r)-rh'(r)g'(r)+h'(r)g(r)=0, \tag{3.2}$$

which has the following solution

$$F_{tr}=h'(r)=\frac{q}{r}g(r), \tag{3.3}$$

where q is an integration constant which is indeed the electric charge of the black hole. Substituting metric (2.9), condition (2.10), and the electric field (3.3) into eq. (2.7), we find

$$l^2rf'-2r^2+2q^2l^2=0, \tag{3.4}$$

$$l^2gf'+3l^2rf'g'+2l^2rfg''+l^2rgf''-4rg=0, \tag{3.5}$$

$$3l^2r^2f'g'+2l^2r^2fg''+l^2r^2gf''-2r^2g-2q^2l^2g=0. \tag{3.6}$$

These equations have the following solutions

$$f(r)=-M-2q^2\ln(r)+\frac{r^2}{l^2}, \tag{3.7}$$

$$g(r)=1+b_1\int\frac{dr}{\sqrt{|r^2-2q^2\ln(r)-Ml^2|^3}}, \tag{3.8}$$

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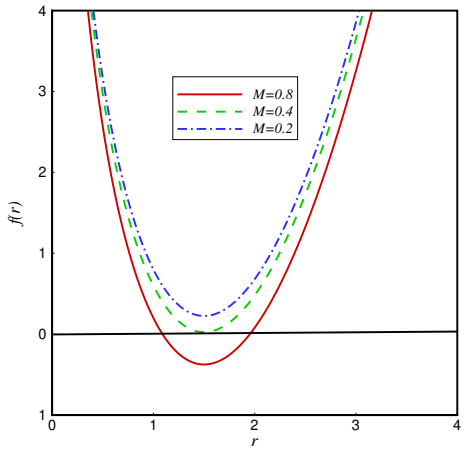

Figure 1. The behavior of $f(r)$ for charged mimetic black holes in 3D and different M. Here we have taken $l = 1$ and $q = 1.5$.

where b_1 is again an integration constant which reflects the imprint of the mimetic field on the spacetime. Clearly for $b_1 = 0$, one recovers the $(2+1)$-dimensional charged black holes of general relativity [16]. One can easily check that these solutions also satisfy eq. (2.8) for the mimetic scalar field. The horizons are given by the roots of $f(r_h) = 0$. Depending on the parameters this equation may have at most two real roots corresponding to inner and outer horizon of the black hole. Of course, one may choose the parameters such that the solutions also describe extremal black hole with one horizon, or a naked singularity (see figure 1). The integrand in expression (3.8) also diverges at r_h indicating that we encounter singularity. However, $r = r_h$ is only a coordinate singularity and both Kretschmann and Ricci scalars have finite values on the horizon. The integral in $g(r)$ function cannot be analytically performed for arbitrary values of r, however, it is instructive to study the large r limit of $g(r)$. In this case, one can write

$$g(r) \approx 1 + b_1 \int \frac{dr}{r^3} \approx 1 - \frac{b_1}{2r^2}, \tag{3.9}$$

$$\begin{aligned} -\mathbf{g}_{tt}(r) &\approx \left(-M - 2q^2 \ln(r) + \frac{r^2}{l^2}\right)\left(1 - \frac{b_1}{2r^2}\right)^2 \\ &\approx -M - \frac{b_1}{l^2} - 2q^2 \ln(r) + \frac{r^2}{l^2} + \frac{2b_1 q^2 ln(r)}{r^2} + \left(Mb_1 + \frac{b_1^2}{4l^2}\right)\frac{1}{r^2} + O\left(\frac{1}{r^4}\right). \end{aligned} \tag{3.10}$$

Therefore, as $r \to \infty$ we have $-\mathbf{g}_{tt} \neq \mathbf{g}^{rr}$, and the remnant of the mimetic field contributes to the metric function $\mathbf{g}_{tt}$ through constant b_1. It is also interesting to take a close look on the roots of $\mathbf{g}_{tt} = 0$ which define the infinite redshift surfaces. Since there is no any physical reason to avoid negative b_1, thus one can choose either $b_1 > 0$ or $b_1 < 0$. For $b_1 \leq 0$, the infinite redshift surfaces r_{s_i} coincide with the horizons $r_h = r_\pm$, namely the roots of $f(r_h) = 0$. However, for $b_1 > 0$, the function $\mathbf{g}_{tt}$ admits an additional root $r_{s_3} = \sqrt{b_1/2}$ (see figure 2). Note that r_{s_3} can be either larger, equal or smaller than r_- depending on the values of b_1. It is easy to show that, for all values of b_1, both Ricci and Kretschmann scalars diverge at $r = 0$, they are finite at $r \neq 0$ and as $r \to \infty$ they tend to constant

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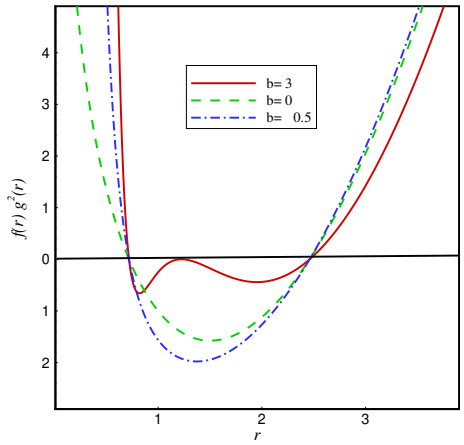

Figure 2. The behavior of $-\mathbf{g}_{tt} = f(r)g^2(r)$ for charged mimetic black holes in 3D and different b. Here we have taken $M = 2$, $l = 1$, $q = 1.5$.

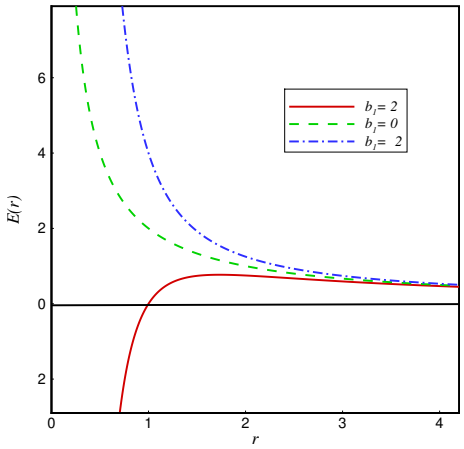

Figure 3. The behavior of the electric field $F_{tr} = E(r)$ for 3D charged mimetic black holes with $q = 2$.

values, $R = -6/l^2$ and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 12/l^4$, similar to 3D solutions of Einstein gravity. This analysis confirms that there is a curvature singularity at $r = 0$, regardless of the value of b_1. The behaviour of the electric field for the large r is also given by

$$F_{tr} \approx \frac{q}{r}\left(1 - \frac{b_1}{2r^2}\right). \tag{3.11}$$

We have also plotted the behavior of the electric field F_{tr} in figure 3. This figure shows that the electric field diverges for small r and goes to zero in large r limit. For $b_1 > 0$, the electric field has a maximum at finite r which can be easily seen from expression (3.11). Expression (3.11) also shows that, compared to the case of Einstein gravity, the electric field of three dimensional charged mimetic black hole get modified due to the presence of mimetic field.

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4 Rotating mimetic black holes in 3D

Now we consider spinning solutions of mimetic gravity in $(2+1)$-dimensions. We take the spinning metric as

$$ds^2 = -f(r)g^2(r)dt^2 + \frac{dr^2}{f(r)} + r^2(JN(r)dt + d\varphi^2)^2, \tag{4.1}$$

where functions $f(r)$, $g(r)$ and $N(r)$ are determined by solving the field equations. Clearly, in the limiting case where the mimetic gravity reduces to Einstein gravity, one expects to have $g(r) = 1$ and $N(r) = -1/2r^2$ [7]. The $(t\phi)$ component of the field equations (2.7) yields

$$rg(r)N''(r) + 3g(r)N'(r) - rN'(r)g'(r) = 0, \tag{4.2}$$

which implies $g(r) = r^3N'(r)$. Combining with (tt) component of the field equation we arrive at

$$l^2J^2 + 2l^2r^3f'(r) - 4r^4 = 0, \tag{4.3}$$

with the following solution

$$f(r) = -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}. \tag{4.4}$$

Finally, the (rr) component of the field equations (2.7) can be solved to give

$$N(r) = -\frac{1}{2r^2} + \frac{b_0}{r^2}\sqrt{|l^2J^2 - 4Ml^2r^2 + 4r^4|}, \tag{4.5}$$

$$g(r) = r^3N'(r) = 1 + \frac{2b_0Ml^2\left(2r^2 - \frac{J^2}{M}\right)}{\sqrt{|l^2J^2 - 4Ml^2r^2 + 4r^4|}}. \tag{4.6}$$

One can easily show that these solutions fully satisfy the field equations (2.7) and (2.8). In the limiting case where $b_0 = 0$, the obtained solutions reduce to the $(2+1)$-dimensional rotating black hole solutions of general relativity [7]. When $J = 0$, they restore the solutions (2.11) and (2.12) provided we define $b = 2b_0Ml^2$. It is a matter of calculations to check that $f(r)$ vanishes for two values of r given by

$$r_\pm = l\sqrt{\frac{M}{2}\left(1 \pm \sqrt{1 - \frac{J^2}{M^2l^2}}\right)}, \tag{4.7}$$

where r_+ is the black hole horizon. In order for the solution to describe a black hole, one must have

$$M > 0, \qquad |J| \leq Ml. \tag{4.8}$$

In the extremal case where $|J| = Ml$, the two roots coincide and we have $r_+ = r_- = l\sqrt{M/2}$.

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The surface of infinite redshift can be given by finding the positive real roots of the following equation

$$\mathbf{g}_{tt}(r) = 4b_0^2 r^2(J^2 - M^2 l^2) + M - 2Mb_0\sqrt{l^2J^2 - 4Ml^2r^2 + 4r^4} - \frac{r^2}{l^2} = 0, \tag{4.9}$$

with following solution

$$r_s = l\sqrt{\frac{M(1+2b_0lJ)}{1+4b_0lJ+4b_0^2l^2(J^2-M^2l^2)}}. \tag{4.10}$$

Therefore, the solution admits a surface of infinite redshift, similar to the (2+1)-dimensional rotating black hole of general relativity [15] which has an infinite redshift surface located at $r_s = l\sqrt{M}$. In the limiting case where $b_0 = 0$ the infinite redshift surface coincides with one of Einstein gravity, while for the extremal case ($|J| = Ml$) the infinite redshift surface becomes

$$r_s^{\text{ext}} = l\sqrt{\frac{M(1+2b_0Ml^2)}{1+4b_0Ml^2}}. \tag{4.11}$$

In general the location of r_s^{ext} depends on the value of b_0Ml^2 and it is easy to check that it is located out of horizon, namely $r_s^{\text{ext}} > r_+$ where $r_+ = l\sqrt{M/2}$ is the horizon radius of extremal case.

Next we study the scalar invariants. It is easy to check that as $r \to 0$, the Ricci and Kretschmann invariants behave as

$$\lim_{r\to 0} R = \frac{2[2b_0lJ^2 + 4b_0l^3M^2 - 3J]}{l^2J(1-2b_0lJ)}, \tag{4.12}$$

$$\lim_{r\to 0} R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{4\left[3J^2(1+4b_0^2l^2J^2) - 16b_0^2l^4M^2(J^2-M^2l^2) - 4b_0lJ(J^2+2M^2l^2)\right]}{l^4J^2(1-2b_0lJ)^2}, \tag{4.13}$$

which have finite values unless for the static case ($J = 0$), where both of them diverge at $r = 0$. This is a very interesting result which reveals that in $(2+1)$-dimensional mimetic gravity, adding the angular momentum J to the spacetime removes the curvature singularity at $r = 0$. Indeed for $J = 0$, the Ricci and Kretschmann invariants reduce to (2.17) and (2.18) with replacement $b = 2b_0Ml^2$ and both of them diverge at $r = 0$. On the other hand, in the asymptotic region where $r \to \infty$ we have still $R = -6/l^2$ and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 12/l^4$ for rotating solutions ($J \neq 0$), which confirms that the solutions are asymptotically AdS similar to $(2+1)$-dimensional rotating solutions of general relativity.

Let us remind that for $b_0 = 0$ the curvature invariants have finite values at the origin for both static and rotating solutions, namely $R = -6/l^2$ and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 12/l^4$ [15]. In mimetic gravity, however, we observed that while for static solutions they diverge at $r = 0$, but for rotating solution they have finite values at $r = 0$. In other words, imprint of mimetic field to the spacetime structure in mimetic theory of gravity makes the spacetime singular at the origin $r = 0$, while the combination of angular momentum J and mimetic field remove the curvature singularity at the origin.

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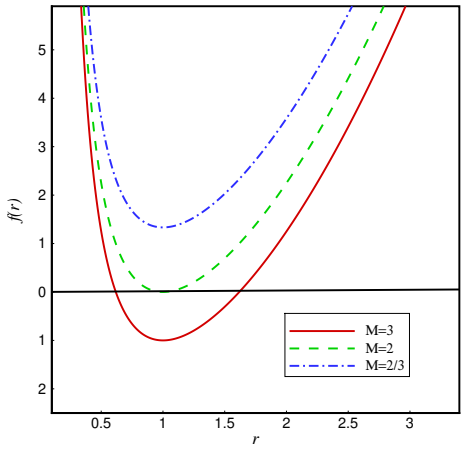

Figure 4. The behavior of $f(r)$ for rotating black hole in 3D with different M, where we have taken $l = 1$ and $J = 2$.

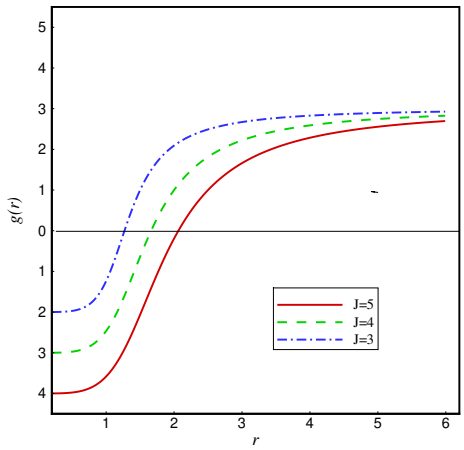

Figure 5. The behavior of $g(r)$ for rotating black hole in 3D with different J, where we have taken $l = 1$, $b_0 = 0.5$ and $M = 2$.

The behavior of the metric functions $f(r)$, $g(r)$ and $N(r)$ for rotating mimetic black holes in three dimensions are plotted in figures 4–8. From figure 4, we see that depending on the metric parameters, our solutions can represent black hole with one horizon, two horizon or naked singularity. Figures 5 and 6 reveal that the metric function $g(r)$ tends to a constant value far from the black hole. The value of this constant depends on b_0, M and l but independent of J. The intersection of all curves in figure 6 is the point in which $g(r) = 1$ which occurs at $r = J/\sqrt{2M}$ independent of b_0. We have also depicted the behavior of $N(r)$ in figures 7 and 8 where it can be seen that $N(r) \to \infty$ as $r \to 0$, while for large values of r we have $N(r) \to 2b_0$.

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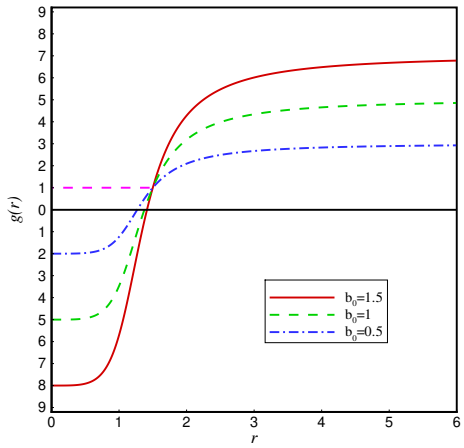

Figure 6. The behavior of $g(r)$ for rotating black hole in 3D with different b_0, where we have taken $l = 1$, $J = 3$ and $M = 2$.

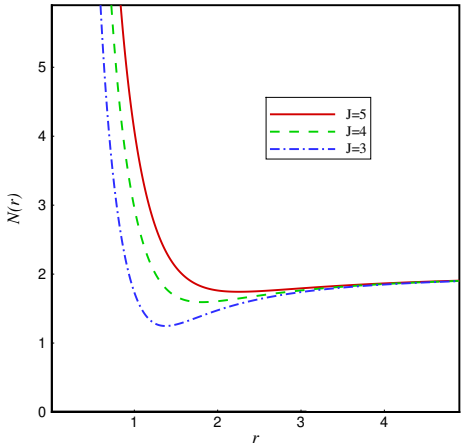

Figure 7. The behavior of $N(r)$ for rotating black hole in 3D with different J, where we have taken $l = b_0 = 1$ and $M = 2$.

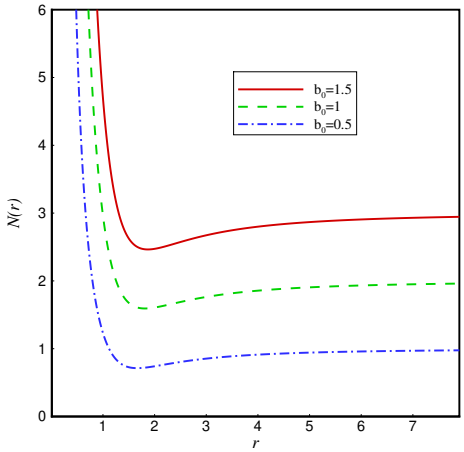

Figure 8. The behavior of $N(r)$ for rotating black hole in 3D with different b_0, where we have taken $l = 1$, $J = 4$ and $M = 2$.

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5 Summary

To sum up, using a scalar mimetic field for isolation the conformal degree of freedom of the gravitational field in a covariant way, it has been demonstrated that the scalar field can encode an extra dynamical longitudinal degree of freedom to the gravitation field which can play the role of mimetic dark matter even in the absence of particle dark matter [34]. In this paper, we have focused on mimetic gravity in $(2+1)$-dimensional spacetime and constructed various static and spinning black hole solutions of this theory. In contrast to the three dimensional solutions of Einstein gravity which has only causal singularity and scalar invariants are constant everywhere, in mimetic gravity the spacetime admits a curvature singularity. We confirmed that both Ricci and Kretschmann curvatures of three dimensional mimetic black holes diverge at $r = 0$ even in the absence of Maxwell field. Interestingly, when the angular momentum is added to the spacetime, the singularity at $r = 0$ disappears and the scalar invariants become constant.

Acknowledgments

I thank Shiraz University Research Council. I am grateful to Max-Planck-Institute for Gravitational Physics (AEI), where this work written and completed, for hospitality. This work has been financially supported by the Research Institute for Astronomy & Astrophysics of Maragha (RIAAM), Iran.

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