$N = 1$ trinification from dimensional reduction of $N = 1$, $10D$ E_8 over $SU(3)/U(1) \times U(1) \times Z_3$ and its phenomenological consequences

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Abstract

We present an extension of the Standard Model that results from the dimensional reduction of the $\mathcal{N} = 1$, $10D$ E_8 group over a $M_4 \times B_0/\mathbf{Z}_3$ space, where B_0 is the nearly-Kähler manifold $SU(3)/U(1) \times U(1)$ and $\mathbf{Z}_3$ is a freely acting discrete group on B_0. Using the Wilson flux breaking mechanism we are left in four dimensions with an $\mathcal{N} = 1$ $SU(3)^3$ gauge theory. Below the unification scale we have a two Higgs doublet model in a split-like supersymmetric version of the Standard Model, which yields third generation quark and light Higgs masses within the experimental limits and predicts the LSP ~ 1500 GeV.

1 Introduction

The origins of our study lie in context of the pioneering work of Forgacs-Manton (F-M) and Scherk-Schwartz (S-S) who studied the Coset Space Dimensional Reduction (CSDR) [1–3] and the group manifold reduction [4], respectively. Roughly, these mechanisms share a common view with another (almost) contemporary framework to them, that of the superstring theories [5], in the sense that they result with GUTs which originate from a spacetime that is extra-dimensional, as, in particular, in the heterotic string [6]. The two approaches found contact in the sense that CSDR incorporated the predictions of the heterotic string, that is the number of extra dimensions and the gauge group of the initial theory. In both, S-S and F-M, mechanisms, in the higher-dimensional theory, the gauge

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and scalar sector are unified and, specifically in the CSDR case, fermions of the higher-dimensional theory lead to Yukawa interactions in the $4D$ one. Also, it is remarkable that it can lead to $4D$ chiral theories [7]. An additional important property of the CSDR breaks the original supersymmetry of a theory, either completely when is reduced over symmetric cosets or softly in the case of the $6D$ non-symmetric [8] ones which are all nearly-Kähler manifolds admitting a connection with torsion [9–11].

Performing the dimensional reduction of an $\mathcal{N}=1$ supersymmetric gauge theory, a very important and desired property is the amount of supersymmetry of the initial theory to be preserved in the $4D$ one. In the present letter we re-examine the dimensional reduction of E_8 over $SU(3)/U(1)\times U(1)\times \mathbf{Z}_3$, where the latter is the non-symmetric coset space $SU(3)/U(1)\times U(1)$ equipped with the freely acting discrete symmetry $\mathbf{Z}_3$ in order that the Wilson flux breaking mechanism to get induced for further reduction of the gauge symmetry of the $4D$ GUT, specifically to $SU(3)^3$ along with two $U(1)$ global symmetries [2,8,9,12] (see also [13]). The potential of the resulting $4D$ theory contains terms that can be identified as F-, D- and soft breaking terms, which means that the resulting theory is a (broken) $\mathcal{N}=1$ supersymmetric theory.

In our case, the compactification and unification scales coincide, leading to a split-like supersymmetry scenario in which some supersymmetric particles are superheavy, while others obtain mass in the TeV region. After the employment of the spontaneous symmetry breaking of the GUT, the model can be viewed as a two Higgs doublet model (2HDM) which is phenomenologically consistent, since it produces masses of the light Higgs boson and the top and the bottom quarks within the experimental range.

2 Dimensional Reduction of E_8 over $SU(3)/U(1)\times U(1)$

In this section we focus directly on the application of the CSDR scheme in which we are interested. For a more complete picture of the geometry of coset spaces see [2,14]. Also, for the main aspects of the CSDR, the generalized methodology of the reduction and the treatment of the constraints, see ref. [2].

Let us now demonstrate an illustrative example of the CSDR scheme, that is the case of an $\mathcal{N}=1$ supersymmetric E_8 YM theory, which undergoes a dimensional reduction over the non-symmetric coset space $SU(3)/U(1)\times U(1)$ [2,8,13]. The $4D$ YM action is:

$$S = C\int d^4x\,\mathrm{tr}\left[-\frac{1}{8}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}(D_\mu\phi_a)(D^\mu\phi^a)\right]+V(\phi)+\frac{i}{2}\bar{\psi}\Gamma^\mu D_\mu\psi-\frac{i}{2}\bar{\psi}\Gamma^a D_a\psi\,, \quad (1)$$

where it has been identified:

$$V(\phi) = -\frac{1}{8}g^{ac}g^{bd}\mathrm{tr}\left(f_{ab}{}^{C}\phi_C - ig[\phi_a,\phi_b]\right)\left(f_{cd}{}^{D}\phi_D - ig[\phi_c,\phi_d]\right) \quad (2)$$

and $\mathrm{tr}(T^iT^j)=2\delta^{ij}$, where T^i are the generators of the gauge group, C is the volume of the coset, $D_\mu=\partial_\mu - igA_\mu$ is the $4D$ covariant derivative, D_a is that of the coset and the coset metric is given (in terms of its radii) by $g_{\alpha\beta}=\mathrm{diag}(R_1^2,R_1^2,R_2^2,R_2^2,R_3^2,R_3^2)$.

The $4D$ gauge group is determined by the way the $R = U(1) \times U(1)$ is embedded in E_8 and is obtained by the centralizer of $R = U(1) \times U(1)$ in $G = E_8$, that is:

$$H = C_{E_8}(U(1)_A \times U(1)_B) = E_6 \times U(1)_A \times U(1)_B \,. \tag{3}$$

Moreover, solving the constraints, the scalar and fermion fields that remain in the $4D$ theory are obtained by the decomposition of the representation 248 -adjoint representation- of E_8 under $U(1)_A \times U(1)_B$. Also, in order to result with the representations of the surviving fields of the $4D$ theory, it is necessary to examine the decompositions of the vector and spinor representations of $SO(6)$ under $R = U(1)_A \times U(1)_B$ (details in [2,8,12]). Therefore, the CSDR rules imply that the surviving gauge fields (those of $E_6 \times U(1)_A \times U(1)_B$) are accommodated in three $\mathcal{N} = 1$ vector supermultiplets in the $4D$ theory. Also, the matter fields of the $4D$ theory end up in six chiral multiplets. Three of them are E_6 singlets carrying $U(1)_A \times U(1)_B$ charges, while the rest are chiral multiplets. The unconstrained fields transforming under $E_6 \times U(1)_A \times U(1)_B$ are:

$$\alpha_i \sim 27_{(3,\frac{1}{2})}, \quad \beta_i \sim 27_{(-3,\frac{1}{2})}, \quad \gamma_i \sim 27_{(0,-1)}, \quad \alpha \sim 1_{(3,\frac{1}{2})}, \quad \beta \sim 1_{(-3,\frac{1}{2})}, \quad \gamma \sim 1_{(0,-1)}$$

and the scalar potential of the theory is:

$$\begin{aligned}
\frac{2}{g^2}V(\alpha^i,\alpha,\beta^i,\beta,\gamma^i,\gamma) &= \frac{2}{5}\left(\frac{1}{R_1^4}+\frac{1}{R_2^4}+\frac{1}{R_3^4}\right) \\
&+\left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\alpha^i\alpha_i+\left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\bar{\alpha}\alpha \\
&+\left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\beta^i\beta_i+\left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\bar{\beta}\beta \\
&+\left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\gamma^i\gamma_i+\left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\bar{\gamma}\gamma \\
+\Bigg[\sqrt{2}80\left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)&d_{ijk}\alpha^i\beta^j\gamma^k+\sqrt{2}80\left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)\alpha\beta\gamma+h.c\Bigg] \\
&+\frac{1}{6}\left(\alpha^i(G^\alpha)_i^j\alpha_j+\beta^i(G^\alpha)_i^j\beta_j+\gamma^i(G^\alpha)_i^j\gamma_j\right)^2 \\
&+\frac{10}{6}\left(\alpha^i(3\delta_i^j)\alpha_j+\bar{\alpha}(3)\alpha+\beta^i(-3\delta_i^j)\beta_j+\bar{\beta}(-3)\beta\right)^2 \\
&+\frac{40}{6}\left(\alpha^i(\tfrac{1}{2}\delta_i^j)\alpha_j+\bar{\alpha}(\tfrac{1}{2})\alpha+\beta^i(\tfrac{1}{2}\delta_i^j)\beta_j+\bar{\beta}(\tfrac{1}{2})\beta+\gamma^i(-1\delta_i^j)\gamma^j+\bar{\gamma}(-1)\gamma\right)^2 \\
&+40\alpha^i\beta^jd_{ijk}d^{klm}\alpha_l\beta_m+40\beta^i\gamma^jd_{ijk}d^{klm}\beta_l\gamma_m+40\alpha^i\gamma^jd_{ijk}d^{klm}\alpha_l\gamma_m \\
&+40(\bar{\alpha}\bar{\beta})(\alpha\beta)+40(\bar{\beta}\bar{\gamma})(\beta\gamma)+40(\bar{\gamma}\bar{\alpha})(\gamma\alpha)\,,
\end{aligned}$$

being also positive definite. In the above expression of the scalar potential, the $F-, D-$ and soft supersymmetry breaking terms are identified. The F-terms emerge from the superpotential:

$$\mathcal{W}(A^i,B^j,C^k,A,B,C) = \sqrt{40}d_{ijk}A^iB^jC^k + \sqrt{40}ABC\,, \tag{4}$$

while the D-terms are structured as:

$$\frac{1}{2}D^\alpha D^\alpha + \frac{1}{2}D_1 D_1 + \frac{1}{2}D_2 D_2\,, \tag{5}$$

where the D quantities are calculated as:

$$\begin{aligned}
D^\alpha &= \frac{1}{\sqrt{3}}\Big(\alpha^i (G^\alpha)_i^j \alpha_j + \beta^i (G^\alpha)_i^j \beta_j + \gamma^i (G^\alpha)_i^j \gamma_j\Big),\\
D_1 &= \sqrt{\frac{10}{3}}\Big(\alpha^i (3\delta_i^j)\alpha_j + \bar{\alpha}(3)\alpha + \beta^i(-3\delta_i^j)\beta_j + \bar{\beta}(-3)\beta\Big)\\
D_2 &= \sqrt{\tfrac{40}{3}}\Big(\alpha^i(\tfrac{1}{2}\delta_i^j)\alpha_j + \bar{\alpha}(\tfrac{1}{2})\alpha + \beta^i(\tfrac{1}{2}\delta_i^j)\beta_j + \bar{\beta}(\tfrac{1}{2})\beta + \gamma^i(-1\delta_i^j)\gamma_j + \bar{\gamma}(-1)\gamma\Big)\,.
\end{aligned}$$

Apart from the terms of the potential of Eq. (4) identified as F- and D- terms, the remaining ones admit the interpretation of soft scalar masses and trilinear soft terms. The gaugino demonstrates a special behaviour compared to the rest soft supersymmetric terms, as can be seen in the following relation[1]:

$$M = (1+3\tau)\frac{R_1^2 + R_2^2 + R_3^2}{8\sqrt{R_1^2 R_2^2 R_3^2}}\,. \tag{6}$$

The next step is the minimization of the potential, which requires at least two of the three singlets to acquire vevs at the compactification scale. For the purposes of our current work we choose the singlets α and β to acquire vevs, while γ remains massless. This results in the breaking of the two $U(1)$, reducing our gauge group from $E_6 \times U(1)_A \times U(1)_B$ to E_6. The two abelian groups remain, however, as global symmetries, which will be very useful in conserving Baryon number, as it will be discussed below.

3 Breaking by Wilson Flux mechanism

In the above section we presented the case in which the CSDR scheme is applied on a higher-dimensional E_8 gauge theory which is reduced over an $SU(3)/U(1)\times U(1)$ coset space and leads to a $4D$ E_6 gauge theory. However, the E_6 group cannot be broken exclusively by the presence of the 27 Higgs multiplet. For this reason, that is to reduce the resulting gauge symmetry, the Wilson flux breaking mechanism is employed [15–17]. The below procedure can be found in detail in [12].

3.1 $SU(3)^3$ produced by Wilson flux

The Wilson flux breaking mechanism, projects the theory in such a way that the surviving fields are those which remain invariant under the action of the freely acting discrete

[1] The relation shows that the gauginos naturally obtain mass at the compactification scale [2]. This, however, is prevented by the inclusion of the torsion [8], which is the case in the following construction.

symmetry, the $\mathbf{Z}_3$, on their gauge and geometric indices. The non-trivial action of the $\mathbf{Z}_3$ group on the gauge indices of the various fields is parametrized by the matrix [18]:

$$\gamma_3 = \mathrm{diag}\{\mathbf{1}_3, \omega\mathbf{1}_3, \omega^2\mathbf{1}_3\}\,, \tag{7}$$

where $\omega = e^{i\frac{2\pi}{3}}$. The latter acts on the gauge fields of the E_6 gauge theory and a non-trivial phase acts on the matter fields. First, the gauge fields that pass through the filtering of the projection are those which satisfy the condition:

$$[A_M, \gamma_3] = 0 \quad \Rightarrow A_M = \gamma_3 A_M \gamma_3^{-1} \tag{8}$$

and the remaining gauge symmetry is $SU(3)_c \times SU(3)_L \times SU(3)_R$. The matter counterpart of Eq. (8) for is:

$$\vec{\alpha} = \omega\gamma_3\vec{\alpha},\;\; \vec{\beta} = \omega^2\gamma_3\vec{\beta},\;\; \vec{\gamma} = \omega^3\gamma_3\vec{\gamma}\,, \qquad \alpha = \omega\alpha,\;\; \beta = \omega^2\beta,\;\; \gamma = \omega^3\gamma\,. \tag{9}$$

where $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are the matter superfields which belong to the 27 representation and α, β, γ the singlets that only carry $U(1)_{A,B}$ charges. The representations of the remnant group, $SU(3)_c \times SU(3)_L \times SU(3)_R$, in which the above fields are accommodated, are obtained after considering the decomposition rule of the 27 representation of E_6 under the new group, $(1, 3, \bar{3}) \oplus (\bar{3}, 1, 3) \oplus (3, \bar{3}, 1)$. Therefore, in the projected theory we are left with the following matter content:

$$\alpha_3 \equiv \Psi_1 \sim (\bar{3}, 1, 3)_{(3,\frac{1}{2})}, \quad \beta_2 \equiv \Psi_2 \sim (3, \bar{3}, 1)_{(-3,\frac{1}{2})}, \quad \gamma_1 \equiv \Psi_3 \sim (1, \bar{3}, 3)_{(0,-1)}, \quad \gamma \equiv \theta_{(0,-1)},$$

where the three former are the leftovers of $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ and together they form a 27 representation of E_6, that means that the leftover content can be identified as one generation. In order to obtain a spectrum consisting of three generations, one may introduce non-trivial monopole charges in the $U(1)$s in R, resulting in a total of three replicas of the above fields (where an index $l = 1, 2, 3$ can be used to specify each of the three families).

The scalar potential of the E_6 (plus the global abelian symmetries) that was obtained after the dimensional reduction of E_8, Eq. (4), can now (that is after the adoption of the Wilson flux breaking mechanism and the projection of the theory) be rewritten in the $SU(3)_c \times SU(3)_L \times SU(3)_R$ language as [12]:

$$V_{sc} = 3 \cdot \frac{2}{5}\Big(\frac{1}{R_1^4} + \frac{1}{R_2^4} + \frac{1}{R_3^4}\Big) + \sum_{l=1,2,3} V^{(l)}\,, \tag{10}$$

in which:

$$V^{(l)} = V_{susy} + V_{soft} = V_D + V_F + V_{soft}\,. \tag{11}$$

From now on, we give up on the generation superscript (l), since our analysis will be focused on the third generation, and it will only be written explicitly when required.

Regarding the D and F-terms, they are identified as:

$$V_D = \frac{1}{2}\sum_A D^A D^A + \frac{1}{2}D_1 D_1 + \frac{1}{2}D_2 D_2, \tag{12}$$

$$V_F = \sum_{i=1,2,3} |F_{\Psi_i}|^2 + |F_\theta|^2, \quad F_{\Psi_i} = \frac{\partial \mathcal{W}}{\partial \Psi_i}, \quad F_\theta = \frac{\partial \mathcal{W}}{\partial \theta}\,, \tag{13}$$

where the F-terms derive from the expression:

$$\mathcal{W} = \sqrt{40} d_{abc} \Psi_1^a \Psi_2^b \Psi_3^c\,, \tag{14}$$

while the D-terms are written explicitly as:

$$D^A = \frac{1}{\sqrt{3}} \langle \Psi_i | G^A | \Psi_i \rangle, \tag{15}$$

$$D_1 = 3\sqrt{\frac{10}{3}} (\langle \Psi_1 | \Psi_1 \rangle - \langle \Psi_2 | \Psi_2 \rangle), \tag{16}$$

$$D_2 = \sqrt{\frac{10}{3}} (\langle \Psi_1 | \Psi_1 \rangle + \langle \Psi_2 | \Psi_2 \rangle - 2\langle \Psi_3 | \Psi_3 \rangle - 2|\theta|^2)\,. \tag{17}$$

Last, the soft supersymmetry breaking terms are written down as:

$$\begin{aligned} V_{soft} = & \left(\frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2}\right) \langle \Psi_1 | \Psi_1 \rangle + \left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2}\right) \langle \Psi_2 | \Psi_2 \rangle \\ & + \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2}\right) (\langle \Psi_3 | \Psi_3 \rangle + |\theta|^2) \\ & + 80\sqrt{2} \left(\frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_1 R_2}\right) (d_{abc} \Psi_1^a \Psi_2^b \Psi_3^c + h.c) \end{aligned} \tag{18}$$

$$= m_1^2 \langle \Psi_1 | \Psi_1 \rangle + m_2^2 \langle \Psi_2 | \Psi_2 \rangle + m_3^2 \Big(\langle \Psi_3 | \Psi_3 \rangle + |\theta|^2 \Big) + (\alpha_{abc} \Psi_1^a \Psi_2^b \Psi_3^c + h.c)\,. \tag{19}$$

The $(G^A)_a{}^b$ are the structure constants of the $SU(3)_c \times SU(3)_L \times SU(3)_R$ and therefore antisymmetric in a and b. According to ref. [19], the vectors of the 27 of E_6 can be written in a more convenient form in the $SU(3)_c \times SU(3)_L \times SU(3)_R$ language, that is in complex 3×3 matrices. Identification of:

$$\Psi_1 \sim (\bar{3}, 1, 3) \to (q^c)_p{}^\alpha, \quad \Psi_2 \sim (3, \bar{3}, 1) \to (Q_\alpha{}^a), \quad \Psi_3 \sim (1, 3, \bar{3}) \to L_a{}^p\,, \tag{20}$$

leads to the following relabeling and assignment of the particle content of the MSSM (and more) in the above representation of the model:

$$q^c = \begin{pmatrix} d_R^{c1} & u_R^{c1} & D_R^{c1} \\ d_R^{c2} & u_R^{c2} & D_R^{c2} \\ d_R^{c3} & u_R^{c3} & D_R^{c3} \end{pmatrix}, \quad Q = \begin{pmatrix} -d_L^1 & -d_L^2 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 \\ D_L^1 & D_L^2 & D_L^3 \end{pmatrix}, \quad L = \begin{pmatrix} H_d^0 & H_u^+ & \nu_L \\ H_d^- & H_u^0 & e_L \\ \nu_R^c & e_R^c & S \end{pmatrix}.$$

It is evident from the above that $d_{L,R}, u_{L,R}, D_{L,R}$ transform as $3, \bar{3}$ under the colour group.

4 Selection of parameters and GUT breaking

With the above-mentioned theoretical framework fully in place, it is time to specify the compactification scale of the theory, as well as other (resulting) quantities, in order to proceed to phenomenology.

4.1 Choice of radii

We will examine the case where the compactification scale is high[2], and more specifically $M_C = M_{GUT}$. Thus for the radii we have $R_l \sim \frac{1}{M_{GUT}}$, $l = 1, 2, 3$.

Without any special treatment, this results in soft trilinear couplings and soft scalar masses around M_{GUT}. However, we can select our third radius slightly different than the other two in a way that yields:

$$m_3^2 \sim -\mathcal{O}(TeV^2), \quad m_{1,2}^2 \sim -\mathcal{O}(M_{GUT}^2), \quad a_{abc} \gtrsim M_{GUT} . \tag{21}$$

In other words, we have supermassive squarks and TeV-scaled sleptons. Thus, supersymmetry is softly broken already at the unification scale, in addition to its breaking by both D-terms and F-terms.

4.2 Further gauge symmetry breaking of $SU(3)^3$

The spontaneous breaking of the $SU(3)_L$ and $SU(3)_R$ can be triggered by the following vevs of the two families of L's.

$$\langle L_s^{(3)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & V \end{pmatrix}, \quad \langle L_s^{(2)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V & 0 & 0 \end{pmatrix} ,$$

where the s index denotes the scalar component of the multiplet. These vevs are singlets under $SU(3)_c$, so they leave the colour group unbroken. If we use only $\langle L_s^{(3)} \rangle$ we get the breaking

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1) , \tag{22}$$

while if we use only $\langle L_s^{(2)} \rangle$ we get the breaking

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times SU(2)'_R \times U(1)' . \tag{23}$$

Their combination gives the desired breaking [20]:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times U(1)_Y . \tag{24}$$

[2]In this case Kaluza-Klein excitations are irrelevant. Otherwise one would need the eigenvalues of the Dirac and Laplace operators in the $6D$ compactification space.

The configuration of the scalar potential just after the breaking gives vevs to the singlet of each family (not necessarily to all three). In our case we have $\langle\theta^{(3)}\rangle \sim \mathcal{O}(TeV)$, $\langle\theta^{(1,2)}\rangle \sim \mathcal{O}(M_{GUT})$.
Electroweak (EW) breaking then proceeds by the vevs [21]:

$$\langle L_s^{(3)}\rangle = \begin{pmatrix} \upsilon_d & 0 & 0 \\ 0 & \upsilon_u & 0 \\ 0 & 0 & 0 \end{pmatrix} .$$

4.3 Lepton Yukawa couplings and μ terms

Although the two $U(1)$s were already broken before the Wilson flux breaking, they still impose global symmetries. As a result, in the lepton sector we cannot have invariant Yukawa terms. However, below the unification scale, an effective term can occur from higher-dimensional operators [12]:

$$L\overline{e}H_d\left(\frac{\overline{K}}{M}\right)^3 , \tag{25}$$

where $\overline{K}$ is the vacuum expectation value of the conjugate scalar component of either $S^{(i)}$, $\nu_R^{(i)}$ or $\theta^{(i)}$, or any combination of them, with or without mixing of flavours. Using similar arguments, one can also have mass terms for $S^{(i)}$ and $\nu_R^{(i)}$, which will then be rendered supermassive.

Another much needed quantity that is missing from our model is the μ term, one for each family of Higgs doublets. In the same way, we can have:

$$H_u^{(i)}H_d^{(i)}\overline{\theta}^{(i)}\frac{\overline{K}}{M} . \tag{26}$$

The first two generations of Higgs doublets will then have supermassive μ terms, while the μ term of the third generation will be at the TeV scale.

In order to avoid confusion, it is useful to sum up the scale of some important parameters in Table 1.

Parameter	Scale
soft trilinear couplings	$\mathcal{O}(GUT)$
squark masses	$\mathcal{O}(GUT)$
slepton masses	$\mathcal{O}(TeV)$
$\mu^{(3)}$	$\mathcal{O}(TeV)$
$\mu^{(1,2)}$	$\mathcal{O}(GUT)$
unified gaugino mass M_U	$\mathcal{O}(TeV)$

Table 1: Approximate scale of parameters.

5 Phenomenological Analysis

Like every GUT, this model considers all gauge couplings to start as one coupling g at M_{GUT}. However, since at the E_8 level there is only one coupling, it is clear that the (quark) Yukawa couplings are equal to g at M_{GUT} as well. This makes the selection of a large $tan\beta$ necessary. We use the unified coupling g as a boundary condition for all the above-mentioned couplings at M_{GUT}.

In our analysis we will use 1-loop beta functions for all parameters included. Below the unification scale they run according the RGEs of the MSSM (squarks included) plus the 4 additional Higgs doublets (and their supersymmetric counterparts) that come from the two extra L multiplets of the first and second generations, down to an intermediate scale M_{int}. Below this scale, all supermassive particles and parameters are considered decoupled, and the RGEs used include only the 2 Higgs doublets that originate from the third generation (and their respective Higgsinos), the sleptons and the gauginos. Finally, below a second intermediate scale that we call M_{TeV}, we run the RGEs of a non-supersymmetric 2HDM.

5.1 Constraints

In our analysis we apply several experimental constraints, which we briefly review in this subsection.
Starting from the strong gauge coupling, we use the experimental value [22]:

$$a_s(M_Z) = 0.1187 \pm 0.0016\,. \tag{27}$$

We calculate the top quark pole mass, while the bottom quark mass is evaluated at M_Z, in order not to induce uncertainties that are inherent to its pole mass. Their experimental values are [22]:

$$m_t^{\rm exp} = (172.4 \pm 0.7)~{\rm GeV}\,, \qquad m_b(M_Z) = 2.83 \pm 0.10~{\rm GeV}\ . \tag{28}$$

We interpret the Higgs-like particle discovered in July 2012 by ATLAS and CMS [23] as the light $\mathcal{CP}$-even Higgs boson of the supersymmetric SM. The (SM) Higgs boson experimental average mass is [22]:

$$M_H^{\rm exp} = 125.10 \pm 0.14~{\rm GeV}\ . \tag{29}$$

5.2 Gauge unification

A first challenge for each unification model is to predict a unification scale, while maintaining agreement with experimental constraints on gauge couplings. The 1-loop gauge β fuctions are given by:

$$2\pi\beta_i = b_i\alpha_i^2\ , \tag{30}$$

where for the three energy regions the b coefficients are given in Table 2.

Scale	b_1	b_2	b_3
M_{EW}-M_{TeV}	$\frac{21}{5}$	-3	-7
M_{TeV}-M_{int}	$\frac{11}{2}$	$-\frac{1}{2}$	-5
M_{int}-M_{GUT}	$\frac{39}{5}$	3	-3

Table 2: b coefficients for gauge RGEs.

The $a_{1,2}$ determine the unification scale and the a_3 is used to confirm that unification is indeed possible. Using a 0.3% uncertainty at the unification scale boundary, we predict the different scales of our model (shown on Table 3), while the strong coupling is predicted within 2σ of the experimental value (Eq. (27)):

$$a_s(M_Z) = 0.1218 \ . \tag{31}$$

It should be noted that although the unification scale is somewhat lower than expected in a supersymmetric theory, there is no fear of fast proton decay, as the $U(1)_A$ remaining global symmetry can be immediately recognised as:

$$U(1)_A = -\frac{1}{9}B \ , \tag{32}$$

where B is the baryon number. Therefore, the unification scale could, in principle, lie even lower without such problems.

Scale	GeV
M_{GUT}	$\sim 1.7 \times 10^{15}$
M_{int}	$\sim 9 \times 10^{13}$
M_{TeV}	~ 1500

Table 3: Scale predicted by gauge unification.

5.3 Higgs potential

We once again turn our focus on the third family. After GUT breaking, the Higgs scalar potential calculated from the D-, F- and soft terms of Sect. 3.1 is given by:

$$\begin{aligned} V_{Higgs} =& \Big(3|\mu^{(3)}|^2 + m_3^2\Big)\Big(|H_d^0|^2 + |H_d^-|^2\Big) + \Big(3|\mu^{(3)}|^2 + m_3^2\Big)\Big(|H_u^0|^2 + |H_u^+|^2\Big) \\ &+ b^{(3)}\Big[(H_u^+ H_D^- - H_u^0 H_D^0) + c.c.\Big] \\ &+ \frac{10}{3}g^2\Big[|H_d^0|^4 + |H_d^-|^4 + |H_u^0|^4 + |H_u^+|^4 + \\ &\qquad 2|H_d^0|^2|H_d^-|^2 + 2|H_d^-|^2|H_u^0|^2 + 2|H_d^0|^2|H_u^+|^2 + 2|H_u^0|^2|H_u^+|^2\Big] \\ &+ \frac{80}{3}g^2\Big[|H_d^0|^2|H_u^0|^2 + |H_d^-|^2|H_u^+|^2\Big] - 20g^2\Big[\overline{H_d^0}H_d^-\overline{H_u^0}H_u^+ + c.c.\Big] \ , \end{aligned} \tag{33}$$

where g is the gauge coupling at the unification scale and it is understood that the RG running has not yet taken place. One can easily compare the above potential with the standard 2 Higgs doublet scalar potential [24–26] and identify:

$$\lambda_1 = \lambda_2 = \lambda_3 = \frac{20}{3}g^2 \ , \qquad \lambda_4 = 20g^2 \ , \qquad \lambda_5 = \lambda_6 = \lambda_7 = 0 \tag{34}$$

The above relations are used as boundary conditions at GUT scale, then all the Higgs couplings run using their RGEs (see [27] for the full expressions), which in turn change appropriately for each energy interval explained above.

5.4 1-loop results

The Higgs couplings λ_i are evolved from the GUT scale down to the EW scale together with the gauge couplings, the top, bottom and tau Yukawas, all at 1 loop. It is useful to remind the reader that all gauge and quark Yukawa couplings use g as boundary condition, while the tau Yukawa emerges from a higher-dimensional operator and has significantly wider freedom. We use the standard tau lepton mass [22] as an input.

We consider uncertainties on the two important boundaries we consider, namely M_{GUT} and M_{TeV}, because of threshold corrections (for a more comprehensive discussion see [28]). For simplicity we have considered degeneracy between all supersymmetric particles that acquire masses at the TeV scale. The uncertainty of the top and bottom Yukawa couplings on the GUT boundary is taken to be 6%, while on the TeV boundary is taken to be 2%. For $\lambda_{1,2}$ the uncertainty is 8% on both boundaries and for $\lambda_{3,4}$ is 7% at GUT and 5% at TeV.

Both top and bottom quark masses are predicted within 2σ of their experimental values (Eq. (28)):

$$m_b(M_Z) = 3.00\ GeV \ , \qquad \hat{m}_t = 171.6\ GeV \ , \tag{35}$$

while the light Higgs boson mass is predicted within 1σ of Eq. (29):

$$m_h = 125.18\ GeV \ . \tag{36}$$

The model features a large $tan\beta \sim 48$. This is necessary, since the Yukawas begin from the same value at the GUT boundary, so a large difference between the two vevs is needed to reproduce the known fermion hierarchy. The pseudoscalar Higgs boson is considered to have mass between $700 - 3000\ GeV$.

The above 1-loop calculation could be subject to larger uncertainties, since they lack the precision of a higher-loop analysis. The prediction of the full (light) supersymmetric spectrum, a 2-loop analysis of the model, the application of more experimental constraints (i.e. B-physics observables) and its discovery potential at present and/or future colliders are planned for future work [29].

6 Conclusions

Starting from an $\mathcal{N}=1$, $10D$ E_8 Yang-Mills theory, we consider a compactified spacetime $M_4 \times B_0/\mathbf{Z}_3$, where B_0 is the non-symmetric manifold $SU(3)/U(1)\times U(1)$ and $\mathbf{Z}_3$ is a freely acting discrete group on B_0. Then we reduce dimensionally the E_8 on this manifold and we employ the Wilson flux mechanism leading in four dimensions to an $\mathcal{N}=1$ $SU(3)^3$ gauge theory. We consider the compactification scale to match the unification scale, a choice that results in a split-like SUSY scenario, where gauginos, Higgsinos (of the third generation) and sleptons all acquire masses at $\sim 1500\ GeV$, and the rest supesymmetric spectrum is superheavy ($\sim M_{GUT}$). The global $U(1)_A$ conserves Baryon number, a fact which allows for the predicted unification scale $\sim 10^{15} GeV$. The 2HDM employed below GUT predicts a light Higgs boson mass within the experimental limits, while the top and bottom quark masses are also in (2σ) agreement with experimental measurements.

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