

News from horizons in binary black hole mergers

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In a binary black hole merger, it is known that the inspiral portion of the waveform corresponds to two distinct horizons orbiting each other, and the merger and ringdown signals correspond to the final horizon being formed and settling down to equilibrium. However, we still lack a detailed understanding of the relation between the horizon geometry in these three regimes and the observed waveform. Here we show that the well known inspiral chirp waveform has a clear counterpart on black hole horizons, namely, the shear of the outgoing null rays at the horizon. We demonstrate that the shear behaves very much like a compact binary coalescence waveform with increasing frequency and amplitude. Furthermore, the parameters of the system estimated from the horizon agree with those estimated from the waveform. This implies that even though black hole horizons are causally disconnected from us, assuming general relativity to be true, we can potentially infer some of their detailed properties from gravitational wave observations. [This document has been assigned the LIGO Preprint number ligo-p2000098.]

Introduction: Starting with the first binary black hole detection in 2015 [1], at least 10 binary black hole mergers have been observed to date [2–7]. For all of these detections, the parameters of the binary system, including the masses and spins of the individual black holes, can be inferred from the observed data [8]. This inference relies crucially on gravitational waveform models meant to represent, with sufficient accuracy, the gravitational wave emission from binary black hole mergers in general relativity [9–11]. Within general relativity, we also have detailed information about properties of curved spacetime around a black hole merger from numerical simulations. Since the first successful merger simulations [12–14], it is now relatively straightforward to evolve black hole binaries through the inspiral, merger and ringdown regimes, at least for moderate mass ratios. Indeed, the waveform models mentioned above are all based on, and ultimately verified by, comparisons with these numerical simulations. The wealth of information contained in the full numerically generated binary black hole spacetimes might plausibly have some imprints in the observed gravitational wave signal.

One might in fact be able to infer properties of spacetime regions hidden behind the event horizon and causally disconnected from us. The signal received at our observatories is generated by the non-linearities and dynamics of the spacetime metric near and around the black holes. These non-linear and dynamical fields are responsible for both the signal seen by us, and also the properties of spacetime inside the event horizon [15–18]. The infalling flux of gravitational waves, representing tidal coupling, is of course part of the energy balance govern-

ing the dynamics of the binary system. Thus, any modifications of the infalling flux will also affect the overall dynamics of the system. Furthermore, in situations when the black holes are spinning sufficiently rapidly, phenomena like superradiance can play an important role (see e.g. [19]). In the regime when the effect of the companion can be treated as a perturbation, these tidal effects can be calculated analytically [20–27]. Here we shall go beyond these calculations, close to the merger where linear perturbation theory is not sufficient. Note also that all of these perturbative calculations refer to the event horizon for which, as a matter of principle, no generally valid non-perturbative quasi-local flux formula can exist. As in almost all such numerical studies, we always work with dynamical horizons which are expected to differ significantly from the event horizon near the merger, and do not suffer from the teleological properties of the event horizon. Moreover, exact flux formulae, valid in full non-linear general relativity, are known for dynamical horizons (see e.g. [28, 29]).

Besides the effects on the orbital motion through tidal coupling, the infalling radiation must be very special for other reasons. The remnant black hole horizon is highly distorted on formation, and it loses its hair to reach its final equilibrium state represented by a Kerr black hole. However, the horizon, being a one-way membrane, cannot “radiate away” its hair. Instead, it approaches its very special final state by absorbing just the right amount of infalling radiation that precisely cancels any hair that it might have when it is initially formed [18]. Thus, the infalling radiation also determines the highly non-trivial issue of the final state within general relativity. The im-

portant question then is this: Assuming that general relativity holds, are there strong correlations between the observed gravitational wave signal, and suitable fields on black hole horizons? If we have a sufficiently detailed understanding of these correlations, we might be able to understand fundamental properties of dynamical strong field gravity and black hole horizons from gravitational wave observations.

The presence of such correlations might be difficult to discern in analytic or numerical studies. The relevant spacetime regions could not be more different: The spacetime we inhabit is very close to flat and extremely well described by linearized general relativity. On the other hand, the region where black hole horizons live could have very high curvature (depending on the mass of the black hole), and non-linearities of the Einstein equations need to be taken into account. It is thus not immediately obvious precisely which fields should be correlated in a gauge invariant manner, and how mathematical results might be proved.

Despite these potential pitfalls, several authors have previously found evidence for correlations between quantities on horizons and in the wave-zone [15, 17, 18, 30]. These works have considered either the post-merger regime or subtleties regarding gravitational wave recoil. Thus far none have considered what is one of the most well known features of binary merger waveforms, namely the inspiral chirp with increasing frequency and amplitude. The evolution of the frequency and amplitude have been calculated to high orders in various post-Newtonian approaches, while accounting for a variety of physical effects such as precession and eccentricity (see, e.g., [31]). Moreover, these post-Newtonian calculations have been combined with numerical relativity merger signals to construct complete waveform models including the inspiral, merger and ringdown regimes as well [32–38]. We shall use these complete waveform models to quantitatively compare gravitational wave signals with horizon fields.

It has been shown previously [18] that for the post-merger signal, the gravitational wave News (in essence the time derivative of the gravitational wave strain), is correlated with the shear on the black hole horizons. Here we will extend this study to the inspiral regime, and show quantitatively that the chirp signal is also extremely well correlated with the News. Remarkably, for reasons that we do not yet fully understand, we shall see that very little effort is required to extract these correlations, and the gauge conditions employed in the simulations do not seem to play an important role.

Basic notions: Our results deal with two surfaces. The first is future null infinity \mathcal{I}^+ , the end point of future null-geodesics which escape to infinity [39, 40]. The second is a dynamical horizon \mathcal{H} [28, 29] obtained by a time evolution of marginally trapped surfaces. These two surfaces might seem initially to be very different. Future null infinity \mathcal{I}^+ is an invariantly defined null surface where out-

going null geodesics end. On the other hand, a dynamical horizon is located inside the event horizon. Nevertheless, both \mathcal{I}^+ and \mathcal{H} are one-way membranes and exact flux formulae hold for both surfaces.

For both cases, we consider spacelike 2-surfaces S of spherical topology, with an intrinsic Riemannian metric q_{ab} . S will be either a cross section of \mathcal{I}^+ (approximated as a large coordinate sphere in the wave-zone enclosing the source), or a section of \mathcal{H} , i.e. a marginally trapped surface. In either case, assuming that we can assign outgoing and ingoing directions, we denote the outgoing future directed null vector normal to S by ℓ^a , and the ingoing null normal as n^a ; we will require $\ell \cdot n = -1$. Let m be a complex null vector tangent to S satisfying $m \cdot \bar{m} = 1$ (the overbar denotes complex conjugation), and $\ell \cdot m = n \cdot m = 0$.

In the wave-zone, spacetime geometry is completely described by the Weyl tensor C_{abcd} . In particular, outgoing transverse radiation is described by the Weyl tensor component [41]

$$\Psi_4 = C_{abcd} n^a \bar{m}^b n^c \bar{m}^d. \quad (1)$$

Ψ_4 can be expanded in spin-weighted spherical harmonics ${}_{-2}Y_{\ell,m}$ of spin weight -2 [42]. Let $\Psi_4^{(\ell,m)}$ be the mode component with $\ell \geq 2$ and $-m \leq \ell \leq m$. The (ℓ, m) component of the News function $\mathcal{N}^{(\ell,m)}$ is defined as [39]

$$\mathcal{N}^{(\ell,m)}(u) = \int_{-\infty}^u \Psi_4^{(\ell,m)} du. \quad (2)$$

The outgoing energy flux is related to the integral of $|\mathcal{N}|^2$ over all angles. In a numerical spacetime it is in principle possible to extract Ψ_4 going out all the way to \mathcal{I}^+ [43], and this is what should be done to reduce systematic errors. We shall follow the common approach of calculating Ψ_4 on a sphere at a finite radial coordinate r and the integral in the previous equation is over time instead of the retarded time coordinate u . The lower limit in the integral is not $-\infty$ but the earliest time available in the simulation. The News function is then a function of time at a fixed value of r , starting from the earliest time available in the simulation. A further time integration of \mathcal{N} yields the gravitational wave strain.

Turning now to the black hole, the basic object here is a marginally outer trapped surface (MOTS), again denoted S . This is a closed spacelike 2-surface with vanishing outgoing expansion $\Theta_{(\ell)}$:

$$\Theta_{(\ell)} = q^{ab} \nabla_a \ell_b = 0. \quad (3)$$

The shear of ℓ^a is defined as

$$\sigma = m^a m^b \nabla_a \ell_b. \quad (4)$$

Both \mathcal{N} and $\bar{\sigma}$ have the same behavior under spin rotations $m \rightarrow me^{i\psi}$, i.e. they have the same spin weight.

q	D/M	p_r/M	p_ϕ/M
1.0	9.5332	0.0	0.099322
0.85	12.0	-0.000529	0.08448
0.75	11.0	-0.000686	0.08828
0.667	11.75	-0.000529	0.08281
0.5	11.0	-0.000572	0.0802
0.25	11.0	-0.000308	0.05794

TABLE I. Initial parameters for non-spinning binary black holes with quasi-circular orbits. $q = M_2/M_1$ is mass ratio, D is the initial separation between the two holes, p_r and p_ϕ are radial and azimuthal linear momenta respectively.

Also, similar to the news function and the Bondi mass-loss formula, $|\sigma^2|$ appears in the energy flux falling into the black hole [44, 45], though in this case the flux also contains other contributions. It is shown in [46] that for the case of a slowly evolving horizon, which is what we are dealing with in the inspiral phase, $|\sigma|^2$ is the dominant part of the flux. Thus, as suggested in [15], we will compare the shear at the horizon with the News.

The numerical simulations: Our numerical simulations are performed using the publicly available Einstein Toolkit framework [47, 48]. The initial data is generated based on the puncture approach [49, 50], which has been evolved through BSSNOK formulation [51–53] using the $1 + \log$ slicing and Γ -driver shift conditions. Gravitational waveforms are extracted [54] on coordinate spheres at various radii between $100M$ to $500M$. The computational grid set-up is based on the multipatch approach using Llama [55] and Carpet modules, along with adaptive mesh refinement (AMR). The various horizons (or more precisely, marginally outer trapped surfaces) are located using the method described in [56, 57]. Quasi-local physical quantities are computed on the horizons following [58, 59].

We consider non-spinning binary black hole systems with varying mass-ratio $q = M_2/M_1$, where $M_{1,2}$ are the component masses (with $M_1 \geq M_2$). We use the GW150914 parameter file available from [60] as our template. For each of the simulations, as input parameters we provide initial separation between the two punctures D , mass ratio q and the radial and azimuthal linear momenta p_r , p_ϕ respectively, while keeping the total mass $M = M_1 + M_2 = 1$. Parameters are listed in table I. We then compute the corresponding initial locations, the x , y , z components of linear momentum for both black holes, and grid refinement levels, etc., before generating the initial data and evolving it. We chose 6 non-spinning cases ranging between $q = 1.0$ to 0.25 , based on the initial parameters listed in [61, 62]. Our simulations match very well with the catalog simulations [63], having merger time discrepancies less than a few percent.

Results: We begin by looking at the complex shears, σ_1 and σ_2 , of the outgoing null normal ℓ^a at the two in-

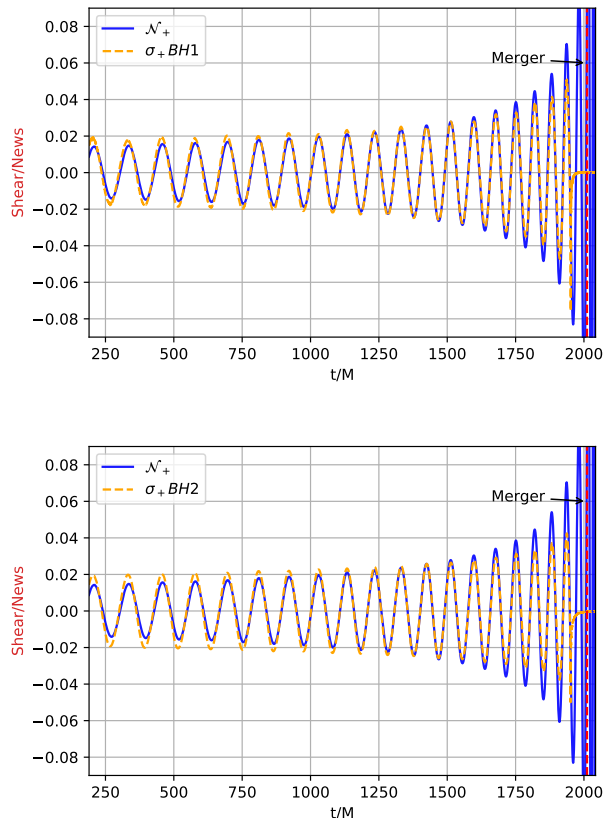


FIG. 1. The real part of the shear, $\sigma_+(t)$, for the two black holes ($BH1$ and $BH2$) for the $q = 0.25$ configuration, compared with the real part of the News function $\mathcal{N}_+^{(2,2)}$. The functions σ_+ and $\mathcal{N}_+^{(2,2)}$ have been suitably aligned and their amplitudes scaled so that they have unit norm over the time interval where the shear is defined. Similar results hold for the \times polarization and the other mass-ratios.

dividual horizons for a particular configuration, namely $q = 0.25$. We write the shear as $\sigma = \sigma_+ + i\sigma_\times$. As in [18], we introduce coordinates (θ, ϕ) on the horizons with the z -axes perpendicular to the orbital plane. Just like the waveform the angular distribution is mostly quadrupolar, i.e. $\sigma \propto -{}_2Y_{2,2}(\theta, \phi)$. It will then be sufficient for our purposes to just look at the values of $\sigma_{1,2}$ on the north poles of the two horizons. This will not suffice for precessing spins or when higher modes become more important. In these more complicated cases the approach suggested in [64] can be followed.

Figure 1 shows the real parts $\sigma_+(t)$ for the two black holes and for the $q = 0.25$ configuration. The plot also shows the News function for the corresponding mode and polarizations, i.e. $\mathcal{N}_+^{(2,2)}$. The plot continues as long as the individual horizons can be found reliably. The time when the common horizon is formed is indicated by a vertical line. The figure shows that the shear has very similar properties as the well known “chirping” gravita-

tional waveform: it has increasing frequency and amplitude. We note also that since the shear is non-vanishing, it follows that the horizon is not isolated and its area is increasing. However, the area is not increasing rapidly and this area increase is not measured reliably in our simulations. See [65, 66] for a more accurate study of the area increase in a black hole merger.

Does the qualitative agreement of the shear with gravitational wave signals shown in Fig. 1 hold quantitatively? To answer this question, we treat the shear as a bonafide gravitational waveform and attempt to estimate its parameters. The chirp mass, $\mathcal{M} = Mq^{3/5}/(1+q)^{6/5}$, determines the frequency evolution of the signal at leading order. For any given simulation, we have then three possible gravitational wave signals: the waveform $h(t)$ extracted in the wave-zone over a large sphere, and the shears $\sigma_{1,2}(t)$ calculated at the individual horizons. Here t is the coordinate time used in the numerical evolution. For all three of these time series, we estimate \mathcal{M} and q using a well tested model for binary mergers known as IMRPhenomPV2 [67, 68]. This waveform model is a development of the so-called phenomenological binary merger models [32–34] and it includes, in principle, precession due to the misalignment of the individual spins with the orbital angular momentum (though this is not relevant here). Other waveforms could also be used [35–38] but we do not expect any significant differences for our purposes.

We obtain three estimates of (q, \mathcal{M}) using, in-turn, the waveform $h_{+, \times}$ extracted in the wave-zone (on an extraction sphere of radius $R_E = 100M$), and the shears $\sigma_{1,2}$ at the two horizons. The waveform $h_{+, \times}$ is matched with the model waveform itself, while $\sigma_{1,2}$ are matched with the News, i.e., the time derivative of the model waveform. In each case we use a sufficiently fine grid in (q, \mathcal{M}) and minimize a standard least-squares figure-of-merit over the relative time-shift and initial phase (which will henceforth be referred to as alignment) as well as the mass parameters. Table II shows the best fit values of \mathcal{M} and q for the real part of the shears. The chirp mass \mathcal{M} is very well measured, with typical errors of $\sim 0.5\%$ for the strain and $\sim 1 - 3\%$ for the shears. The uncertainties in the mass-ratio are much larger (as expected), with errors of $\sim 9\%$ for the strain and $\sim 17 - 24\%$ for the shears. We have chosen to present our results in terms of (\mathcal{M}, q) as independent parameters, though we could have used the total mass M as well. It is easy to check that the best fit values of M turn out to be very close to unity as they should.

Motivated by this excellent agreement, we postulate that given the News, it should be possible to predict the horizon shears. First comparing the amplitudes and the phases of σ_1 and σ_2 (after aligning them with the News), we find that to a very good approximation the phase

\hat{q}	q	\mathcal{M}	q_1	\mathcal{M}_1	q_2	\mathcal{M}_2
1.0	1.0	0.432	0.800	0.439	1.200	0.439
0.85	1.0	0.433	0.770	0.434	0.770	0.434
0.75	0.78	0.428	0.630	0.426	0.870	0.426
0.67	0.779	0.427	0.867	0.433	0.747	0.429
0.50	0.498	0.403	0.580	0.410	0.580	0.410
0.25	0.222	0.328	0.330	0.345	0.250	0.333

TABLE II. Best fit values of the mass ratio and chirp mass for i) the waveform extracted in the wave-zone (denoted q and \mathcal{M}), ii) the shear of the first black hole (q_1 and \mathcal{M}_1), and iii) the shear of the second black hole (q_2 and \mathcal{M}_2). The mass ratio is nominally \hat{q} for the puncture initial data.

difference between them is very small, and

$$\frac{M_2 \sigma_2}{M_1 \sigma_1} \approx q^{-0.7}. \quad (5)$$

We multiply the shears by the respective masses to make them dimensionless. Similarly, comparing the News and one of the shears, say σ_1 , we find that the phase difference is again small and their amplitudes are related as follows:

$$R_E |\mathcal{N}| \approx 0.5(1+q)M_1 |\sigma_1|. \quad (6)$$

Here $R_E = 100M$ is the extraction radius. With these relations, given the observed gravitational wave strain, one can estimate the amplitude and frequency content of the horizon shears for binaries consisting of non-spinning black holes.

Conclusions: We have shown quantitatively that in a black hole merger, the shear of the horizons behaves just like gravitational wave signals seen in the wave-zone. This adds an important ingredient to the idea that there are strong correlations between gravitational wave signals seen by gravitational wave detectors and suitable fields in the strong field dynamical region near the black holes.

Future work will extend this study in many directions, e.g. allowing spinning black holes, precession effects, and possibly super-radiance in the non-linear merger regime. The horizon shears are related to the variation of the horizon multipole moments. One might therefore be able to relate the radiative multipole moments to the horizon moments. On dynamical horizons, various balance laws are known relating the change in the horizon multipole moments to fluxes across the horizon [64]. Combining this with the idea of slowly evolving horizons [46, 69] might provide an interesting route to relate properties of waveforms with horizons [70, 71] and to perhaps build better waveform models. Note that in Fig. 1 the difference between the shear and the News becomes larger near the merger where the additional terms in the horizon flux law start to matter. It will be important to compare the full flux at the horizon with the News.

A deeper mathematical understanding of these observed correlations is still lacking. In particular, it is

important to identify the precise spacetime region and the non-linearities that generate the gravitational waves seen at the horizons and in the wave-zone. Elucidating the precise relationship of our results with the perturbative calculations of tidal coupling [20–24] is of great interest as well. In particular we mention the work by O’Sullivan & Hughes [23, 24] that studies high mass-ratio systems perturbatively, and especially the effect on the geometry of the event horizon. They find a strong correlation between the shear of the horizon with the particle orbit (and thus with the observed waveform) which is broadly consistent with our results.

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