TOROIDAL CONFINEMENT (THEORY)

Classical Diffusion in Tokomak

W. Feneberg, Institut für Plasmaphysik GmbH, 8046 Garching near Munich, Federal Repuolic of Germany

Abstract: The mass flow perpendicular to the magnetic surfaces is calculated for rotational symmetric systems. The influence of an induced electric field on the classical diffusion is included. The formula will be simplified for a special self-consistent Tokomak case with magnetic surfaces of nearly circular cross section. It will be shown that a stationary equilibrium with vanishing mass flow has no physical meaning for realistic Tokomak conditions.

The influence of an induced electric field produced by ohmic heating in a toroidal discharge has not been taken into account in the work of Pfirsch-Schlüter on classical diffusion [1]. The effect of this electric field consists in an additional $\vec{E} \times \vec{B}$ drift which decreases the classical diffusion. If we want to calculate this effect we must start with the magnetohydrodynamic equation

$$\vec{i} = -\frac{c}{B^2}[\nabla p \times \vec{B}] + \lambda\vec{B} \qquad \text{- - - - - (1)}$$

From the condition $\nabla\vec{i} = 0$ there results the differential equation for λ,

$$\frac{\partial\lambda}{\partial\varrho}\frac{\partial F}{\partial z} - \frac{\partial\lambda}{\partial z}\frac{\partial F}{\partial\varrho} = -\frac{2c\varrho}{B_0 R}\frac{dp}{dF}\frac{\partial F}{\partial z} \qquad \text{- - - - - - (2)}$$

which is to be solved by the general solution:

$$\lambda = -\frac{dp}{dF}\frac{c\,\varrho^2}{B_0 R} + \lambda_0(F) \qquad \text{- - - - - - - (3)}$$

Here we have used cylindrical coordinates (ϱ, θ, z), with ϱ - the distance from the torus axis, θ - the direction of the main magnetic field $B_\theta = B_0\frac{R}{\varrho}$, R - the distance of the center of the discharge vessel from the torus axis, z - the distance from the torus plane. We shall also introduce a system of polar coordinates:

$$z = r\sin\vartheta$$
$$\varrho - R = -r\cos \qquad \text{- - - - - - - (4)}$$

It is known that the magnetic field can be generated by a scalar function F [2],

$$B_r = -\frac{1}{\varrho r}\frac{\partial F}{\partial\vartheta}, \quad B_\vartheta = \frac{1}{\varrho}\frac{\partial F}{\partial r} \qquad \text{- - - - - - - (5)}$$

where the lines F = const coincide with the lines p = const. The contribution of the magnetic field of the diamagnetic currents to the main magnetic field B_θ is neglected.

$$\beta_\theta = \frac{8\pi p}{B_0^2} \ll 1, \text{ p - the plasma pressure} \qquad \text{- - - - - (6)}$$

In equation (2) the assumption was made $|B| = |B_\theta|(1+\frac{B_M^2}{B_\theta^2})^{1/2} \approx |B_\vartheta|$, where $B_M^2 = \frac{1}{\varrho^2}\left(\left(\frac{\partial F}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial F}{\partial\vartheta}\right)^2\right)$. This assumption is not used in the following equations.

The free function $\lambda_0(F)$ is determined by Ohm's law:

$$\vec{E} + \frac{1}{c}[\vec{v}\times\vec{B}] = \frac{m}{ne^2\tau}\left(-\frac{c}{B^2}[\nabla p\times\vec{B}] + \frac{1}{2}\lambda\vec{B}\right) + \frac{1}{2ne}\nabla p \qquad \text{- - - - - (7)}$$

Ohm's law in this form can be received only for constant temperature.

(That means: $\vec{v}$ - Mass velocity, n - particle density, e - electron charge, m - electron mass, τ - electron-ion collision time.) The difference between the conductivity parallel and perpendicular to the magnetic field is taken into account by a factor 2. The electric field

$$\vec{E} = -\nabla\phi + \frac{1}{c}\frac{\partial\vec{A}}{\partial t} \qquad \text{- - - - - - - (8)}$$

consists of a scalar field in the meridional (ϱ, z) plane and the induced field in direction $-\theta$.

$$\left(\frac{1}{c}\frac{\partial\vec{A}}{\partial t}\right)_\theta = E_\theta\,,\quad E_\vartheta = E_0\frac{R}{\varrho} \qquad (9)$$

If we multiply equation (7) scalar with $\vec{B}$ and integrate over a magnetic surface F = const, we obtain:

$$\lambda_0(F) = \frac{c}{B_0 R}\frac{dp}{dF}\frac{\int_0^{2\pi} r\varrho\left(\frac{\partial F}{\partial r}\right)^{-1}d\vartheta}{\int_0^{2\pi} r\varrho^{-1}\left(\frac{\partial F}{\partial r}\right)^{-1}d\vartheta} + \frac{2ne^2\tau E_0}{m B_0}\left\{1 - \frac{\int_0^{2\pi} r\varrho^{-1}\frac{B_M^2}{B_\theta^2}\left(\frac{\partial F}{\partial r}\right)^{-1}d\vartheta}{\int_0^{2\pi}\left(\frac{\partial F}{\partial r}\right)^{-1} r\varrho^{-1}d\vartheta}\right. \qquad \text{- - - - - (10)}$$

The formula for the mass flow M is obtained using the vector product of equation (7) with $\vec{B}$:

$$M = \int_0^{2\pi}\left(n\vec{v}, \frac{\nabla F}{|\nabla F|}\right)2\pi\varrho r\frac{|\nabla F|}{\left|\frac{\partial F}{\partial r}\right|}d\vartheta =$$

$$-\frac{2\pi mc^2}{e^2\tau B_0^2}\frac{dp}{dF}A(F) - \frac{\pi mc^2}{e^2\tau}\frac{dp}{dF}B(F) - \frac{2\pi ncE_0}{B_0^2 R}C(F) \qquad \text{- - (11)}$$

with

$$A(F) = \int_0^{2\pi} r\varrho\frac{(\nabla F)^2}{\left|\frac{\partial F}{\partial r}\right|}d\vartheta,\quad B(F) = \int_0^{2\pi} r\varrho^3\left|\frac{\partial F}{\partial r}\right|^{-1}d\vartheta - \frac{\left(\int_0^{2\pi} r\varrho\left(\frac{\partial F}{\partial r}\right)^{-1}d\vartheta\right)\left(\int_0^{2\pi} r\varrho\left|\frac{\partial F}{\partial r}\right|^{-1}d\vartheta\right)}{\int_0^{2\pi} r\varrho^{-1}\left(\frac{\partial F}{\partial r}\right)^{-1}d\vartheta}$$

$$C(F) = \frac{\left(\int_0^{2\pi}\left(\frac{\partial F}{\partial r}\right)^{-1} r\varrho^{-1}(\nabla F)^2 d\vartheta\cdot\int_0^{2\pi} r\varrho\left|\frac{\partial F}{\partial r}\right|^{-1}d\vartheta\right)}{\int_0^{2\pi} r\varrho^{-1}\left(\frac{\partial F}{\partial r}\right)^{-1}d\vartheta} \qquad \text{- - - - (12)}$$

The first term A(F) accounts for the classical diffusion in a straight cylinder, the second term B(F) contains the Pfirsch-Schlüter diffusion and the third term C(F) gives the contribution of the electric field.

In the following now we shall look for a special simple Tokomak case. The Maxwell equation for the meridional magnetic field is:

$$\mathrm{rot}\,\vec{B}_M = \frac{4\pi}{c}i_\theta \qquad \text{- - - - - - - - (13)}$$

This equation will be solved in the approximation of a high discharge current.

$$i_\theta \approx \frac{2ne^2\tau}{m}E_\theta = \frac{2ne^2\tau E_0}{m}\frac{R}{\varrho} \qquad (14)$$

We apply a homogeneous vertical magnetic field

$$B_z = \frac{Ja^2}{Rc\,b^2} \qquad \text{- - - - - - - - (15)}$$

(J - total current, a-plasma radius) and obtain the solution for the magnetic surfaces:

$$F = \frac{\alpha}{4}\left(x^2 - \frac{\varepsilon}{4}x^3\cos\vartheta\right),\quad x = \frac{r}{b},\quad \alpha = \frac{8\pi ne^2\tau E_0 Rb^2}{mc} \qquad (16)$$

In this configuration with nearly circularly shaped surfaces we can use a simplified expression for the mass flow:

$$M = -\frac{4\pi^2 Rr\,mc^2}{e^2\tau B_0^2}\left(1+\frac{4\pi^2}{\iota^2}\right)\frac{dp}{dr} - 4\pi^2r^2\frac{E_0cn}{B_0}\left(\frac{\iota}{2\pi}\right),\quad \frac{\iota}{2\pi} = \frac{\alpha}{2b^2B_0} \qquad \text{- - (17)}$$

For M = 0 it follows for the density profile:

$$n = n_0\left(1 - \frac{\gamma r^2}{2n_0}\right),\ \gamma = \frac{\iota^3 eE_0ne^2\tau B_0}{16\pi^3 Rmc\,kT} \qquad \text{- - - - - - (18)}$$

For the conditions realized in T-3 we have $\frac{\gamma}{n_0} \approx 10^3$, that means that the electric field in realistic cases is too small to prevent the plasma diffusion. An interesting stationary stage with M = 0 can be achieved only when a temperature gradient is considered [3, 4].

References

1 Pfirsch, D., Schlüter, A.: MPI/PA 7/62

2 Lüst, R., A. Schlüter: Zeitschr.f.Naturforschg.12a,850(1957)

3 Shafranov, V.D., Reviews of Plasma Physics, Vol. 2, p.145

4 Feneberg, W., IPP 3/84

This work was performed as part of the agreement between the Institut für Plasmaphysik GmbH, Munich-Garching, and Euratom to conduct joint research in the field of plasma physics.