Effect of Inertia on Losses from a Plasma in Toroidal Equilibrium

B.J. Green, H.P. Zehrfeld, Institut für Plasmaphysik GmbH, Garching near Munich, Federal Republic of Germany

We investigate the magnetohydrodynamic equilibrium of a resistive, low-density plasma in a model stellarator field. The effect of inertia on plasma motion is treated exactly, and its influence on plasma loss determined. It is shown that the losses due to inertia are limited by the conditions for the existence of an equilibrium.

One of the basic theoretical problems in the fusion programme is the calculation of plasma loss from particular containment devices. However, because of the complexity of the problem, brought about by the choice of plasma model and the complicated geometry of realistic devices, to say nothing of boundary conditions, it has not been possible to carry through a completely satisfactory calculation. As an example we can consider the Pfirsch-Schlüter calculation [1] for plasma loss from a model stellarator or Levitron device. In this calculation the plasma flow velocity parallel to the magnetic field can become very large, and this casts strong doubts on the assumption that the plasma inertia is negligible.

In the present calculation we have included the effect of plasma inertia and investigated the modified losses from a model configuration. We describe the plasma by means of the one-fluid equations (see below, where "standard" notation is used), and treat the flows and geometry exactly, but consider resistive effects as a perturbation to the perfectly conducting plasma motion.

$$\rho \underline{v}\cdot\nabla\underline{v} = \underline{J}\times\underline{B} - c^2\nabla\rho$$

$$-\nabla\phi + \underline{v}\times\underline{B} = \eta \underline{J}$$

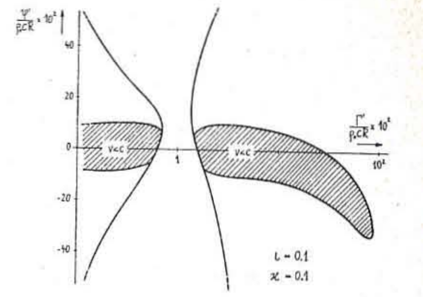
$$\nabla\cdot\rho\underline{v} = Q$$

$$\nabla\cdot\underline{J} = 0$$

The model magnetic field used, is the axisymmetric Pfirsch - Schlüter field where we used the cylindrical co-ordinate system R, Z, ζ and in a meridional cross-section, the polar co-ordinates r, θ.

Two assumptions deserve explicit mention a) isothermal plasma (i.e. c the sound speed is constant), and b) Q the mass source term that is necessary to replace losses, as we are considering stationary solutions. Closer investigation of the equations reveals that the differential part of them can be brought into a form $\frac{\partial G}{\partial \theta} = \eta R(G, \theta)$, where G indicates a column of four known algebraic functionals of quantities to be determined. Expansion in resistive effects leads to easily solvable differential equations $\frac{\partial G^{(n)}}{\partial \theta} = \eta R(G^{(n-1)}, \theta)$, so that the nth order solution is given by quantities in the order n-1, together with surface functions $S^{(n)}$ i.e. $G^{(n)} = \eta \int_0^\theta d\theta' R(G^{(n-1)}, \theta') + S^{(n)}(r)$.
These surface functions in nth order are determined by the condition that the (n+1)th order solution is periodic in θ i.e. $\int_0^{2\pi} d\theta R(G^{(n)}, \theta) = 0$. Hence, once we have the zeroth order we have a systematic procedure to obtain all higher orders. What we have done is to solve the zeroth order, which corresponds to non-linear ideal MHD flows, and to derive the plasma loss expression in the resistive first order.

It is interesting to note that we are led to restrictions on the ideal plasma flow. In terms of the mass fluxes the long and the short way, these are shown in the figure. One can see (for definite values of aspect ratio and rotational transform), the possible departures from the quasistatic case, which corresponds to the neighbourhood of the origin. The quantities plotted are related to the mass fluxes Γ the short and Ψ the long way. The two regions where stationary solutions are possible, can be shown to correspond to super and subsonic meridional flow, where the appropriate sound speed is not c, but fc.



In first order the mass loss rate through a surface (where Q is located at r = 0) is

$$W^{(1)} = \int_{r(\zeta)=r_s} \rho \underline{v}^{(1)}\cdot d\underline{S}$$
$$= -\frac{4\pi^2 q r R c^2}{B_0^2 f^2}\Big\{\Big\langle \rho\frac{\partial \rho}{\partial r}N^2\Big\rangle - \frac{(1-\kappa^2)^{1/2}}{1+f^2}\langle\rho N\rangle\Big\langle N\frac{\partial\rho}{\partial r}\Big\rangle$$
$$-\frac{1}{r}\Big[\Big\langle\rho^2N^2\Big(\frac{v_0^2}{c^2}+\kappa\frac{\cos\theta}{N}\frac{v_s^2}{c^2}\Big)\Big\rangle - \frac{(1-\kappa^2)^{1/2}}{1+f^2}\langle\rho N\rangle\Big\langle\rho N\Big(\frac{v_0^2}{c^2}+$$
$$\kappa\frac{\cos\theta}{N}\frac{v_s^2}{c^2}\Big)\Big\rangle\Big] + \Big\langle\rho N\frac{\partial}{\partial r}\Big(\frac{1}{f}\rho N^2\frac{v_0}{c}\cdot\frac{v_s}{c}\Big)\Big\rangle - (1-\kappa^2)^{1/2}\langle\rho N\rangle\Big\langle\frac{1}{N}\frac{\partial}{\partial r}\Big(\frac{1}{f}\rho N^2\frac{v_0}{c}\cdot\frac{v_s}{c}\Big)\Big\rangle\Big\}$$

where each line can be identified with a particular force. Note that this expression is given in terms of zeroth order quantities, which can in principle be calculated from conditions on the first order solution, as we have pointed out earlier. These conditions are differential equations and their solution is a boundary value problem. We can, however, estimate losses without a full solution of these equations. Different estimates will be presented which indicate that in representative situations the "extra" losses caused by plasma flow are of the same order as the classical (quasi-static) resistive losses.

[1] D.Pfirsch, A.Schlüter: MPI/PA/7/62 (unpublished).

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