Full Wave Simulation of RF Waves in Cold Plasma with the Stabilized Open-Source Finite Element Tool ERMES

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Abstract. When RF waves are applied in tokamaks with metal walls, sheath rectification effects associated with the fields induced in the scrape-off layer (SOL) may lead to enhanced plasma-wall interactions (i.e. heat-loads in the limiters, RF-induced impurity sources) which can endanger the integrity of the machine and limit the RF power. Currently, some numerical tools are being used to simulate the RF antenna near fields in the presence of magnetized plasmas, but they have their limitations. Some neglect completely the interaction of the RF waves with the low-density plasma close to the antenna. Others take into account these interactions, but generate numerical spurious oscillations around the Lower Hybrid Resonance (LHR). Simplifications to reach convergence had also been tried (e.g. neglect gyrotropy, increase electron density to avoid the LHR), but the fields obtained with these simplifications can be very different to the real ones and this difference can affect the accuracy of derived magnitudes which use these fields as an input (e.g. sheath rectification effects). In this work we try to overcome all the limitations mentioned above by customizing the open-source finite element code ERMES. This code implements a finite element formulation which allows to simulate the near fields of the RF antenna in a continuous gyrotropic non-homogeneous media without limits in the minimum value of the plasma density and provides stable solutions even in the presence of the LHR. Benchmarking of this approach is underway and comparison against measurements, semi-analytical approaches and other codes will be presented.

INTRODUCTION

Radio-Frequency (RF) electromagnetic waves are commonly employed as an auxiliary heating system in tokamak nuclear fusion devices. The RF waves are sent to the plasma core from antennas situated on the tokamak walls and, on its way to the core, they pass through the Scrape-Off Layer (SOL), a low-density plasma region situated between the antenna and the last closed magnetic surface. Unfortunately, in the SOL of a tokamak with metal walls the RF waves can induce sheath rectification effects which intensify the plasma-wall interactions. These enhanced interactions can heat the limiters to the point of melting and increase the contamination (and consequent cooling) of the plasma core with impurities coming from the increased heavy ion wall sputtering. Therefore, the RF waves in the SOL can endanger the integrity of the machine and limit the maximum RF heating power that can be safely sent to the plasma. In particular, it is especially important for the forthcoming DT experiments at the Joint European Torus (JET) where the avoidance of impurities in the plasma will be of key importance.

Although many codes are available to describe the wave-particle physics in the plasma core, the modelling of the RF waves interactions with the low-density plasma in the SOL is much less explored since the RF physics describing the involved mechanisms is not yet fully understood and the solution of the problem is numerically demanding due to the excitation of waves with wavelengths of just a few millimetres and the close interaction of these waves with the complex antenna and wall geometries. Moreover, the presence of resonances challenges the numerical formulations to its limits.

Currently, some numerical tools are being used to simulate the RF antenna near fields in the presence of magnetized plasmas, but they have their limitations. For instance, the well-known in-house code TOPICA [1], which is typically used to couple realistic antenna geometries with the hot plasma inside the reactor, needs a vacuum buffer area of separation between the antenna and the plasma. Therefore, it neglects all the physical phenomena related to the interaction of the RF waves with the low-density plasma close to the antenna surface. Other codes, such as the commercial software package COMSOL [2], have been customized to take into account the close interaction of the near-fields with the low-density plasma. Unfortunately, they produce spurious oscillations around the Lower Hybrid Resonance (LHR) due to numerical instabilities associated with the finite element method (FEM) formulation implemented inside them [3]. Simplifications can be used to reach convergence (neglect gyrotropy, increase electron density to avoid the LHR), but the fields obtained can be very different to the real ones (even if the input impedance of the antennas are similar to the ones measured) and this difference can affect the accuracy of derived magnitudes, as the sheath rectification effects, which use these fields as an input.

In this work we try to overcome all the limitations mentioned above by customizing the open-source finite element code ERMES [4]. We adapted ERMES for reading measured plasma density profiles and magnetic fields from files, incorporate these measurements in a realistic three-dimensional CAD representation of the RF antennas and tokamak walls and calculate the near-fields and other relevant magnitudes in the presence of a cold magnetized plasma. ERMES implements a stabilized finite element formulation [5] which allows to simulate the near fields of the antenna in a continuous gyrotropic non-homogeneous media without limiting the minimum value of the plasma density. As it is shown in [3] and at the end of this paper, ERMES obtains stable solutions near the LHR (although the work to demonstrate that this stable solution is the physical one is still on going). In the following, we show why the standard FEM approach is unstable close to the LHR and how ERMES stabilizes the numerical solution.

MODELLING RF FIELDS IN THE SOL WITH FEM

The most widespread finite element formulation for solving the Maxwell's equations in frequency domain is the so-called double-curl formulation [6]. This approach discretizes with edge elements the weak form of

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{E} \right) - \omega^2 \varepsilon \, \mathbf{E} = i\omega \mathbf{J}, \tag{1}$$

where $\mathbf{E}$ is the electric field, μ is the magnetic permeability, ε is the electric permittivity, ω is the angular frequency, i is the imaginary unit and $\mathbf{J}$ is a divergence-free current source. The double-curl formulation is the one used by commercial codes as CST, HFSS and COMSOL or in-house codes as [7]. In the SOL, the permittivity ε takes the form of the cold plasma permittivity tensor given in [8], which contains the parameters S, D and P that are functions of the position, electron density, magnetic field, the species composing the plasma and the frequency ω. Close to the LHR, we have that

$$\nabla \sim i\mathbf{k} \rightarrow \infty \ \ , \ \ S \rightarrow 0 \ \ , \tag{2}$$

where $\mathbf{k}$ is the wave vector of the incident field. As a consequence of Equation 2, the operators in Equation 1 satisfy

$$\nabla \times \nabla \times \mathbf{E} \ \gg \ \omega^2 \varepsilon \mu \, \mathbf{E}, \tag{3}$$

which means that, close to the LHR resonance, we are effectively solving:

$$\nabla \times \nabla \times \mathbf{E} \ \approx \ 0. \tag{4}$$

This equation is ill-posed and produces ill-conditioned matrices which are difficult to solve. Equation 4 has an infinity set of possible solutions, indeed, any field coming from a gradient $\mathbf{E} = -\nabla\phi$ is a solution. In practical terms, this translates in the presence of numerical spurious oscillations that pollute the calculated fields at the vicinity of the LHR, as can be observed in the COMSOL simulations shown in [3]. We can try to overcome this difficulty by increasing the plasma density above the resonance, add some artificial damping mechanism (e.g. add an imaginary frequency) or even try to emulate the plasma behaviour with metamaterials (e.g. a multilayer set of anisotropic materials). But, even though, we can obtain reflection coefficients similar to the measurements, fields calculated in such a ways can be completely different to the physical ones, and this will affect derived magnitudes that use these fields as an input (e.g. induced rectification voltages on plasma sheaths).

ERMES

Instead of the double-curl formulation shown above, ERMES implements the so-called regularized formulation [5, 4]. This approach discretizes with nodal (Lagrangian) elements the weak form of

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{E} \right) - \bar{\varepsilon}\, \nabla \left(\frac{1}{\|\bar{\varepsilon}\varepsilon\|\, \mu} \nabla \cdot (\varepsilon \mathbf{E}) \right) - \omega^2 \varepsilon \mathbf{E} = i\omega \mathbf{J}, \tag{5}$$

where, for the cold plasma case, $\bar{\varepsilon}$ is the complex conjugate of the permittivity tensor given in [8] and $\|\bar{\varepsilon}\varepsilon\|$ is the Frabenious norm of the matrix product $\bar{\varepsilon}\varepsilon$. Close to the LHR, now we have that

$$\nabla \times \nabla \times \mathbf{E} - \bar{\varepsilon}\, \nabla \left(\frac{1}{\|\bar{\varepsilon}\varepsilon\|\, \mu} \nabla \cdot (\varepsilon \mathbf{E}) \right) \;\gg\; \omega^2 \varepsilon \mu\, \mathbf{E}, \tag{6}$$

which means that, close to the LHR resonance, we are effectively solving:

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{E} \right) - \bar{\varepsilon}\, \nabla \left(\frac{1}{\|\bar{\varepsilon}\varepsilon\|\, \mu} \nabla \cdot (\varepsilon \mathbf{E}) \right) \;\approx\; 0. \tag{7}$$

This operator has a null-kernel and it only admits one solution, which eliminates the spurious solutions around the LHR. We will show this with two examples in the next section.

VALIDATION EXAMPLES

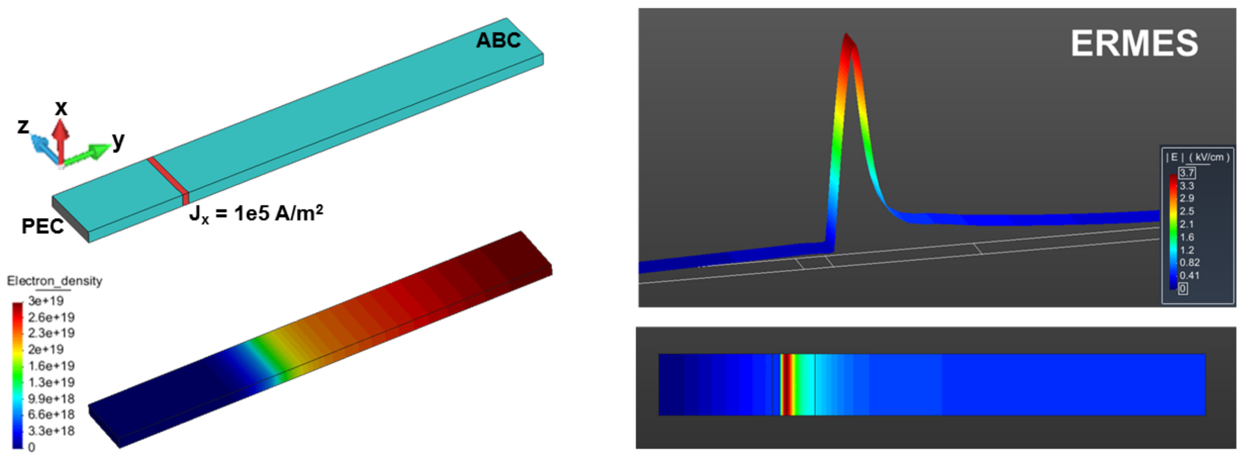

FIGURE 1. ERMES simulations of a LHR in front of a strap antenna. The problem consists in a vertical current density J_x = 1e5 A/m^2, oscillating at f = 42 MHz, applied to the red volume shown in the top left figure, embedded in a deuterium cold plasma. The electron density distribution is shown at the bottom left figure. The external magnetic flux density varies almost linearly from B_z = 2.0 T on the left side to B_z = 2.4 T on the right side. A Perfect Electric Conductor (PEC) boundary condition is imposed on the left surface, which is separated 16 cm from the current source. An Absorbing Boundary Condition (ABC) is applied at the right surface to avoid reflections. Periodic boundary conditions are used on the remaining surfaces. ERMES results are shown on the right panel. We tried to solve the same problem with COMSOL but it failed to find a convergent solution.

In Figure 1 is shown a simple strap RF antenna inside a continuous plasma density variation that presents a LHR in front of the strap. The same problem was solved with ERMES and COMSOL. ERMES results are shown in Figure 1. COMSOL failed to find a solution (error message: Failed to find a solution. Very ill-conditioned preconditioner. The relative residual is more than 1000 times larger than the relative tolerance. Returned solution is not converged).

The Large Plasma Device (LAPD) at UCLA [9] is an experimental facility that allows the direct measure of the electric field created by a RF antenna in a magnetized cold plasma. Figure 2 shows ERMES simulations of the LAPD and measurements results. The plasma presents a LHR close to the antenna. Figure 2 shows that the spurious solution around the LHR are avoided with ERMES. The comparison with experiments in Figure 2 is qualitative, more detailed validation work against experiments is on-going.

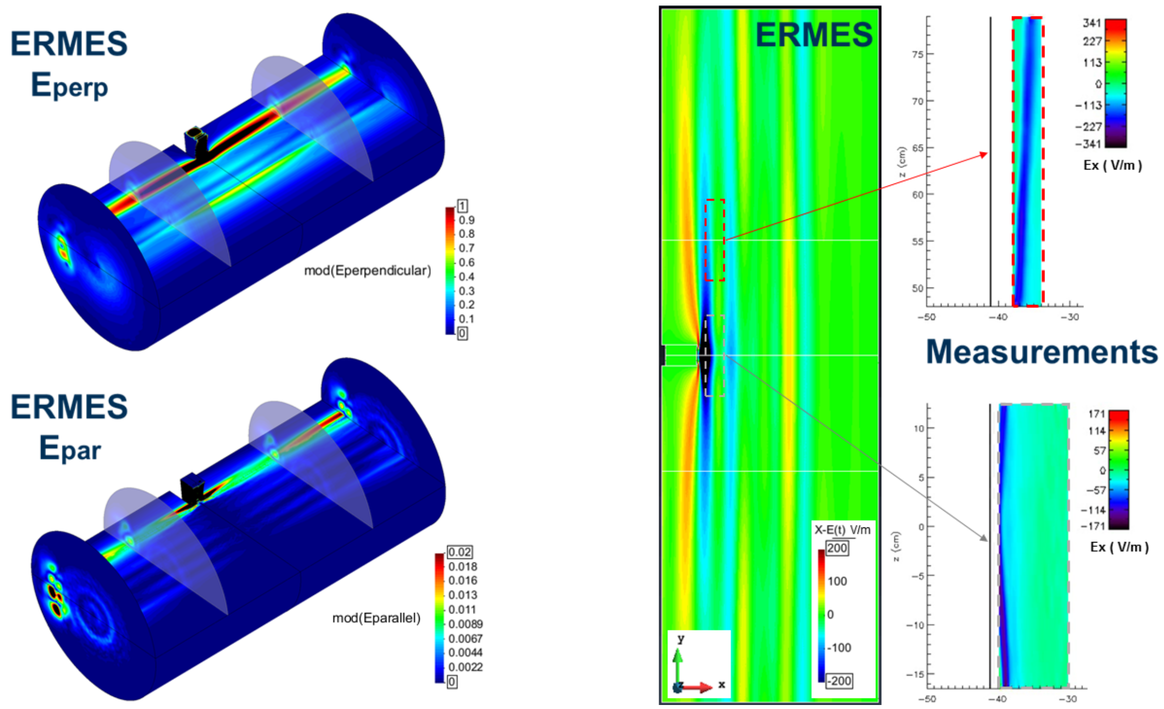

FIGURE 2. ERMES simulations and LAPD [9] experimental measurements of the electric field generated by a RF antenna in a magnetized cold plasma. The external magnetic field is applied along the axis of the LAPD. *Left:* ERMES simulations showing the antenna positioning and 3D electric field distribution (parallel and perpendicular components with respect to the external magnetic field) for a normalized output power. *Right:* Electric field perpendicular component at the LAPD middle plane calculated with ERMES compared against LAPD measurements on the rectangular marked areas.

SUMMARY

We have presented a customization of the open-source code ERMES for RF-cold plasma interaction modelling which implements a finite element formulation that avoids the numerical instabilities on the lower hybrid resonance associated with the standard double-curl formulation. Work is on-going to further validate the approach and more stabilized formulations are being investigated.

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