On the characterisation of the wave field in the problem of ship response

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The optimisation of ship response to waves requires the specification of the ensemble of input wave spectra. The concept of a design spectrum cannot adequately characterise the real ensemble of widely varying sea states. An example based on JONSWAP data illustrates the strong dependence of the response statistics on the assumed sea state ensemble. Wave spectrum statistics can be constructed from wind-field data using numerical wave forecasting models. However, parametrisation techniques need to be developed to reduce the information content of high-resolution forecasting models to the level required for statistical applications.

1. Introduction

Ocean waves represent a random field which to a good approximation may be regarded as dynamically linear and statistically stationary, homogeneous and Gaussian. To this approximation the wave field is completely characterised by the two-dimensional spectrum $F(\omega, \Theta)$ of the variance $\overline{\zeta^2}$ of the surface elevation ζ with respect to wave frequency ω and propagation direction Θ,

$$\overline{\zeta^2} = \iint F(\omega,\Theta)\, d\omega\, d\Theta$$

Most of the nonlinear, non-Gaussian deviations from the linearised state are small in the mean und can be evaluated by perturbation methods. The corrections derived in this way can be expressed again in terms of the wave spectrum, so that for practical purposes the wave spectrum may in fact be regarded as a complete representation of the real wave field, including nonlinearities.

The immediate goal of investigations of ship response — or of any system subjected to wave action — is to relate the statistical properties of the responding system to the statistics of the wave field, i. e. the wave spectrum. Thus if $\beta_1, \beta_2, \ldots$ denote statistical parameters or functions of the system (e. g. variances and covariances of degrees of freedom or stresses, variance and covariance spectra, higher order moments if the system is nonlinear, etc.) the system response is characterised by a set of functional relations

$$\beta_j = \beta_j(F(\omega,\Theta)) \tag{1}$$

Equations (1) provide the basis for optimising the system with respect to wave response. To solve the optimisation problem, however, the input wave spectrum must be specified. For this purpose a "design spectrum" is often adopted, usually in the form of a fully developed spectrum

$$F(\omega,\Theta) = E(\omega)\, s(\omega,\Theta) \tag{2}$$

where the one-dimensional frequency spectrum E (ω) is given by the Pierson-Moskowitz (1964) spectrum

$$E(\omega) = E_{PM}(\omega) = \alpha g^2 \omega^{-5} \exp\left[-1{,}25\left(\frac{\omega_m}{\omega}\right)^4\right] \qquad (3)$$

with $\alpha = 0.0081$ and $\omega_m = \left(\frac{0.74}{1.25}\right)^{1/4} \cdot g/U$, ($\omega_m$ = peak frequency, U = wind speed, g = acceleration of gravity), and $s(\omega, \Theta)$ is a directional spreading factor normalised such that $\int_{-\pi}^{\pi} s \, d\Theta = 1$. Measurements yield typically half-plane $\cos^2 \Theta$ or $\cos^4 \Theta$ spreading factors with somewhat broader distributions at higher frequencies than near the peak.

The spectral form (3) was derived by Pierson and Moskowitz from wave data for a small subset of wind fields, selected from a much larger ensemble, which corresponded as closely as possible to the "ideal" case of a stationary, spatially uniform wind field.

Rather than taking as input a single "design" spectrum, it appears more realistic to consider instead an ensemble of wave spectra corresponding to a variety of possible wind fields. The system response may then be optimised with respect to the probability distribution of this ensemble. For this approach the concept of a fully developed spectrum is not very useful, as it is too specialised to provide a basis for reconstructing the full ensemble of possible wave fields. Two methods for obtaining wave spectrum statistics appear feasible: long-term measurements, and numerical hindcasting using past meteorological data. The former method will often be impracticable on account of the long measuring periods involved — usually several years for meaningful statistics — and can be applied only at a finite number of stations. The latter may require extensive computations, but is fast (once the capability is developed) compared with real-time data collection. It is also capable of providing statistics for large areas of the ocean simultaneously*).

By either method the derivation of wave spectrum statistics is a nontrivial task. On the other hand, it must be recognised that reliable wave spectrum statics are a prerequisite for the practical application of the considerable work which has been devoted to wave response studies in the past years. A crude characterisation of the sea state probability distribution in terms of a few parameters such as significant wave height and period will normally be inadequate for purposes of system optimisation, which can depend on widely differing properties of the wave spectrum ensemble. Economical ship operation, for example, is governed by weighted averages over the entire wave spectrum ensemble, whereas safety considerations depend on the probability of occurence of a small percentage of extreme sea states. In order to fully utilise the progress achieved in the understanding of the response relations (1), a comparable effort will need to be devoted to the problem of characterising the input wave spectrum.

2. *Statics of wave spectra for limited fetches*

A simple example may serve to illustrate the significance of wave-spectrum statistics in evaluating system response.

The spectrum ensemble is based on wave-growth data obtained during the Joint North Sea Wave Project (JONSWAP, Hasselmann et al., 1973), in which a large number of wave spectra were measured at various fetches and wind speeds during stationary offshore wind conditions. The observed spectra were parameterised by best fitting a spectrum of the form

$$F(\omega, \Theta) = E_J(\omega) \, s(\Theta)$$

*) *A break through in remote sensing of sea state from satellites could also yield ocean-wide observational data. Although several techniques appear promising, the full wave spectrum has not get been measured from satellits.*

where

$$s(\Theta) = \begin{cases} \frac{2}{\pi} \cos^2 \Theta, & |\Theta| \leq \pi/2 \\ 0, & \frac{\pi}{2} < |\Theta| < \pi \end{cases}$$

and the one-dimensional frequency spectrum is given by

$$E_J(\omega) = \alpha g^2 \omega^{-5} \exp\left[-1{,}25\left(\frac{\omega_m}{\omega}\right)^4\right] \gamma^P, \text{ with}$$

$$p = \exp\left[-\frac{(\omega - \omega_m)^2}{2\sigma^2 \omega_m^2}\right], \quad \sigma = \begin{cases} \sigma_a \text{ for } \omega < \omega_m \\ \sigma_b \text{ for } \omega > \omega_m \end{cases} \qquad (4)$$

The spectrum contains five free parameters α, ω_m, γ, σ_a and σ_b.

Equation (4) includes the Pierson-Moskowitz spectrum as the special case $\alpha = 0.0081$, $\gamma = 1$.

For finite fetches the JONSWAP spectra differ considerably from the Pierson-Moskowitz spectrum. The spectral peaks are generally sharper, as expressed by "peak-enhancement" factors $\gamma > 1$ ($\gamma = E_J(\omega_m) / E_{PM}(\omega_m)$, where the same peak frequency ω_m and α-value are taken for both spectra). Also, the equilibrium-range constant α was found to vary with wind speed and fetch. At small fetches α is larger than the Pierson-Moskowitz value 0.0081 by factors of 2 to 5. Similar results have been reported by Mitsuyasu (1968, 1969) and other workers.

In accordance with Kitaigorodskii's (1962) dimensional hypothesis, the fetch and wind dependence of the spectrum after non-dimensionalising by g and U were found to depend only on the single nondimensional fetch parameter $\tilde{x} = gx/U^2$. The scale parameters α and ω_m followed power law relations

$$\alpha = 0{,}076 \, \tilde{x}^{-0{,}22} \qquad (5)$$

$$\omega_m = 7\pi \, g/U \, \tilde{x}^{-0{,}33} \qquad (6)$$

whereas the shape parameters showed some scatter but no significant $\tilde{x}$-dependence about their mean values

$$\gamma = 3.3, \; \sigma_a = 0{,}07, \; \sigma_b = 0{,}09 \qquad (7)$$

Fig. 1 shows a comparison of the mean JONSWAP spectrum and Pierson-Moskowitz spectrum, both scaled with the same α and ω_m-values.

The limited-fetch relations (4) — (6) apply only to idealised cases of stationary, offshore wind fields. Thus they cannot be used as a basis for the rigorous construction of a general wave spectrum ensemble for arbitrary wind fields. Nevertheless, they represent a generalisation of the Pierson-Moskowitz spectrum through the inclusion of the fetch as an additional parameter besides the wind. The fetch-limited data may therefore be used to construct an example illustrating the effect of

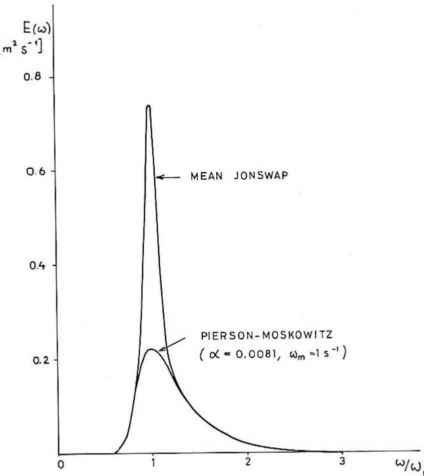

Figure 1: Pierson-Moskowitz and mean JONSWAP spectrum

successively improving the statistical representation of the wave spectrum ensemble. We consider three cases, where each case is derived from the previous case by adding a further parameter in the characterisation of the spectrum ensemble:

(a) the wave spectrum is given by the Pierson-Moskowitz form (3) and depends only on the wind speed,

(b) the spectrum depends on wind-speed and fetch in accordance with the mean JONSWAP relations (4) — (7),

(c) the ensemble of JONSWAP spectrum is generalised to include the statistical variability of the peak enhancement factor γ. The empirical probability distribution p (γ) (Fig. 2, from JONSWAP data) is assumed to be independent of U and x.

The computation of the wave-spectrum ensemble is based in all cases on the same probability density of wind fields, which we take as the simple analytical form

$$p(U,x) = \frac{U}{U_0^2}\exp\left(\frac{-U^2}{2U_0^2}\right)\cdot\frac{x}{x_0^2}\exp\left(\frac{-x^2}{2x_0^2}\right) \qquad (8)$$

The mean values and variances of the distribution are given by $\sqrt{\pi}U_0$, $\sqrt{\pi}x_0$ and $0.86\ U_0^2$, $0.86\ x_0^2$, respectively. In interpreting (8) for an arbitrary ensemble of wind fields x may be regarded in the usual approximate sense as the distance travelled by the principal waves within the region of uniform wind speed U.

As system response we take a single degree of freedom q characterised by a directionally independent transfer function T (ω) with a sharply peaked resonance at ω_r. If the system is linear, the response is Gaussian and is completely characterised by the variance $v = \overline{q^2}$.

For a sharply tuned system the variance is proportional to the input spectrum at the resonance frequency,

$$v = \int |T(\omega)|^2 E(\omega) d\omega \approx K E(\omega_r) \qquad (9)$$

where

$$K = \int |T|^2 d\omega$$

Each realisation ϕ of the wind field yields a variance $v(\phi)$ of the response through equation (9), given the dependence of the wave spectrum on the wind field. If the ensemble of the wind fields is characterised generally by a probability distribution dP (ϕ), the mapping of the wind field ensemble into a probability density p (v) with respect to the variance is given by

$$P(v') = \int \delta(v' - v(\phi))\, dP(\phi) \qquad (10)$$

where δ represents Dirac's function.

In examples (a) and (b), dP (ϕ) = p (U, x) dU · dx; in case (c) the additional statistical variable γ yields dP (ϕ) = p (U, x) p (γ) dUdxdγ. (A common symbol p is used for all probability densities, different functions being distinguished by their arguments).

Figs. 3, 4 and 5 show the probability distributions p (v) computed according to (10) for the three cases (a), (b) and (c).

The wind field ensemble can be characterised by two nondimensional parameters, a mean nondimensional fetch $\tilde{x}_0 = g x_0 / U_0^2$ and the ratio ω_r/ω_0, where $\omega_0 = 7\pi\,(g/U_0)\,\tilde{x}_0^{-0.33}$ is a mean peak wave frequency. The limited-fetch relations (5) — (7) apply for field data in the range $10^2 \leq \tilde{x} \leq 3.10^4$, the upper nondimensional fetch limit representing the transition to the fully developed spectrum. Figs. 1 and 2 ($\tilde{x}_0 = 3.10^2$)

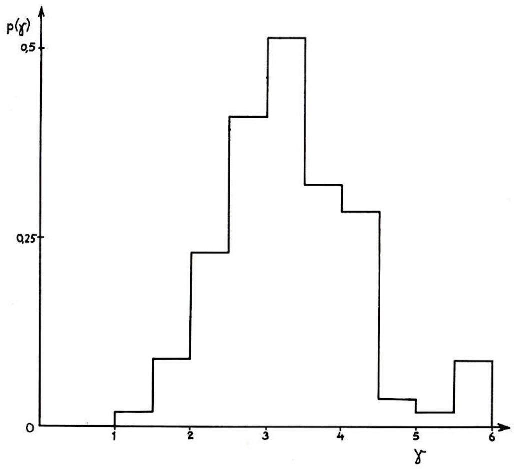

Figure 2: Probability distribution of peak enhancement factor γ

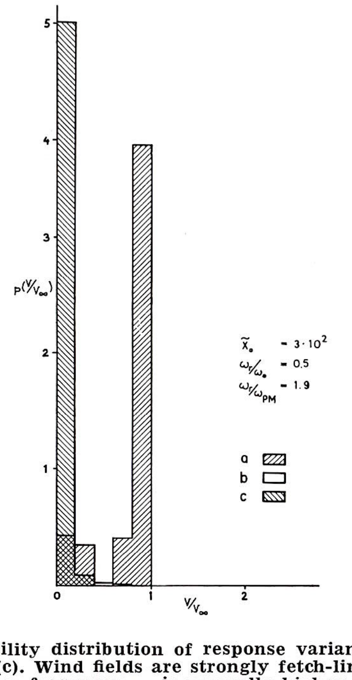

Figure 3: Probability distribution of response variance v for models (a), (b) and (c). Wind fields are strongly fetch-limited. In model (a) the response frequency ω_r is generally higher, in models (b) and (c) lower than the peak frequency of the spectrum.

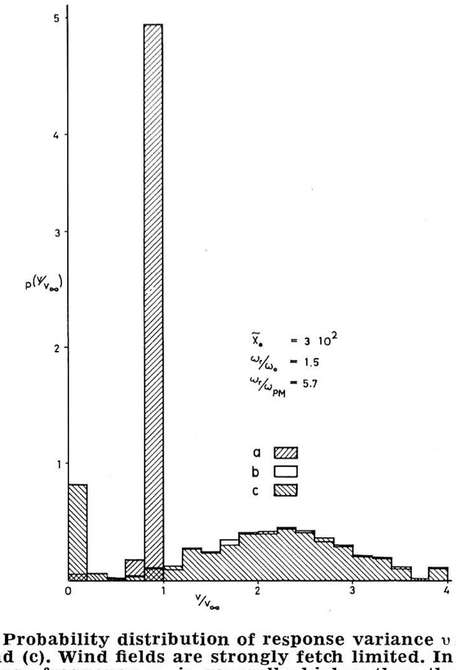

Figure 4: Probability distribution of response variance v for models (a), (b) and (c). Wind fields are strongly fetch limited. In all models the response frequency ω_r is generally higher than the peak frequency of the spectrum.

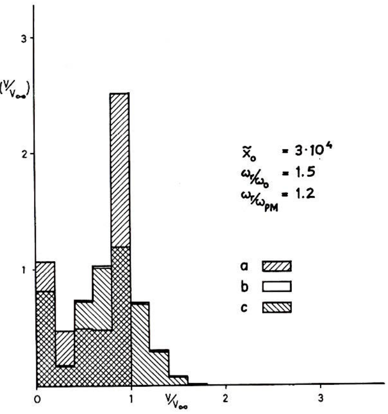

Figure 5: Probability distribution of response variance υ for models (a), (b) and (c). Wind fields lie close to the fully developed case. In all models the response frequency is generally slightly higher than the peak frequency of the spectrum.

correspond to an ensemble of strongly fetch-limited wind fields, whereas the ensemble of wind fields in fig. 3 ($\tilde{x}_0 = 3.10^4$) lies on the border to the fully developed case.

As expected, the variations between models (a), (b) and (c) in fig. 5 are small.

However, figs. 3 and 4 show pronounced differences between the models. The ratio $\omega_r/\omega_0 = 0.5$ in fig. 1 implies that for most of the wind fields the resonance peak ω_r lies to the left of the fetch-limited spectral peak. This yields a high probability for small responses υ in models (b) and (c). For $\tilde{x} = 300$, the ratio $\omega_r/\omega_0 = 0.5$ corresponds to a ratio $\omega_r/\omega_{PM} = 1.9$ where $\omega_{PM} = (0.74/1.25)^{1/4}\, g/U_0$ is the mean peak frequency of the ensemble of Pierson-Moskowitz spectra. Thus for most U the resonance peak ω_r lies well to the right of the Pierson-Moskowitz peak. In this case the Pierson-Moskowitz spectrum at ω_r is close to the asymptotic value $E_\infty = \alpha_{PM}\, g^2\, \omega_r^{-5}$ for $U \to \infty$. Consequently the probability distribution p (υ) for model (a) has a pronounced peak at the maximum value $\upsilon_\infty = KE_\infty$.

In fig. 4 the ratio $\omega_r/\omega_0 = 1.5$, corresponding to $\omega_r/\omega_{PM} = 5.7$, yields a similar but still more peaked distribution p (υ) for model (a). However, the distribution p (υ) in cases (b) and (c) is now radically altered. Small values of υ are less probable, since the resonance frequency is normally higher than the fetch-limited spectral peak at ω_m. For $\omega_r > \omega_m$, the response υ is determined largely by α, and the high values of α at small fetches $\tilde{x}$ (cf. equation (5)) yield υ-values considerably greater than the cut-off value υ_∞ of the fully developed model.

In all examples, the statistical variations of γ in model (c) yield only minor modifications of the fetch-limited model (b). However, the differences between the fully-developed model (a) and the fetch-limited models (b) and (c) are considerable for fetch-limited wind fields. It is of interest that the fetch-limited model can yield appreciably higher responses than the fully developed model (fig. 4), although this depends strongly on the peak frequency of the response (cf. figs. 3 and 4). For safety considerations the fully-developed model is inadequate, since it predicts a maximal possible response υ_∞ which, as the fetch-limited models demonstrate, can in fact be significantly exceeded.

3. *Construction of wave spectrum statistics from wind fields by numerical hindcasting*

The marked differences between wave statistics based on the finite-fetch and the fully developed model demonstrate the limitations of deriving spectrum statistics from idealised empirical formulae. Although more general than the fully developed spectrum, the finite-fetch JONSWAP relations also apply only to a special class of wind fields and cannot provide a realistic basis for deriving general sea state probability distributions. The only available method for constructing such statistics — apart from extensive observations — is to compute the ensemble of wave spectra from the known ensemble of wind fields using numerical hindcasting techniques.

These are based on the integration of the spectral energy balance equation (Gelci et al., 1957; Hasselmann, 1960; Pierson et al., 1966; Darbyshire and Simpson, 1967; Inoue, 1967; Barnett, 1968; Gelci and Devillaz, 1970; Ewing, 1971)

$$\frac{\partial}{\partial t} F(\omega, \theta; \underset{\sim}{x}, t) + (\underset{\sim}{v} \cdot \nabla) F = S \qquad (11)$$

Here υ denotes the group velocity of the waves and S the net source function. S may be divided generally into three terms

$$S = S_{in} + S_{nl} + S_{ds}$$

where S_{in} represents the input by the wind, $S_{n\ell}$ the nonlinear transfer across the spectrum by conservative wave–wave interactions, and S_{ds} the energy loss due to linear or nonlinear dissipative process such as white capping.

Some insight into the structure of S was obtained from JONSWAP. In particular, it was found that the nonlinear transfer plays a dominant role in controlling the shape of the spectrum, the degree of overshoot, the sharp cut-off at low frequencies and the shift of the peak from high to low frequencies with increasing fetch. However, the terms S_{in} and S_{ds} still remain largely unknown. Various theories predicting the general structure of S_{in} or S_{ds} have been suggested (cf. Phillips, 1966; Hasselmann, 1968, 1973), but verification through detailed field tests has hitherto been lacking. Existing numerical wave prediction models use expressions for these terms based partly on theory, partly on empirical hindcasting experience. Further extensive field investigations will be needed to provide experimental support for the various hypotheses which still enter in the prediction models.

The application of deterministic prediction models to generate wave spectrum statistics poses additional problems not encountered in straightforward prediction applications. Formally a statistical ensemble of wave spectra represents a probability distribution in infinite-dimensional function space. To reduce this to a manageable probability density in a finite dimensional space, the wave spectra must be parametrised. The usual discretisation into a number of frequency and directional bands involves far more degrees of freedom than can usefully be incorporated in a multidimensional probability distribution. The optimal projection of the information generated by prediction models (or obtained from measurements) onto the much smaller set of parameters appropriate for a statistical representation is an important problem for the statistical treatment of sea state which has not yet been seriously addressed.

A related problem is the large computing time requirement for the generation of reliable statistics from high resolution wave prediction models. A possible solution is to develop numerical models which predict the evolution of the basis parameters of the statistical representation directly rather than the full set of frequency and directional parameters used in present prediction schemes (cf. Hasselmann et al., 1973). This would also be in accordance with the parametrisation requirements for the nonlinear source term $S_{n\ell}$, which is too complex to be computed rigorously at each time step in the numerical integration of the energy balance equation (11).

It is clear that considerable work, both experimental and theoretical, is still needed before reliable wave spectrum statistics can be generated numerically from recorded wind field data. However, the goal does not appear unrealistic with existing field instrumentation and computer technology. Considerable progress has been made in the past years in the understanding of ship and structural response to wave forces. In order for these results to have full impact on optimal system design, more work will be needed to apply the parallel advances in the field of wave dynamics and wave prediction to the common problem of wave spectrum statistics.

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