

EQUATION OF STATE CONSTRAINTS FROM NUCLEAR PHYSICS, NEUTRON STAR MASSES, AND FUTURE MOMENT OF INERTIA MEASUREMENTS

S. K. GREIF,^{1,2} K. HEBELER,^{1,2} J. M. LATTIMER,³ C. J. PETHICK,^{4,5} AND A. SCHWENK^{1,2,6}

¹Institut für Kernphysik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany

²ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, D-64291 Darmstadt, Germany

³Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794-3800, USA

⁴The Niels Bohr International Academy, The Niels Bohr Institute,

University of Copenhagen, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

⁵NORDITA, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, SE-10691 Stockholm, Sweden

⁶Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, D-69117 Heidelberg, Germany

ABSTRACT

We explore constraints on the equation of state (EOS) of neutron-rich matter based on microscopic calculations up to nuclear densities and observations of neutron stars. In a previous work we showed that predictions based on modern nuclear interactions derived within chiral effective field theory and the observation of two-solar-mass neutron stars result in a robust uncertainty range for neutron star radii and the EOS over a wide range of densities. In this work we extend this study, employing both the piecewise polytrope extension from Hebeler et al. as well as the speed of sound model of Greif et al., and show that moment of inertia measurements of neutron stars can significantly improve the constraints on the EOS and neutron star radii.

1. INTRODUCTION

Recently, there has been significant progress in our understanding of the equation of state (EOS) of dense matter. This was triggered by advances in nuclear theory, new constraints from precise measurements of heavy neutron stars, as well as astrophysical observations from the LIGO/Virgo (Abbott et al. 2018, 2019) and NICER (Miller et al. 2019; Raaijmakers et al. 2019; Riley et al. 2019) collaborations. These offer complementary insights to the EOS. While nuclear theory provides reliable predictions for neutron-rich matter up to densities around saturation density ($\rho_0 = 2.8 \times 10^{14} \text{ g cm}^{-3}$), observations of neutron stars, and neutron star mergers probe the EOS over a higher range of densities but provide indirect constraints.

In nuclear physics the development of chiral effective field theory (EFT) has revolutionized our approach to nuclear forces. The description of the interactions between neutrons and protons, both particles with a complex substructure, has been a challenge in nuclear theory for decades. Pioneered by the seminal works of Weinberg (1990, 1991), chiral EFT has now become the only known framework that allows a systematic expansion of nuclear forces at low energies (Epelbaum et al. 2009; Machleidt & Entem 2011; Hammer et al. 2013) based on the symmetries of quantum chromodynamics (QCD), the fundamental theory of the strong interaction. In addition, chiral EFT allows one to derive systematic estimates of uncertainties for observables. Incorporating such chi-

ral EFT interactions in microscopic many-body frameworks makes it possible to compute uncertainty bands for the pressure and energy density of matter (Hebeler & Schwenk 2010; Carbone et al. 2013; Holt et al. 2013; Tews et al. 2013; Wellenhofer et al. 2014; Drischler et al. 2016, 2019, 2020; Lynn et al. 2016). As any effective low-energy theory, chiral EFT contains an intrinsic breakdown scale. When approaching this breakdown scale with increasing energy or density the convergence of the effective expansion becomes slower until eventually it breaks down. This breakdown scale translates into an upper density limit for such calculations. The precise value for this upper density limit is still unknown, and also depends on details of the interactions. In a previous work (Hebeler et al. 2013), we chose an upper density limit of $1.1 \rho_0$ for neutron-rich matter. This limit represents a rather conservative choice and it might be possible to push this limit to somewhat higher densities (Tews et al. 2018), although a full understanding of the implied uncertainties is still an open problem. Finally, for very high densities ($\rho \gtrsim 50 \rho_0$), there are model-independent constraints from perturbative QCD calculations of quark matter (Kurkela et al. 2010).

Neutron star observations provide powerful constraints on the EOS beyond the densities accessible by nuclear theory as well as laboratory experiments (Tsang et al. 2012). In particular, the precise mass measurements of the pulsars PSR J1614-2230 and PSR J0348+0432 with masses of $1.928 \pm 0.017 M_\odot$ (Fonseca et al. 2016)

and $2.01 \pm 0.04 M_{\odot}$ (Antoniadis et al. 2013) turned out to be a key discovery, as the existence of such heavy neutron stars puts tight constraints on the EOS and the composition of matter, ruling out a large number of EOSs with simple inclusion of exotic degrees of freedom like hyperons or deconfined quarks. Recently, the mass of the pulsar PSR J0740+6620 was measured to be $2.14^{+0.10}_{-0.09} M_{\odot}$ (Cromartie et al. 2019), which further tightens these constraints.

In this work, we study the EOS constraints that can be achieved from future moment of inertia measurements, in addition to the heavy mass constraint discussed above. The moment of inertia has been suggested to provide complementary constraints for the EOS (Ravenhall & Pethick 1994; Lyne et al. 2004; Lattimer & Schutz 2005). It can be obtained from measurements of the rate of advance of the periastron, $\dot{\omega}$ (Damour & Schäfer 1988). This advance is mainly caused by the relativistic spin-orbit coupling in a binary system (Barker & O’Connell 1975; Wex 1995; Kramer & Wex 2009), and the magnitude of the advance depends sensitively on the orbital period and the compactness of the binary system. In 2003, the double neutron-star system PSR J0737–3039 was discovered (Burgay et al. 2003; Lyne et al. 2004). This system is particularly promising for such measurements, as it is extremely compact with an orbital period of only 2.4 hr (Burgay et al. 2003, 2005; Lyne et al. 2004). In addition, due to the high orbital inclination (Burgay et al. 2003, 2005), the masses of the two neutron stars have been determined very precisely to be $1.3381(7) M_{\odot}$ and $1.2489(7) M_{\odot}$ (Kramer & Wex 2009). Due to the compactness of the system, the moment-of-inertia correction to $\dot{\omega}$ is estimated to be an order of magnitude larger for PSR J0737-3039A (the heavier of the two pulsars) than for other systems like PSR B1913+16 (Lyne et al. 2004). Such a moment of inertia measurement has to be performed over a long period of time and an increase of timing precision would be beneficial (Kramer & Wex 2009). Based on this, it was argued that a moment of inertia measurement with a relative uncertainty of about 10% may be achievable (Damour & Schäfer 1988; Lattimer & Schutz 2005; Kramer & Wex 2009).

Previous works studied to what extent such measurements are able to provide constraints for different types of EOS (Morrison et al. 2004; Bejger et al. 2005; Lattimer & Schutz 2005). In particular, Ravenhall & Pethick (1994) showed that the moment of inertia can be parameterized efficiently as a function of the compactness parameter, and Lattimer & Schutz (2005) demonstrated that a universal relation between the moment of inertia and the compactness parameter exists, which can be used to provide constraints on neutron star radii. More recently, Steiner et al. (2015), Gorda (2016), and Lim et al. (2019) studied the moment of inertia based on neutron star ob-

servations and EOS constraints, and Raithel et al. (2016) investigated the inference of neutron star radii from moment of inertia measurements.

In this work, we study how microscopic calculations based on chiral EFT interactions combined with neutron star masses and a future moment of inertia measurement can provide novel predictions for the EOS and neutron star radii. In Section 2, we briefly review our approach employing both the piecewise polytrope extension from Hebeler et al. (2013) as well as the speed of sound model of Greif et al. (2019) and present uncertainty ranges for neutron star observables such as the mass, the radius, and the moment of inertia. In Section 3, we present our results for neutron star radii, and how these can improve upon information from the neutron star merger GW170817 (Abbott et al. 2019). Moreover, we discuss the resulting EOS constraints and explore scaling relations for the dimensionless moment of inertia. Finally, we conclude in Section 4.

2. CONSTRAINTS FROM NUCLEAR THEORY AND NEUTRON STAR MASSES

In Hebeler et al. (2010, 2013) we combined constraints from nuclear physics and neutron star masses to derive constraints for the EOS for all densities relevant for neutron stars. We briefly review the strategy of this work and refer to Hebeler et al. (2013) for details:

a) The first constraint results from microscopic calculations of neutron-rich matter up to density $\rho_1 = 1.1 \rho_0$ based on modern nuclear interactions derived from chiral EFT (Hebeler & Schwenk 2010; Tews et al. 2013). These calculations resulted in uncertainty bands for the energy density and pressure. For densities below $\rho_{\text{crust}} = 0.5 \rho_0$ the BPS crust EOS of Baym et al. (1971) and Negele & Vautherin (1973) was used. Remarkably, around the transition density ρ_{crust} both EOSs overlap smoothly, so that our final results are insensitive to the particular choice for ρ_{crust} .

b) Based on the constraints from nuclear physics at low densities the EOS was extended in a general way to higher densities using piecewise polytropes, $P(\rho) = K_i \rho^{\Gamma_i}$, with the adiabatic indices Γ_i and constants K_i (see also Read et al. (2009)). The values for Γ_i are allowed to vary freely, whereas the values of K_i are fixed by the constraint that the EOS should be continuous as a function of density. For the extension beyond ρ_1 , three polytropes characterized by exponents Γ_1, Γ_2 (beyond ρ_{12}), and Γ_3 (beyond ρ_{23}) allow one to control the softness or stiffness of the EOS in a given density region, and the transition densities ρ_{12} and ρ_{23} between polytropes are allowed to vary as well. Sampling all possible EOSs using the step size $\Delta\Gamma_i = 0.5$ and $\Delta\rho_{12,23} = \rho_0/2$ results in a very large number of possible EOSs (for details, see Hebeler et al. (2013)), which include construc-

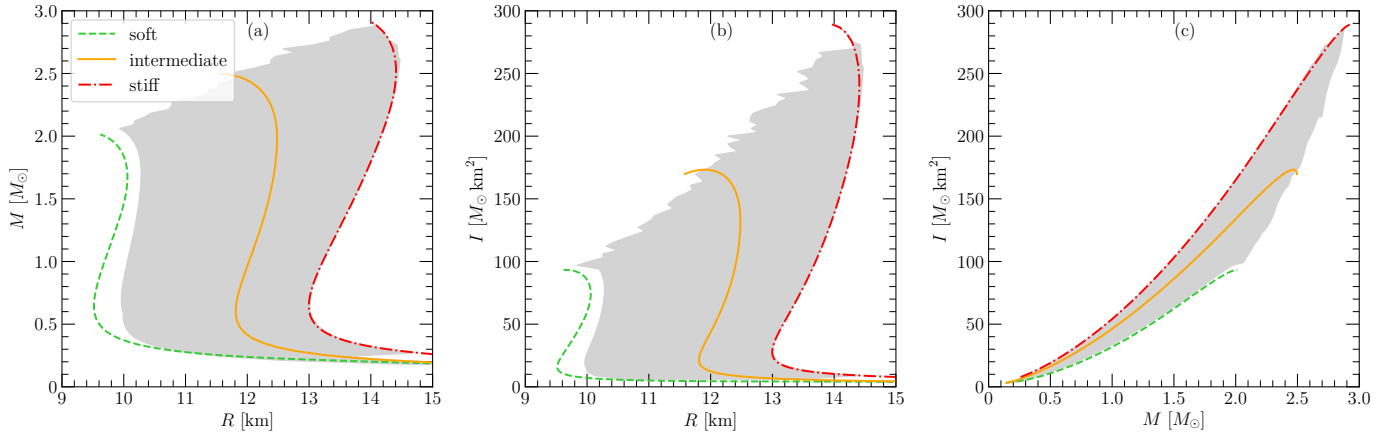


Figure 1. Results for mass M , radius R , and moment of inertia I of neutron stars based on the EOS constraints (bands) derived with the piecewise polytrope model based on chiral EFT calculations up to density $\rho_1 = 1.1\rho_0$, the new mass constraint $M_{\text{obs}} \geq 2.05 M_\odot$, and causality constraints. The individual panels (a), (b), and (c) show the mass-radius, moment of inertia-radius, and moment of inertia-mass results, respectively. The green (dashed), yellow (solid), and red (dotted-dashed) lines correspond to the three representative EOS (soft, intermediate, and stiff, respectively) from [Hebeler et al. \(2013\)](#). Note that the latter are for the old mass constraint $M_{\text{obs}} \geq 1.97 M_\odot$, so that the soft EOS leads to smaller radii.

tions that mimic first-order phase transitions. The values of Γ_i , ρ_{12} , and ρ_{23} are then constrained by the condition that each EOS must be able to support a neutron star of at least $M_{\text{obs}} = 2.05 M_\odot$, which we take as the 68% lower limit of the mass of the heaviest precisely known pulsar ([Cromartie et al. 2019](#)). This mass constraint provides an update compared to the 1σ lower limit ($1.97 M_\odot$) of the mass of PSR J0348+0432 ([Antoniadis et al. 2013](#)) used in [Hebeler et al. \(2013\)](#).

c) As the final constraint we require that the speed of sound, c_s , remain smaller than the speed of light, c , for all densities: $c_s/c = \sqrt{dP/d\mathcal{E}} \leq 1$, where P is the pressure and \mathcal{E} is the energy density. Each EOS is followed in density until causality is violated or the maximum neutron star mass is reached when $dM/dR = 0$.

The combination of these three conditions leads to mass-radius constraints on neutron stars shown in panel (a) of Fig. 1. In general, the boundaries of the band are spanned by a large number of different EOSs, but to distinguish soft and stiff EOSs, we show the three representative EOSs (soft, intermediate, and stiff) of [Hebeler et al. \(2013\)](#), which span the radius range as shown in Fig. 1, while the soft EOS leads to somewhat smaller radii due to the previous mass constraint $M_{\text{obs}} \geq 1.97 M_\odot$. For a typical $M = 1.4 M_\odot$ star, the update gives a radius range of $R = 10.2\text{--}13.6$ km (taking the chiral EFT constraints from renormalization-group-evolved interactions, which have improved many-body convergence; [Hebeler et al. \(2013\)](#)).

In order to explore the sensitivity to details of the high-density extension, we also employ the speed of sound model of [Greif et al. \(2019\)](#) in addition to the piece-

wise polytrope extension. The speed of sound model is based on the same crust EOS and chiral EFT band, but uses a parameterization of the speed of sound to high densities, which includes a maximum in the speed of sound $c_s^2/c^2 > 1/3$ and an asymptotic convergence to the conformal limit from below, for very high densities ($\rho \gtrsim 50\rho_0$) suggested by the perturbative QCD calculations ([Kurkela et al. 2010](#)). The two different extensions lead to small changes in the predicted ranges, e.g., for the radius of a neutron star. These differences result from the choice of three polytropes and the particular functional form chosen for the speed of sound parameterization, and would be diminished for arbitrarily fine discretizations of the high-density part of the EOS.

In this work we build on our past mass-radius results ([Hebeler et al. 2013](#); [Greif et al. 2019](#)) and investigate how future moment of inertia measurements of neutron stars will be able to further constrain the EOS and neutron star radii. To this end, we investigate rotating neutron stars and use the Hartle–Thorne slow-rotation approximation ([Hartle 1967](#); [Hartle & Thorne 1968](#)). Later studies have been more conservative, verifying the applicability of this treatment to frequencies up to $f \approx 200$ Hz ([Benhar et al. 2005](#); [Cipolletta et al. 2015](#)). The heavier neutron star of the system PSR J0737–3039 has a period of about 23 ms ([Burgay et al. 2003](#); [Lyne et al. 2004](#)) and can hence reliably be treated within the slow-rotation approximation.

Panels (b) and (c) of Fig. 1 show the results for the moment of inertia I as a function of neutron star mass and radius based on our EOS bands from the piecewise polytrope extension. The moment of inertia can reach values

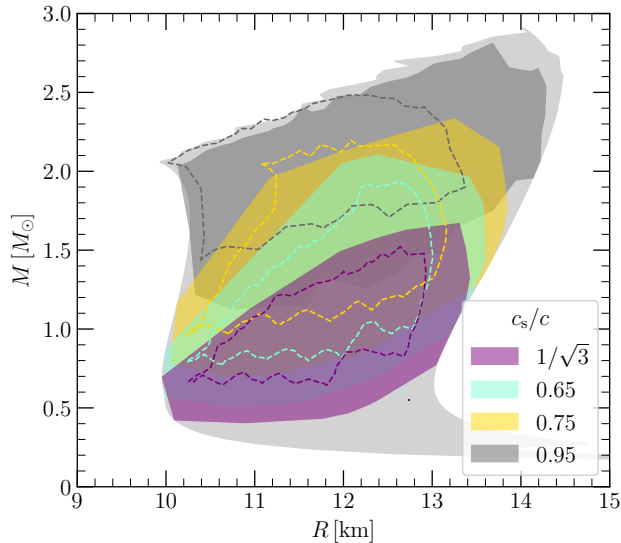


Figure 2. Mass M as a function of radius R . The gray area depicts the entire region allowed by the general EOS construction using the piecewise polytrope extension. The highlighted areas represent M – R pairs that reach values for the speed of sound $c_s/c \leq 1/\sqrt{3}$ (purple), 0.65 (blue), 0.75 (orange), and 0.95 (dark gray). The dashed lines mark the corresponding regions for the speed of sound model.

up to about $290 M_\odot \text{ km}^2$ for very heavy neutron stars, where the maximal values are clearly correlated with the stiffness of the EOS. In addition, it is manifest that the three EOSs which are representative with respect to the radius are also representative with respect to the moment of inertia and practically span the full moment-of-inertia range (with only minor modifications for the soft EOS due to the new mass constraint). For the pulsar PSR J0737-3039A with $M = 1.338 M_\odot$ we find the moment of inertia to be in the range $I = 53.2\text{--}85.7 M_\odot \text{ km}^2$. Our predicted range is significantly smaller than that of Raithel et al. (2016), where $I = 21.1\text{--}113.2 M_\odot \text{ km}^2$, and similar to the range obtained by Gorda (2016) with $I = 60.3\text{--}90.5 M_\odot \text{ km}^2$.

In addition, we show the speed of sound c_s reached in our general EOS bands. In Fig. 2 the highlighted areas represent M – R pairs that reach particular values for c_s/c . Note that c_s/c is small at low densities in the nonrelativistic chiral EFT calculations and reaches $1/\sqrt{3} \approx 0.577$ from below in the perturbative QCD regime (Kurkela et al. 2010). Figure 2 clearly demonstrates that c_s/c has to reach values of around 0.65 to be compatible with two-solar-mass neutron stars. In particular, if one demands that $c_s/c \leq 1/\sqrt{3}$ for all densities in neutron-star matter, no EOS exists in our general construction that is compatible with the observed heavy neutron stars. This has also been pointed out by Bedaque & Steiner (2015) and is consistent with the findings of

Table 1. Radius Constraints Resulting from Mass and Moment of Inertia Measurements for the Same Star, Assuming the Mass Uncertainty Is Negligible and Using the Piecewise Polytrope Extension

M	I_c	$R(\pm 10\%)$	$R(\pm 20\%)$	R	
1.338	I_{low}	55	10.2–11.4	10.2–12.0	10.2–13.6
	I_{int}	70	11.3–12.9	10.6–13.4	10.2–13.6
	I_{high}	85	12.5–13.6	11.8–13.6	10.2–13.6
2.0	I_{low}	95	10.1–11.0	10.1–11.7	10.1–14.2
	I_{int}	135	11.6–13.5	10.8–14.0	10.1–14.2
	I_{high}	165	13.1–14.2	12.3–14.2	10.1–14.2
2.4	I_{low}	–	–	–	11.6–14.4
	I_{int}	170	11.6–13.2	11.6–13.8	11.6–14.4
	I_{high}	220	13.3–14.4	12.4–14.4	11.6–14.4

Note. The columns give the assumed values for M (in units of M_\odot) and central value I_c of the moment of inertia (in units of $M_\odot \text{ km}^2$), as well as the resulting radius ranges from Fig. 3 (in units of km), assuming a relative uncertainty of $\Delta I = \pm 10\%$ and $\pm 20\%$, respectively. The last column gives the radius range in the absence of a moment of inertia measurement. For each assumed mass, we consider three values of I_c that approximately correspond to the soft, intermediate, and stiff EOS: I_{low} , I_{int} , and I_{high} , respectively

Table 2. Same As Table 1 but Corresponding to Fig. 4 Using the Speed of Sound Model to Extrapolate to Higher Densities

M	I_c	$R(\pm 10\%)$	$R(\pm 20\%)$	R	
1.338	I_{low}	55	10.4–11.5	10.4–12.0	10.4–13.2
	I_{int}	70	11.3–12.9	10.7–13.2	10.4–13.2
	I_{high}	85	12.6–13.2	11.8–13.2	10.4–13.2
2.0	I_{low}	95	10.1–11.0	10.1–11.6	10.1–13.6
	I_{int}	135	11.7–13.4	10.8–13.6	10.1–13.6
	I_{high}	165	13.2–13.6	12.3–13.6	10.1–13.6
2.4	I_{low}	–	–	–	11.1–13.5
	I_{int}	170	11.3–13.0	11.1–13.5	11.1–13.5
	I_{high}	220	13.3–13.5	12.3–13.5	11.1–13.5

Tews et al. (2018) and Greif et al. (2019).

3. IMPROVED CONSTRAINTS FROM MOMENT OF INERTIA MEASUREMENTS

Based on the frameworks discussed in Section 2, we now investigate to what extent moment of inertia measurements can improve these constraints. To this end, we assume that it is possible to measure simultaneously the neutron star mass (with negligible uncertainty) and the moment of inertia with central value I_c and relative uncertainty of $\Delta I = \pm 10\%$ and $\pm 20\%$, respectively. We consider three different masses, $M = 1.338 M_\odot$, $2.0 M_\odot$, and $2.4 M_\odot$, and for each mass, three possible central values I_c , given by I_{low} , I_{int} , and I_{high} , which approximately correspond to the moment of inertia given by the three representative EOSs shown in panel (c) of Fig. 1. The

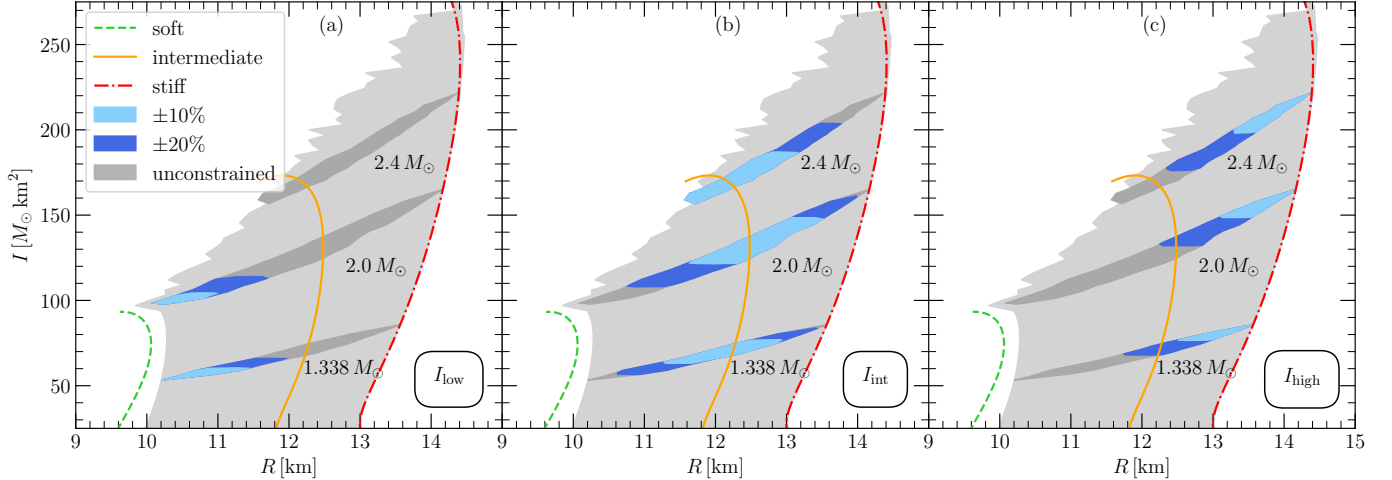


Figure 3. Moment of inertia I as a function of radius R . The gray band gives the allowed I – R range resulting from the general EOS band for the piecewise polytrope extension as shown in Fig. 1. The dark gray, light blue, and dark blue areas show the allowed I – R values for the particular neutron star masses indicated, where the dark gray area includes all possible I – R pairs for each mass, and the light blue (dark blue) area corresponds to an assumed measurement of the moment of inertia with central value I_c given in Table 1 with a relative uncertainty of $\Delta I = \pm 10\%$ ($\pm 20\%$). The three panels assume central values I_c that approximately correspond to the soft (a), intermediate (int) (b), and stiff (c) EOS, see Table 1. Note that for a $2.4 M_\odot$ neutron star, the soft EOS is ruled out and thus no compatible I_c exists in this case.

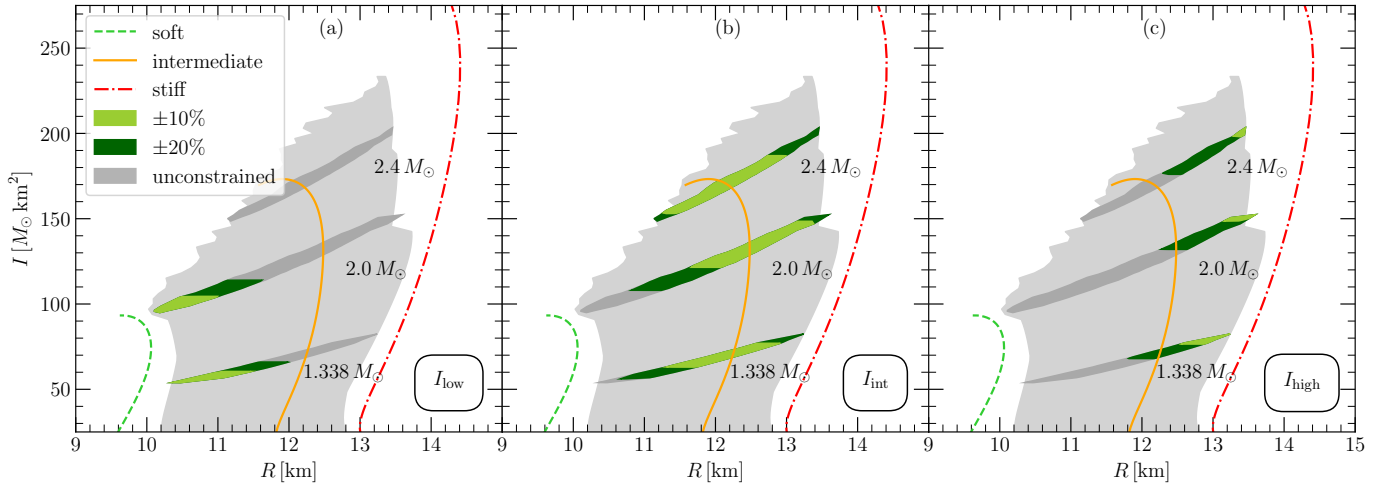


Figure 4. Same as Fig. 3 but using the speed of sound model from Greif et al. (2019) to extrapolate to high densities.

values of I_c for these assumed measurements are listed in Table 1, where we also give the improved radius ranges resulting from such a simultaneous measurement. In addition, we show the allowed I – R areas in Fig. 3, where the three panels correspond to the low, intermediate, and high I_c cases. For a $2.4 M_\odot$ neutron star, the soft EOS is ruled out (see Fig. 1), and no low I_c scenario exists in this case. We also note that the EOS can have a more intricate behavior in the general EOS band, e.g., going from soft to stiff and vice versa with higher slopes in the M – R diagram (see Fig. 10).

Moreover, we show in Table 2 and Fig. 4 how these

radius constraints change if one uses the speed of sound model instead of the piecewise polytrope extension. The results show that the radius constraints are remarkably consistent, with the largest differences due to the underlying allowed bands (see the gray regions versus the area within the representative EOS in Fig. 4), occurring for heavy mass neutron stars and the high I_c case.

Figures 3 and 4 clearly show that a measurement of I_c with a relative uncertainty of $\Delta I = \pm 10\%$ ($\pm 20\%$) in (almost) all cases significantly improves the constraints on neutron star radii. For a $\pm 10\%$ measurement, if the measured value of I_c is located close to the center of the EOS

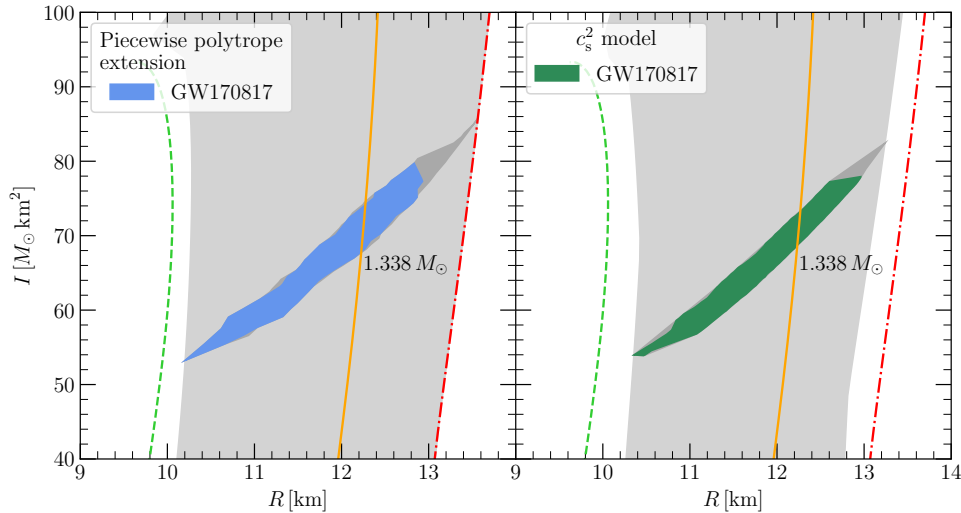


Figure 5. Allowed values for the moment of inertia I as a function of radius R (gray bands) resulting from the general EOS construction using the piecewise polytrope extension (left panel) and the speed of sound model (right panel). The darker gray regions indicate the $I - R$ pairs that are consistent with a $1.338 M_{\odot}$ neutron star, whereas the blue and green highlighted areas include in addition the GW170817 constraints for the chirp mass, mass ratio, and binary tidal deformability from LIGO/Virgo (Abbott et al. 2019), see the text for details.

band, the radius range decreases by about 50%, whereas the radius becomes even more narrowly predicted when I_c is close to low or high values. In the latter cases, the radius spread in Table 1 is only 0.9–1.2 km for the piecewise polytrope extension and 0.2–1.1 km for the speed of sound model.

Next, we focus on the neutron star PSR J0737–3039A with mass $1.338 M_{\odot}$, which is the target of a future moment of inertia measurement. In Fig. 5 we show the allowed values for the moment of inertia as a function of radius resulting from the piecewise polytrope extension (left panel) and the speed of sound model (right panel), where the darker gray regions indicate the $I - R$ pairs that are consistent with a $1.338 M_{\odot}$ star. The impact of an accurate I measurement is clear from the representative cases in Tables 1 and 2. Figure 5 shows again that the tightest radius constraints would result from I_c values toward the extremes of our general EOS bands.

In addition, we explore the constraints from the gravitational-wave signal of the neutron star merger GW170817 (Abbott et al. 2018, 2019). In Fig. 5, we have highlighted the $I - R$ regions in blue (green) for the general EOS construction based on the piecewise polytrope extension (speed of sound model) that are consistent with the LIGO/Virgo results (Abbott et al. 2019) for the chirp mass $\mathcal{M} = 1.186 \pm 0.001 M_{\odot}$, the mass ratio $q = 0.73 - 1.00$, and the binary tidal deformability $\tilde{\Lambda} = 300^{+420}_{-230}$ (for the 90% highest posterior density interval). These ranges are compatible with the analysis of De et al. (2018), suggesting that they are robust with respect to assumptions about the underlying EOS and deformability priors. The comparison to the general

EOS regions without the GW10817 constraints (darker gray vs. blue and green regions) in Fig. 5 shows that the GW170817 observation is consistent with the general EOS band based on nuclear physics and the observation of $2 M_{\odot}$ neutron stars.

In addition to the radius measurement, we can also study the corresponding constraints for the EOS. The different I_c and mass scenarios for the piecewise polytrope extension (corresponding to the radius constraints of Fig. 3 and Table 1) are shown in Fig. 6. The gray region is again the general EOS band of Hebeler et al. (2013) (updated for the maximum mass constraint), whereas the different panels show the constraints for the assumed simultaneous measurements of the mass (different rows) and the moment of inertia (different columns). Naturally, we find that the constraints on the EOS are the strongest for those cases that also give the strongest radius constraints. In addition, small values of I tend to give stronger constraints on the EOS at higher densities, whereas large values for I provide stronger constraints at lower densities. Moreover, measurements of heavy neutron stars provide stronger constraints on the EOS than the scenarios for typical neutron stars. Further, we give in Fig. 7 the EOS constraints for the speed of sound model (corresponding to the radius constraints of Fig. 4 and Table 2). This shows very similar constraints on the EOS, as for the piecewise polytrope extension.

Several studies based on different phenomenological EOS have shown that the dimensionless moment of inertia I/MR^2 correlates with the compactness M/R to a good approximation (Lattimer & Prakash 2001; Bejger &

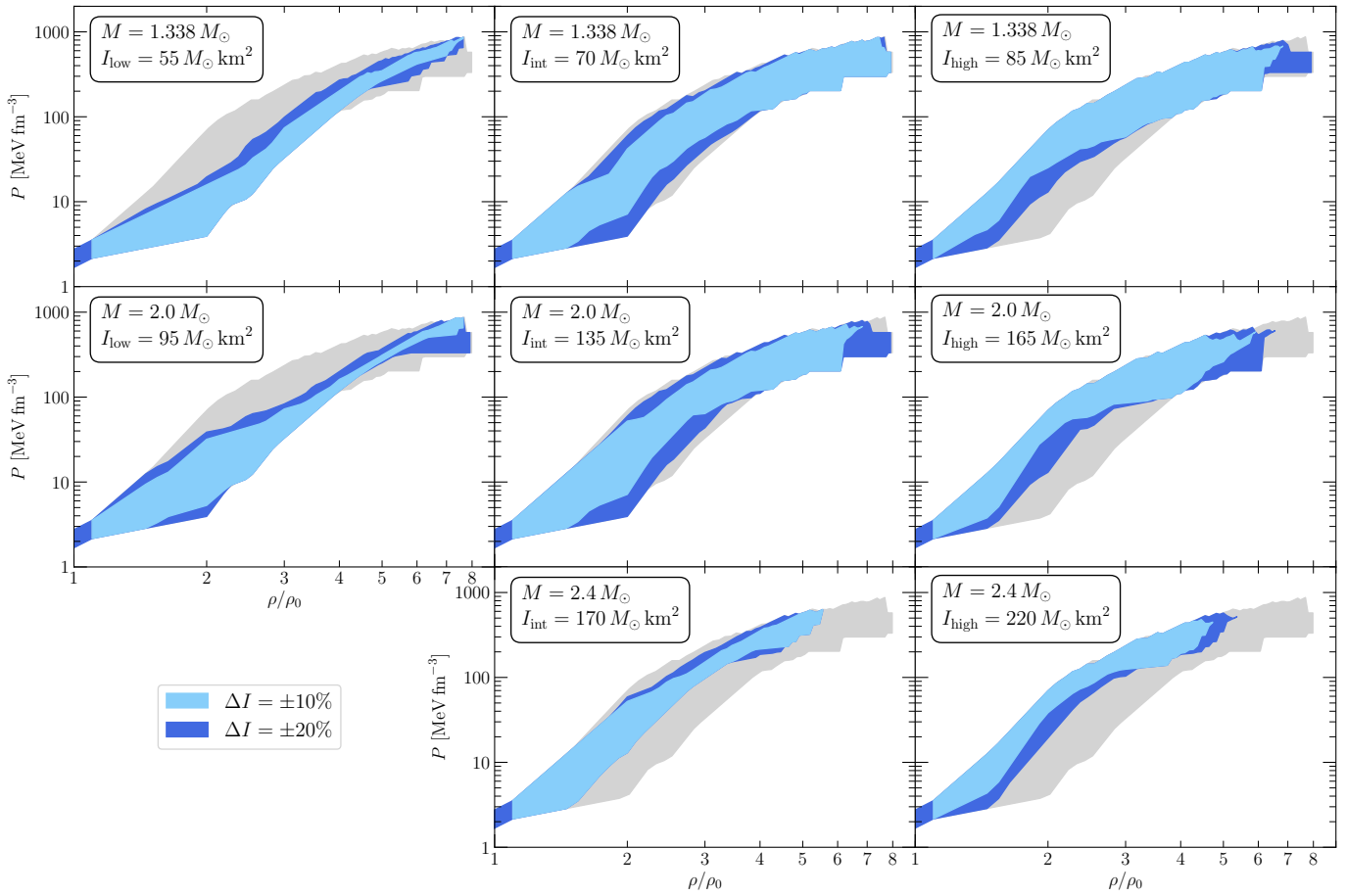


Figure 6. Pressure P as a function of mass density ρ/ρ_0 in units of the saturation density. The gray region is the general EOS band based on the piecewise polytrope extension. The light and dark blue areas show the allowed EOS range for assumed simultaneous measurements of the mass (different rows) and the moment of inertia (different columns), as in Fig. 3 and Table 1, with a relative uncertainty of $\Delta I = \pm 10\%$ ($\pm 20\%$).

Haensel 2002; Lattimer & Schutz 2005; Breu & Rezzolla 2016). In Fig. 8 we present our results for the piecewise polytrope extension (color coded) and the speed of sound model (black dashed line) for the dimensionless moment of inertia, which yield a very similar correlation band, and compare these to the bands from Steiner et al. (2016) and Breu & Rezzolla (2016). Our results agree reasonably well with these for $M/R > 0.15 M_\odot/\text{km}$, while we find a deviation for smaller compactness parameters and also a somewhat larger band for $M/R > 0.2 M_\odot/\text{km}$. This shows that, e.g., predictions for neutron stars with small mass and large radii based on the former correlation bands are not compatible with the general EOS band. This is most likely due to low-density assumptions made that are incompatible with modern nuclear physics.

In addition, we show in the lower panel of Fig. 8 the three representative EOSs (soft, intermediate, and stiff) of Hebeler et al. (2013). These are representative with respect to radius and moment of inertia for all masses (see Fig. 1) but, as is clear from Fig. 8, they do not capture

the extremes of the dimensionless moment of inertia. In order to investigate the band for the dimensionless moment of inertia in more detail, we determined the individual EOSs that represent the limits of the band in Fig. 8 for the piecewise polytrope extension, which provides the more conservative estimate. To this end, we discretized M/R for $M/R \geq 0.1 M_\odot \text{ km}^{-1}$ and determined the χ^2 of each EOS for the deviation of I/MR^2 from the lower (upper) band. The results for the individual EOSs with the minimal χ^2 values are shown as red (blue) lines in the lower panel of Fig. 8.

The corresponding EOSs for these extreme cases are shown in Fig. 9. We observe that the EOSs with a minimum χ^2 with respect to the lower boundary of the dimensionless moment of inertia I/MR^2 (red lines) tend to be rather stiff at nuclear densities and soft at high densities, whereas the EOSs leading to large values of I/MR^2 tend to be soft at nuclear densities and stiff at high densities (blue lines). These trends are also reflected in the results for the mass, radius, and moment of inertia in Fig. 10, where these individual EOSs are clearly extreme

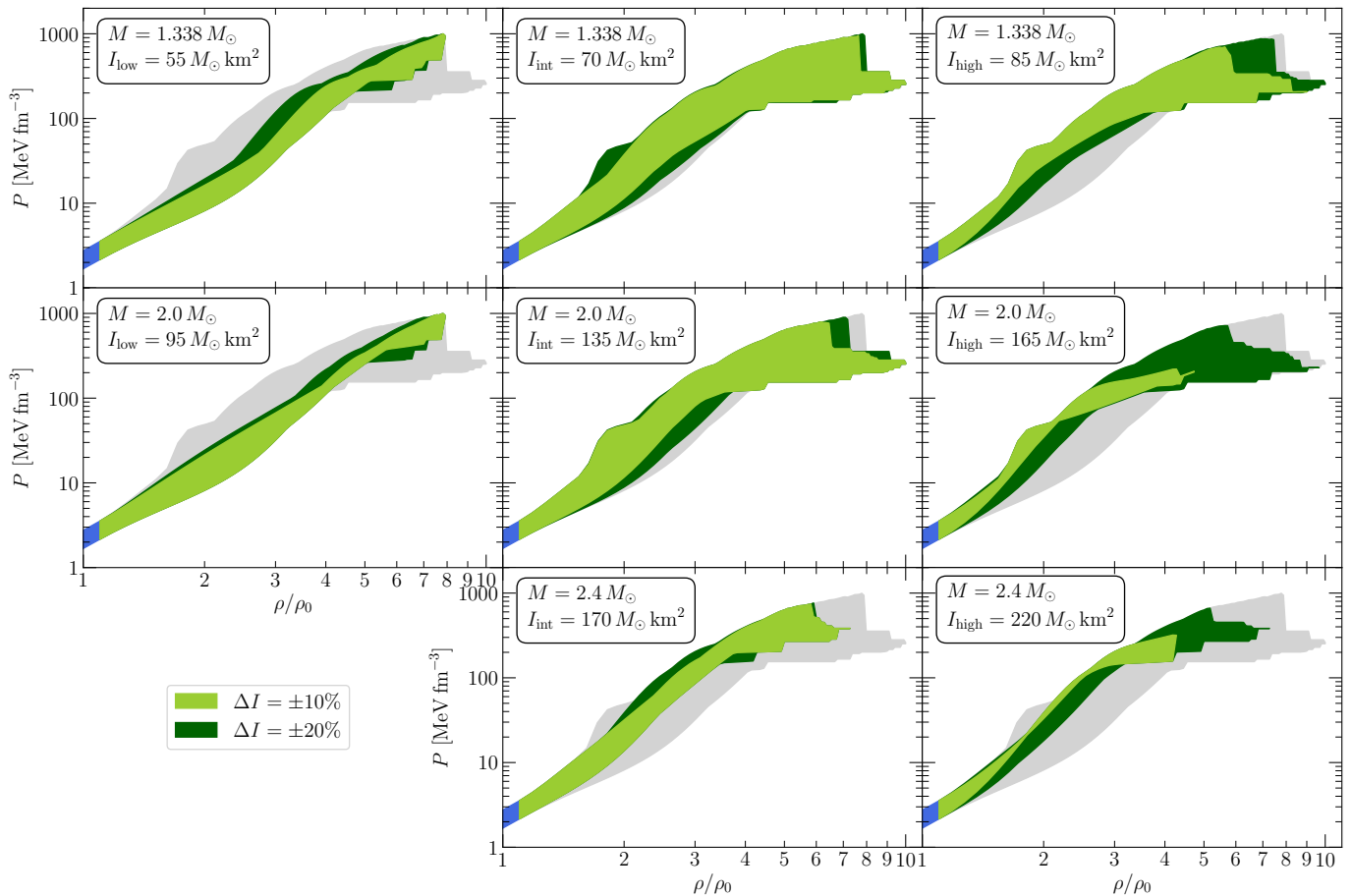


Figure 7. Same as Fig. 6 but using the speed of sound model, corresponding to Fig. 4 and Table 2.

but nevertheless very interesting cases. The EOSs with the low values for the dimensionless moment of inertia predict large radii at small masses (and moment of inertia) and small radii at larger masses (red lines), while the ones corresponding to large values for I/MR^2 show the opposite trend.

4. SUMMARY AND OUTLOOK

We have explored new and improved constraints for the EOS of neutron-rich matter and neutron star radii. Our work is based on four inputs: (a) microscopic calculations of the EOS up to $1.1 \rho_0$ based on state-of-the-art nuclear interactions derived from chiral EFT combined with the piecewise polytrope or speed of sound extension to high densities following Hebeler et al. (2013) and Greif et al. (2019), respectively, (b) the precise measurement of the mass of PSR J0740+6620 with $2.14^{+0.10}_{-0.09} M_\odot$ (Cromartie et al. 2019), (c) causality constraints at all densities and an asymptotic behavior of the speed of sound consistent with perturbative QCD calculations at very high densities for the c_s^2 model, and (d) constraints from future measurements of the mass and moment of inertia of the same star. Note that this analysis does not rely on any assumptions regarding the composition and properties of

matter beyond the density $1.1 \rho_0$, and within the space of the piecewise polytrope and speed of sound extension includes EOS that mimic regions with a first-order phase transition.

For the moment of inertia measurements we considered different scenarios by assuming various values and uncertainties for the moment of inertia. We find that measurements with an uncertainty of 10% lead to a reduction of the radius range by about 50% compared to the general EOS band from Hebeler et al. (2013) and Greif et al. (2019) when the moment of inertia corresponds to an intermediate EOS. If the moment of inertia corresponds to values predicted by a soft or stiff EOS the radius range is reduced by a factor of 3 or more. For all $\pm 10\%$ measurements, the resulting radius range is smaller than 1.9 km for all considered masses $M = 1.338, 2.0,$ and $2.4 M_\odot$. Specifically, for a $1.338 M_\odot$ star, we find radius ranges of $R = 10.2\text{--}11.5$ km for low values of the moment of inertia ($I_{\text{low}} = 55 M_\odot \text{ km}^2$ with $\Delta I = \pm 10\%$; combining the ranges from the piecewise polytropic and speed of sound extensions), $R = 11.3\text{--}12.9$ km for intermediate values ($I_{\text{int}} = 70 M_\odot \text{ km}^2$), and $R = 12.5\text{--}13.6$ km for high values ($I_{\text{high}} = 85 M_\odot \text{ km}^2$). These ranges need to be compared with $R = 10.2\text{--}13.6$ km based on the com-

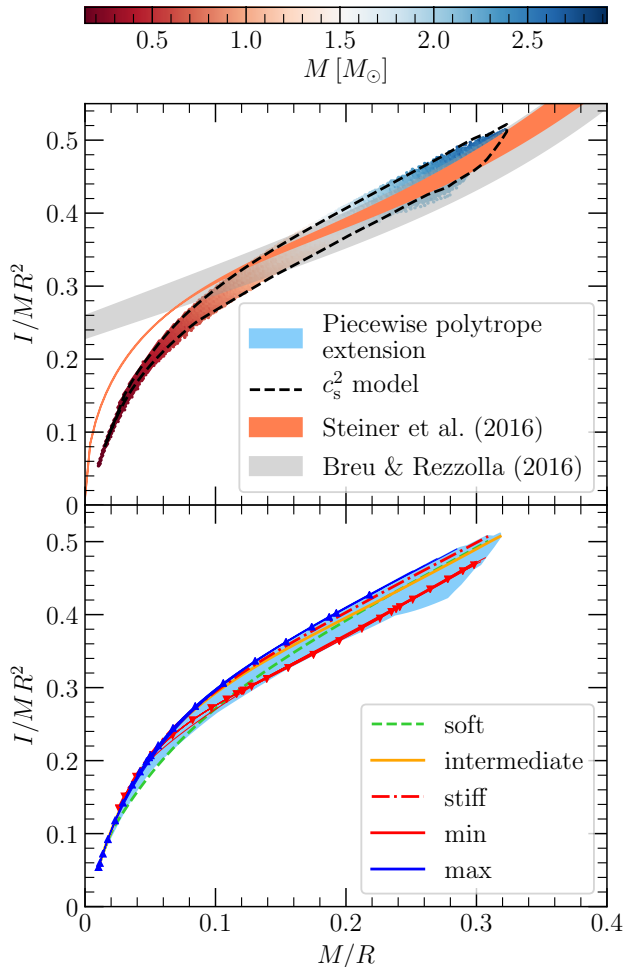


Figure 8. Dimensionless moment of inertia I/MR^2 as a function of compactness M/R . The red-to-blue region in the upper panel and the light blue region in the lower panel show our results for the general EOS band using the piecewise polytrope extension, with color coding according to the neutron star mass in the upper panel. In addition, we also show the results for the speed of sound model as the region enclosed by the black dashed lines. In the upper panel, this is compared to correlation bands from [Steiner et al. \(2016\)](#) in orange as well as [Breu & Rezzolla \(2016\)](#) in gray. In the lower panel, we also show the three representative EOS (soft, intermediate, and stiff) of [Hebeler et al. \(2013\)](#). The red (blue) lines with down (up) triangle points are the individual EOS within the piecewise polytrope extension with minimal χ^2 of I/MR^2 with respect to the lower (upper) boundary (from fits for $M/R \geq 0.1$).

bined general EOS bands for this mass, when no information about the moment of inertia is used. We have also investigated the corresponding constraints for the EOS. We found that large values for the moment of inertia provide stronger constraints at lower densities, whereas small values tend to constrain the EOS at higher den-

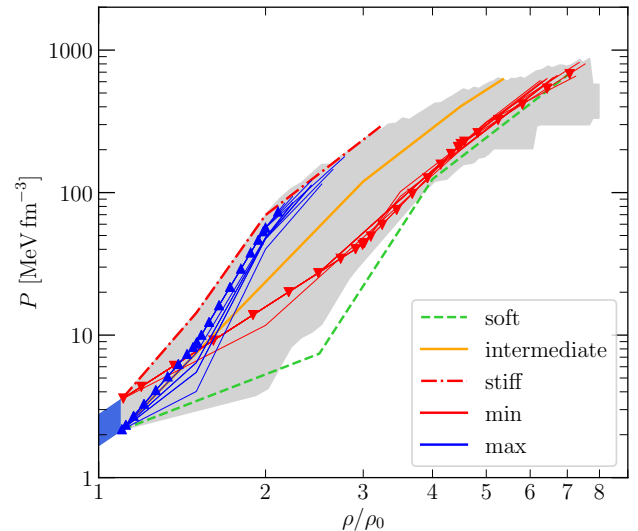


Figure 9. Pressure P as a function of mass density ρ/ρ_0 in units of the saturation density. The gray region is the general EOS band based on the piecewise polytrope extension. The lines correspond to the individual EOS shown in the lower panel of Fig. 8, where the red and blue lines extremize the I/MR^2 -compactness correlation.

sities. Moreover, measurements of heavy neutron stars provide overall stronger constraints. In addition, we have studied the dimensionless moment of inertia I/MR^2 and established the full uncertainty ranges based on our general piecewise polytrope and speed of sound extension. We find very interesting extreme EOSs at the boundaries of the correlation with the compactness, which have not been considered before.

Finally, we showed that the gravitational-wave constraints from the neutron star merger GW170817 ([Abbott et al. 2018, 2019](#)) are consistent with the general EOS bands explored here (see also [Raaijmakers et al. \(2020\)](#)). We found that the latest analysis of GW170817 ([Abbott et al. 2019](#)) only slightly reduces the radius range predicted by the general EOS bands from the piecewise polytrope and speed of sound extension, and only weakly narrows the range for the predicted moment of inertia for a $1.338 M_\odot$ star. Therefore, additional future detections from LIGO/Virgo, as well as NICER and other X-ray timing observations ([Watts et al. 2016](#)), combined with measurements of neutron star masses and in particular the moment of inertia, are a powerful avenue to further constrain the EOS of dense matter in a model-independent way.

ACKNOWLEDGMENTS

We thank G. Raaijmakers, I. Tews, and A. Watts for useful discussions. This work was supported in part by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – Project-ID 279384907 –

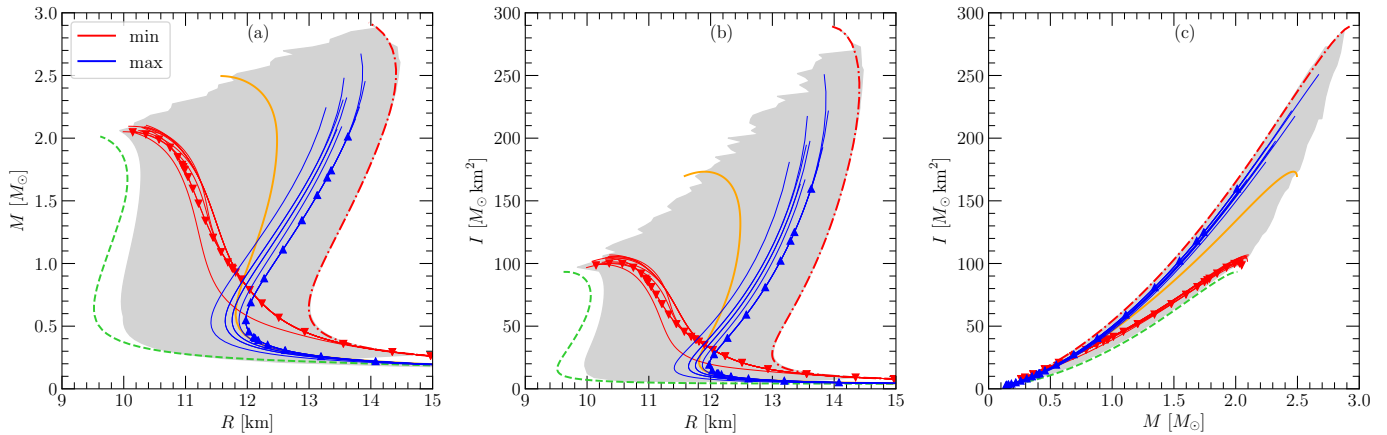


Figure 10. Same as Fig. 1, but including the individual EOS shown in the lower panel of Fig. 8, where the red and blue lines extremize the I/MR^2 –compactness correlation.

SFB 1245, and J.M.L. acknowledges support from NASA through grant 80NSSC17K0554 and the U.S. DOE from grant DE-FG02-87ER40317.

REFERENCES

- Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2018, *Phys. Rev. Lett.*, 121, 161101
- . 2019, *Phys. Rev. X*, 9, 011001
- Antoniadis, J., Freire, P. C. C., Wex, N., et al. 2013, *Science*, 340, 1233232
- Barker, B. M., & O’Connell, R. F. 1975, *Phys. Rev. D*, 12, 329
- Baym, G., Pethick, C., & Sutherland, P. 1971, *Astrophys. J.*, 170, 299
- Bedaque, P., & Steiner, A. W. 2015, *Phys. Rev. Lett.*, 114, 031103
- Bejger, M., Bulik, T., & Haensel, P. 2005, *Mon. Not. Roy. Astron. Soc.*, 364, 635
- Bejger, M., & Haensel, P. 2002, *Astron. Astrophys.*, 396, 917
- Benhar, O., Ferrari, V., Gualtieri, L., & Marassi, S. 2005, *Phys. Rev. D*, 72, 044028
- Breu, C., & Rezzolla, L. 2016, *Mon. Not. Roy. Astron. Soc.*, 459, 646
- Burgay, M., D’Amico, N., Possenti, A., et al. 2003, *Nature*, 426, 531
- . 2005, *ASP Conf. Ser.* 328, *Binary Radio Pulsars*, ed. F. A. Rasio & I. H. Stairs (San Francisco, CA: ASP), 328, 53
- Carbone, A., Polls, A., & Rios, A. 2013, *Phys. Rev. C*, 88, 044302
- Cipolletta, F., Cherubini, C., Filippi, S., Rueda, J. A., & Ruffini, R. 2015, *Phys. Rev. D*, 92, 023007
- Cromartie, H. T., Fonseca, E., Ransom, S. M., et al. 2019, *Nat. Astron.*, 4, 72
- Damour, T., & Schäfer, G. 1988, *Nuovo Cim.*, B101, 127
- De, S., Finstad, D., Lattimer, J. M., et al. 2018, *Phys. Rev. Lett.*, 121, 091102
- Drischler, C., Furnstahl, R., Melendez, J., & Phillips, D. 2020, <https://arxiv.org/abs/2004.07232>
- Drischler, C., Hebeler, K., & Schwenk, A. 2016, *Phys. Rev. C*, 93, 054314
- . 2019, *Phys. Rev. Lett.*, 122, 042501
- Epelbaum, E., Hammer, H.-W., & Meißner, U.-G. 2009, *Rev. Mod. Phys.*, 81, 1773
- Fonseca, E., Pennucci, T. T., Ellis, J. A., et al. 2016, *Astrophys. J.*, 832, 167
- Gorda, T. 2016, *Astrophys. J.*, 832, 28
- Greif, S. K., Raaijmakers, G., Hebeler, K., Schwenk, A., & Watts, A. L. 2019, *Mon. Not. Roy. Astron. Soc.*, 485, 5363
- Hammer, H.-W., Nogga, A., & Schwenk, A. 2013, *Rev. Mod. Phys.*, 85, 197
- Hartle, J. B. 1967, *Astrophys. J.*, 150, 1005
- Hartle, J. B., & Thorne, K. S. 1968, *Astrophys. J.*, 153, 807
- Hebeler, K., Lattimer, J. M., Pethick, C. J., & Schwenk, A. 2010, *Phys. Rev. Lett.*, 105, 161102
- . 2013, *Astrophys. J.*, 773, 11
- Hebeler, K., & Schwenk, A. 2010, *Phys. Rev. C*, 82, 014314
- Holt, J. W., Kaiser, N., & Weise, W. 2013, *Prog. Part. Nucl. Phys.*, 73, 35
- Kramer, M., & Wex, N. 2009, *Class. Quant. Grav.*, 26, 073001
- Kurkela, A., Romatschke, P., & Vuorinen, A. 2010, *Phys. Rev. D*, 81, 105021
- Lattimer, J. M., & Prakash, M. 2001, *Astrophys. J.*, 550, 426
- Lattimer, J. M., & Schutz, B. F. 2005, *Astrophys. J.*, 629, 979
- Lim, Y., Holt, J. W., & Stahulak, R. J. 2019, *Phys. Rev. C*, 100, 035802
- Lyne, A. G., Burgay, M., Kramer, M., et al. 2004, *Science*, 303, 1153
- Lynn, J. E., Tews, I., Carlson, J., et al. 2016, *Phys. Rev. Lett.*, 116, 062501
- Machleidt, R., & Entem, D. R. 2011, *Phys. Rep.*, 503, 1
- Miller, M. C., Lamb, F. K., Dittmann, A. J., et al. 2019, *Astrophys. J.*, 887, L24
- Morrison, I. A., Baumgarte, T. W., Shapiro, S. L., & Pandharipande, V. R. 2004, *Astrophys. J.*, 617, L135
- Negele, J. W., & Vautherin, D. 1973, *Nucl. Phys. A*, 207, 298
- Raaijmakers, G., Riley, T. E., Watts, A. L., et al. 2019, *Astrophys. J.*, 887, L22
- Raaijmakers, G., Greif, S. K., Riley, T. E., et al. 2020, *Astrophys. J. Lett.*, 893, L21
- Raithel, C. A., Özel, F., & Psaltis, D. 2016, *Phys. Rev. C*, 93, 032801
- Ravenhall, D. G., & Pethick, C. J. 1994, *Astrophys. J.*, 424, 846
- Read, J. S., Lackey, B. D., Owen, B. J., & Friedman, J. L. 2009, *Phys. Rev. D*, 79, 124032
- Riley, T. E., Watts, A. L., Bogdanov, S., et al. 2019, *Astrophys. J.*, 887, L21
- Steiner, A. W., Gandolfi, S., Fattoyev, F. J., & Newton, W. G. 2015, *Phys. Rev. C*, 91, 015804
- Steiner, A. W., Lattimer, J. M., & Brown, E. F. 2016, *Eur. Phys. J. A*, 52, 18
- Tews, I., Carlson, J., Gandolfi, S., & Reddy, S. 2018, *Astrophys. J.*, 860, 149

- Tews, I., Krüger, T., Hebel, K., & Schwenk, A. 2013, Phys. Rev. Lett., 110, 032504
- Tsang, M. B., Stone, J. R., Camera, F., et al. 2012, Phys. Rev. C, 86, 015803
- Watts, A. L., Andersson, N., Chakrabarty, D., et al. 2016, Rev. Mod. Phys., 88, 021001
- Weinberg, S. 1990, Phys. Lett. B, 251, 288
- . 1991, Nucl. Phys. B, 363, 3
- Wellenhofer, C., Holt, J. W., Kaiser, N., & Weise, W. 2014, Phys. Rev. C, 89, 064009
- Wex, N. 1995, Class. Quant. Grav., 12, 983