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CORRECTION

Correction to: A Polyakov Formula for Sectors

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Let S_α denote a finite circular sector of opening angle $\alpha \in (0, \pi)$ and radius one, and let $e^{-t\Delta_\alpha}$ denote the heat operator associated to the Dirichlet extension of the Laplacian. Based on recent joint work [2] and [3], we discovered an extra contribution to the variational Polyakov formula in [1] coming from the curved boundary component of the sector. Theorems 3 and 4 of [1] should have an added term $+\frac{1}{4\pi}$. This calculation will appear in [2]. The corrected statements of these theorems are given below.

Theorem 1 (Theorem 3 [1]) *Let $S_{\pi/2} \subset \mathbb{R}^2$ be a circular sector of opening angle $\pi/2$ and radius one. Then the variational Polyakov formula is*

$$\frac{\partial}{\partial\gamma}\left(- \log(\det(\Delta_{S_\gamma}))\right)\Big|_{\gamma=\pi/2} = \frac{-\gamma_e}{4\pi} + \frac{2}{3\pi},$$

where γ_e is the Euler-Mascheroni constant.

Theorem 2 (Theorem 4 [1]) *Let $0 < \alpha < \pi$, and let*

$$k_{min} = \left\lceil \frac{-\pi}{2\alpha} \right\rceil, \text{ and } k_{max} = \left\lfloor \frac{\pi}{2\alpha} \right\rfloor \text{ if } \frac{\pi}{2\alpha} \notin \mathbb{Z}, \text{ otherwise } k_{max} = \frac{\pi}{2\alpha} - 1,$$

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and $W_\alpha = \left\{k \in \left(\mathbb{Z} \bigcap [k_{min}, k_{max}]\right) \setminus \left\{\frac{\ell\pi}{\alpha}\right\}_{\ell\in\mathbb{Z}}\right\}$. *Then*

$$\mathcal{S}(\alpha) := \frac{\partial}{\partial\gamma}\left(-\log(\det(\Delta_\gamma))\right)\Big|_{\gamma=\alpha} = \frac{1}{3\pi} + \frac{\pi}{12\alpha^2} + \sum_{k\in W_\alpha} \frac{-2\gamma_e + \log(2) - \log\left(1-\cos(2k\alpha)\right)}{4\pi(1-\cos(2k\alpha))} - (1-\delta_{\alpha,\frac{\pi}{n}})\,\frac{2}{\alpha}\sin(\pi^2/\alpha)\int_{-\infty}^{\infty}\frac{\gamma_e + \log(2) - \log(1+\cosh(s))}{16\pi(1+\cosh(s))(\cosh(\pi s/\alpha) - \cos(\pi^2/\alpha))}ds,$$

where $n \in \mathbb{N}$ *is arbitrary and* $\delta_{\alpha,\frac{\pi}{n}}$ *denotes the Kronecker delta.*

It therefore follows that the list of examples given following Theorem 4 in [1] should be revised accordingly:

(1) $\alpha = \frac{\pi}{4}$, $W_{\frac{\pi}{4}} = \{-2, \pm 1, \}$, $\mathcal{S}(\frac{\pi}{4}) = \frac{-5\gamma_e}{4\pi} + \frac{\log(2)}{2\pi} + \frac{5}{3\pi} \sim 0.411167$
(2) $\alpha = \frac{\pi}{3}$, $W_{\frac{\pi}{3}} = \{-1, 1\}$, $\mathcal{S}(\frac{\pi}{3}) = \frac{13}{12\pi} - \frac{2\gamma_e}{3\pi} + \frac{\log(4/3)}{3\pi} \sim 0.252871$
(3) $\alpha = \frac{\pi}{2}$, $W_{\frac{\pi}{2}} = \{-1\}$, $\mathcal{S}(\frac{\pi}{2}) = \frac{-\gamma_e}{4\pi} + \frac{2}{3\pi} \sim 0.166273.$
(4) For $\alpha \in]\frac{\pi}{2}, \pi[$, $W_\alpha = \emptyset$, but $\sin(\pi^2/\alpha) \neq 0$. If $\alpha = \frac{2\pi}{3}$, the integral converges rapidly, and a numerical computation gives an approximate value of 0.0075015. Hence $\mathcal{S}(\frac{2\pi}{3}) \sim \frac{1}{3\pi} + \frac{3}{16\pi} + \frac{3}{\pi}(0.0075015) \sim 0.1729498.$

1.1 Misprint

There is a two missing in Equation (1.3) of [1]. That equation should be:

$$\partial_t \log\det(\Delta_{g_t}) = -\frac{1}{12\pi}\int_M \sigma'(t)\,\mathrm{Scal}_t\, dA_{g_t} + \partial_t \log \mathrm{Area}(M, g_t).$$

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