ASYMPTOTIC MAGNETIC SURFACES *)

by

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Toroidal magnetic fields $B = \sum_{\nu \geq 0} \varepsilon^{\nu} \vec{B}_{\nu}$ where the unperturbed field $\vec{B}_o$ has zero rotational transform are considered. On rather weak assumptions it is shown that single-valued formal solutions $F = \sum_{\nu \geq 0} \varepsilon^{\nu} F_{\nu}$ of the equation $\vec{B} \cdot \nabla F = 0$ exist, and that the asymptotic magnetic surfaces F= const are unique to all orders. Recursion formulae are derived which allow any order of F to be calculated from its lower orders, and explicit expressions are given for F_o. These depend on the order of the rotational transform , which can either be the same as, or higher than, the order of the perturbing field. In the former case, i.e. if $\vec{B} - \vec{B}_o \sim \iota$, $F_o(X)$ is simply the flux of the lowest order perturbing field through the closed line of force of the unperturbed field passing through the point X. The latter case occurs if, and only if, this flux is a constant.

For the special cases of stellarator-like vacuum fields, i.e. for vacuum fields having circular lines of force in the limit $\varepsilon \to o$, it is shown that the asymptotic magnetic surfaces can be toroidally closed without encircling any current carrying wires only if ι is of higher order than the perturbing field. This necessary condition is fulfilled by classical stellarators with alternating helical windings, but it is violated by any torsatron configurations.

As another application the lowest order of an adiabatic invariant is constructed for the longitudinal guiding centre motion. This invariant applies if ι is comparable with the usual expansion parameter of the adiabatic orbit theory. It reduces to the usual longitudinal invariant if ι is much smaller. If $\iota \sim \vec{B} - \vec{B}_o$, then it agrees with an adiabatic invariant previously constructed by Hastie et al. on the assumption that exact magnetic surfaces exist, taking an entirely different form in general.

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