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The Ancestors of Two-Component Neutrino Theories

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Abstract: The two-component theory of neutrino was developed in 1957, immediately after the discovery of parity violation in weak interactions. Its officially acknowledged precursor is identified with Hermann Weyl, who - in an attempt to get rid of the negative-energy solutions to the Dirac equation - had coined the term *Zweikomponententheorie*. This contribution will sacrifice the pedagogical simplified linearity of textbooks for a more complex approach to the subject, that integrates in the story another character, who also spent efforts to achieve the elimination of negative-energy wavefunctions: Ettore Majorana. His alternative interpretation of the theory will be retraced and the consequences of the co-existence of his approach with the one built upon Weyl's work will be investigated.

Keywords: Majorana, Weyl, Two-component theory, Neutrino.

1. Two-component theory of neutrino

Two-component theory of neutrino represents the theoretical framework, thanks to which the elusive particle, whose existence had first been reluctantly suggested by Wolfgang Pauli in a desperate attempt to prevent the law of energy conservation from being given up, found a dignified position within the walls of that majestic building, that we know nowadays as the Standard Model. The portrait of neutrino offered by the theory – namely, of an elementary particle deprived of its mass, uncapable of changing flavour and distinct from its antipartner – plays in stark contrast to the status obtained by the neutral lepton after the experimental observation of neutrino oscillations (Fukuda *et al.* 1998). The birth of this seemingly *conservative* concept is usually dated to the year 1957, when a well-defined *helicity* was assigned to the neutrino, the term *helicity* meaning the projection of spin on the direction of momentum.

The idea of depicting this ghostly fundamental entity as a massless left-handed[1] "screw", whose originally four-component spinor interacts in the Hamiltonian of the beta decay with only its upper pair of components, was proposed, at the same time, independently, by the Chinese-American physicists Tsung-Dao Lee and Chen-Ning Yang (Lee, Yang 1957), the Soviet scientist Lev Landau (Landau 1957b) and the Pakistani theoretician Abdus Salam (Salam 1957). The spin and velocity of the antineutrino would then lead to its characterization as a physical object with opposite *chirality*[2] with respect to the neutrino, providing therefore a tool for discrimination between the two particles.

[1] The actual physical value of the helicity became clear only after the experiment of Goldhaber, Grodzins and Sunyar was perfomed at Brookhaven National Laboratory (Goldhaber et al. 1958).
[2] Note that helicity and chirality coincide in the case of a massless particle.

The reason of this interest in a novel theory of neutrino, arisen in distant places across the globe, is to be found in the discovery of parity violation in weak interactions, that had occurred during the previous year at the hands of Lee and Yang (Lee, Yang 1956), earning them the Nobel Prize "for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles" [Nobel Prize 1957].

The history of two-component approaches is usually told as if it were a path towards the resurrection of the old *Zweikomponententheorie*, originally proposed by Hermann Weyl and soon abandoned because of its intrinsic breaking of reflection symmetry. This contribution aims to go beyond that unidirectional interpretation and to offer a more heterogeneous and complete portrait of the theory considered, by including, in the reconstruction, another actor whose perspective used to clash with the concept of a neutrino *polarized* in a privileged direction. The character that plays a key role in the story is the Italian physicist Ettore Majorana: besides building an alternative version of the two-component theory, his approach provides us with innovative tools to reconcile the *static* depiction of neutrino – developed by the aforementioned theoreticians in the Fifties – with a more complex idea of mixing, closer to the contemporary image of *oscillating flavour neutrino*.

2. The traditional ancestor: Hermann Weyl

As recalled above, the conventional narrative of two-component theory revolves around the idea that the neutrino represents the *Totengräber der Parität*[3] – as Wolfgang Pauli's assistant and collaborator, Markus Fierz, referred to it in his attempt to show that, in Lee-Yang's interpretation, parity violation was viewed as inherent in the definition of neutrino itself (Pauli 2005). At this point of our reconstruction, however, the thread connecting parity non-conserving interactions with two-component theory of neutrino might still look thin. To dispel this impression, let us take a step back and add that, even though the aforementioned approach to neutrino physics only found fertile ground in 1957, it was actually rooted in a paper published by the German mathematician Hermann Weyl almost three decades earlier, where the word *Zweikomponententheorie* first made its appearance (Weyl 1929).

Despite being perceived as the traditional ancestor of the theory, Weyl elaborated it out of motivations that were totally alien to the neutrino, whose existence was not even taken into account back then. His purpose indeed was to construct a unified theory of gravitation, matter and electricity, which could be realized by first combining the Pauli-Dirac theory of electron with spin and general relativity. However, Dirac's outcome gave twice as many energy levels as the ones truly required, this having been identified within the physics community with the problem of negative-energy levels. As Weyl stated in his revised German edition of *Gruppentheorie und Quantenmechanik*, completed in 1930,

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[3] "Gravedigger of parity".

> The electron will possess, in addition to its positive energy levels, negative ones as well, the latter arising from the positive energy levels of the positive electron on changing signs [...]. Obviously, something is wrong here; we should be able to get rid of these negative energy levels of the electron. (Weyl 1930, p. 225)

The problem addressed by the mathematician born in Elmshorn coincided with the question that worried Dirac, namely how to deal with the negative energy levels, seemingly devoid of any physical meaning. Dirac's reaction had been to maintain the whole amount of energy levels and to construct a brand new concept of vacuum, the *Dirac sea*. Weyl, on the other hand, suggested to prevent the bad apples from spoiling the bunch, by halving the components of the wave function: this could be done by embracing Pauli's spin theory and, at the same time, retaining relativistic invariance. Indeed, as Charles P. Enz – Pauli's biographer and former collaborator – claimed, "Weyl's idea was that, by the reduction of the Dirac ψ from four to two components, the negative-energy states could be avoided" (Enz 2002, p. 254). The price that he had to pay, in order to do so, was hardly affordable, since he gave up the mass.

Nevertheless, Hermann Weyl relied on gravitation to compensate the fact that his theory was only suitable to describe a massless particle, in the hope that this lack could be filled up with some new tool provided by the theory of gravitation. After all, Weyl's work built upon the quest for a unified theory; what could not be explained by Dirac's theory of matter would be accounted for within the context of gravitation. According to the mathematician, the actual problem of his suggested two-component theory was of different nature, it concerned in fact the violation of parity: "Nur diese tatsächlich in der Natur bestehende Symmetrie von rechts und links wird uns zwingen, ein zweites Paar von ψ-Komponenten einzuführen" (Weyl 1929, p. 334).[4]

This set Weyl's theory against what experiments had shown about atomic spectra – where invariance under space reflection seemed to hold – and caused it to be partly forgotten, at least until the events of 1957 led to its re-exhumation.

To describe in few sentences the technical details of the procedure adopted by Hermann Weyl, we shall say that he had managed to reformulate the four coupled equations appearing in Dirac's relativistic system as two independent equation pairs, but was then forced to keep both in order to preserve parity. Each of the equation pairs had indeed only two solutions, both characterized by a specific relation between the sign of the free-particle Hamiltonian and the direction of the spin (in other terms, by a precise value of the helicity). Abandoning one of those systems would then be equivalent to sacrificing symmetry under parity inversion, due to the different behaviours that the resulting couple of equations would present in the real world and in its mirrored image.[5] These aspects lead us to believe that Weyl's original goal to eliminate the negative-energy solutions to the

[4] Only this left-right symmetry that exists in nature will force us to introduce another pair of components for the wavefunction ψ.

[5] For example, a left-handed neutrino (characterised by helicity -1) would become right-handed, after performing a parity transformation, which is not possible because the first equation pair has only two solutions: one corresponding to positive energy (neutrino) with spin down and one to negative energy (antineutrino) with spin up. Therefore, there exists no solution that could be identified with a right-handed neutrino, because only left-handed neutrinos and right-handed antineutrinos are admitted.

relativistic wave equation had not been accomplished, since each pair of solutions contained one corresponding to the negative-energy eigenfunction of the Hamiltonian.

From the formalistic details, we can draw the conclusion that the discovery of 1956 moved the goalposts, taking power out of the constraint of parity conservation, that had previously represented an obstacle to the spread of Weyl's idea: Lee and Yang, Landau, and Salam were then allowed to get rid of one of the two equation pairs. Borrowing Lev Landau's words,

> In the usual theory it is impossible to restrict attention to one pair of equations, since the two pairs are interchanged by a space-inversion. But if we require only invariance under combined inversion, then we can suppose that the neutrino is described by a single pair of equations. (Landau 1957b, p. 337)

3. The forgotten ancestor: Ettore Majorana

The approach presented in the previous paragraph is the one traditionally adopted by textbooks, when it comes to giving some historical insights to two-component theory of neutrino. By looking into the original papers and retracing the correspondence among the physicists of the time, nonetheless, the linearity and unambiguity emerging from the standard standpoint need to be put aside in favour of an unorthodox path, namely the one outlined by Ettore Majorana. Let us now therefore dig deeper into the history of physics, in order to bring to light the too often neglected version of two-component approaches, embodied by this physicist.

Like Weyl's theory, Majorana's also built upon an unsatisfaction towards Dirac's negative energy states and, especially, towards his odd explanation of the latter. According to the theoretician from Catania, the British physicist had constructed a symmetric theory of electrons and positrons, obtaining a full symmetrisation by applying some *controversial* methods, such as the one of removing divergences by cutting out infinite constants. The setting, upon which Dirac's interpretation of negative energy states was based, was asymmetric since the vacuum itself – i.e. the Dirac sea – consisted of an infinite number of negative-energy electrons, which generated, according to Maxwell equations, an infinite electric field that then required complicated artifices in order to be removed.

To develop a truly symmetric theory of electron and positron, one should avoid making compromises: similarly to Weyl, Majorana aimed to give up negative energy levels, which – in his view – represented the cause of divergence. This was exactly the purpose of 1937 Majorana's well-known paper "Teoria simmetrica dell'elettrone e del positrone", published in the Italian magazine *Il Nuovo Cimento* and remained for a long time untranslated. His new theory, which he derived from a generalization of the variational principle, had to be suitable to further "possible extensions by analogy" (Majorana 2006, p. 219) to neutral particles. By knocking down the notion of negative energy states, indeed, it would become possible, according to Majorana, to pave the way for a theory of elementary neutral particles, such as neutrinos. Therefore, by reformulating Dirac equation in a new basis, where the gamma matrices were chosen to be imaginary, and letting it act onto the real part of the wave function, he actually succeeded in his purpose to get rid of the negative-energy solutions: he ended up having only two wave functions, representing a particle with both possible spin orientations.

The paths followed by Weyl and Majorana seemed to strongly diverge, at least as far as it concerns the conservation of parity, the mass of the particle involved – namely, whether it had to be set to zero or not – and the existence of a charge (since Weyl's theory could describe the physical behaviour of charged particles, while Majorana's only applied to "truly neutral" entities). Even the spinors differed from each other: two-component complex objects for Weyl; four-component real-valued entities for Majorana. However, after the two-component theory of neutrino was developed in 1957, some physicists – among them the American James A. McLennan Jr. from Lehigh University and Kenneth M. Case, former Ph.D. student of Julian Schwinger, and Pauli's assistant Markus Fierz – pointed out the similarity of the processes considered and proved their mathematical equivalence (McLennan 1957; Case 1957; Pauli 2005, letter February 6th, 1957).

4. Conclusive remarks

As stated above, two-component theorists in 1957 claimed to have the final say in the definition of the nature of the neutrino: the refusal to identify neutrinos with "truly neutral" particles – or "Majorana particles", in Columbia University's fashion – because of their being longitudinally polarized, their masslessness and well-determined chirality were common traits of Lee-Yang's, Salam's, and Landau's papers: this led to the ultimate conviction that neutrinos should be considered as "Dirac particles". However, the attempts at showing the equivalence of two-component theory of neutrino and Majorana's theory undermined these apparent certainties and left room for the alternative interpretation. The approach pursued by the Italian theoretician, indeed, represented the last bulwark of defence against a left-handed neutrino, endowed with a trademark (i.e., its helicity) that made it distinguishable from its counterpart, thus paving the way for a more blurred definition, that took into account the possibility of a partial nonconservation of lepton charge (known nowadays as "flavour quantum number"). The initial purpose of this contribution was to provide a novel view on two-component theory of neutrino, integrated with an unaccustomed variation on the theme, inspired by Majorana's 1937 work. Far from being a merely formalistic issue, this is closely related to the long-standing dilemma (of Dirac *versus* Majorana particles) concerning the nature of neutrino, which – alongside its own existence, that had remained unproven for decades – is still surrounded by an aura of mystery. Large experiments are going on, whose goal is to provide evidence for neutrinoless double beta decay, according to which two beta processes could occur in the same nucleus without the emission of neutrinos (or antineutrinos), and therefore supporting the theoretical scaffolding previously discussed.

On the other hand, this is not the only fact that makes Majorana's hypothesis so tantalising. His symmetric theory paved the way for a concept of "mixing" in neutrino physics, since – by performing a Bogoliubov transformation *ante litteram* – he defined his novel "truly neutral" particle in terms of a linear combination of a field and its charge conjugate, applying the quantum mechanical principle of superposition. One could say that, in addition to being an ancestor of two-component theories, Majorana was also a precursor of neutrino mixing, a notion he presented in a purely mathematical fashion and that would later acquire physical concreteness with B. Pontecorvo's studies.

From these last remarks, we can notice how the original goal of this paper has slowly been overturned, transforming the history of two-component neutrino theory – from the coinage of the word by Weyl to the application of the concept to neutrino physics in 1957 – into the narrative of the reception and interpretation of Majorana's paper, whose complexity and richness have not so far ceased to inspire physicists.

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