



Origin and growth of primordial black holes

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ABSTRACT

Building on the insight that primordial black holes can arise from the formation and subsequent gravitational collapse of bound states of stable supermassive elementary particles during the early radiation era, we offer a comprehensive picture describing the evolution and growth of the resulting mini-black holes through both the radiation and matter dominated phases, until the onset of (small scale) inhomogeneities. This is achieved by means of an exact metric solving Einstein's equations throughout both phases. We show that, thanks to a special enhancement effect producing an effective horizon above the actual event horizon, this process can explain the observed mass values of the earliest giant black holes. Unlike other proposals, it also predicts a lower limit on the mass of supermassive black holes.

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1. Introduction

In very recent work a new mechanism was proposed to explain the origin of supermassive black holes in the early Universe by means of the condensation of superheavy elementary particles during the early radiation phase [1]. Accordingly, the existence of primordial black holes would be due to the gravitational collapse of such bound states, shortly after their formation, to small black holes, whose masses must lie above a certain critical value to evade Hawking evaporation. Their subsequent growth during the radiation era can be modeled by an exact metric solving Einstein's equation, such that towards the end of the radiation era the emerging macroscopic black holes can grow to nearly solar mass objects.

In this Letter we discuss the complete evolution of such primordial black holes throughout both the radiation and matter dominated eras, and show that the proposed mechanism can indeed explain the observed mass values of supermassive black holes, as reported in [2]. This completes the argument given in [1], where we did not follow the evolution of the emergent macroscopic black holes beyond equilibrium time t_{eq} , and did not provide mass estimates for the large black holes that emerge at the time of the formation of small scale inhomogeneities. Here we close this crucial gap by offering a much more comprehensive picture, modeling the growth of mini-black holes into giant black holes 'from beginning to end'. The fact that this can be done by means of a closed form metric solving the Einstein equations that encompasses both

the radiation and the matter dominated phase is an important input in our analysis.

As we have explained in [1], superheavy gravitinos can serve as microscopic seeds for generating mini-black holes if their mass is sufficiently large so that their gravitational attraction exceeds the repulsive or attractive electric forces between them. Furthermore, these seed particles must be stable against decay into Standard Model matter. Although other kinds of particles with similar properties might serve the same purpose, we have argued in [1] that the gravitinos of maximal ($N = 8$) supergravity are distinguished in view of a possible unification of the fundamental interactions (however, as explained there, the underlying theory must transcend $N = 8$ supergravity). This follows from the structure of the fermionic sector of the maximal $N = 8$ supermultiplet [3]: identifying the 48 non-Goldstino spin- $\frac{1}{2}$ fermions of the $N = 8$ supermultiplet with three generations of quarks and leptons of the Standard Model of particle physics (including right-chiral neutrinos), one is left with eight massive gravitinos with the properties described in [3,1]. These properties are radically different from those of the more familiar sterile gravitinos of low energy $N = 1$ supergravity models; in particular, unlike the latter, superheavy gravitinos do participate in Standard Model interactions.

Although our proposal is thus mainly motivated by unification, we emphasize again that, except for the properties listed below in section 2, our considerations are largely independent of the precise nature of the "seed particles" that produce primordial black holes. Evidently, our proposal differs in several important ways from other scenarios aiming to explain the origin of primordial black holes, which we cannot review here for lack of space. See,

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however [4,5] for alternative ansätze, and [6] for a comprehensive survey of the present state of the art and a discussion of the relative merits of different proposals.

2. Basic considerations

We refer readers to [1] for a more detailed explanation of the basic motivation and assumptions underlying our proposal. As argued there, the gravitino mass M_g must lie between M_{BPS} and M_{Pl} , where the ‘BPS-mass’ M_{BPS} is the mass for which the electrostatic repulsion between two (anti-)gravitinos of the same charge equals their gravitational attraction. M_{Pl} is the reduced Planck mass $\sim 4.34 \cdot 10^{-9}$ kg (it corresponds to the Planck time $t_{\text{Pl}} = 2.70 \cdot 10^{-43}$ s). For numerical estimates we will take $M_{\text{BPS}} \sim 0.01 \cdot M_{\text{Pl}}$, so that

$$0.01 \cdot M_{\text{Pl}} < M_g < M_{\text{Pl}} \quad (1)$$

This ensures that the force remains attractive also between gravitinos of the same electric charge. The minimal seed mass $M_{\text{seed}} \sim NM_g$ for a primordial black hole in the early radiation phase is determined by asking the total energy of a bound system of N (anti-)gravitinos to be negative, viz.¹

$$\langle E_{\text{kin}}(t) \rangle + \langle E_{\text{pot}}(t) \rangle = NT_{\text{rad}}(t) - N^2 \frac{GM_g^2}{\langle d(t) \rangle} \stackrel{!}{<} 0 \quad (2)$$

where $\langle d(t) \rangle$ is the (time-dependent) average distance between two gravitinos in the ambient hot radiation plasma. As we explain in [1], the cosmic time t drops out in this inequality upon substituting the relevant quantities with their time dependence. We then find

$$N \gtrsim \frac{T_{\text{eq}}}{GM_g^2} \cdot 10^2 \text{ m} \sim 10^{12} \quad (3)$$

for the minimum number of (anti-)gravitinos in a bound state for gravitational collapse to occur, where $T_{\text{eq}} \sim 1$ eV and we take $M_g \sim 10^{-9}$ kg as an exemplary value [1]. Since the cosmic time t drops out in the derivation of this inequality, the value of N remains the same throughout the radiation phase. If the bound state is meta-stable, the collapse can be delayed in such a way that an even larger number N of (anti-)gravitinos can accrue before gravitational collapse occurs, in which case the seed mass could be even larger. The minimum mass of a black hole resulting from gravitational collapse of such a bound state is therefore

$$M_{\text{seed}} \sim 10^{12} M_g \sim 10^3 \text{ kg} \Rightarrow GM_{\text{seed}} \sim 10^{-24} \text{ m} \quad (4)$$

Now, a black hole of such a small mass would be expected to decay immediately by Hawking radiation [7]: from the well known formula for the lifetime of a black hole (see e.g. [8]) we have

$$\tau_{\text{evap}}(m) = t_{\text{Pl}} \left(\frac{m}{M_{\text{Pl}}} \right)^3 \quad (5)$$

This is the result which would hold in empty space. However, during the early radiation phase this is not the only process that must be taken into account, because of the presence of extremely hot and dense radiation, which can ‘feed’ black hole growth. The absorption of radiation thus provides a competing process which can stabilize the black hole against Hawking decay, such that with the

initially extremely high temperatures of the radiation era mass accretion can overwhelm Hawking evaporation *even for very small black holes*. The details of this process are complicated, because a proper treatment would require generalizing the original Hawking calculation to the time-dependent space-time background given by (18) below, something that remains to be done. However, there is a simple approximate criterion for accretion to overcome the rate for Hawking radiation for a black hole of given mass m , which reads

$$T_{\text{rad}}(t) > T_{\text{Hawking}}(m) = \frac{1}{8\pi Gm} \quad (6)$$

The break-even point is reached when the radiation temperature equals the Hawking temperature, at time $t_0 = t_0(m)$ when $T_{\text{rad}}(t_0) \sim T_{\text{Hawking}}(m)$. For larger times $t > t_0$ (and lower radiation temperatures) a black hole of mass m will decay. Imposing this equality, or alternatively using eqn.(26) of [1] we deduce the relevant mass at time t , which gives

$$m^4(t) \simeq \frac{M_{\text{Pl}}^3}{t_{\text{Pl}}} \cdot \frac{1}{G^2 \rho_{\text{rad}}(t)} = \frac{32\pi M_{\text{Pl}}^3}{3Gt_{\text{Pl}}} \cdot t^2 \quad (7)$$

When read from right to left this equation tells us which is the latest time for a mini-black hole of given mass m to remain stable against Hawking decay during the radiation phase. This is the case for $t < t_0 \equiv t(m) \propto m^2$, after which time the black hole will decay. Conversely, for a given time t any mini-black hole of initial mass greater than $m(t)$ will be able to survive and can start growing, whereas those of smaller mass decay. With (4) as the reference value we thus take the initial mass to be $\sim M_{\text{seed}}$, and assume that the time range available for the formation of such a mini-black hole is

$$t_{\text{min}} = 10^8 \cdot t_{\text{Pl}} \simeq 10^{-34} \text{ s} < t < t_{\text{max}} \simeq 10^{-18} \text{ s} \quad (8)$$

During this time interval a black hole of initial mass (4) can survive and start growing by accreting radiation. While the upper bound is thus determined by setting $t_{\text{max}} \equiv t(M_{\text{seed}})$, the lower bound has been chosen mainly to stay clear of the quantum gravity regime and a possible inflationary phase.

Once we have a stable mini-black hole we can study its further evolution through the radiation phase by means the exact solution derived in [1], until matter starts to dominate over radiation at time $t \sim t_{\text{eq}} \sim 42000$ yr, when these objects have grown into macroscopic black holes. With (8) we get the following range of masses

$$10^{-12} M_{\odot} \lesssim m(t_{\text{eq}}) \lesssim 10^{-3} M_{\odot} \quad (9)$$

However, the solution in [1] does *not* apply to the matter dominated phase. To investigate the further evolution one would conventionally switch to a different description by invoking the Edington formula [2,9]

$$m(t) = M_0 \exp\left(\frac{4\pi G m_p t}{\epsilon \sigma_T}\right) \simeq M_0 \exp\left(\frac{t}{45 \text{ Myr}}\right) \quad (10)$$

where m_p is the proton mass, σ_T is the Thompson cross section, and ϵ is the fraction of the mass loss that is radiated away (usually taken as $\epsilon = 0.1$). Unfortunately, because of the exponential dependence this formula is extremely sensitive to the precise value of ϵ and the choice of ‘final’ time t – surely, exponential growth does not persist into the present epoch!

We also note that this formula was originally developed to describe the evolution of luminous stars [9]. Its derivation relies on the Newtonian approximation and is based on a simple equilibrium condition, balancing the rate of mass absorption against the

¹ We adopt units with $\hbar = 1$, $c = 1$, $k_B = 1$, so that for instance $1 \text{ eV} = 1.16 \cdot 10^4 \text{ K}$, etc. The final formulas are then re-expressed in convenient units (eV, kg, m, s, or K).

luminosity of infalling matter, where the luminosity is assumed to grow linearly with the mass of the star. It thus appears doubtful whether one can use it in the present context, and we therefore prefer to refrain from a ‘blind’ application of (10). Instead we here propose a general relativistic treatment of black hole evolution in a dense environment by means of an exact solution of Einstein’s equations, which seems superior to (10) even though it does not (yet) take into account rotation and matter self-interactions. Furthermore, unlike (10), our final formula does not rely on exponential growth.

3. Black hole evolution from radiation to matter dominated era

To present this new solution we employ conformal coordinates, with conformal time η , instead of the cosmic time coordinate t used above. One main advantage of this coordinate choice is that the causal structure of the space-time is often easier to analyze (for the solution to be presented below it is the same as that of the Schwarzschild solution). Secondly, we wish to exploit the fact that the use of conformal time allows us to exhibit a simple closed form solution that encompasses *both* the radiative and the matter dominated phase. With conformal time η , the Friedmann equations read (for a spatially flat universe and vanishing cosmological constant)

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^4, \quad a\ddot{a} - \dot{a}^2 = -\frac{4\pi G}{3}(\rho + 3p)a^4 \quad (11)$$

where

$$\dot{a} \equiv \frac{da}{d\eta}, \quad dt = a(\eta)d\eta. \quad (12)$$

The requisite exact solution of (11) is (see e.g. [10]).

$$a(\eta) = A\eta + B^2\eta^2 \left(\Rightarrow t = \frac{1}{2}A\eta^2 + \frac{1}{3}B^2\eta^3 \right) \quad (13)$$

together with the density and pressure

$$8\pi G\rho(\eta) = \frac{3A^2}{a^4(\eta)} + \frac{12B^2}{a^3(\eta)}, \quad 8\pi Gp(\eta) = \frac{A^2}{a^4(\eta)} \quad (14)$$

The relevant numbers A and B can be calculated from known data, up to rescaling $\eta \rightarrow \lambda\eta$, $A \rightarrow \lambda^{-2}A$, $B \rightarrow \lambda^{-3/2}B$, $a \rightarrow \lambda^{-1}a$. The latter scale is conventionally fixed by setting $a(t_0) = 1$, where $t_0 \simeq 13.8 \cdot 10^9$ yr is the present time. Taking this as the reference value we make use of the fact that at equilibrium between radiation and matter [11]

$$a(\eta_{eq}) \simeq \frac{1}{3400}, \quad t_{eq} \simeq 1.5 \cdot 10^{12} \text{ s} \quad (15)$$

and at the last scattering [11]

$$a(\eta_{LS}) \simeq \frac{1}{1090}, \quad t_{LS} \simeq 1.2 \cdot 10^{13} \text{ s} \quad (16)$$

This gives

$$A = 2.1 \cdot 10^{-20} \text{ s}^{-1}, \quad B = 6.2 \cdot 10^{-19} \text{ s}^{-1}. \quad (17)$$

for our Universe (starting from nucleosynthesis).

For the new metric ansatz we now substitute (13) into²

$$ds^2 = a(\eta)^2 \left[-\tilde{C}(r)d\eta^2 + \frac{dr^2}{\tilde{C}(r)} + r^2 d\Omega^2 \right] \quad (18)$$

² This ansatz is somewhat similar to, but actually different from, the McVittie solution [12–16] and the Lemaître-Tolman-Bondi metric [17]. This can be seen for instance from the fact that for our solution the black hole mass grows with time, cf. (21) below, and has a non-vanishing heat flow vector $q_\mu \neq 0$ in (25).

Here the *a priori* unknown function $\tilde{C}(r)$ is uniquely fixed by imposing two physical requirements corresponding to the two limiting cases of pure matter and pure radiation. For pure radiation ($B = 0$) we demand the trace of the energy-momentum tensor resulting from (18) to vanish

$$T^\mu{}_\mu \stackrel{!}{=} 0 \Rightarrow \frac{d^2}{dr^2}(r^2\tilde{C}(r)) \stackrel{!}{=} 2. \quad (19)$$

With the standard form of the energy-momentum tensor for a perfect fluid (i.e. (25) below for $q_\mu = 0$), this is equivalent to the statement that $\rho = 3p$ throughout the radiation era. For the other limiting case of pure matter ($A = 0$), we require the pressure to vanish: $p = 0 \Rightarrow (r\tilde{C})' \stackrel{!}{=} 1$. This leads to the unique solution

$$\tilde{C}(r) \equiv C(r) := 1 - \frac{2Gm}{r} \quad (20)$$

which we will use in the following. The essential new feature here is that the metric (18) allows us to evolve the black hole through *both* the radiative and matter dominated periods, with a smooth transition between the two.

In (20) we use a different font for the fixed mass parameter because m is *not* the physical mass, unlike $m(t)$ above. This is most easily seen by replacing

$$\frac{Gm}{r} \rightarrow \frac{Gm(\eta)}{ra(\eta)} \equiv \frac{Gm(\eta)}{r_{phys}} \Rightarrow m(\eta) = ma(\eta) \quad (21)$$

Using (4), (8) and the above relation with $\eta_{min} = 10^{-7}$ s and $\eta_{max} = 10$ s, as well as $Gm_{min} = GM_{seed}/a_{max}$ and $Gm_{max} = GM_{seed}/a_{min}$ we get

$$Gm_{min} \sim 5 \cdot 10^{-6} \text{ m}, \quad Gm_{max} \sim 5 \cdot 10^2 \text{ m} \quad (22)$$

For the metric ansatz (18) with $C(r)$ from (20) the non-vanishing components of the Einstein tensor, hence the associated energy-momentum tensor, are given by:

$$\begin{aligned} 8\pi GT_{\eta\eta} &= \frac{3\dot{a}^2}{a^2} = \frac{3(A + 2B^2\eta)^2}{(A\eta + B^2\eta^2)^2} \\ 8\pi GT_{r\eta} &= \frac{2Gm}{r^2 C(r)} \cdot \frac{\dot{a}}{a} = \frac{2Gm}{r^2 C(r)} \cdot \frac{A + 2B^2\eta}{A\eta + B^2\eta^2} \\ 8\pi GT_{rr} &= \frac{\dot{a}^2 - 2a\ddot{a}}{a^2 C(r)^2} = \frac{1}{C(r)^2} \cdot \frac{A^2}{(A\eta + B^2\eta^2)^2} \end{aligned} \quad (23)$$

together with³

$$T_{\theta\theta} = C(r)r^2 T_{rr}, \quad T_{\varphi\varphi} = \sin^2\theta T_{\theta\theta} \quad (24)$$

Now, to elevate (23) beyond the status of a mere identity, we must endow it with physical meaning by interpreting the r.h.s. in terms of physical sources of energy and momentum, that is, a proper energy-momentum tensor, appropriate to radiation and matter. To this aim we re-express the r.h.s. of (23) in the standard form [18]

$$T_{\mu\nu} = pg_{\mu\nu} + (p + \rho)u_\mu u_\nu - u_\mu q_\nu - u_\nu q_\mu \quad (25)$$

³ We take this opportunity to correct two misprints in [1]: the extra factor of C in (24) below is missing in (46) there. Furthermore, in eqn.(50) of [1] it should read

$$8\pi Gp(\eta, r) = \frac{r}{A^2\eta^4(r - 2Gm)}$$

Here we neglect higher derivatives in u_μ and matter self-interactions (viscosity, *etc.*). For the density and pressure to match between (25) and (23) we must include an extra inverse factor $C(r)$ in comparison with (14) to account for the curvature

$$8\pi G\rho(\eta, r) = \frac{1}{C(r)} \left(\frac{3A^2}{a^4(\eta)} + \frac{12B^2}{a^3(\eta)} \right)$$

$$8\pi Gp(\eta, r) = \frac{1}{C(r)} \frac{A^2}{a^4(\eta)} \quad (26)$$

again with $a(\eta)$ from (13). The 4-velocity is⁴

$$u_\mu = -\frac{a(\eta)}{C(r)^{1/2}} (C(r) \cosh \xi, \sinh \xi, 0, 0) \quad (27)$$

while the heat flow vector is given by

$$8\pi Gq_\mu = -\frac{2Gm\dot{a}(\eta)}{r^2 C(r)^{3/2} a(\eta)^2} (C(r) \sinh \xi, \cosh \xi, 0, 0) \quad (28)$$

These vectors obey $u^\mu u_\mu = -1$ and $u^\mu q_\mu = 0$. The parameter $\xi = \xi(\eta, r) > 0$ is determined from

$$\tanh \xi = \frac{Gm\eta}{r^2} \cdot \left(1 - \frac{B^4\eta^2}{A^2 + 3AB^2\eta + 3B^4\eta^2} \right) \quad (29)$$

The signs in (27) and (28) are chosen such that for the contravariant components of the 4-velocity we have $u^\eta > 0$ and $u^r < 0$, hence *inward* flow of matter. (Choosing the opposite sign for the components of u_μ would correspond to a shrinking white hole.)

To keep ξ real and finite we must demand $\tanh \xi < 1$. It is readily seen that

$$\tanh \xi \sim \frac{Gm\eta}{r^2} \text{ for } B^2\eta \ll A \quad (\text{radiation})$$

$$\sim \frac{2}{3} \frac{Gm\eta}{r^2} \text{ for } B^2\eta \gg A \quad (\text{matter}) \quad (30)$$

The representation (25) is valid as long as all quantities remain real and finite. This requires $r^2 > \mathcal{O}(1)Gm\eta$, with a strictly positive $\mathcal{O}(1)$ prefactor. When r reaches the value for which $\tanh \xi = 1$ the components of u_μ and q_μ diverge, and the expansion (25) breaks down. For the external observer the average velocity of the infalling matter then reaches the speed of light, so for all practical purposes everything happening inside this shell is shielded from the outside (even though light rays can still escape from this region, as long as $r > 2Gm$). As we are not concerned with $\mathcal{O}(1)$ factors here we define

$$r_H(\eta) := a(\eta) \sqrt{Gm\eta} \quad (31)$$

and interpret the associated outward moving shell as an *effective* horizon (or ‘pseudo-horizon’) that lies above the actual event horizon; note that $r_H(\eta)$ is invariant under the coordinate rescalings mentioned after (17). Physically, we expect the matter inside the shell $r_{phys} \lesssim r_H(\eta)$ to be rapidly sucked up into the black hole, once the outside region $r_{phys} > r_H(\eta)$ gets depleted of ‘fuel’ due to the formation of inhomogeneities. The extra matter inside the shell $r_{phys} \lesssim r_H(\eta)$ thus enhances the growth substantially, beyond the linear growth with the scale factor implied by (21).

At the onset of inhomogeneities, we must stop using the metric (18) because the growth of the black hole gets decoupled from the growth of the scale factor $a(\eta)$, after which the black hole evolves

in a more standard fashion by much slower accretion (for this reason there is also no point in extending the metric ansatz (18) into the present epoch, which is dominated by Dark Energy). To estimate its mass we take the value of r_H at that particular time to define an effective Schwarzschild radius, thus equating the mass with the maximum energy that can possibly fit inside a shell of radius r_H . This approximation appears justified not only because of the apparent divergent kinetic energy of the infalling matter near r_H , but also because of the strong increase of the density and pressure inside this shell, due to the extra factor $C^{-1}(r)$ in (26). A more detailed investigation of the evolution inside the shell in view of eliminating the firewall (= infinite density and pressure) and the curvature singularity at $r = 2Gm$ would require modifying the metric ansatz (18) for $r \lesssim r_H(\eta)$, for instance replacing $C(r)$ by $C(r, \eta)$.

4. Mass estimates

We can now apply the above formulas to estimate the resulting black hole mass at the onset of (small scale) inhomogeneities, *i.e.* the onset of star formation. To be sure, there are still uncertainties about the actual numbers, but it is reassuring that we do end up the right orders of magnitude. The relevant time t at which to evaluate $r_H(\eta(t))$ lies well after decoupling, since the inhomogeneities in the CMB are still tiny, of order $\mathcal{O}(10^{-5})$. Rather, we take $t_{inhom} \simeq 10^8 \text{ yr} \simeq 3.2 \cdot 10^{15} \text{ s}$, which is the time when the first stars are born [19]. This corresponds to $\eta_{inhom} \simeq 2.7 \cdot 10^{17} \text{ s} \Rightarrow a(\eta_{inhom}) \simeq 0.034$. Substituting (22) into (31) and using $r_S(M_\odot) = 3 \text{ km}$ we can calculate the range of possible black hole masses, for instance taking $t \sim 100 \text{ Myr}$ as an approximate reference value:

$$10^5 M_\odot \lesssim m_{BH} \lesssim 2 \cdot 10^9 M_\odot \quad (32)$$

which is consistent with observations [2]. To reach such large mass values the replacement of Gm by $\sqrt{Gm\eta}$ in (31), as advocated in this paper, is evidently of crucial importance.

Observe that, as a consequence of the Hawking evaporation of seed black holes with too small mass, our calculation also provides a *lower bound* in (32), in contradistinction to other proposals where there is no such lower bound on the mass range. It is thus a prediction of the present mechanism that the black holes formed from gravitinos should belong to a very different mass category than the black holes formed from stellar collapse and subsequent mergers, a prediction that can also serve to discriminate our proposal against alternative ones. From the present point of view, the existence of such a gap in the mass distribution of black holes in the Universe would thus constitute indirect observational evidence for the existence of Hawking radiation. At least so far, this expectation seems to be in accord with observation, as no such objects (intermediate mass black holes = ‘IMBHs’) have been found until now [20].

Note added: After this paper was accepted for publication we became aware of earlier work [21] which discusses a metric ansatz similar to (18). Possible astrophysical applications different from the ones considered here have very recently been considered in [22].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

⁴ There is a second solution with the same ρ and p , but $u_r = q_\eta = 0$, which we discard as unphysical because it would imply the absence of motion of matter other than the co-motion with the cosmic frame.

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