Effect of Wall Stabilisation on Free Boundary $m = 2$ Modes in Toroidal $\ell = 2$ Stellarators with Small Shear

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1. Introduction.

It has been shown[1] that results on the normalized eigenvalues $(\gamma R_T/v_A)^2$ of unstable fixed boundary $m = 2$ modes in straight $\ell = 2$ stellarator configurations with vanishing longitudinal net current computed with the asymptotic STEP code[2] (based on the stellarator expansion and averaging) are in good agreement with results obtained with the HERA helically symmetric eigenvalue code[3] and the BETA 3D code[4]. In the present paper the asymptotic STEP code has been applied to investigate free-boundary $m = 2, n = 1$ modes in $\ell = 2$ stellarator configurations with vanishing longitudinal net current. The $\ell = 2$ configurations were selected so that the rotational transform ι (twist) is in the range $0.36 < \iota_{ax} < 0.58$, where the $m = 2, n = 1$ mode is resonant: $k_{res} \approx mh\iota_p$, $\iota_p = \iota/M$, M is the number of equilibrium field periods of length L_p, $ha = 2\pi/L_p$, a is the minor radius of the free plasma boundary, b is the minor radius of the conducting wall, ka the wave number of the unstable mode, m its poloidal node number, n the longitudinal mode number, $v_A = (B_0^2/\rho_0)^{\frac{1}{2}}$ the Alfvén velocity, and B_0 the main magnetic field; the shear $(\iota_b - \iota_{ax})/\iota_b$ is typically 0.2.

2. Model.

The $\ell = 2$ configuration consists of $M = 5$ field periods of length L_p/a which is continuously bent into a torus with torus curvature $\epsilon = a/R_T$. If the toroidal curvature is $\epsilon = 0.13$, the configuration is a closed toroidal system with $ha = M\epsilon$. The vacuum magnetic field is given in a pseudo-cylindrical coordinate system (r, θ, z) by $\vec{B} = B_0[\vec{e_z} + \frac{\delta}{h}\nabla I_2(hr)\sin(2\theta - hz)]$, where the Bessel function $I_2(hr)$ is a solution of the Laplace equation in a straight system (Bessel model); δ describes the helical $\ell = 2$ field amplitude giving a twist ι_{ax} on magnetic axis of $\iota_{ax} = M\delta^2/16$ (asymptotic value). The pressure profile $p = p_0(1 - \Psi)$ is approximately parabolic in the minor radius r of the flux surfaces (Ψ is the poloidal flux

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inside a magnetic surface). In all cases one wave length of the instability fits onto $M = 5$ field periods: $k = h/M$. A rather small average $\langle\beta\rangle \approx \beta_0/2 \approx 0.8\%$ is chosen so that the change of the ι-profile in the toroidal configurations with zero net current is small (e.g. for $\delta = 1.2$ the twist varies in the range $0.45 \leq \iota(r) \leq 0.55$ for $\epsilon = 0$ and $0.46 \leq \iota(r) \leq 0.53$ for $\epsilon = 0.13$). The $\iota(r)$-profile is approximately a parabolic function of r. No additional vertical field is applied ($B_v = 0$).

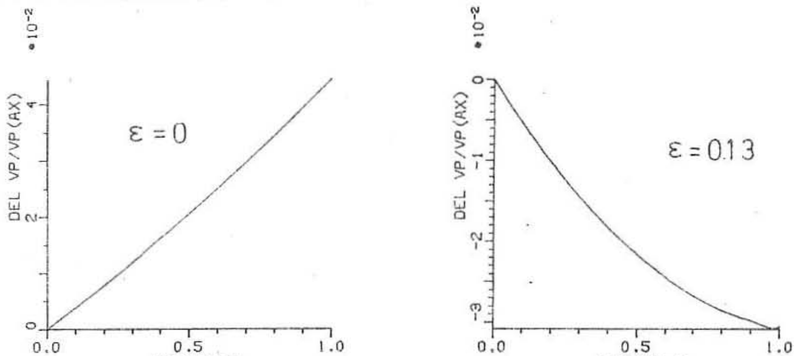

Fig.1. Normalized specific volume as function of r^2 for $\epsilon = 0, 0.13$.

The normalized specific volume $(V_b' - V_{ax}')/V_{ax}'$ as function of r^2 is plotted in Fig.1 for $\epsilon = 0, 0.13$. A rather deep magnetic well ($V'' < 0$) is created at $\epsilon = 0.13$ for $\langle\beta\rangle = 0.8\%$ which affects the stability of the configuration. The corresponding vacuum field has a magnetic hill ($V'' > 0$). Applying an additional vertical field ($B_V > 0$ causes a radially inward shift of the plasma column) the magnetic well can be diminished and therefore the stability properties are changed.

3. Results.

Using the STEP code it is shown that in toroidal $\ell = 2$ stellarators the resonant free boundary $m = 2$ mode can be stabilized by a conducting wall being close enough even for those $\ell = 2$ configurations where the twist at the plasma boundary is $\iota = 0.5$ being resonant to that mode. In straight $\ell = 2$ configurations there is no wall stabilization effect on that resonant $m = 2$ mode. Introducing toroidal curvature creates a magnetic well at finite β and the $m = 2$ fixed-boundary mode is completely stabilized. The absolute eigenvalue of the free-boundary $m = 2$ mode is diminished as well. These results are shown in Figs.2 - 5 where the eigenvalues of $m = 2, n = 1$ free-boundary and fixed-boundary modes are plotted as function of the twist ι_{ax} on axis for various positions b/a of the conducting wall. Figure 2 shows the results for the straight $\ell = 2$ configurations ($\epsilon = 0$). At $0.44 < \iota_{ax} < 0.52$ one observes a stabilizing effect due to the wall scaling approximately like $(a/b)^4$ with the inverse wall distance. At $\iota_{ax} \approx 0.408$ the corresponding twist value at the free-boundary is $\iota_b = 0.50$ which is resonant to the $m = 2, n = 1$ mode, no stabilizing effect due to the wall can be observed. Increasing the curvature to $\epsilon = 0.065$ (Fig.3) the resonance phenomenon is less distinct and obviously a stabilizing effect due to the wall is observed even at a wall position of $b/a = 1.2$.

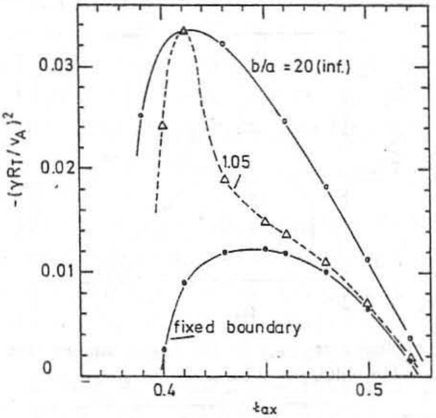

Fig.2. Normalized eigenvalues versus t_{ax} for the free-boundary ($b/a = 1.05, 1.20, \infty$) and the fixed-boundary mode (straight, $\epsilon = 0$).

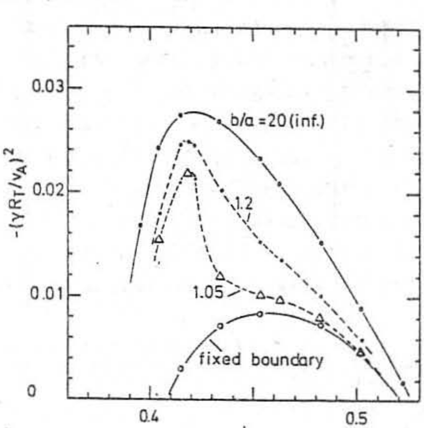

Fig.3. Normalized eigenvalues versus t_{ax} for the free-boundary ($b/a = 1.05, 1.20, \infty$) and the fixed-boundary mode (toroidal, $\epsilon = 0.065$).

The wall stabilization effect on the free-boundary mode depending on the wall distance b/a is even more obvious at $\epsilon = 0.13$ (Fig.4). In this case the fixed-boundary mode could not be found. The left edge of the eigenvalue curves is shifted to higher ι_{ax}-values as ϵ is increased. Figure 5 shows the maxima of the eigenvalue curves as function of the torus curvature ϵ for the free-boundary mode at various wall distances [$b/a = 20$ (infinity) and 1.2] and the fixed-boundary modes. The eigenvalues scale approximately like ϵ^2.

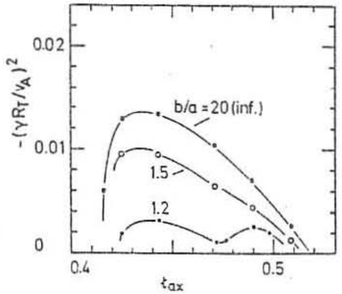

Fig.4. Normalized eigenvalues versus ι_{ax} for the free-boundary ($b/a = 1.20, 1.50, \infty$) and the fixed-boundary mode (toroidal, $\epsilon = 0.13$).

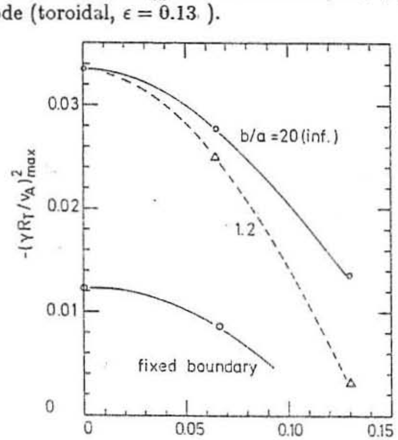

Fig.5. Scaling of the maxima of the eigenvalue curves as function of toroidal curvature $\epsilon(\epsilon_{max} = 0.13)$.

According to this model weakly unstable $m = 2, n = 1$ free-boundary modes can be stabilized by a conducting wall in a toroidal $\ell = 2$ configuration (e.g. for $\langle\beta\rangle \approx 0.8\%$ the wall position is about $b/a \approx 1.1$ to stabilize this mode).