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Entropy trade-offs in artistic design: A case study of Tamil ko m

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Abstract

From an evolutionary perspective, art presents many puzzles. Humans invest substantial effort in generating apparently useless displays that include artworks. It ese vary greatly from ordinary to intricate. From the perspective of signaling theory, the e investments into highly complex artistic designs can reflect information and out individuals and their social standing.

Using a large corpus of kolam art from South India (N = 3,139 κ lar from 192 women), we test a number of hypotheses about the ways in which soil stratification and individual differences affect the complexity of artistic design.

Consistent with evolutionary signaling theories of constant dispersion, we 20 find that kolam art tends to occupy a "sweet spot" at "hich a tistic complexity, as 21 measured by Shannon information entropy, remains a privile constant from small to 22 large drawings. This stability is maintained rough an observable, apparently unconscious trade-off between two standard in rm. ion-theoretic measures: richness and evenness. Although these drawings are in a highly stratified, caste-based society, 25 we do not find strong evidence the artistic complexity is influenced by the caste boundaries of Indian society. Ratler, the rade-off is likely due to individual-level 27 aesthetic preferences and differences as similar similar similar similar similar similar similar as well as the 28 fundamental constraints of hu an e-gnition and memory. 29

31 Keywords: Art, Sign. ng, El tropy, Skill, Material Culture, Bayesian inference

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Media Sum. ar: South Indian Tamil kolam drawings indicate successful artistic traditions hit: complexity "sweet spot".

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Introduction 37

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X From the perspective of human evolution, art is mysterious. People in all know. 38 populations invest substantial time, energy and effort into generating abstrac paterns 39 and performances (Brown, 1991), to no obvious benefit. In biology, the start of 40 seemingly non-functional traits in social communication relies on the evolutonary theory of signaling, a framework for understanding how reproductive trade-offs produce phenomena such as warning displays, mating calls, and specialized adaptations such as bright, colorful plumage (Zahavi, 1975). It is currently unclear who her human art is comparable to signaling behaviours, what features they make in common with each other, or if art is even something that can be usefully unlerstood using an evolutionary approach. In recent years, the availability of large and datasets has enabled large-scale 48 quantitative analysis (Liu et al., 2018; Winters, 2018; Sigaki, Perc, & Ribeiro, 2018), which is the cornerstone of the "popu. tion thinking" approach characteristic of evolutionary thinking in modern bio v (Mayr, 1994). Here we present such an 51 analysis of a large corpus of mater. Lart from South India: the kolam drawings created by the women of Tamil Nady in or h India. Because this long-standing artistic 53 tradition follows systema: rule, amenable to quantification, statistical models allow us to characterize the strategary pursued by individual artists, detect the existence of a 55 theoretically-derived entropy trade-off between richness and evenness, and weigh the importance of paticula constraints on the flow of information within an artistic 57 community.

Theoretical Background

evolutionary theory, signals can successfully coordinate behaviour between 60 org, isms by reliably indicating skill (Hawkes & Bird, 2002), commitment (Bulbulia & Sosis, 2011; Soler, 2012), social status (Smith, Bird, & Bird, 2003), strength (Sosis, 62 Kress, & Boster, 2007) and cooperativeness (Gintis, Smith, & Bowles, 2001; Granito,

Tehrani, Kendal, & Scott-Phillips, 2019). Courtship behaviours, such as the ornate nest structures built by bowerbirds, often have no practical use, but their great cost it self is a signal of underlying phenotypic quality and potential mate value (Madden, 2003; Schaedelin & Taborsky, 2009; Zahavi, 1975). Some human behaviours, sych a inefficient and unnecessarily difficult spearfishing in Meriam communities Blace Bro & Douglas, 2002), have been nominated as having a similar purpose, to only one a signaler's social status and thus mating success (Bird, Smith, & Bird, 2001) More generally, costly, public signals can lead to improved status and epicional standing (Power, 2017), reproductive success (Smith et al., 2003) or included support 72 (Bird, Scelza, Bird, & Smith, 2012). Beyond latent property of the individuals, signals can evolve to indicate persistent group memberships which a come the basis for cooperative assortments. Especially in multi-ethnic opuntions ethnic marker theory has become substantial to understand how jadivitures coordinate their norms and 76 behaviors with others using identity or group new bership signals (Boyd & Richerson, 1987). These signals referred to as ethn. ma. have evolved to prevent individuals from interacting with others with different norms in coordination games (Granito et al., 2019; McElreath, Boyd, & Richers n, 2 11; Moffett, 2013). 80 As a medium of commuraca on, numan art might reflect fitness-relevant qualities 81

As a medium of communication, numan art might reflect fitness-relevant qualities and capacities (e.g., preference skills or personality traits such as patience, creativity, commitment) as well as prometer social standing and mating qualities (e.g., health and fertility) (Davies, 2011 Gras eni, 2018). The signal is manifested as the aesthetic appeal or value of the artwork and as such, it makes sense to see artists compete with each other in proceeding, the most appealing and aesthetically pleasing artwork (Grasseni, 2-18; G stafsson, 2018; Varella & Fernández, 2015) that reflects their qualities and recial status. Information on an artist's capacities, their social standing or posting qualities are judged by the apparent costs of the artistic production reflected in its property (Grasseni, 2018; Varella & Fernández, 2015).

A number of quantitative approaches have been used to measure cultural diversity on some distribution of traits. In economics and anthropology, a popular distributional

measure is the Gini index of inequality (Ravallion, 2014; Zoli, 1999). A Gini index value of 0 represents a state of total equality, while a value of 1 represents total inequality. In ecology, three common methods of biological diversity are the richness (the number of 95 unique variants present), evenness (the relative abundance of variants) and Sh. mon information entropy, which weights richness by the relative abundance. For a 97 low-entropy, low-diversity state, the representation of alternative variants is ighly unequal, and in the limiting case in which only one variant is present entroly is 0. At 99 the other extreme, all n variants are represented equally, maxim zing enness, and so 100 the entropy is also maximized to the value of $\log(n)$ (Jost, 2007). Entropy has 101 also been used in several recent papers quantifying artistic versity, where an artwork 102 can be represented by an empirical probability distribution of variants (Müller & 103 Winters, 2018; Pavlek, Winters, & Morin, 2019; Winters & Morin, 2019). 104 Although the Gini index in economics and civersity in ecology quantify the 105 relative abundance in a very similar way, to or keep wledge no systematic relationship 106 has been described between the Gini inc. van. annon information entropy, richness, 107 or evenness. If we define evenness as v = 1 - g, for a given Gini index g, numerical 108 simulations show the relationship 'etwe n Shannon information entropy, richness and 109 evenness is quite strict, so that the primum entropy \widehat{H} is given by evenness v and 110

$$\operatorname{exp}(\widehat{H}) \approx n - (n-1)v^{1 + \frac{2}{2+a} + \frac{a}{a+n}} \tag{1}$$

where a = xp/J.51390.28) (see SM for more details). This approximation allows us to detect $arcop_g$ ade-offs between evenness and richness, which we use as analog to fitness trade if a d can be applied to the study of any well-defined artistic system.

Kolam A. of Southern India

richness n as

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Kolam drawings are geometric art practiced by women in the Kodaikanal region of Tamil Nadu, Southern India (Layard, 1937). A kolam consists of one or more loops drawn around a grid of dots (in Tamil called pulli). On a typical morning, a Tamil

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woman will prepare a grid of dots on the threshold of her home, and then draw a kolam 119 with rice powder or chalk. During the day the drawing weathers away, and a new k lam 120 is created the next day. Kolam drawings are historically traditions of matrilines but 121 more recently are also a topic of cultural education in Tamil schools. Girls in mil Nadu begin practicing kolam-making from an early age, and competency in the art. 123 considered necessary for the transition into womanhood (Nagarajan, 2018). \text{1}though the primary medium is the threshold of the home, women practice $k e^{i}$ am-m king in 125 notebooks, and it is common for artists to share, copy and embedish shother's kolam 126 designs. Such unrestrained artistic exchange is fostered by the fact that kolam designs 127 are not considered to belong to any one person, but rather be a type of community 128 knowledge (Nagarajan, 2018). However, the ability to ucces fully draw aesthetically 129 pleasing (i.e., diverse, complex, large) kolam drawing is soid to reflect certain qualities of a woman (e.g., her degree of traditionalness or patiente), and as such her capacity to 131 run a household and become a good wife and not er (Laine, 2013; Nagarajan, 2018). 132

Kolam drawings further broadcast moningrul information about a household to neighbors and visitors. Nagarajan (2018) argues that the presence or absence of kolam drawings help mark important events and the emotional or physical state of the artist and its household. Auspicious events, such as weddings or community festivals, warrant unusually large and complex mamarawings, while inauspicious events such as death or illness are marked by the absemption of kolam drawings, and might communicate the inability to receive or jost voitors or the need for social support (Laine, 2013; Nagarajan, 2018)

Overal' kolam-making plays an integral role in Tamil community and is deeply embedded in the Tamil culture with playful or even large-scale competitions among women Nagar jan, 2018, p. 179-203). Women often come together to carefully examine that the Tamil each others kolam drawings in terms of aesthetic qualities (e.g., geometric contactive or density Nagarajan, 2018, p. 189) or consult each other on designs to optimally showcase their skills (Nagarajan, 2018, p. 182). Contemporary interpretations of the kolam in Tamil movies even use "the motif of the heroine's beautiful kolam in

attracting the male gaze of the hero. The romance is either initiated by a strikingly
beautiful kolam or sustained during the nocturnal hours when a kolam is being n 2d2 by
the heroine [....].", (Nagarajan, 2018, p. 179-267)

51 Current Study

Kolam drawings are highly diverse, and contain multiple disting article families. 152 Here we study the ner pulli nelevu or sikku kolam family because f itsque form. 153 Because sikku kolam drawings represent an unusually strict system of satisfic 154 expression, kolam drawings can be mapped onto a small identifiant set of gestures and 155 are therefore well-suited to systematic, quantitative and as a naturalistic model 156 system of cultural evolution. A given kolam's gesture's ruence can be characterized by 157 a number of informative summary statistics which can be aspects of kolam itself: the 158 sequence length (i.e., the total number of ge. 1. 2s), discrete canvas size (measured 159 by the grid of dots, or pulli), the gestured ensity per unit canvas area, and gesture diversity as measured by evenness (here, th. Gini index), richness, and Shannon 161 information entropy.

With the ability to calculate standard measures and properties to describe
artworks derived from information theory, we can explore the possible functions of
signaling in *kolam* drawn. Specifically, we wish to understand better the social and
strategic landscape within which artists work. Moreover, we seek to understand how
realized *kolam* drawing result from the conflicting pressures of the need to
communicate social against, and various constraints on artistic production, among them
the skill and experience of the artist, and the social system she lives within.

Since these trade-offs are properties of the design space of the art itself, a substantial amount of variation may be explained simply by understanding strategic recent conscious or unconscious, made by the artist. Thus, two major research que ions arise: first, can a trade-off model explain the pattern of variation among kolam drawings, as is commonly done in behavioural ecology? And second, can we relate structural and information-theoretic properties of kolam designs to underlying

social and cognitive constraints operating on individual artists?

177 Methods

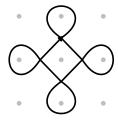
178 Kolam Dataset

We (TW) interviewed 312 artists in the Kodaikanal region in Tami. Jadu in 2009, collecting a total of 6,393 kolam drawings from the ner pulli nelevu (sikk kolam family, along with details of each woman's education, kolam-making experience, place of origin and household demographic background, including caste.

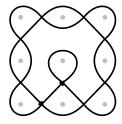
Using the lexicon of 29 *kolam* gestures developed in Waring 2012b), each *kolam* was digitally transcribed into a sequence of gestures, ar a conserved into a database using the *kolam* R package (see http://github.com/n. ran93/kolam for more details).

An example of transcribed *kolam* drawings an become in Figure 1. The geometry of the *kolam* can be divided into three geometry, spaces (orthogonal, diagonal, transitional) with their specific corresponding gentures. Each set of gestures is represented by a letter (O, D, T, respectively) while special variations of these moves are given special letters (C, H, P). To cologically, diagonal and transitional gestures are chiral with distinct left and right version because rotations of these gestures in space cannot yield their exact mirror in age (Waring, 2012b). The detailed lexicon of gestures can be consulted in the boundaries.

We excluded 674 non n drawings that could not be matched to an artist, 695 kolam drawings becaus the included non-lexical gestures and another 17 kolam drawings due to ray ser ption errors. We further excluded 120 women because their survey data has incomplete with substantial missing data in key variables: age, GPS, duration of protice or caste membership. In total, 3, 139 kolam drawings (on average 16 kola per voman) from 192 artists were included in the analysis (age: M = 31.83, $\gamma = 10^{-0.2}$ vears, range = 15 - 60; married: 75%). The artists are from 19 different cas's, spanning from low-, middle- to high-castes. Of the 3, 139 kolam drawings, 1801 kolam drawings came from artists of a low-caste, 593 kolam drawings from artists of a middle-caste and 745 kolam drawings from artists of a high-caste.



sequence: o4 o1 o4 o1 o4 o1 o4 o1



sequence 1: o4 o1 o3 o2 o3 o2 o3 o2 o3 o1 sequence 2: o2 o1 o2 o1 o2 o1 o2 o1

Figure 1. Example of two orthogonal kolam drawings and their corresponding using a lexicon of gestures.

204 Information-theoretic measures

We use Shannon information entropy H(p) as asure of artistic complexity or 205 diversity for each kolam drawing j and probabilities or each possible, discrete gesture i, computed as the average log-probability: $H(p)_i = -\sum_i^n p_i \log(p_i)$. Entropy as a 207 measure for complexity is continuous, addn. e and increases as the number of possible gestures increases. While the lexico. of 29 gestures (Waring, 2012b) decomposed the 209 diagonal and transitional gesture opes it to distinct left and right versions, we did not distinguish between them because they are a property of the transcription and not of 211 the artist. Thus, inform. in-the concic measures were computed based on 18 distinct 212 gestures (with each chiral pair sunted as only one) and the theoretical upper bound of 213 the entropy in our and vses i $\sum_{i=18}^{18} \frac{1}{18} \log(\frac{1}{18}) = 2.89 \log \text{ units.}$ In contrast, the theoretical lower poy id of earropy is 0 for a kolam that consists only of one gesture (see 215 2). 216

Richnes, represents the number of unique gestures (accounting for chirality)

present n a $k\epsilon$ am drawing and evenness represents the relative abundance of each

We computed evenness v using the Gini index of inequality g: v = 1 - g, where $g(n) \sum_{i=1}^{n} \sum_{j=1}^{n} |p_i - p_j|$, where n is the richness and p the frequency of specific variants or

gestures. Figure 2 illustrates how these properties or information-theoretic measures

correspond to specific kolam drawings.

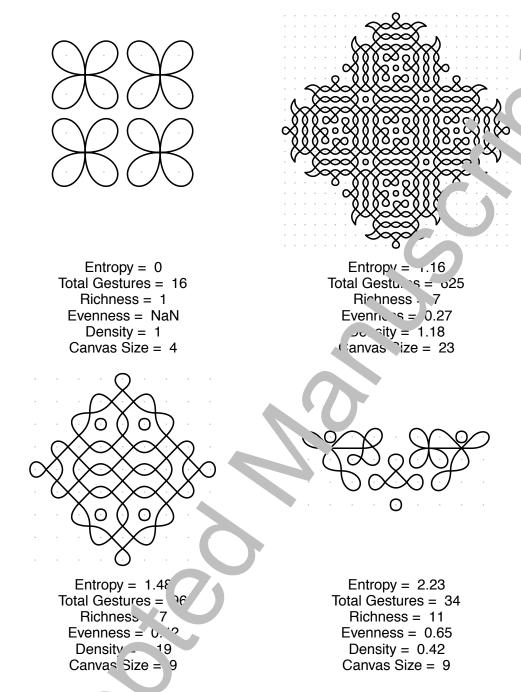


Figure 2. Struct and in formation-theoretic properties of *kolam* drawings. The Figure shows four *sola i* examples and their respective information-theoretic measures and structural properties.

Statistic 1 Analysis

To investigate the scope for viewing *kolam* art as a signaling system for aesthetic value, we modeled five information measures of each kolam in our sample using a variety of predictor variables. The five properties used as dependent variables to

describe a kolam drawing were the canvas size, the gesture density per unit canvas area, 227 evenness, richness, and Shannon information entropy. The canvas size of a kolan is a discrete count variable measured by the grid of dots, or pulli, and captures the 229 dimension of the kolam. Since kolam drawings always start with an initial squee grid of dots, the canvas size is equal to the width or length of this initial dot mat ix, a gard. ss 231 of whether the resulting kolam is not maximally spanning both the width and length of this grid. The gesture density reflects the number of gestures by can's are: 233 $density = \frac{sequence length}{canvas size^2}$. Age, duration of practice and caste were use in predictor variables to explain individual variation. Age and duration of raction were 235 standardized to be centered on zero with a standard deviat. To of the. 236

Since our data contains repeated observations for artists and castes (i.e., multiple kolam drawing from an artist or from any given costs. We partially pooled information across these two units using hierarchical moding it or er to account for imbalances in sampling and to yield more reliable and precise estables (Efron & Morris, 1977).

While information was pooled across artists to as aid over-dispersed parameter estimates, we estimated a random intercept (i.e., offset) for each artist. Caste is comprised of 19 different categories and as modeled as a varying effect to estimate individual offsets for each casts are tegory.

Evenness and rich. So at releved to entropy by a mathematical identity (shown 245 in the derivation in the (1) and ubject to an optimization process. This theoretical guide motivates the specific phoice of predictor variables in our statistical models, which 247 is why we would not no luce e.g. canvas size as predicted by richness. Including these 248 predictors we'll not address our larger question of modeling information entropy or 249 mapping its otent al trade-offs, nor would such an analysis add an adequate potential 250 alternative explanation of the invariance in entropy and the richness/ evenness 251 offs because the system does not prevent artists from drawing kolams with 252 mi i al or maximum entropy.

The statistical models were implemented in the probabilistic programming language Stan (v2.18) (Carpenter et al., 2017), using 6000 samples in four independent

chains. We applied an iterative process of model building, inference, model checking 256 and evaluation, and model expansion to ensure a principled and robust Bayesian 257 workflow (Gabry, Simpson, Vehtari, Betancourt, & Gelman, 2019; Talts, Betancourt, 258 Simpson, Vehtari, & Gelman, 2018). Prior predictive simulations and fitted in dels to simulated data were used to determine reasonable and regularizing priors or the 260 parameters that respects the parameter type's bounds. We present a compact description of the statistical models and the priors in the SI. Analyse were performed 262 in R (R Core Team, 2019). Data and analyses can be found her http://github.com/nhtran93/kolam_signaling. All \hat{R} value we less than 1.01, 264 and visual inspection of trace plots, rank histograms and pars plots indicated 265 convergence of all models. Visual MCMC diagnostics can be found in the SI. 266

267 Results

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kolam drawings concentrate around an entry of 1.17 log units regardless of their size, 269 they systematically vary in evenness and richness as they increase in size (see Figure 3). Larger kolam drawings employ a greater achness of gestures, on average, but also have 271 greater inequality between ge cur s in such a way that entropy remains tightly bounded between 1.1 and 1.4. As "astr. of aurther in Panel A and C in Figure 4, evenness 273 decreases with increasing canvas size, while richness increases with increasing canvas size. 275 In characte izi g he as ist-level variation, we also find similar patterns. Figure 4 276 illustrates ar' so onsets on the different structural and information-theoretic properties 277 on kolam dra rings. Artist means cluster between an entropy of 1.1 and 1.4 log-units. 278 Thus, v ry pla n (entropy < 1) as well as highly complex kolam drawings (entropy >279 very rare. Individuals who draw larger kolam drawings tend to use more 280 different gestures but in turn repeat a few gestures disproportionately (Figure 4, panel 281 B). 282

As indicated by Figure 5, there is also some small distinct variation between

Consistent with the entropy trad fis implied by equation 1, we find that as

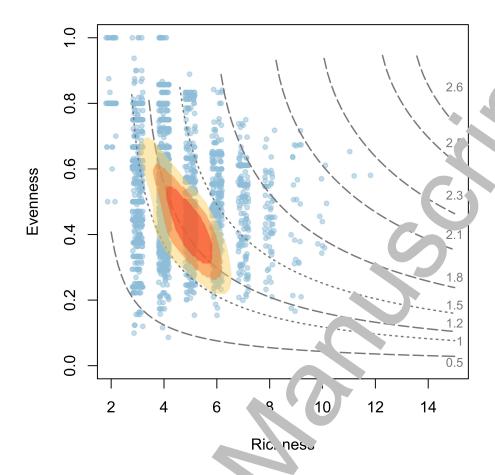


Figure 3. Trade-off between the L enness and the Richness. The grey lines measure maximum entropy isoclines. The raw ko, m data are jittered and illustrated in blue (light blue = low density, dark 'l' e = mgh density). The (90%, 75%, 50%) kernel-density of the average is ones and evenness for each canvas size of the data are depicted in the orange area (inth) orange to dark orange).

artists on the average entropy of their kolam drawings $\sigma_{artist} = 0.04$, 90% CI [0.02, 0.05]). This between dist variability is most pronounced in canvas size ($\sigma_{artist} = 0.15$, 90% CI [0.1, 0.17] and in density ($\sigma_{artist} = 0.10$, 90% CI [0.08, 0.11]) with individuals showing differences in the average canvas size and density of their kolam drawing.

288 Detween-incovidual variation the evenness ($\sigma_{artist} = 0.05$, 90% CI [0.04, 0.06]) and in the richness ($\sigma_{artist} = 0.01$, 90% CI [0.00, 0.03]) were estimated with high certainty to be now-zero, but very small (see right panel in Figure 5).

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We detected very small effects of caste membership on density, evenness, richness,

and entropy, with varying-effect deviations estimated near zero with high certainty as illustrated in Figure 5 (density $\sigma_{caste} = 0.02$, 90% CI [0.00, 0.04]; evenness $\sigma_{caste} = 0.03$, 90% CI [0.02, 0.05]; and richness $\sigma_{caste} = 0.01$, 90% CI [0.00, 0.03]; entropy $\sigma_{cast} = 0.05$, 90% CI [0.01, 0.05] respectively). However, evidence for caste differences in carvas sizes of kolam drawings was more pronounced ($\sigma_{caste} = 0.11$, 90% CI [0.06, 0.17).

Evidence for an effect of age and an effect of duration of practice on the five outcomes is also very weak. Figure 5 shows that both predictor variables have a very small effect on the five outcome variables. Age and the duration of the tice are estimated with high uncertainty to be close to zero across the tree models.

Only a small amount of variation in the information statistics we employed can be 301 accounted for by variation in artists, their age, years o pract, 2 and caste membership: 302 about 15% for canvas size, 13% of the evenness, 1¹/₂ of the variation in the gesture 303 density, 0.01% for the richness and 0.03% for entropy as measured by the Interclass 304 Correlation Coefficient (Gelman & Hill, 2006) (see M for more details). Residential 305 proximity and regional origin of artists hardly as bunts for any variation in the 306 structural and information-theore is properties (see SM). In contrast, the residual 307 variance of the outcomes is large and dominates model inference more than the variation explained by our fixed and a soom effects combined. 309

Discussion

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Viewed at the populat on scale, the complexity of kolam drawings is quite 311 invariant, sugges ing the existence of an entropy "sweet spot" at which most artists and 312 most kolam rawings center around, regardless of the design's size or gesture richness. 313 The observed increase in gesture richness in bigger kolam drawings is compensated for 314 almost vactly by a corresponding decrease in gesture evenness, such that as kolam 315 increase in size, richness is traded off against evenness so as to maintain nearly 316 con ant entropy. Our findings are consistent with the general view of signaling in 317 behavioural ecology as an attempt at optimization under constraints and lend support 318 that entropy is optimized through an observable and apparently unconscious trade-off 319

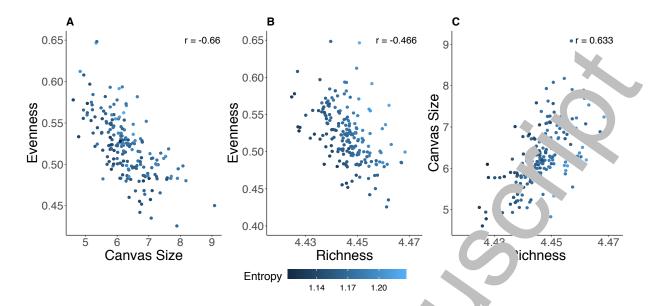


Figure 4. Scatter plot of posterior estimates of individual interprets (sum of individual offsets and population mean). The posterior estimates of individual variation of two models are plotted against each other to illustrate the parrelation between outcomes. The blue colour gradient reflects the posterior estimates of individual variation of entropy. Pearson's correlation r between the pasterior estimates of the two variables is shown on the upper left for each panel. A. The canvas size and the evenness model. B. The evenness and the richness model.

between richness and evennes (S) wn theoretically and empirically).

In this interpretation kommunating fawings that are generally more diverse are more valuable art products (Marajan, 2018, p. 189). For this reason, we see very few kolam drawings with an entropy be ow one, which would be unusually simplistic or repetitive, regardless of the sing. Conversely, artists seem to hit an upper entropy constraint around 1.5 legandless of the size of the kolam, which suggests some form of constraint on more complex (and more valuable) artwork.

A hough the nature and origin of these constraints are unclear, our analysis can the theorem of the possibilities. Almost no meaningful information about caste stratification is vable in the information metrics we employ. Members of different caste categories tended to create distinct *kolam* drawings of different canvas sizes, but no clear differences in other major structural or information-theoretic properties. Indeed, our

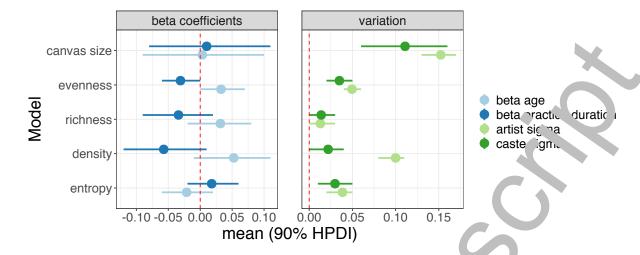


Figure 5. Prior-Posterior Coefficient Plots. All panels have the time y-axis indicating the five models. The left panel (beta coefficients) illustrates the extimated beta coefficients for the two predictors, duration of practice (dark time) and artist's age (light blue) for each model. The right panel (variate) illustrates the estimated population level standard deviation for the treet of tasts (dark green) and the estimated individual variation (light green) for each model. The 90% Highest Posterior Density Interval (HPDI) was computed to each posterior.

findings are consistent with ethnoraphic occounts of *kolam* as a form of community knowledge, and suggest that, as a public art form drawn on a home's threshold, *kolam* drawings enjoy a relatival vega arian information flow even in a stratified, multiethnic society (Waring, 2012a)

Based on the above we believe that complexity in kolam design is more likely 336 constrained by a sthatic preferences and cognitive limitations, rather than by 337 information etworks or social hierarchies. Although we were able to observe variation 338 in average en, one between artists, with some highly complex kolam above an entropy 339 score of 1.5 log units, we were not able to map this variation to patterns of age or 340 This could reflect cultural selection pressures to make traditional practices 341 of wistic ornamentation and design, such as kolam art more learnable or transmissable (Kirby, Cornish, & Smith, 2008; Müller & Winters, 2018; Ravignani, Delgado, & Kirby, 343 2017; Tamariz & Kirby, 2015; Tylén et al., 2020) or limitations in procedural and 344

working memory capacities (Oberauer, 2010; Oberauer & Kliegl, 2006) unrelated to the action of experiential memory or cognitive senescence (Gurven et al., 2017).

An overly complex and large *kolam* with rich and diverse gestures might be to 347 difficult, time-consuming or too risky to execute successfully because options for revisions and corrections are limited. Artists might want to avoid highly omple. kolam 349 drawings because they draw them in front of their house and hesitation, pauses or 350 corrections could be interpreted by the audience as imperfection or as lack of skill 351 (Nagarajan, 2018, p. 53; p.156). This avoidance of maximally complex artistic designs due to increased risk of deficiency and failure might also be relevant to other practices 353 of ornamentation or decorations where mistakes often last and another rectified easily 354 (e.g., polychrome bowl designs, Bowser, 2000 or Angol sona drawings, Gerdes, 1990). 355 Alternatively, it might also be that more diverse / sta drawings are simply not as aesthetically appealing to artists and their a vience because individuals often tend to 357 prefer a certain extent of regularity and repetit on 1, ther than complete randomness 358 and thus highly complex kolam drawings (Juang et al., 2018; Voloshinov, 1996). Other 359 artistic design such as loop pattern, for decorations in Japan or Angolan sand drawings have already been known to be influenced by the aspiration for symmetry (Gerdes, 1990; 361 Nagata, 2015). Therefore, the are t's aesthetic preferences are the final constraint. 362 363 rotation, and repetitio , and can correspond to distinct complexity measures (Sigaki et al., 2018). Aesthetic pr fere ces can determine these distinct structural properties and 365 reflect shared at ent on and learning (Tomasello, Kruger, & Ratner, 1993). Artists can 366 deliberately moose to impose structural constraints according to their and consumers' 367 preferences of to on artwork. For instance, artists can strive for symmetry, only use the 368 same type of variants (i.e., gesture types) or decide to primarily use the same two 369 (i.e., gestures) and only add very low frequencies of other, special variants as 370 dec ation. All these decisions underlie the time, skills and the aesthetic preferences of 371 the artist and can profoundly shape the distribution of information-theoretic properties 372 of the resulting artwork (Grasseni, 2018; Gustafsson, 2018). Beyond measures of 373

entropy, we do not have direct ratings of the aesthetic quality of *kolam* drawings or
whether the artist has employed a particularly appealing style. Other informatio
metrics, such as bilateral or rotational symmetry, or fractal scaling, might reveal energy
details beyond diversity or complexity and should be an endeavor for future states.

While the observed patterns in *kolam* art imply a certain degree of a variance in 378 complexity across different canvas sizes and only small traces of individual viriation and 379 social stratification, they exhibit what has been called "equifinal" behaviour (Barrett, 380 2018; von Bertalanffy, 1969). Equifinality means that inferring the gergrative processes 381 that might have given rise to the observed cultural frequer v date 3 difficult because 382 we only have cross sectional data (Barrett, 2018; Kand'a Powell, 2015). Temporal data could allow us to narrow the subset of causal mech. Pisms that underlie the 384 observed distribution of information-theoretic proper ie. Generative simulations could 385 approximate temporal data and provide mor in depun understanding on how artistic 386 traditions could have theoretically evo' spec scally in regards to the diversity or the complexity and the stability of the kolam in be population across time. In order to 388 infer the underlying generative process, a probabilistic model, in which the hypothesized causal mechanisms (1.2., cornitive constraints, aesthetic preferences or 390 other potential constraints) are explicitly defined, needs to be built (Kandler & Powell, 391 2015). Such a probabilist model an allow us to repeatedly simulate datasets with 392 known parameters and con pare the resulting distribution with observed data to infer the most likely hypothesized causal mechanisms. Furthermore, measuring the signaling 394 value of specific vole n notifs for coordinating using classification tasks (Bell, 2020) 395 could be a possising endeavor to explain the role of kolam art for social coordination. 396 A comparison of the signaling value of culturally salient kolam motifs between the 397 Tamil population in South India and the Tamil diaspora in the U.S. could further reveal 398 we. * functions of kolam art for different communities. Another promising future 399 end over could be to focus specifically on how kolam drawings are perceived and 400 whether the processing efforts of kolam drawings (visual complexity measured by 401 perimetric complexity or algorithmic complexity) (Miton & Morin, 2019; Pelli, Burns, 402

Farell, & Moore-Page, 2006) are in alignment with the actual production efforts (e.g.,
gesture complexity measured by Shannon entropy) invested in kolams. These
perception and processing efforts of a consumer or learner of kolams could even have
implications on the transmission of *kolam* knowledge in terms of learning and
reproduction (Tamariz & Kirby, 2015).

Our results on entropy trade-offs and various constraints on complexity, operating
on *kolam* art encourage us to distance ourselves from underspecified and varie attempts
to explain the evolution of art (Miller, 2011; Pinker, 2003) and minimal epply about

on kolam art encourage us to distance ourselves from underspecified and variate attempts 409 410 artistic traditions in terms of evolutionary signaling theories of constrained 411 optimization. Further investigations of how evolutionary sig. ling theories of 412 constrained optimization could be applied to other art forms other communities, such 413 as Vanuatuan sand art (Lind, 2017; Zagala, 2004) gola. sand drawings (Gerdes, 414 1988, 1993) or Islamic geometric art (Abdu' hi & F nb 2013), could advance our 415 evolutionary understanding of investments in an enstraints on art. A careful synthesis 416 of evolutionary signaling theory with etc. ograp can help us understand individual's 417 strategic investments into mastery of specific a tistic skills and how they optimize their 418 artistic displays (e.g., size, novelties, con ur diversity) within certain constraints (e.g., 419 aesthetic preferences, cognitive constraints or motor constraints), allowing us to 420 elucidate properties of art. In partarily, evaluating evolutionary constraints on cultural 421 productions beyond function 1 sufficiency is integral to understand how cultural 422 productions have evol ed (e. ., motor constraints in music production Miton, Wolf, 423 Vesper, Knoblich & Sper. or, 2020). All these future direction will be time consuming and computation $\forall y$ ch llenging, but we believe that the long-term gains for an 425 evolutionary under tanding of artistic traditions will make this enterprise worthwhile. 426

427 Conclusion

Using quantitative measures to systematically study material art in a large-scale
anthropological dataset, our findings inform discussions on entropy trade-offs and
various constraints on complexity operating on artistic traditions.

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In the case study of the hand-drawn Tamil artistic tradition, our findings are 431 consistent with evolutionary signaling theories of constrained optimization and lead 432 support that artistic complexity, measured by Shannon information entropy, is 433 optimized through an observable, apparently unconscious trade-off between two standard ecological and information-theoretic measures: richness and ever less. This 435 trade off between richness and evenness can potentially be explained by cog. 'tive constraints and aesthetic preferences. Variation in structural and informatic n-theoretic 437 properties of kolam drawings are small, and evidence of social st uct as reflected in the information measures we employ, are weak. This corroborates vir understanding of 439 kolam art as signal that does not primarily communicate so ial st atification or individual differences in age or practice, but rather aes hetic references, dedication, 441 time and skill, as well as constraints of human cogration and memory.

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Author Contributions

T. (W co lected the data, designed the *kolam* lexicon and wrote the Netlogo

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Conflict of Interest

The authors declare that the research was conducted in the absolve of the commercial or financial relationships that could be construed as a potential conflict of interest.

Research Transparency and Repreducibility

The *kolam* data and code for this study are available and can be found on GitHub:

http://github.com/nhtran93/kolam_signaling The revackage to analyse kolam

drawings can be further found on GitHub: / tp://itlub.com/nhtran93/kolam.

References

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Abdullahi, Y., & Embi, M. R. B. (2013). Evolution of Islamic geometric patterns.
```

- Frontiers of Architectural Research, 2(2), 243–251. doi: 10.1016/j.foar.20.3.03. (2)
- 472
- Barrett, B. J. (2018). Equifinality in empirical studies of cultural transm. ion.
- Behavioural Processes. doi: 10.1016/j.beproc.2018.01.011
- Bell, A. V. (2020). A measure of social coordination and group signaling in the wild.
- Evolutionary Human Sciences, 2, e34. doi: 10.1017/ehs 2020.24
- Bird, R. B., Scelza, B., Bird, D. W., & Smith, E. A. (2012) The Lerarchy of virtue:
- mutualism, altruism and signaling in Martu wom an cooperative hunting.
- Evolution and Human Behavior, 33(1), 64-78. do.
- 480 10.1016/j.evolhumbehav.2011.05.007
- Bird, R. B., Smith, E. A., & Bird, D. W. (2011) The nunting handicap: Costly
- signaling in human foraging stra' . Be avioral Ecology and Sociobiology,
- 483 50(1), 9–19. doi: 10.1007/s002650100
- Bliege Bird, R., & Douglas, W. (201) Constraints of Knowing or Constraints of
- Growing? Fishing and Collecting by the Children of Mer. Human Nature, 13(2),
- 486 239–267. doi: 10.1007/s1211.0-\02-1009-2
- Bowser, B. J. (2000). Free Poully to Politics: An Ethnoarchaeological Study of
- Political Faction and Etanicity, and Domestic Pottery Style in the Ecuadorian
- Amazon. Journa. of A chaeological Method and Theory, 7(3).
- Boyd, R., & Riclers n, P. J. (1987). The Evolution of Ethnic Markers. Cultural
- 491 Anthre $ology \gtrsim (1)$, 65–79. doi: 10.1111/j.1468-1331.2008.02145.x
- Brown, D (191) Human Universals. McGraw-Hill Professional.
- Bulbuli J., & Sosis, R. (2011). Signalling theory and the evolution of religious
- 494 Apperation. Religion, 41(3), 363–388. doi: 10.1080/0048721X.2011.604508
- Car enter, B., Gelman, A., Hoffman, M., Lee, D., Goodrich, B., Betancourt, M., ...
- Riddell, A. (2017). Stan: A probabilistic programming language. Journal of
- 497 Statistical Software, Articles, 76(1), 1–32. doi: 10.18637/jss.v076.i01

- Davies, S. (2012). The Artful Species: Aesthetics, Art, and Evolution. Oxford 498 University Press. 499 Efron, B., & Morris, C. (1977). Stein's paradox. Scientific American, 236(5), 110-127. 500 doi: 10.1038/scientificamerican0577-119 501 Gabry, J., Simpson, D., Vehtari, A., Betancourt, M., & Gelman, A. (2019). 502 Visualization in bayesian workflow. Journal of the Royal Statistical Society: Series 503 A (Statistics in Society), 182(2), 389-402. doi: 10.1111/rssa.12.78 504 Gelman, A., & Hill, J. (2006). Data analysis using regression ar t m c'evel/hierarchical models. Cambridge University Press. doi: 10.1017/CBC. 7805_790942 506 Gerdes, P. (1988). On possible uses of traditional angolan sand drawings in the 507 mathematics classroom. Educational Studies in Mathematics, 19(1), 3–22. doi: 508 10.1007/BF00428382 Gerdes, P. (1990). On ethnomathematical r search and ymmetry. Symmetry Cult. Sci., 510 1, 154–170. 511 Gerdes, P. (1993). Exploring Angolan and ings (sona): Stimulating Cultural 512 Awareness in Mathematics Teachers. The Radical Teacher (43), 18–24. 513 Gintis, H., Smith, E. A., & Bowles S. (2001). Costly signaling and cooperation. 514 Journal of Theoretical Biology, [9,1] (1), 103–119. doi: 10.1006/jtbi.2001.2406 515 Granito, C., Tehrani, J., Ken 'a' J., & Scott-Phillips, T. (2019). Style of pictorial 516 representation is shap by intergroup contact. Evolutionary Human Sciences, 1. 517 doi: 10.1017/ehs 2019. 518 Grasseni, C. (2018) Beaves as skill and 'common sensing'. In S. Bunn (Ed.), 519 Anthropologiand leauty - from aesthetics to creativity (1st Ed ed., p. 502). 520 Londo: Rou ledge. 521 Gurven, M., Fuerstenberg, E., Trumble, B., Stieglitz, J., Beheim, B., Davis, H., & 522 Ka. lan. I. (2017). Cognitive performance across the life course of bolivian 523
- 160–176. doi: 10.1037/dev0000175 Gustafsson, A. (2018). Beauty as a capacity. In S. Bunn (Ed.), Anthropology and beauty

524

525

*Tag r-farmers with limited schooling. Developmental Psychology, 53(1),

```
- from aesthetics to creativity (1st Ed ed., p. 502). London: Routledge.
527
   Hawkes, K., & Bird, R. B. (2002). Showing off, handicap signaling, and the evolution of
528
         men's work. Evolutionary Anthropology, 11, 58–67. doi:
529
         https://doi.org/10.1002/evan.20005
530
   Huang, Y., Xue, X., Spelke, E., Huang, L., Zheng, W., & Peng, K. (2018) Th
531
         aesthetic preference for symmetry dissociates from early-emerging attention to
         symmetry. Scientific Reports, 8(1), 1–8. doi: 10.1038/\text{s}41598-0 8-245/8-x
533
   Jost, L. (2006). Entropy and diversity. Oikos, 113(2), 363-375. 10i
534
         10.1111/j.2006.0030-1299.14714.x
535
   Jost, L. (2009). Mismeasuring biological diversity: Response to he fimann and hoffmann
536
         (2008). Ecological Economics, 68(4), 925 - 928. (oi:
537
         https://doi.org/10.1016/j.ecolecon.2008.10.01
538
   Kandler, A., & Powell, A. (2015). Inferring learning stritegies from cultural frequency
539
         data. In A. Mesoudi & K. Aoki (Eds.), Year ing strategies and cultural evolution
540
         during the Palaeolithic (pp. 85–10. Springer Japan.
541
   Kirby, S., Cornish, H., & Smith, K. (2008). Comulative cultural evolution in the
         laboratory: An experimental app. ach to the origins of structure in human
543
         language. Proceedings of the Arical Academy of Sciences, 105(31),
544
         10681–10686. doi: 10.1 73 pn; s.0707835105
   Laine, A. (2013). Kolam pa. terns as materialisation and embodiment of rhythms.
546
         Journal of Anth ovision. doi: https://doi.org/10.4000/anthrovision.607
547
   Layard, J. (1937) Toby. In ritual in south india: Threshold and tattoo designs.
548
         Folklore, 4, (5), 1, 5–182. doi: 10.1080/0015587X.1937.9718684
549
   Lind, C. (2017). The beauty of sand-drawing in Vanuatu: Kinship and continuity on
550
         Paria Island. Anthropology and Beauty: From Aesthetics to Creativity, 418–433.
551
   Liu, L., Vang Y., Sinatra, R., Giles, C. L., Song, C., & Wang, D. (2018). Hot streaks
552
         ranstic, cultural, and scientific careers. Nature, 559 (7714), 396–399. doi:
553
         10.1038/s41586-018-0315-8
554
   Madden, J. R. (2003). Bower decorations are good predictors of mating success in the
```

spotted bowerbird. Behavioral Ecology and Sociobiology, 53(5), 269–277. doi: 556 10.1007/s00265-003-0583-6Mayr, E. (1994). Typological versus population thinking. Conceptual issues in 558 evolutionary biology, 157–160. McElreath, R., Boyd, R., & Richerson, P. J. (2011). Shared norms and the evolution of 560 ethnic markers. Current Anthropology, 44(1), 122-130. doi: $10.1^{\circ}6/2^{\circ}143$ Miller, G. (2011). The mating mind: How sexual choice shaped the endution of human 562 nature. Anchor. Miton, H., & Morin, O. (2019). When iconicity stands in the vy clusbreviation: No 564 Zipfian effect for figurative signals. PLoS ONE, 14(8), 1–19 doi: 565 10.1371/journal.pone.0220793 566 Miton, H., Wolf, T., Vesper, C., Knoblich, G., & Spriber, D. (2020). Motor constraints influence cultural evolution of rhythm. Motor one traints on culture. Proceedings 568 of the Royal Society B: Biological Sciences, 197(1937). doi: 569 10.1098/rspb.2020.2001rspb20202 570 Moffett, M. W. (2013). Human Identity and the Evolution of Societies. Human Nature, 571 24(3), 219–267. doi: 10.1007 s12.10-013-9170-3 572 Müller, T. F., & Winters, J. (2018). Compression in cultural evolution: Homogeneity 573 and structure in the emergence and evolution of a large-scale online collaborative art project. *PLoS CN* 7 13(9). doi: 10.1371/journal.pone.0202019 575 Nagarajan, V. (2018) Feea. g a Thousand Souls: Women, Ritual, and Ecology in 576 India- An Emborate of the Kolam. Oxford University Press. 577 Nagata, S. (2015) Loo Patterns in Japan and Asia. Forma, 19–33. doi: 10.504 /forn .2015.004 579 Oberaue, K. (2010). Declarative and procedural working memory: Common principles, con mor capacity limits? Psychologica Belgica, 50(3-4), 277–308. 581 Chera er, X., & Kliegl, R. (2006). A formal model of capacity limits in working 582

memory. Journal of Memory and Language, 55(4), 601 - 626. doi:

https://doi.org/10.1016/j.jml.2006.08.009

583

- Pavlek, B., Winters, J., & Morin, O. (2019). Ancient coin designs encoded increasing amounts of economic information over centuries. *Journal of Anthropologica'***Archaeology, 56(October), 101103. doi: 10.1016/j.jaa.2019.101103

 **Pelli, D. G., Burns, C. W., Farell, B., & Moore-Page, D. C. (2006). Feature a ection and letter identification. *Vision Research*, 46(28), 4646–4674. doi: 10.1016/j.visres.2006.04.023
- ⁵⁹¹ Pinker, S. (2003). How the mind works. UK: Penguin.
- Power, E. A. (2017). Discerning devotion: Testing the signaling the of religion.
- Evolution and Human Behavior, 38(1), 82–91. doi:
- 594 10.1016/j.evolhumbehav.2016.07.003
- ⁵⁹⁵ R Core Team. (2019). R: A language and environmen for stistical computing.
- Vienna, Austria: R. F. for S. Computing, Ed
- Ravallion, M. (2014). Income inequality in the developing world. *Science*, 344 (6186), 851–855. doi: 10.1126/science.1251875
- Ravignani, A., Delgado, T., & Kirby, S. Vor. Iusical evolution in the lab exhibits rhythmic universals. *Nature Human Ben viour*. doi: 10.1038/s41562-016-0007
- Schaedelin, F. C., & Taborsky, M. (200°). Extended phenotypes as signals. *Biological reviews of the Cambridge Phut are ical Society*, 84(2), 293–313. doi:
- 10.1111/j.1469-185X.20 8 / 007 5.х
- Sigaki, H. Y., Perc, M., & r beiro, H. V. (2018). History of art paintings through the
 lens of entropy and co. plexity. Proceedings of the National Academy of Sciences
 of the United Ctate. America, 115 (37), E8585–E8594. doi:
- 607 10.1073/pn < 1800)83115
- Smith, E. A, Bird, R. B., & Bird, D. W. (2003). The benefits of costly signaling:
- Mean burde hunters. Behavioral Ecology, 14(1), 116-126.
- 610 Soler, M (2012). Costly signaling, ritual and cooperation: Evidence from Candomblé,
- 611 Evolution and Human Behavior, 33(4), 346–356. doi:
- 612 10.1016/j.evolhumbehav.2011.11.004
- Sosis, R., Kress, H. C., & Boster, J. S. (2007). Scars for war: evaluating alternative

- signaling explanations for cross-cultural variance in ritual costs. Evolution and 614 Human Behavior, 28(4), 234–247. doi: 10.1016/j.evolhumbehav.2007.02.007 615 Talts, S., Betancourt, M., Simpson, D., Vehtari, A., & Gelman, A. (2018). Validating 616 Bayesian inference algorithms with simulation-based calibration. arXiv, eprint Tamariz, M., & Kirby, S. (2015). Culture: Copying, compression, and corvens, value 618 Cognitive Science, 39(1), 171-183. doi: 10.1111/cogs.12144 619 Tomasello, M., Kruger, A. C., & Ratner, H. H. (1993). Cultural lear ing. I chavioral 620 and Brain Sciences, 16(3), 495?511. doi: 10.1017/S0140525X0000123X 621 Tylén, K., Fusaroli, R., Rojo, S., Heimann, K., Fay, N., Johan, en, J. N., ... Lombard, 622 M. (2020). The evolution of early symbolic behavior 1. hom sapiens. *Proceedings* 623 of the National Academy of Sciences. doi: 10.1073/pn. 1910880117 624 Varella, M. A. C., & Fernández, A. M. (2015). Evel tion of Artistic and Aesthetic 625 Propensities through Female Competitive O. no ner tation (Vol. 1; M. L. Fisher, 626 Ed.). Oxford University Press. doi: 10.195/xfordhb/9780199376377.013.46 627 Voloshinov, A. V. (1996). Symmetry as sup inciple of science and art. Leonardo, 628 29(2), 109-113.629 von Bertalanffy, L. (1969). Gener Sycom Theory: Foundations, Development, 630 Applications. George Braziller. 631 Waring, T. M. (2012a). Cool er dioi Dynamics in a Multiethnic Society: A Case Study 632 from Tamil Nadu. Va. rent Anthropology, 53(5), 642-649. doi: 10.1086/666942 633 Waring, T. M. (2012). Sequential Encoding of Tamil Kolam Patterns. Forma, 27, 634 83-92. 635 (2019). From Context to Code: Information Transfer Winters, J., & M v n, (). 636 Constrains the Emergence of Graphic Codes. Cognitive Science, 43(3). doi: 637 $10.11/\omega_{8} s.12722$ 638 Zagala, (20/4). Vanuatu sand drawing. Museum International, 56(1-2), 32–35. doi: 639 J.1.1/j.1350-0775.2004.00455.x
- Zahari, A. (1975). Mate selection-a selection for a handicap. Journal of theoretical biology, 53(1), 205–214.

Zoli, C. (1999). Intersecting generalized Lorenz curves and the Gini index. Social Choice and Welfare, 16(2), 183-196. doi: 10.1007/s003550050139