Fermat et les débuts modernes de la géométrie
By Roshdi Rashed. Hildesheim (Olms). ISBN 978-3-487-15685-9. 2018. viii/320.

The book *Fermat et les débuts modernes de la géométrie* by Roshdi Rashed offers a reconstruction and a highly refined commentary on Fermat's geometrical works. It counts, at present, as the only volume dedicated to Fermat's geometry in its entirety. The book may be seen as the follow-up to previous works by Rashed dealing with Fermat's results in arithmetic and algebra.[1]

The volume is divided into five thematic chapters, arranged in roughly chronological order, which ex-plain, almost theorem by theorem, all of Fermat's results in geometry. The book is self-contained but takes the form of a commentary and the reader would be well advised to have to hand, when reading it, Fermat's own works, to which Rashed makes constant reference.[2]

The book begins (pp. 1–52) with a discussion of Fermat's early essays dedicated to the "restitution" of classical works. In these essays Fermat, following the example of Viète, Snell and Ghetaldi, attempted to reinvent some lost works of Euclid, Apollonius and Aristaeus the Elder, building on the fragments of these works mentioned in Pappus' *Collectiones*. We have therefore two books by Fermat on Apollonius' *loci plani* and one essay on the latter's *De contactibus sphaericis*; an introduction to Aristaeus' lost *loci solidi*; and fragments of a reconstruction of Euclid's *porismata* and *loci ad superficiem*. Rashed offers a masterful presentation of these difficult texts and guides the reader in the exploration of the complex relations between Pappus' original statements, the discussions of these works which had already occurred in the Islamic Middle Ages and the European Renaissance, and the other "restitutions" of the 17th century. A particular emphasis is given to Fermat's use of pointwise transformations of geometrical curves. This is a theme which Rashed has already discussed at length in his previous works, and which has produced a remarkable number of studies by the scholars working alongside him, who have identified the birth of the notion of a geometrical transformation in the Arabic reception of Apollonius and, in particular, in the works by al-Haytham and al-Sijzi.[3] In Fermat, claims Rashed, we see the fully developed outcome of this medieval revolution in mathematical practice.

The second chapter (pp. 53–86) discusses at length the application of algebra to geometry. Rashed takes up once again the discussion of the *Isagoge ad locos planos* and its *appendix* in order to show the role of algebraic constructions in these latter texts; he then goes on to deal with Fermat's more explicitly algebraic *Dissertatio tripartita* and with his method of elimination (the *Novus usus in analyticis*). The chapter offers a confrontation with Descartes' own algebraic geometry and shows the remarkable differences between the two approaches by also documenting the disputes between the two mathematicians. Rashed's bold historio-graphical claim (p. 76), based on his previous studies in the theory of transformations and the development

[1] Rashed, R., Houzel, C., Christol, G., 1999. Pierre Fermat. La théorie des nombres, Blanchard, Paris. Rashed, R., 2013. Histoire de l'analyse diophantienne classique: d'Abū Kāmil à Fermat. De Gruyter, Berlin.

[2] Tannery, P., Henry, C., De Waard, C., 1891–1922. Oeuvres de Fermat. Gauthier-Villars, Paris.

[3] See for instance Rashed, R., Bellosta, H., 2000. Ibrāhīm ibn Sinān. Logique et géométrie au X^{e} siècle. Brill, Leiden; Rashed, R., 2002. Les mathématiques infinitésimales du IX^{e} au XI^{e} siècle. Vol. 4. Al-Furquān, London; Crozet, P., 2010. De l'usage des transformations géométriques à la notion d'invariant: la contribution d'al-Sijzī. Arabic Sciences and Philosophy, 20, 53–91.

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of Diophantine analysis, is that the account usually given of the genesis of Fermat's algebraic geometry has to be revised. This latter cannot be considered, according to Rashed, to be a development of the Apollonian techniques put to work in Fermat's "restitutions" together with the innovations of Viète's algebra. Rather, it should be understood as the meeting of a threefold tradition: the theory of pointwise transformations, the rediscovery of Diophantus, and the influence of Descartes' *Géométrie*. Compared to previous interpretations,[4] then, Rashed's own reading greatly scales down the role of Viète, which is here understood to be only one of several elements of Fermat's interpretation of Diophantine analysis. An interpretation of this sort also draws out a quite specific aspect of the reception of Apollonius (i.e. the theory of transformations, which was not explicit in the Greek text), and insists, more than others have, on Descartes' contribution to Fermat's own understanding of geometry.

The third chapter (pp. 87–120) continues to trace out the disputes between Fermat and Descartes and delves into Fermat's "infinitesimal methods." The notion on which the discussion centers here is that of *adequality*, which Fermat borrowed from Diophantus and which has generated a substantial controversy among historians of mathematics. Rashed agrees that the word means "an approximate equality" (p. 89, *contra* Breger[5] and others), but seems not to believe that the term is susceptible of exact definition. It is rather applied by Fermat with different meanings and functions in different mathematical contexts (pp. 92–93). Rashed, therefore, offers plenty of examples of Fermat's equalities and adequalities, be it in number theory, the theory of tangents, or the variational problems. It should be mentioned that the chapter contains a highly remarkable digression (pp. 98–113) on the history of the notion of tangency, the interest of which extends beyond the main topic of the book and is developed as a stand-alone essay on the subject.

Variational problems are taken up again, and at greater length, in chapter four (pp. 121–200), dealing with Fermat's theory of *maxima* and *minima*. This is a long chapter, full of technical details. Rashed reconstructs dozens of Fermat's arguments bearing on the application of his celebrated theory to several different problems, from centers of gravity to the path of light rays. The chapter concludes (pp. 184–200) with an insightful sketch of the diffusion of Fermat's methods through Beaugrand's work, the controversy with Descartes, the reaction of Huygens, and other important episodes in the development of 17th-century mathematics.

The last chapter is even longer (pp. 201–298) and deals with the most difficult results obtained by Fer-mat, namely, his rectifications and quadratures. A great deal of the chapter is devoted, therefore, to a careful examination of the *Propositions* to Lalouvère and the *De linearum curvarum*, where Fermat provided the rectification of the cycloid, thus disproving a famous claim by Descartes regarding the latter's impossibility. Rashed also deals with Fermat's theory of areas, showing how Fermat rejected Cavalieri's method of indivisibles and attempted, instead, to develop a new foundational strategy making use of infinitesimal surfaces. Given the importance of the cycloid in these first analytic results and the many controversies that surrounded these new techniques, the chapter closes with a useful section on the history of the mathematical treatments of this curve in the 17th century (pp. 281–298).

The above list of topics, however, does not do justice to the richness of Rashed's book. In fact, its most striking aspect consists in the enormous quantity of references to the history of mathematics as a whole. These range from lengthy discussions of Greek authors and classical works through to accounts of early modern mathematicians or of more contemporary mathematical constructions. Not surprisingly, a large portion of the book is dedicated to the comparison of Fermat's results with those obtained in the Arabic world. The extent and depth of these comparisons are impressive and unmistakably signal the work of a master in the field of the history of mathematics. The present book, indeed, will be the sixty-second volume on the history of mathematics written by Rashed. The reader, therefore, should not expect an *a solo* by Fermat,

[4] E.g. Mahoney, M., 1973. The Mathematical Career of Pierre de Fermat. Princeton University Press, Princeton.

[5] Breger, H., 1994. The mysteries of Adaequare. A vindication of Fermat. Archive for History of Exact Sciences, 46, 193–219.

nor even a trio or quartet together with Viète, Roberval or Descartes, but rather a complex symphony in which every theorem resonates with thoughts and themes from Apollonius, Proclus, Abū Kāmil, Khayyām, Tūsi, Stevin, Carcavi, Saint-Vincent, Bézout, and many, many others. The frequent comparisons with other mathematical results or geometrical styles serve to throw light on Fermat's results, by pointing up their peculiarities and their limits. I think that this richness of references and connections is the strongest merit of the book, which offers us an opportunity to immerse ourselves in the history of mathematics as a whole and manages to frame with precision Fermat's role in the slow unfolding of this history. At the same time, the very detailed mathematical discussions of every theorem proven by Fermat excludes any risk of empty generality. The result is an in-depth study of several fundamental geometrical propositions from a point of view encompassing two-thousand years of mathematical theories.

At the same time, Rashed's book is likely to give rise to some discussion about methodology. Rashed's reconstructional approach, which he has also applied in his previous works, consists in presenting the mathematical results in modern fashion and making use of modern symbolism, notions and tools. In this respect, Rashed's presentation of Fermat's work, although expounded in a highly detailed way, proposition after proposition, does not resemble the original text very closely but appears rather as a rational reconstruction of this latter. Such modernization helps, of course, the comparison with other authors and with different periods and mathematical practices. The richness of Rashed's network of cross-references among differ-ent mathematicians could, indeed, hardly be achieved without altering the form of Fermat's mathematics. Modernization in the historiography of mathematics, however, has been strongly criticized in the last few decades (at least starting from a famous 1975 paper by Sabetai Unguru) and nowadays its detractors are more numerous than its advocates.[6] In opposition to these ideas, Rashed consciously consents to an em-brace of anachronism as a proper method of historical explanation. He freely employs integral calculus, trigonometric functions, homographies and other projective transformations as well as a series of modern theorems in order to explain the early modern results. Whereas Unguru strongly insisted that introducing modern algebra into a classical geometrical demonstration would disrupt our understanding of the original proof, since in historical reconstruction "the form is the content,"[7] Rashed reconstructs (for example) one of Fermat's demonstration and then openly claims that "*la démonstration de Fermat est équivalente au calcul precedent, mais sans l'utilisation de l'algèbre*" (p. 13).

Moreover, Rashed's numerous and enlightening comparisons between different mathematical traditions are often devoid of any actual historical connection. The greater part of the Arabic authors, for example, who are mobilized in the book so as to outline the development of one or another mathematical notion (such as the definition of a tangent or the idea of a geometrical transformation, etc.) were largely unknown in Europe in the 17th century and how they could have exerted an influence on Fermat, be it direct or in-direct, remains difficult to fathom. The connections highlighted in the book, therefore, are often decidedly ahistorical and serve really only to help us to understand some deep similarities between different math-ematical practices – similarities, however, which could surely not have been recognized by the historical actors themselves. In this respect, then, Rashed offers us more a *morphology* than a diachronic description of the development of mathematics. In other works, Rashed has insisted that this sort of morphological comparison helps our historical understanding of texts insofar as it shows what is really new in a mathe-matician's results. In the present work, indeed, this methodology prompts to several reconsiderations and reassessments regarding Fermat's results, and his contributions to the (ideal) development of mathematics.

[6] Unguru, S., 1975. On the need to rewrite the history of Greek mathematics. Archive for History of Exact Sciences, 15, 67–114. A recent essay documenting the wide acceptance of Unguru's thesis in the scholarly community is Sidoli, N., 2003 Research on ancient Greek mathematical sciences, 1998–2012. In: N. Sidoli, G. Van Brummelen (Eds.), From Alexandria, Through Baghdad: Surveys and Studies in the Ancient Greek and Medieval Islamic Mathematical Sciences in Honor of J.L. Berggren. Springer, Berlin, pp. 25–50.

[7] *Ibidem*, p. 111.

The reader of Rashed's monumental work on Fermat should be aware of the author's bold and uncompromising views on the epistemology of historiography. However, scholars coming from different methodological perspectives will surely not fail to recognize the importance of the present book, which offers a wealthy harvest of remarkable mathematical insights into the history of geometry in general and into the works of Fermat in particular.

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Kurt Gödel: *Collected Works*. Vol. IV: *Correspondence A–G*. Vol. V: *Correspondence H–Z*

Edited by Solomon Feferman, et al. Oxford (Clarendon Press). 2003. ISBN 978-0-19-968961-3. Paperback 2014. Vol. IV (pbk). xxii + 662 pp. + 12 ill. Vol. V: ISBN 978-0-19-968962-0 (pbk). xxvi + 664 pp. + 15 ill.

This review concerns the paperback edition of the magnificent selection of Kurt Gödel's scientific correspondence in his *Collected Works*. It includes almost 400 letters exchanged with fifty individuals mostly from the Gödel *Nachlass* which alone comprises 3500 of Gödel's letters. The edition covers the period from 1928 to 1977. Criterion for inclusion was "that letters should either possess intrinsic scientific, philo-sophical or historical interest or should illuminate Gödel's thoughts or his personal relationships with others" (CW IV, p. v).

This edition uses the groundbreaking editorial conception of the other volumes of the *Collected Works*. The letters are edited in their original language with facing English translations if their language is not English. The editorial apparatus for the letters is small, restricted basically to cross-references to other let-ters and bibliographic references. Each correspondence is opened by an introductory essay of varying length of, e.g., 11 pages (Wilfried Sieg on the correspondence with Jacques Herbrand), 17 pages (Charles Parsons on the correspondence with Hao Wang), or even 37 pages (Solomon Feferman on the correspondence with Paul Bernays). These introductions provide important biographical information about the correspondents and they connect the topics treated in the letters to their historical and systematic contexts.

Although the edition is carefully corrected, mistakes could not completely be avoided. A sentence, for example, in the shorthand draft of Gödel's presumably unsent answer to Bernays's letter of 7 September 1942 runs in its final version (respecting crossed out words and including an editor's conjecture): "Wegen des Ersetzungsaxioms müsste man sich auf den Fall beschränken, in dem Ordinalzahl [in dem die Sinne der Höhe der Iteration der Pot.mengenbildung] eine isolierte Zahl ist" (CW IV, p. 140). This sentence makes no sense in German. A "die" should be inserted before "Ordinalzahl" and the editor's conjecture to add "die" before "Sinne" should be withdrawn, resulting in "im Sinne", i.e. "in the sense of", correctly translated in the English version.

The letters treat a wide variety of topics. They refer to incidents in the history of modern logic and to discussions of Gödel's results. They document his intellectual development—for example, his growing interest in philosophical questions in his later years—and they highlight his influence on the network of people working on mathematical logic and foundations.

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