Chiral Exchange Drag and Chirality Oscillations in Synthetic Antiferromagnets

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Long-range interactions between quasiparticles give rise to a "drag" that affects the fundamental properties of many systems in condensed matter physics[1-11]. Drag typically involves the exchange of linear momentum between quasiparticles and strongly influences their transport properties. Here, we show a new form of drag that involves the exchange of angular momentum between two current-driven magnetic domain walls. The motions of the domain walls are correlated and determined by the strength of the drag. When the drag is below a threshold value, the domain walls move together at a constant intermediate velocity with a steady leakage of angular momentum from the faster to the slower. However, we find that when the exchange coupling between the domain walls is sufficiently weak, a different dynamic can take place in which the faster domain wall's magnetization oscillates synchronously with a precessional motion of the slower domain wall's magnetization, and angular momentum is continuously transferred between them. Our findings demonstrate a method for delivering spin angular momentum remotely to magnetic entities that

otherwise could not be manipulated directly by current, for example, by coupling domain walls or other non-collinear spin textures in metallic and insulating media.

Domain walls (DWs) in perpendicularly magnetized nanowires that have chiral Néel structures, stabilized by an interfacial Dyzaholshinsky-Moriya exchange interaction (DMI)[12,13], can be driven at high speeds along the nanowires by current via chiral spin orbit torques[14-19]. Coupled domain walls that are formed in nanowires composed of a synthetic antiferromagnet (SAF) move at even higher velocities when driven by current, due to a giant exchange torque[20]. Here we show that the collective response of such DWs is governed by the physics of "drag" in which the DW in one sub-layer of the SAF can drag the DW in the other sub-layer by exchange of angular momentum between the two antiferro magnetic (AF) exchange coupled sub-layers. When this drag exceeds a threshold value, the coupled DWs velocity drops precipitously to a very small value or even zero and leads to a synchronous precession and oscillation of the respective DWs at high frequencies. The physics of this "*chiral exchange drag* (CED)" shares some features of electrical transport in two-dimensional electron or hole gases coupled by Coulomb interactions[21,22], or magnon propagation in ferromagnetic layers coupled by long-range magnetic dipolar interactions[11], but angular momentum rather than linear momentum is at the heart of the phenomenon that we observe. The CED arises from angular momentum conservation via exchange interactions between two DWs that is the counterpart of linear momentum conservation via Coulomb interactions in Coulomb drag (see Methods).

The current induced DW motion was measured in nanowires patterned from films composed of a lower magnetic layer, LM, formed from t_{Co} (=1.5, 1.8, 2.1, and 2.5) Co |

7 Ni | 1.5 Co and an upper magnetic layer, UM, formed from 1.5 Co | 7 Ni | 1.5 Co, that are separated by a layer of 8 Ru (Fig. 1a). The Ru layer provides the AF coupling between LM and UM. All thicknesses are given in Å (here and throughout the paper). From polar magneto-optical Kerr hysteresis loop measurements of unpatterned SAF films, the ratios $\frac{m_U}{m_L}$ ($m_{L(U)}$: LM (UM) moment), that can be varied by varying t_{Co} were extracted (Fig. 1a). Kerr microscopy is used to image the DW motion along the 2 µm wide and 50 µm long nanowires in response to sequences of 5 ns long current pulses (see Fig. 1b and Methods). Typical results for the dependence of the coupled DWs velocity, ν, on current density, J, are shown in Fig. 1c. As $\frac{m_U}{m_L} \to 1$, the exchange coupling torque becomes larger thereby increasing ν for the same current density.

The response of DW velocity ν to an in-plane magnetic field oriented along the nanowire, H_x, has previously been shown to give valuable insight into the mechanisms by which chiral DWs are moved by current[14]. Plots of ν versus H_x (Fig. 1d) show modest decreases in ν for positive H_x but sharp decreases in ν at small negative H_x. This behavior depends sensitively on $\frac{M_U}{M_L}$ ($M_{L(U)}$: LM (UM) DW magnetization) and the magnitude of the AF exchange coupling J^{ex}. The latter can be varied simply by varying t_{Ru} slightly. The sharp drop in ν at small negative H_x disappears when t_{Ru} is reduced such that the AF exchange coupling becomes stronger ($t_{Ru} = 4$ Å, Fig. 1e) or when the coupling becomes ferromagnetic ($t_{Ru} = 2$ Å, Fig. 1f). We note that, by contrast, ν versus H_y curves do not show a dramatic drop in ν (Fig. 1g).

To understand these results we first discuss micromagnetic simulations with input parameters mostly determined from experiments (see Methods). The simulations were carried out on a $\uparrow_L\downarrow_L$ DW configuration in LM that lies between magnetic domains along x with magnetizations directed along $+z$ ($\uparrow_L$) and $-z$ ($\downarrow_L$). The simulated results can well account for our experimentally obtained v versus H_x and the dependence of these observations on J^{ex} (Fig 2a). Only when J^{ex} is sufficiently weak do we reproduce the sharp drop in v with $-H_x$. The input parameters that were used include a much larger DMI field in the LM layer than in UM due to the Pt/Co interface in the former and the Ru/Co interface in the latter (see Ref. 20 and Methods). This means that the DW dynamics are dominated by the DW moment in the LM layer. In this case the sign and magnitude of v are correlated with the orientation of $\vec{M}_L$. For example, when $\vec{M}_L$ is in the 3rd quadrant, considering that the DMI field in LM, H_L^{DM}, is along $-x$, the DW moves along $+x$, i.e., $v > 0$ (see Fig. 2d).

From the simulations we can determine the detailed dependence versus time of the DW's displacement along the nanowire, q. Exemplary data is shown in Fig. 2b for three values of H_x indicated by A, B and C in Fig. 2a. For positive values of H_x, $q(t)$ increases at a constant rate showing that the DW's velocity is constant. On the other hand, when H_x is negative and exceeds a critical value, the DW's velocity is reduced and is no longer constant. The DW's position now oscillates backwards and forwards as the DW moves along the wire. We discuss this further later.

Time resolved snapshots of the nanowire's magnetization in both layers corresponding to H_x at A, B and C reveal the detailed motion of the LM and UM DWs (Fig. 2d-f). In case A both DWs move together at a steady state velocity of $v\sim300\ m/s$ (Fig. 2d). In case B $\vec{M}_L$ oscillates between the 3rd and 2nd quadrants synchronously with the precession of $\vec{M}_U$ (Fig. 2e) and, thus, the DW oscillates backwards and forwards resulting in the sharp drop in ν. We find that there exists a transition region between A and B with an even more complex DW dynamical behavior (Fig. 2f). As before, $\vec{M}_L$ oscillates between the 2nd and 3rd quadrants while, in this case, $\vec{M}_U$ both precesses and oscillates (Fig. 2f). Since this precession/oscillation period is larger than in region B, the average DW velocity is larger here (see Fig. 2b).

We now introduce a 1D analytical model (see SI Section 1) to gain further insight. Note that in a SAF wire the DWs in LM and UM move at the same velocity since their positions are locked to each other although they are subject to different torques. Here we dub these coupled DWs a *composite-domain-wall* (c-DW). We also introduce fictitious DWs, *quasi-domain-walls* (q-DWs) in each sublayer that are postulated to move freely at different velocities with no position-locking (see Methods for details). In our analytical model q-DWs and c-DW are described by (Q_i, Ψ_i) and (q, ψ_i), respectively, where Q_i, Ψ_i are the positions and azimuthal angles of the q-DWs, while q, and ψ_i are the position and azimuthal angles of the c-DW as shown in Fig. 3a ($i = L$ and U).

The c-DW is related to the q-DWs as follows (see Methods):

$$\dot{q} = \frac{M_L \dot{Q}_L + M_U \dot{Q}_U}{M_L + M_U},$$

$$\dot{\psi}_L = \dot{\Psi}_L - \frac{1}{\alpha_L \Delta_L} \frac{M_U}{M_L + M_U} \left(\dot{Q}_L - \dot{Q}_U \right),$$

$$\dot{\psi}_U = \dot{\Psi}_U - \frac{1}{\alpha_U \Delta_U} \frac{M_L}{M_L + M_U} \left(\dot{Q}_L - \dot{Q}_U \right).$$

(1a,b,c)

Here α_i and Δ_i are the Gilbert damping and the DW width parameters, respectively. Thus, from eqs. (1), the c-DW velocity $\dot{q}$ is the average of the q-DWs' velocities $\dot{Q}_i$ weighted by M_i while the c-DW azimuthal velocities $\dot{\psi}_i$ are the q-DWs' azimuthal velocities $\dot{\Psi}_i$ modified by the term $-\frac{1}{\alpha_i\Delta_i}\frac{M_j}{M_L+M_U}(\dot{Q}_L-\dot{Q}_U) \equiv -\zeta_i\mathcal{D}$ where $\mathcal{D} \equiv \dot{Q}_L - \dot{Q}_U$, $\zeta_i \equiv \frac{1}{\alpha_i\Delta_i}\frac{M_j}{M_L+M_U}$ and $(i,j) = (L,U)$ or (U,L). The terms, $-\zeta_i\mathcal{D}$, describe the chiral exchange drag (CED) phenomenon. Here $\mathcal{D}$ and ζ_i correspond to the CED strength (i.e. the q-DW velocity disparity) and the angular momentum transfer coefficient, respectively. Consequently, when $\mathcal{D}$ becomes sufficiently large, exceeding a threshold value, $\mathcal{D}^{th}$, then $-\zeta_i\mathcal{D}$ becomes so large that the steady state condition $\dot{\psi}_i = 0$ cannot be satisfied (since $\dot{\Psi}_i$ are mathematically compact in eqs. (1b,c)). In this mode the magnetizations of both DWs precess or oscillate thus leading to significant drops in ν (see Supplementary Fig. 3c and Supplementary Movies 1c and 1d). We describe this dynamical motion as the *CED anomaly* for which eqs. (1b,c) can more simply understood by rewriting them as follows:

$$\Pi_i = -\zeta_i\mathcal{D} \tag{2}$$

where $\Pi_i \equiv \dot{\psi}_i - \dot{\Psi}_i$, which shows that $\mathcal{D}$ generates angular momentum transfer torques that result in rotational motions of the azimuthal angles, ψ_i. However, when $\mathcal{D}$ is small, the steady-state c-DW velocity $\dot{q}$ is given by the average of the q-DW velocities, i.e., $\dot{q} = \left(\frac{M_L\dot{Q}_L+M_U\dot{Q}_U}{M_L+M_U}\right)_{steady\ state}$, that results from torques given by the transfer of angular momentum from the faster q-DW to the slower one (see Supplementary Fig. 3b and Supplementary Movies 1a,b).

For the case when $M_L > M_U$, $\dot{\Psi}_U$ deviates away from $\zeta_U \mathcal{D}$ more than $\dot{\Psi}_L$ does from $\zeta_L \mathcal{D}$. Consequently, in the CED anomaly it is found from eq. (2) that there exists no values of ψ_i for which $\dot{\psi}_U = 0$ while there are values of ψ_i for which $\dot{\psi}_L$ can go through zero due to a large $\mathcal{D}$. This, therefore, leads to the continual precession of ψ_U, but an oscillatory variation of ψ_L (Supplementary Fig. 3c and Supplementary Movie 1d). An important point is that the precessional motion of ψ_U always results in a periodic change in its chirality, but the oscillatory behavior of ψ_L, in all cases that we have considered, does not. Note that the precession/oscillation frequency f_D increases as $\mathcal{D}$ is increased (see Fig. 3e).

As discussed earlier the application of H_x is a simple way to tune $\mathcal{D}$ for chiral DWs in a SAF (see Fig. 1d). When $-J^{ex}$ is small, the chiral nature of the q-DWs gives rise to a nearly linear variation of $\dot{Q}_i$ ($i = L$ or U) but with different slopes and crossing fields ($\dot{Q}_i = 0$) due to the different spin Hall torques and DMI fields, H_i^{DM} (see Fig. 3b)[14]. Most importantly in a SAF wire with $\uparrow_L \leftarrow_L \downarrow_L$ and $\downarrow_U \rightarrow_U \uparrow_U$ DW configurations in LM and UM respectively, H_x always results in an increase in the velocity of one q-DW and a decrease in the other q-DW, thereby increasing $\mathcal{D}$, as shown in Fig. 3b. Note that as shown in Fig. 3b the average DW velocity increases monotonically with increasing $-H_x$ (orange dashed curve in Fig. 3b). In contrast, when two q-DWs are bound and $\mathcal{D} \geq \mathcal{D}^{th}$, the CED transfers angular momentum from LM q-DW to UM q-DW via exchange coupling giving rise to the CED anomaly and consequently leading to the sharp reduction in v (black line with open circles in Fig. 3b).

It is found that there are distinct regions within the CED anomaly (red and yellow regions in Fig. 3b) that correspond to strong ($\mathcal{D} \geq \mathcal{D}^{th2}$) and weak ($\mathcal{D}^{th1} \leq \mathcal{D} < \mathcal{D}^{th2}$) CED anomalies, respectively, as is also clearly seen in the micromagnetic simulations: B (Fig. 2e) and C (Fig. 2f). First, when $-H_x \geq -H_x^{th2}$, then $\mathcal{D} \geq \mathcal{D}^{th2}$, and it is always the case that $\dot{\Psi}_U < \zeta_U \mathcal{D}$ (strong CED anomaly). Then, following eq. 1b, $\dot{\psi}_U$ will always be negative so that $\vec{M}_U$ precesses in a clockwise direction (blue curves in Fig. 3f where $H_x = -2\ kOe$) while $\dot{\psi}_L$ periodically becomes zero at which points in time $\vec{M}_L$ changes direction of rotation and thus oscillates (blue curves in Fig. 3d and Supplementary Movie 1d). On the other hand, when $-H_x$ is decreased so that $-H_x^{th2} > -H_x \geq -H_x^{th1}$, in a transition region (weak CED anomaly), then $\mathcal{D}$ is reduced, thereby allowing both $\dot{\psi}_U$ and $\dot{\psi}_L$ to be zero but not simultaneously. Consequently, there still exists no steady state DW motion but rather oscillation of $\vec{M}_L$ and precession/oscillation of $\vec{M}_U$ (cyan curves Fig. 3d,f and Supplementary Movie 1c), which is distinct from the strong CED anomaly region discussed above in which $\vec{M}_L$ oscillates while $\vec{M}_U$ precesses only.

Let us investigate how the CED anomaly is influenced by key parameters. First, the CED increases with J since $\dot{Q}_i$ are proportional to J which means that $\mathcal{D}$ is also proportional to J. Consequently, when $J \geq J^{th}$, the CED anomaly turns on, ν decreases dramatically (Fig. 4a), and f_D is almost linearly proportional to J (Fig. 4b). Next as shown in Fig. 3b, an increase in $-H_L^{DM}$ enhances $\dot{Q}_L$ but does not change $\dot{Q}_U$, thereby increasing $\mathcal{D}$. As a result, when $-H_L^{DM} > -H_L^{DM\ th}$, ν sharply decreases (Fig. 4c) and f_D becomes non-zero (Fig. 4d).

With regard to the dependence of the CED on J^{ex}, the CED anomaly is not observed in an SF wire ($J^{ex} > 0$) since H_x causes much smaller changes in $\mathcal{D}$ so that the threshold drag to observe a CED anomaly cannot be achieved (see Fig. 4e,f and Fig. 1f for experiment and Supplementary Figure 6a). Returning to SAF structures, note that the CED anomaly does not monotonically evolve with increasing $-J^{ex}$. f_D vanishes at $J^{ex} \sim -0.2\, erg\, cm^{-2}$. This is because, as $-J^{ex}$ increases, firstly, the DW magnetizations become tightly locked and secondly the exchange coupling torque becomes dominant and the scheme described in Fig. 3b is no longer valid (Fig. 1e).

Lastly, as M_L increases, $\zeta_U \mathcal{D}$ increases since ζ_U increases with M_L. Consequently, ν is dramatically reduced at a threshold value, $\left(\frac{M_R}{M_S}\right)^{th}$, and f_D almost linearly increases with $\frac{M_R}{M_S}$ at $\frac{M_R}{M_S} > \left(\frac{M_R}{M_S}\right)^{th}$ (Fig. 4g,h). The precession of the DW caused by the CED anomaly results in chirality oscillations in M_U. Similar to Dirac fermions, the chirality oscillates when M_R is away from zero while the chirality is preserved when $M_R \sim 0$ (Fig. 4h). Note that the nearly linear variation of f_D with M_R is analogous to massive Dirac fermions (Fig. 4h).

The chiral exchange drag observed here is likely also to play a role in more complex non-collinear spin textures such as skyrmions and skyrmion lattices, where one can anticipate even more complex dynamical behaviors derived from the transfer of spin angular momentum between remote objects using local or non-local currents.

Methods

Sample preparation

SAF films were grown using magnetron sputtering on ~250 Å thick thermally oxidized SiO_2 layer from Si(100) wafer. A seed layer of 100 Al_2O_3 | 20 TaN was deposited by reactive magnetron sputtering of Al_2O_3 using an 93% Ar-3% O_2 atomic mixture followed by a layer of TaN formed by reactive sputtering of Ta in an 90% Ar-10% N_2 mixture. The lower and upper magnetic layers consist of t_{Co} Co | 7 Ni | 1.5 Co and 1.5 Co | 7 Ni | 1.5 Co, respectively, that are separated by an 8 Ru spacer layer. In all cases, a 50 TaN capping layer was grown on top of the film stack to protect the film against oxidation. Using photolithographic techniques and Ar-ion milling, nanowire mesas were etched from the blanket films and their sidewalls were protected by immediate deposition of an Al_2O_3 layer of the same thickness as the film thickness on the etched region.

Polar Magneto-optical Kerr effect and Kerr microscopy

Polar magneto-optical Kerr effect is used to characterize the magnetic properties of grown films that are perpendicularly magnetized. By sweeping the applied field along the easy axis, the strength of magnetic anisotropy can be estimated. The exchange coupling strength in SAF can be evaluated by investigating the magnetic hysteresis loops since the shape of magnetic hysteresis loops are sensitive to the exchange coupling strength and anisotropy.

Kerr microscopy measurements to record DW motion are carried out in differential mode using a xenon lamp source, thus recording the change in nanowire magnetization after application of a sequence of current pulses. Changes in the magnetization state appear as dark or bright regions depending on whether the magnetization in those regions is increased or decreased, i.e., an upward pointing region which becomes displaced by a downward pointing region appears as bright and correspondingly, a downward pointing region which becomes displaced by an upward point region appears as a dark region.

Angular momentum conservation via exchange interaction

Let us consider the magnetization dynamics of two DW moments constrained to the two sublayers in a SAF that are exchange coupled via an exchange interaction energy, $\sigma = -J^{ex}\hat{M}_L \cdot \hat{M}_U = -J^{ex}\cos(\psi_L - \psi_U)$, where M_i, J^{ex}, and ψ_i are, respectively, the DW sub-layer moments per unit area, the exchange coupling constant, and the DW magnetization angles for the lower magnetic, LM ($i = L$), and upper magnetic, UM ($i = U$), layers. Then the exchange fields acting on each DW are given by $\vec{H}_L^{ex} = -\frac{\partial\sigma}{\partial\vec{M}_L} = -\frac{J^{ex}}{M_L}\sin(\psi_L - \psi_U)\,\hat{\psi}_L$ and $\vec{H}_U^{ex} = -\frac{\partial\sigma}{\partial\vec{M}_U} = \frac{J^{ex}}{M_U}\sin(\psi_L - \psi_U)\,\hat{\psi}_U$, where $\hat{\psi}_L$ and $\hat{\psi}_U$ are dynamical unit vectors that are orthogonal to $\hat{M}_L$ and $\hat{M}_U$, respectively, and lie within the plane defined by $\hat{M}_L$ and $\hat{M}_U$ ($\hat{M}_i = \vec{M}_i/M_i$). Consequently, $\frac{d\vec{M}_L}{dt} = -\gamma\vec{M}_L \times \vec{H}_L^{ex} = \gamma J^{ex}\sin(\psi_L - \psi_U)\,\hat{M}_L \times \hat{\psi}_L$ and $\frac{d\vec{M}_U}{dt} = -\gamma\vec{M}_U \times \vec{H}_U^{ex} = -\gamma J^{ex}\sin(\psi_L - \psi_U)\,\hat{M}_U \times \hat{\psi}_U$. Since $\hat{M}_L \times \hat{\psi}_L = \hat{M}_U \times \hat{\psi}_U = \hat{n}$ that is a unit vector normal to the $\hat{M}_L$ and $\hat{M}_U$ plane, we obtain $\frac{d}{dt}\left(\vec{M}_L + \vec{M}_U\right) = 0$, i.e., the total angular momentum is conserved. When the DW moment in one layer changes its direction due to its exchange interaction with the DW moment in the other layer, the exchange field itself is changed, thereby reciprocally affecting the DW moment in the first layer. Consequently, we find that $\frac{d\vec{M}_L}{dt} = -\frac{d\vec{M}_U}{dt}$ thus showing that the angular momentum of one DW moment is transferred to the corresponding DW moment in the other layer and, thereby, generating an angular momentum transfer torque via the AF exchange interaction.

Micromagnetic simulations of chiral exchange drag in SAF nanowires

Micromagnetic simulations are an important tool to understand magnetic domains and domain walls. The magnetization is described by a continuous function, a classical vector that represents an average over a small volume of the magnetization density[23]. Micromagnetic simulations performed in this study were carried out using the LLG micromagnetics simulator[24]. Most parameters used as inputs were obtained from experimental measurements of the sample 15 Pt | 1.8 Co | 7 Ni | 1.5 Co | 8 Ru | 1.5 Co | 7 Ni | 1.5 Co (red line and symbols in Fig. 1d,g) whereas a few are from past studies[14,20,25]. All simulations are performed with a current density J of 1.1 × 10^8 A/cm^2. $J's$ were set to be uniform in all layers since it is a good approximation to consider the current to flow uniformly along each layer for the following reasons: (1) the measured resistivities of 500 Å thick films of Pt (21 μΩ-cm), Co (14 μΩ-cm), Ni (12 μΩ-cm) and Ru (21 μΩ-cm) are within a factor of two of each other, (2) each layer thickness is comparable with or smaller than the electron mean free path, and (3) the electron reflection and transmission coefficients at each interface are not known. M_L ($630\, emu\, cm^{-3}$) and M_U ($390\, emu\, cm^{-3}$) were determined from superconducting quantum interference device-vibrating sample magnetometer (SQUID-VSM) measurements of the SAF film (Supplementary Fig. 4). The disparity in the values of $M_{L,}$ and M_U, despite their similar thicknesses, is largely the result of the proximity induced moment of the lower magnetic layer due to the Pt underlayer as well as some interdiffusion into the upper magnetic layer from the TaN capping[25]. Value of $\theta_L^{SH} = 0.073$ is based on harmonic Hall measurements[26] of the stack 15 Pt | 3 Co | 7 Ni | 1.5 Co | TaN structure (to be published elsewhere). A previous study has reported a similar number for a Pt/Co bilayer system[27]. The spin Hall torque on the UM layer is mostly due to the spin current that is generated and attenuated after traveling across the LM layer and Ru layer[20] with an attenuation factor $\exp\left(-\frac{t_L}{\lambda_L}-\frac{t_{Ru}}{\lambda_{Ru}}\right)$ since the spin Hall effect in Ru is small[28]. Here λ_L and λ_{Ru} correspond to the spin decoherence length of LM layer ($\sim 1\ nm$)[29] and the spin diffusion length of Ru ($\sim 4\ nm$)[30], respectively.

The DMI constant for the LM layer, $D_L = 0.43\ erg\ cm^{-2}$, was determined by measuring H_x dependent measurements of v on a sample with a film stack identical to the SAF film but without the top magnetic layer (100 Al_2O_3 | 20 TaN | 15 Pt | 1.8 Co | 7 Ni | 1.5 Co | 8 Ru | 50 TaN) from which $H_L^{DM} = 1.02\ kOe$ is obtained (see Supplementary Fig. 4b). Note that the application of an H_x equal in magnitude and opposite to the internal DMI field at which DW stops moving leads to a transition from a Néel to a Bloch DW configuration[14]. The DMI constant for the UM layer, $D_U = 0.1\ erg\ cm^{-2}$, was used from $H_U^{DM} = 0.3\ kOe$ obtained in our earlier study[20].

The anisotropy constant for LM, $K_L = 4.33 \times 10^6\ erg\ cm^{-3}$, is based on the anisotropy field $H_L^K = 7\ kOe$ measured by VSM, while the anisotropy field for UM, H_U^K, is assumed to be same as H_L^K, i.e., $H_U^K = 7\ kOe$. The exchange stiffnesses $A_{L,U} = 1 \times 10^{-6}\ erg\ cm^{-1}$ are obtained from the weighted average of those for Co and Ni (cf. $A_{Co} = 1.8 \times 10^{-6} erg\ cm^{-1}$ and $A_{Ni} = 0.8 \times 10^{-6} erg\ cm^{-1}$). The Gilbert damping parameters, $\alpha_{L,U} = 0.1$, are taken from our earlier studies which are consistent with experimental determinations from time-resolved MOKE measurements on the same system[31]. Since the conventional volume STT contributions are very small and negligible, compared to spin-orbit torques from our past studies[14], we set them to zero.

The RKKY exchange coupling constant, J_{ex} is set to be $-0.15\ erg\ cm^{-2}$ which is similar to our previous study[20]. Based on our modelling results, there is a narrow window of J_{ex} in which the CED anomaly is observed (see Fig. 2a and Fig. 4e,f). For $-J^{ex} < -0.05\ erg\ cm^{-2}$, the DWs become decoupled during motion[32]. For $-J^{ex} > -0.2\ erg\ cm^{-2}$, the CED anomaly is found to be absent. The value of J_{ex} thus chosen is likely close to the actual value of the system. Layer thicknesses used in the micromagnetic simulations are 1.5 nm Pt, 1 nm LM, 0.8 nm Ru and 1 nm UM. Unless stated, the simulations were carried out on 10 nm wide wires. The SAF wire width dependence of the CED in micromagnetic simulations is discussed in Supplementary Section 3 (see also Supplementary Fig. 5).

Quasi-domain-walls (q-DWs) and composite-domain-walls (c-DWs) in synthetic antiferromagnets

In an analytical model for a single isolated magnetic sub-layer, two parameters, the DW position q and the azimuthal magnetization angle ψ, fully describe the DW dynamics that result from spin-orbit torques (and spin transfer and field-driven torques). ψ is the in-plane rotation of the magnetization of the wire about the perpendicular direction to the layer plane, as shown in Fig. 3a. In the absence of any pinning potential or any out-of-plane magnetic field, the equations of motion can be written as follows,

$$\dot{q} = \frac{1}{M_s} f_{\pm}(\psi)$$

$$\dot{\psi} = \frac{\alpha}{M_s \Delta} g_{\pm}(\psi)$$

(3a,b)

where $\alpha,\ M_s$ and Δ are the Gilbert damping, magnetization and DW width parameter, respectively. The functions $\left(f_+(\psi), g_+(\psi)\right)$ and $\left(f_-(\psi), g_-(\psi)\right)$ correspond to ↑←↓ and ↓→↑ DW configurations, respectively, and they are

$$\begin{aligned} f_{\pm}(\psi) = \frac{M_s}{1+\alpha^2}\Big[&-(1+\alpha\beta)u \\ &\mp \gamma\Delta\left(\frac{H_k}{2}\sin 2\psi - \frac{\pi}{2}H_x \sin\psi + \frac{\pi}{2}H_y\cos\psi - \frac{\pi}{2}H_{DM}\sin\psi\right) \\ &\mp \frac{\gamma\pi}{2}\alpha\Delta H_{SH}\cos\psi\Big], \end{aligned}$$

$$\begin{aligned} g_{\pm}(\psi) = \frac{M_s}{1+\alpha^2}\Big[&\mp\frac{\beta-\alpha}{\alpha}u \\ &+ \gamma\Delta\left(\frac{H_k}{2}\sin 2\psi - \frac{\pi}{2}H_x \sin\psi + \frac{\pi}{2}H_y\cos\psi - \frac{\pi}{2}H_{DM}\sin\psi\right) \\ &- \frac{\gamma\pi}{2\alpha}\Delta H_{SH}\cos\psi\Big] \end{aligned}$$

where β is the non-adiabatic spin-transfer torque parameter, H_k is the DW shape anisotropy field, H_y is the external field applied along transverse direction to the wire, and H_{SH} is the parametrized effect field that represents the spin Hall effect induced spin current. Note that $\left(f_{\pm}(\psi), g_{\pm}(\psi)\right)$ are closed and bounded, i.e., compact with respect to ψ since they are simple combinations of constants, and sine and cosine functions. Consequently, $(\dot{q}, \dot{\psi})$ are compact. The chiral structure of the DWs means that the magnetization rotates from ↑ (up) to ↓ (down) or ↓ to ↑ in a plane perpendicular to the domain wall with an in-plane moment in the middle of the DW that is aligned along ←

and →, respectively (or → and ← depending on the chirality set by the DMI). Note that the functions $f_{\pm}(\psi)$ and $g_{\pm}(\psi)$ implicitly include all the torques that drive DWs. Eqs. (3) show that the DW dynamics are determined only by ψ, an independent variable. In steady state conditions, $\dot{\psi} = 0$, corresponds to $g_{\pm}(\psi) = 0$. Thus, there may be values of $\psi = \{\psi_{zero}\}$, for which $g_{\pm}(\psi) = 0$, and, consequently, $\dot{q}$ is constant, which corresponds to steady-state motion of the DW. In this case as $|\dot{\psi}|$ decreases towards zero, $(\dot{q}, \dot{\psi})$ approaches $\left(\frac{1}{M_s} f_{\pm}(\psi_{zero}), \frac{\alpha}{M_s\Delta} g_{\pm}(\psi_{zero}) = 0\right)$ (see Supplementary Fig. 3a). On the other hand, when there exists no value of ψ that allows for $g_{\pm}(\psi) = 0$, q oscillates and ψ precesses, since $\dot{\psi}$ is always either positive or negative while $\dot{q}$ keeps on changing its sign.

Now we extend the analytical model to the SAF case with ↑→↓ and ↓←↑ DW configurations for lower (LM) and upper sub-layer (UM) magnetic configurations, respectively (Fig. 3a). For the special case when $\alpha_L \cong \alpha_U$ and $\Delta_L \cong \Delta_U$, the following eqs. of motion can be obtained:

$$\dot{q} = \frac{1}{M_L + M_U} F(\psi_L, \psi_U)$$

$$\dot{\psi}_L = \frac{\alpha_L}{M_L\Delta_L} G_L(\psi_L, \psi_U)$$

$$\dot{\psi}_U = \frac{\alpha_U}{M_U\Delta_U} G_U(\psi_L, \psi_U)$$

(4a,b,c)

where q, ψ_i, M_i, α_i and Δ_i are the DW position, azimuthal magnetization angles, sub-layer moments per unit area, Gilbert damping, and the DW width parameters for LM

($i = L$) and UM ($i = U$) sub-layers, respectively. The functions $F(\psi_L, \psi_U)$ and $G_i(\psi_L, \psi_U)$ are

$$
\begin{aligned}
F(\psi_L, \psi_U) = \frac{1}{1+\alpha_L^2} \Bigg\{ & M_L \Bigg[-(1+\alpha_L\beta_L)u_L \\
& + \gamma\Delta_L \left(\frac{H_L^k}{2}\sin 2\psi_L - \frac{\pi}{2}H_x \sin\psi_L + \frac{\pi}{2}H_y\cos\psi_L - \frac{\pi}{2}H_L^{DM}\sin\psi_L \right. \\
& \left. + \frac{\Delta_U}{\Delta_L}\frac{\xi J^{ex}}{M_L}\sin(\psi_L - \psi_U) \right) - \frac{\gamma\pi}{2}\alpha_L\Delta_L H_L^{SH}\cos\psi_L \Bigg] \\
& + M_U \Bigg[-(1+\alpha_U\beta_U)u_U \\
& - \gamma\Delta_U \left(\frac{H_U^k}{2}\sin 2\psi_L - \frac{\pi}{2}H_x\sin\psi_L + \frac{\pi}{2}H_y\cos\psi_L - \frac{\pi}{2}H_U^{DM}\sin\psi_L \right. \\
& \left. - \frac{\xi J^{ex}}{M_L}\sin(\psi_L - \psi_U) \right) + \frac{\gamma\pi}{2}\alpha_U\Delta_U H_U^{SH}\cos\psi_U \Bigg] \Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
G_i(\psi_L,\psi_U) = \frac{M_i}{1+\alpha_i^2}\Bigg[&\mp\frac{\beta_i-\alpha_i}{\alpha_i}u_i \\
&+\gamma\Delta_i\left(\frac{H_L^k}{2}\sin 2\psi_i-\frac{\pi}{2}H_x\sin\psi_i+\frac{\pi}{2}H_y\cos\psi_i-\frac{\pi}{2}H_i^{DM}\sin\psi_i\right. \\
&\left.+\frac{\xi J^{ex}}{M_i}\sin(\psi_L-\psi_U)\right)-\frac{\gamma\pi}{2\alpha_i}\Delta_i H_i^{SH}\cos\psi_i\Bigg] \\
&-\frac{M_j}{M_L+M_U}\Bigg\{\frac{1}{1+\alpha_L^2}\Bigg[-(1+\alpha_L\beta_L)u_L \\
&-\gamma\Delta_L\left(\frac{H_L^k}{2}\sin 2\psi_L-\frac{\pi}{2}H_x\sin\psi_L+\frac{\pi}{2}H_y\cos\psi_L-\frac{\pi}{2}H_L^{DM}\sin\psi_L\right. \\
&\left.+\frac{\Delta_U}{\Delta_L}\frac{\xi J^{ex}}{M_L}\sin(\psi_L-\psi_U)\right)-\frac{\gamma\pi}{2}\alpha_L\Delta_L H_L^{SH}\cos\psi_L\Bigg] \\
&-\frac{1}{1+\alpha_U^2}\Bigg[-(1+\alpha_U\beta_U)u_U \\
&+\gamma\Delta_U\left(\frac{H_L^k}{2}\sin 2\psi_U-\frac{\pi}{2}H_x\sin\psi_U+\frac{\pi}{2}H_y\cos\psi_U-\frac{\pi}{2}H_U^{DM}\sin\psi_U\right. \\
&\left.-\frac{\xi J^{ex}}{M_U}\sin(\psi_L-\psi_U)\right)+\frac{\gamma\pi}{2}\alpha_U\Delta_U H_U^{SH}\cos\psi_U\Bigg]\Bigg\}
\end{aligned}
$$

where $(i,j)=(L,U)$ or (U,L), $\xi=\int_{-\infty}^{\infty}\operatorname{sech}x\operatorname{sech}\left(\frac{\Delta_U}{\Delta_L}x\right)dx$. Note that one dependent variable q and two independent variables ψ_i define the position and the magnetization angles of the sub-layers in a c-DW. ψ_L and ψ_U thus determine the DW dynamics in a SAF just as ψ does for a single sub-layer. Most importantly, we find that eqs. (4a,b,c) can be rewritten as,

$$F(\psi_L,\psi_U)=f_+^*(\psi_L)+f_-^*(\psi_U)$$

$$G_L(\psi_L,\psi_U)=g_+^*(\psi_L)-\frac{M_U}{M_L+M_U}\left[\frac{1}{M_L}f_+^*(\psi_L)-\frac{1}{M_U}f_-^*(\psi_U)\right]$$

$$G_U(\psi_L, \psi_U) = g_-^*(\psi_U) - \frac{M_L}{M_L + M_U}\left[\frac{1}{M_L} f_+^*(\psi_L) - \frac{1}{M_U} f_-^*(\psi_U)\right]$$

It should be noted that $f_\pm^*$ and $g_\pm^*$ for the SAF case have identical forms to $f_\pm$ and $g_\pm$, respectively, for the single sub-layer case, except for an additional term, that we will name the exchange coupling torque cross-term, $h(\psi_L - \psi_U)$, as follows:

$$f_+^*(\psi_L) = f_+(\psi_L) - h(\psi_L - \psi_U)$$

$$f_-^*(\psi_U) = f_-(\psi_U) - h(\psi_L - \psi_U)$$

$$g_+^*(\psi_L) = g_+(\psi_L) + h(\psi_L - \psi_U)$$

$$g_-^*(\psi_U) = g_-(\psi_U) - h(\psi_L - \psi_U)$$

where $h(y) = \frac{1}{1+\alpha_L^2} \gamma \Delta_U J^{ex} \sin y \int_{-\infty}^{\infty} \operatorname{sech} x \operatorname{sech}\left(\frac{\Delta_U}{\Delta_L} x\right) dx$, γ is the gyromagnetic ratio, and J^{ex} is the exchange coupling constant ($J^{ex} < 0$ for SAF). We can rewrite $f_\pm^*$ and $g_\pm^*$ just as for the single sub-layer in eqs. (3) as follows,

$$\dot{Q}_L \equiv \frac{1}{M_L} f_+^*(\psi_L) = \frac{1}{M_L^{Qeff}} f_+(\psi_L)$$

$$\dot{Q}_U \equiv \frac{1}{M_U} f_-^*(\psi_U) = \frac{1}{M_U^{Qeff}} f_-(\psi_U)$$

$$\dot{\Psi}_L \equiv \frac{\alpha_L}{M_L \Delta_L} g_+^*(\psi_L) = \frac{\alpha_L}{M_L^{\Psi eff} \Delta_L} g_+(\psi_L)$$

$$\dot{\Psi}_U \equiv \frac{\alpha_U}{M_U \Delta_U} g_-^*(\psi_L) = \frac{\alpha_U}{M_U^{\Psi eff} \Delta_U} g_-(\psi_U)$$

(5a,b,c,d)

where

$$
\begin{aligned}
M_L^{Qeff} &\equiv M_L \frac{f_+(\psi_L)}{f_+(\psi_L) - h(\psi_L - \psi_U)} \\
M_U^{Qeff} &\equiv M_U \frac{f_-(\psi_U)}{f_-(\psi_U) - h(\psi_L - \psi_U)} \\
M_L^{\Psi eff} &\equiv M_L \frac{g_+(\psi_L)}{g_+(\psi_L) + h(\psi_L - \psi_U)} \\
M_U^{\Psi eff} &\equiv M_U \frac{g_-(\psi_U)}{g_-(\psi_U) - h(\psi_L - \psi_U)}
\end{aligned}
\tag{6a,b,c,d}
$$

By comparing eqs. (5) and (6) with eqs. (3) we can see that (Q_i, Ψ_i) represent the dynamics of fictitious DWs that move as if they were independent DWs in a single sub-layer with *effective magnetizations* $\left(M_i^{Qeff}, M_i^{\Psi eff}\right)$ that differ from uncoupled DW magnetizations (M_i, M_i) by the renormalization factors $\left(\frac{f_\pm}{f_\pm - h}, \frac{g_\pm}{g_\pm \pm h}\right)$, respectively. These “dressed” magnetizations have analogues to the motion of dressed quasi-particles (QPs) by electric field in semiconductors or correlated electron systems whose properties can be described by an effective mass[33]. We will call these fictitious DWs *quasi-domain-walls* (*q-DWs*). Note that the q-DWs have anisotropic effective magnetizations since $M_i^{Qeff} \neq M_i^{\Psi eff}$, thus, implying that the q-DWs feel different effective magnetizations for polar rotations (i.e., DW displacements that correspond to Q_i) and azimuthal rotations (Ψ_i). $\left(M_i^{Qeff}, M_i^{\Psi eff}\right)$ vary dynamically depending on the value of $\psi_L - \psi_U$. The anisotropic effective magnetizations are characteristic nature of q-DWs and are analogous to anisotropic effective masses of quasi-particles that travel in e.g. layered or tetragonal crystals[33]. Note that $\left(\dot{Q}_i, \dot{\Psi}_i\right)$ are compact just as are $\left(\dot{q}, \dot{\psi}\right)$.

Let us consider the case of SAF nanowires with positive spin Hall angles and counter-clockwise chirality. Then $f_\pm > 0$ and $h < 0$ which results in $M_i^{Qeff} < M_i$. This implies that exchange coupling torque (ECT)[20] moves q-DWs in sub-layers of an SAF faster than does the chiral spin–orbit torque[14] alone in a single layer. It is obvious that $\left(M_i^{Qeff}, M_i^{\Psi eff}\right) = (M_i, M_i)$ when $h(\psi_L - \psi_U) = 0$, i.e., no dressing of the magnetizations.

Data availability

The data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request

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Author contributions

S.H.Y. and S.S.P.P. conceived and designed these studies. S.H.Y. grew the films and patterned devices. C.G. measured devices, analyzed the experimental data and carried out the micromagnetic simulations. S.H.Y. interpreted the results and developed the model. S.H.Y. and S.S.P.P. wrote the manuscript. All authors discussed the results and made contributions to the manuscript.

Figure Captions

Figure 1 | Magnetic property of SAF films, and current driven domain motion in SAF wires as a function of $\boldsymbol{J}$, $\boldsymbol{H_x}$ and $\boldsymbol{H_y}$ for various $\boldsymbol{t_{Co}}$. **a,** Normalized polar Kerr hysteresis loops measured from unpatterned 20 TaN | 15 Pt | t_{Co} Co | 7 Ni | 1.5 Co | 8 Ru | 1.5 Co | 7 Ni | 1.5 Co | 50 TaN deposited on Si(100) wafers covered with ~250 SiO_2 (bottom left inset) along out-of-plane direction (easy axis). m_U/m_L, that are obtained from the hysteresis loops, are plotted as a function of t_{Co} in the top right inset. **b**, Kerr microscope images of a single DW moving along a nanowire formed from 20 TaN | 15 Pt | 3 Co | 7 Ni | 1.5 Co | 8 Ru | 1.5 Co | 7 Ni | 1.5 Co | 50 TaN. Dark and bright areas correspond to up (⊙ or ↑) and down (⊗ or ↓) domains, respectively. Each image (top to bottom) is taken after applying a train of current pulses (current density $J = 1.37 \times 10^8\ A\ cm^{-2}$) composed of $10 \times 10\ ns$ (upper panel) and $8 \times 10\ ns$ (lower panel). **c,** v vs. J measured from 2 µm wide 50 µm long SAF nanowires with ↑↓ configuration in LM formed from the samples described in **a.** Device image measured by optical microscope is shown in the inset. **d,** Plots of v vs. H_x measured with devices patterned from the samples described in **a** and **c**. $J = 1.1 \times 10^8\ A\ cm^{-2}$ is used. **d-f,** Plots of v vs. H_x measured from the samples with $t_{Ru} = 8$ (**d**), 4 (**e**), and 2 (**f**). **g,** Plot of v vs. H_y measured from the samples described in **a** and **c**. $J = 1.1 \times 10^8\ A\ cm^{-2}$ (**d,e,g**) and $7.5 \times 10^7\ A\ cm^{-2}$ (**f**) are used. Normalized polar Kerr hysteresis loop is plotted in the inset (**e,f**). Black, red, green, and blue symbols/lines correspond to $t_{Co} = 1.5, 1.8, 2.1,$ and 2.5, respectively (**a**,**c-g**). Error bars in plots **d-g** correspond to one standard deviation. All thicknesses are in Å.

Figure 2 | Micromagnetic simulations of chiral exchange drag dynamics in SAF wire. **a,** v vs. H_x plot of micromagnetic simulations of DW motion for different values of $J^{ex} = -0.15$ (black solid line), -0.3 (black dotted line) and $-0.5\ erg\ cm^{-2}$ (black dash dotted line) and comparison with experimental data for sample with $t_{Co} = 1.8$ (red line and symbols). Here J^{ex} was varied while all other parameters were kept constant. **b,** Time-resolved evolution of DW position, q for $J^{ex} = -0.15\ erg\ cm^{-2}$ and three different fields: $H_x = +0.6\ kOe$ (A), $-1.5\ kOe$ (B), and $-0.6\ kOe$ (C). **c,** v vs. H_y plot of micromagnetic simulations for $J^{ex} = -0.15\ erg\ cm^{-2}$ and comparison with experimental data. Error bars correspond to one standard deviation. **d-f,** Series of time-resolved 2D snapshots taken from the micromagnetic simulations for the cases A (**d**), B (**e**) and C (**f**). and covering a time interval indicated in **b**. Successive snapshots are stacked vertically for the upper and lower layer magnetization in the SAF wire. Note that the DW positions are locked and coupled to each other in all three cases. Panels on the right zoom in to focus on the magnetization direction inside the DW for each of the snapshots. The areas of red and blue colors correspond to ↑ and ↓ domains, respectively, thus representing out-of-plane (z) components of magnetizations only. To display the in-plane components of magnetizations, white arrows are overlapped on top. The 1st, 2nd, 3rd and 4th quadrants along with x and y −axes are displayed in the inset of **d**. See the Method for the used parameters in calculations.

Figure 3 | Analytical model simulations of chiral exchange drag dynamics in SAF wire. **a,** Schematic of key variables and parameters used in the analytical model to describe the CED in SAF nanowires of the top view. Upper and lower two panels correspond to the c-DWs and the q-DWs, respectively. Note that the positions of q-DWs in LM and UM are not bound to each other while the DW positions in composite-DW are locked. **b,** Schematic description of CED induced dramatic DW velocity collapse as a function of longitudinal field H_x. Note that when $\mathcal{D}$ is small, i.e, $\mathcal{D} < \mathcal{D}^{th1}$, no CED anomaly is induced and the steady state is allowed. In contrast, when $\mathcal{D} \geq \mathcal{D}^{th1}$, the CED anomaly is activated; $\mathcal{D}^{th1} \leq \mathcal{D} < \mathcal{D}^{th2}$ → weak CED anomaly (yellow region); $\mathcal{D}^{th2} \leq \mathcal{D}$ → strong CED anomaly (red region). H_L^{DM} and H_U^{DM} are DMI fields for LM and UM, respectively, and their negative values, $-H_i^{DM}$, correspond to the crossing fields of $\dot{Q}_i$ with x-axis. Black lines with open circles: $\dot{q}$, orange dashed lines: $\dot{Q}$, red lines with solid triangles: $\dot{Q}_L$, and blue lines with open squares $\dot{Q}_U$. **c,** Calculated v vs. H_x (red curve and symbols) and H_y (blue curve) with analytical model. **d**, Time evolution of $\dot{\psi}_L$. **e,** DW precession/oscillation frequencies f_D that are obtained from the calculation of **c** (red curve and symbols: H_x, blue curve: H_y). Time evolution of q versus time t is plotted in the inset. **f**, Time evolution of $\dot{\psi}_U$. Green, red, cyan, and blue symbols and lines correspond to $H_x = 2, -1.05, -1.1$ and $-2\ kOe$ (**d-f**). The region of open symbols correspond to the CED anomaly (**c,d**). Red and yellow shaded regions correspond to the strong and weak CED anomalies, respectively. Note that the input parameters for the analytical model are same as those used in micromagnetic simulations except that DW width parameters $\Delta_L = 3.7$ nm and $\Delta_U = 3.69$ nm are used in the analytical model. The converted input parameters for the analytical model from the micromagnetic simulations are: $u_L = u_U = 0$, $\alpha_L = \alpha_U = 0$, $\beta_L = \beta_U = 0$, $H_L^k = 0.81\ kOe$, $H_U^k = 0.39\ kOe$, $H_L^{SH} = 0.84\ kOe$, $H_U^{SH} = 0.4\ kOe$, $M_L = 630\ emu\ nm\ cm^{-3}$, $M_U = 390\ emu\ nm\ cm^{-3}$, $H_L^{DM} = -0.102\ kOe$, $H_U^{DM} = 0.3\ kOe$, and $J^{ex} = -0.15\ erg\ cm^{-2}$. The details of input parameter conversions are discussed in the Supplementary Section 4.

Figure 4 | CED dynamics as a function of $\boldsymbol{J}$, $\boldsymbol{H_L^{DM}}$, $\boldsymbol{J^{ex}}$ and $\boldsymbol{\frac{M_R}{M_S}}$. Calculated ν (right panels) and f_D (left panels) as a function of J (**a**,**b**), H_L^{DM} (**c**,**d**), J^{ex} (**e**,**f**), and $\frac{M_R}{M_S}$ (**g**,**h**). The open symbols correspond to the CED anomaly regions where ν is suppressed and f_D's are finite. $M_R = M_L - M_U$ and $M_S = M_L + M_U$ are remanent and saturation moments, respectively (**g**,**h**). CED anomaly is observed when J is larger than a threshold current density $J^{th} \sim 6 \times 10^7\ A\ cm^{-2}$ (**a**,**b**), $-H_L^{DM}$ is larger than a threshold DMI field $-H_L^{DM\,th} \sim 0.47\ kOe$ (**c**,**d**), $-J^{ex}$ is smaller than a threshold exchange coupling strength $-J^{ex\,th} \sim 0.2\ erg\ cm^{-2}$ (**e**,**f**), $\frac{M_R}{M_S}$ is larger than a threshold value of $\left(\frac{M_R}{M_S}\right)^{th} \sim 0.03$ (**g**,**h**). The yellow and red regions corresponds to weak and strong CED anomaly, respectively. Note that $H_x = -2\ kOe$ is applied for all cases.

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