Supplementary Information for "Periodic dynamics in superconductors induced by an impulsive optical quench"

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Supplementary Note 1: Dimensionless units

At equilibrium, it is convenient to define dimensionless quantities as, cf. Ref. [3]:

$$\boldsymbol{r} = \lambda_{ab}\tilde{\boldsymbol{r}}, \quad t = \frac{\tilde{t}}{|\alpha|}, \quad \psi = \sqrt{\frac{|\alpha|}{\beta}}\tilde{\psi}, \quad \boldsymbol{A} = \sqrt{2}\lambda_{ab}H_c\tilde{\boldsymbol{A}}, \tag{S1}$$

$$\boldsymbol{E} = \frac{\sqrt{2}\lambda_{ab}H_c|\alpha|}{c}\tilde{\boldsymbol{E}}, \quad \rho = \frac{\sqrt{2}|\alpha|H_c}{4\pi c}\tilde{\rho}, \quad \chi = \frac{\sqrt{2}H_c}{4\pi c}\tilde{\chi}, \quad \sigma = \frac{c^2}{4\pi\lambda_{ab}^2|\alpha|}\tilde{\sigma}, \tag{S2}$$

where $\lambda_{ab} = \sqrt{m_{ab}c^2\beta/4\pi(e^*)^2|\alpha|}$ defines the unit of length. $\lambda_c = \gamma\lambda_{ab}$ is the magnetic penetration depth for currents along the z-axis. $\xi_{ab} = \sqrt{1/2m_{ab}|\alpha|}$ and $\xi_c = \sqrt{1/2m_c|\alpha|}$ are the two coherence lengths. The Ginzburg parameter is:

$$\kappa = \frac{\lambda_{ab}}{\xi_c} = \frac{\lambda_c}{\xi_{ab}}. \tag{S3}$$

$H_c = \sqrt{4\pi\alpha^2/\beta}$ is the thermodynamic critical field. We also introduce $r_0 = 2\pi(e^*)^2/\lambda_{ab}m_{ab}c^2$, which is nothing but the classical Cooper pair radius in the unit of λ_{ab}.

When it comes to dynamics, here we are primarily interested in quenches in the Landau coefficient $\alpha(t)$. We therefore fix all the dimensionless quantities in Eq. (S2) at some reference α_0 (for example, it can be chosen to be the $T = 0$ value $\alpha(T = 0)$ or the pre-pulse equilibrium value) and write the equations of motion in the dimensionless units as (for notational simplicity, here and below, we drop the tilde sign for dimensionless quantities):

$$\tau(\partial_t + i\chi^{-1}\nabla\cdot\boldsymbol{E})\psi = -(-i\gamma\kappa^{-1}\nabla_{ab} - \boldsymbol{A}_{ab})^2\psi - \gamma^{-2}(-i\gamma\kappa^{-1}\partial_z - A_z)^2\psi - \alpha\psi - |\psi|^2\psi + \eta, \tag{S4}$$

$$\partial_t\boldsymbol{A} = -\boldsymbol{E}, \tag{S5}$$

$$\tau_E\partial_t\boldsymbol{E} = \nabla\times\nabla\times\boldsymbol{A} - \hat{\sigma}(\boldsymbol{E} - \gamma\kappa^{-1}\chi^{-1}\nabla(\nabla\cdot\boldsymbol{E})) - \frac{1}{2}\hat{m}^{-1}(\psi^*(-i\gamma\kappa^{-1}\nabla - \boldsymbol{A})\psi + c.c.) + \boldsymbol{\xi}, \tag{S6}$$

where $\hat{m}^{-1} = \mathrm{diag}(1, 1, \gamma^{-2})$ reflects the anisotropy of the superconductor. The thermal noise terms in the Lagnevin equations (S4) and (S6) obey:

$$\langle\eta^*(\boldsymbol{r}, t)\eta(\boldsymbol{r}', t')\rangle = 4T\tau r_0\delta(\boldsymbol{r} - \boldsymbol{r}')\delta(t - t'), \tag{S7}$$

$$\langle\xi_\alpha(\boldsymbol{r}, t)\xi_\beta(\boldsymbol{r}', t')\rangle = 2T\sigma_{\alpha\beta}r_0\delta(\boldsymbol{r} - \boldsymbol{r}')\delta(t - t'), \tag{S8}$$

such that the fluctuation-dissipation theorem is satisfied [4]. Here $\tau_E = \lambda_{ab}^2\alpha^2/c^2$. Below we define $\Gamma_E = \tau_E^{-1}$.

Supplementary Note 2: Mean-field analysis of equilibrium collective modes

We turn to discuss collective equilibrium excitations in the symmetry broken phase. To this end, we neglect the noise terms, linearize the above equations on top of $\psi = \bar{\psi}(1 + i\theta) + \delta\Delta$ ($\bar{\psi} = \sqrt{-\alpha}$), and obtain the spectrum of collective modes. We find that even though the dynamics of the order parameter amplitude $\delta\Delta$ is overdamped, the dynamics of the phase θ is not, and therefore, we focus on these modes. We consider the cases of isotropic and anisotropic superconductors separately.

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A. Isotropic superconductors

The linearized supercurrent reads: $\boldsymbol{j}_s \approx \bar{\psi}^2(\kappa^{-1}\nabla\theta - \boldsymbol{A})$. We decompose all vectors in momentum space into the transverse and longitudinal components: $\boldsymbol{A}(\boldsymbol{k},\omega) = \boldsymbol{A}_\perp(\boldsymbol{k},\omega) + \boldsymbol{A}_\parallel(\boldsymbol{k},\omega)$, where $\boldsymbol{A}_\parallel$ points along $\boldsymbol{k}$ and $\boldsymbol{A}_\perp$ is orthogonal to $\boldsymbol{k}$. We find that the transverse sector decouples from the rest of the system, and its spectrum is given by:

$$\omega_\perp(k) = \sqrt{(k^2+\bar{\psi}^2)\Gamma_E - \frac{1}{4}\sigma^2\Gamma_E^2} - \frac{i}{2}\sigma\Gamma_E, \tag{S9}$$

i.e. the non-zero expectation value $\bar{\psi}$ opens up the plasmon gap, in accordance with the Anderson-Higgs mechanism and the Meissner effect. The primary role of the normal conductivity is to provide damping and redshift these transverse plasmon excitations.

Longitudinal waves are coupled to the dynamics of the order-parameter phase θ:

$$\Big[\tau_E\omega^2 + i\omega\sigma\Big(1+\frac{k^2}{\kappa\chi}\Big) - \bar{\psi}^2\Big]A_\parallel + \frac{ik\bar{\psi}^2\theta}{\kappa} = 0, \tag{S10}$$

$$\Big[-i\omega\tau + \frac{k^2}{\kappa^2}\Big]\theta - \Big[\frac{\omega\tau}{\chi} - \frac{i}{\kappa}\Big]kA_\parallel = 0. \tag{S11}$$

By solving this system analytically, we obtain the following quadratic equation defining the spectrum of the longitudinal modes:

$$\tau_E\omega^2 + i\omega\Big[\sigma\Big(1+\frac{k^2}{\kappa\chi}\Big) + \tau_E\Gamma\frac{k^2}{\kappa^2}\Big] - \Big(1+\frac{k^2}{\kappa\chi}\Big)\Big[\bar{\psi}^2 + \sigma\Gamma\frac{k^2}{\kappa^2}\Big] = 0. \tag{S12}$$

Similarly to the transverse sector, the longitudinal modes also open up the same plasmon gap. Because of the Coulomb screening and the coupling to the order parameter phase, the longitudinal excitations are more damped for $k \neq 0$ than the transverse ones.

B. Anisotropic superconductors

The linearized supercurrent modifies to

$$\boldsymbol{j}_s \approx \bar{\psi}^2\Big(\frac{\gamma}{\kappa}\nabla_{ab}\theta - \boldsymbol{A}_{ab}\Big) + \frac{\hat{\boldsymbol{z}}\bar{\psi}^2}{\gamma^2}\Big(\frac{\gamma}{\kappa}\partial_z\theta - A_z\Big).$$

Due to the anisontropy, we now decompose all vectors as: $\boldsymbol{A}(\boldsymbol{k}) = A_\parallel\hat{\boldsymbol{k}}_{ab} + A_\perp\hat{\boldsymbol{k}}_{ab}\times\hat{\boldsymbol{z}} + A_z\hat{\boldsymbol{z}}$, etc. Here $\boldsymbol{k} = (\boldsymbol{k}_{ab}, k_z)$. We find that the transverse sector (with vectors pointing along $\hat{\boldsymbol{k}}_{ab}\times\hat{\boldsymbol{z}}$) decouples from the rest of the system, and its spectrum is given by:

$$\omega_\perp(\boldsymbol{k}) = \sqrt{(k^2+\bar{\psi}^2)\Gamma_E - \frac{1}{4}\sigma_{ab}^2\Gamma_E^2} - \frac{i}{2}\sigma_{ab}\Gamma_E. \tag{S13}$$

As for the isotropic case, the transverse sector exhibits opening of the plasmon gap, expected to be a large energy scale.

The remaining linearized EM equations read:

$$\tau_E\omega^2A_\parallel = k_z^2A_\parallel - k_{ab}k_zA_z - \bar{\psi}^2\Big(\frac{\gamma}{\kappa}ik_{ab}\theta - A_\parallel\Big) - i\omega\sigma_{ab}\Big(A_\parallel + \frac{\gamma}{\kappa\chi}k_{ab}(k_{ab}A_\parallel + k_zA_z)\Big), \tag{S14}$$

$$\tau_E\omega^2A_z = k_{ab}^2A_z - k_{ab}k_zA_\parallel - \frac{\bar{\psi}^2}{\gamma^2}\Big(\frac{\gamma}{\kappa}ik_z\theta - A_z\Big) - i\omega\sigma_c\Big(A_z + \frac{\gamma}{\kappa\chi}k_z(k_{ab}A_\parallel + k_zA_z)\Big). \tag{S15}$$

Linearized equation for the order parameter phase θ is:

$$\Big[-i\omega\tau + \frac{\gamma^2k_{ab}^2}{\kappa^2} + \frac{k_z^2}{\kappa^2}\Big]\theta - \frac{\omega\tau}{\chi}(k_{ab}A_\parallel + k_zA_z) + \frac{i\gamma}{\kappa}(k_{ab}A_\parallel + \gamma^{-2}k_zA_z) = 0. \tag{S16}$$

To find the spectrum of collective modes, one can solve Eqs. (S14)-(S16) numerically. Here we are mostly interested in the limit $\boldsymbol{k} \to 0$:

$$\omega_{\parallel}(\boldsymbol{k}=0) = \sqrt{\bar{\psi}^2\Gamma_E - \frac{1}{4}\sigma_{ab}^2\Gamma_E^2} - \frac{i}{2}\sigma_{ab}\Gamma_E, \tag{S17}$$

$$\omega_z(\boldsymbol{k}=0) = \sqrt{\frac{\bar{\psi}^2}{\gamma^2}\Gamma_E - \frac{1}{4}\sigma_c^2\Gamma_E^2} - \frac{i}{2}\sigma_c\Gamma_E. \tag{S18}$$

We conclude that all of the collective modes are gapped. Most importantly, though, we find that the c-axis gap is factor of γ smaller than the gap of the other two plasmons (for a related discussion in layered superconductors, see Ref. [1]). In anisotropic superconductors, such as cuprates, this gap is in the terahertz range, rendering the c-axis Josephson plasmons to be the primary low-energy excitations. We also note that their lifetime is large, since the out-of-plane conductivity σ_c is small.

Supplementary Note 3: Equations of motion within the Gaussian approximation

The equations of motion in the dimensionless units and in momentum space read:

$$\begin{aligned}
\tau\partial_t\psi_1(\boldsymbol{k},t) &= i\tau\chi^{-1}\int_{\boldsymbol{q}} \boldsymbol{q}\cdot\boldsymbol{E}(\boldsymbol{q},t)\psi_2(\boldsymbol{k}-\boldsymbol{q},t) - (\alpha(t) + \gamma^2\kappa^{-2}k_{ab}^2 + \kappa^{-2}k_z^2)\psi_1(\boldsymbol{k},t) \\
&+ i\gamma\kappa^{-1}\int_{\boldsymbol{p}} (\boldsymbol{k}+\boldsymbol{p})\hat{m}^{-1}\boldsymbol{A}(\boldsymbol{k}-\boldsymbol{p},t)\psi_2(\boldsymbol{p},t) \\
&- \int_{\boldsymbol{p}_1,\boldsymbol{p}_2} [\boldsymbol{A}(\boldsymbol{p}_1,t)\hat{m}^{-1}\boldsymbol{A}(\boldsymbol{p}_2,t) + \psi_1(\boldsymbol{p}_1,t)\psi_1(\boldsymbol{p}_2,t) + \psi_2(\boldsymbol{p}_1,t)\psi_2(\boldsymbol{p}_2,t)]\psi_1(\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2,t) + \eta_1(\boldsymbol{k},t), \qquad \text{(S19)}\\
\tau\partial_t\psi_2(\boldsymbol{k},t) &= -i\tau\chi^{-1}\int_{\boldsymbol{q}} \boldsymbol{q}\cdot\boldsymbol{E}(\boldsymbol{q},t)\psi_1(\boldsymbol{k}-\boldsymbol{q},t) - (\alpha(t) + \gamma^2\kappa^{-2}k_{ab}^2 + \kappa^{-2}k_z^2)\psi_2(\boldsymbol{k},t) \\
&- i\gamma\kappa^{-1}\int_{\boldsymbol{p}} (\boldsymbol{k}+\boldsymbol{p})\hat{m}^{-1}\boldsymbol{A}(\boldsymbol{k}-\boldsymbol{p},t)\psi_1(\boldsymbol{p},t) \\
&- \int_{\boldsymbol{p}_1,\boldsymbol{p}_2} [\boldsymbol{A}(\boldsymbol{p}_1,t)\hat{m}^{-1}\boldsymbol{A}(\boldsymbol{p}_2,t) + \psi_1(\boldsymbol{p}_1,t)\psi_1(\boldsymbol{p}_2,t) + \psi_2(\boldsymbol{p}_1,t)\psi_2(\boldsymbol{p}_2,t)]\psi_2(\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2,t) + \eta_2(\boldsymbol{k},t), \qquad \text{(S20)}\\
\partial_t\boldsymbol{A}(\boldsymbol{k},t) &= -\boldsymbol{E}(\boldsymbol{k},t), \qquad \text{(S21)}\\
\tau_E\partial_t\boldsymbol{E}(\boldsymbol{k},t) &= k^2\boldsymbol{A}(\boldsymbol{k},t) - (\boldsymbol{k}\cdot\boldsymbol{A}(\boldsymbol{k},t))\boldsymbol{k} - \hat{\sigma}_{\alpha\beta}(\delta_{\beta\gamma} + \gamma\kappa^{-1}\chi^{-1}k_\beta k_\gamma)E_\gamma(\boldsymbol{k},t) - \boldsymbol{j_s}(\boldsymbol{k},t) + \boldsymbol{\xi}(\boldsymbol{k},t), \qquad \text{(S22)}
\end{aligned}$$

where ψ_1 and ψ_2 are the real and imaginary components of the real-space order parameter $\psi(\boldsymbol{r},t) = \psi_1(\boldsymbol{r},t) + i\psi_2(\boldsymbol{r},t)$. $\langle\eta_a(\boldsymbol{k},t)\eta_b(\boldsymbol{k}',t')\rangle = 2T\tau r_0\delta_{ab}\times(2\pi)^3\delta(\boldsymbol{k}+\boldsymbol{k}')\delta(t-t')$, $\langle\xi_\alpha(\boldsymbol{k},t)\xi_\beta(\boldsymbol{k}',t')\rangle = 2\sigma_{\alpha\beta}Tr_0\times(2\pi)^3\delta(\boldsymbol{k}+\boldsymbol{k}')\delta(t-t')$. Here $\int_{\boldsymbol{q}} \equiv \int\frac{d^3\boldsymbol{q}}{(2\pi)^3}$. The superconducting current density in momentum space reads:

$$\begin{aligned}
\boldsymbol{j_s}(\boldsymbol{k},t) &= i\gamma\kappa^{-1}\hat{m}^{-1}\int_{\boldsymbol{p}}(2\boldsymbol{p}-\boldsymbol{k})\psi_1(\boldsymbol{k}-\boldsymbol{p},t)\psi_2(\boldsymbol{p},t) \\
&- \int_{\boldsymbol{p}_1,\boldsymbol{p}_2} [\psi_1(\boldsymbol{p}_1,t)\psi_1(\boldsymbol{p}_2,t) + \psi_2(\boldsymbol{p}_1,t)\psi_2(\boldsymbol{p}_2,t)]\hat{m}^{-1}\boldsymbol{A}(\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2,t). \qquad \text{(S23)}
\end{aligned}$$

We note that Eqs. (S19)-(S22) are stochastic first-order differential equations. They can be rewritten into a single first-order Fokker-Planck equation on the cumulative distribution functional $\mathcal{P}[t;\psi_1,\psi_2,\boldsymbol{A},\boldsymbol{E}]$. Here we assume that $\mathcal{P}$ is a Gaussian distribution, and derive the corresponding equations on various correlation functions. For a related discussion in incommensurate charge density waves, see Ref. [2]. We also invoke translational symmetry. Specifically, below we introduce: $\psi_1(t) = \langle\psi_1(\boldsymbol{k}=0,t)\rangle$, $\mathcal{D}_{11}(\boldsymbol{k},t) = \langle\psi_1(-\boldsymbol{k},t)\psi_1(\boldsymbol{k},t)\rangle_c$, $\mathcal{D}_{22}(\boldsymbol{k},t) = \langle\psi_2(-\boldsymbol{k},t)\psi_2(\boldsymbol{k},t)\rangle_c$, $\pi_\alpha(\boldsymbol{k},t) = \langle E_\alpha(-\boldsymbol{k},t)\psi_2(\boldsymbol{k},t)\rangle_c$, $a_\alpha(\boldsymbol{k},t) = \langle A_\alpha(-\boldsymbol{k},t)\psi_2(\boldsymbol{k},t)\rangle_c$, $\Phi_{\alpha\beta}(\boldsymbol{k},t) = \langle A_\alpha(-\boldsymbol{k},t)A_\beta(\boldsymbol{k},t)\rangle_c$, $K_{\alpha\beta}(\boldsymbol{k},t) = \langle A_\alpha(-\boldsymbol{k},t)E_\beta(\boldsymbol{k},t)\rangle_c$, and $\Pi_{\alpha\beta}(\boldsymbol{k},t) = \langle E_\alpha(-\boldsymbol{k},t)E_\beta(\boldsymbol{k},t)\rangle_c$. Other correlators: $\langle\psi_2(\boldsymbol{k}=0,t)\rangle$, $\langle\psi_1(-\boldsymbol{k},t)\psi_2(\boldsymbol{k},t)\rangle_c$, $\langle E_\alpha(-\boldsymbol{k},t)\psi_1(\boldsymbol{k},t)\rangle_c$, $\langle A_\alpha(-\boldsymbol{k},t)\psi_1(\boldsymbol{k},t)\rangle_c$ – turn out not to develop within the presented framework and, thus, can be neglected. We omit writing explicit dependence on time of the dynamical variables in the right-hand side of each of the equations below, unless it is needed.

SC sector. The order parameter dynamics and its fluctuations follow:

$$\tau\partial_t\psi_1(t) = \int_{\boldsymbol{p}} i\boldsymbol{p}\cdot(\gamma\kappa^{-1}\hat{m}^{-1}\boldsymbol{a}(\boldsymbol{p}) - \tau\chi^{-1}\boldsymbol{\pi}(\boldsymbol{p})) - \psi_1\Big(\alpha + \psi_1^2 + \int_{\boldsymbol{p}}(\mathrm{tr}(\hat{m}^{-1}\Phi(\boldsymbol{p})) + 3\mathcal{D}_{11}(\boldsymbol{p}) + \mathcal{D}_{22}(\boldsymbol{p}))\Big), \tag{S24}$$

$$\partial_t\mathcal{D}_{11}(\boldsymbol{k},t) = 2\Gamma Tr_0 - 2\Gamma\mathcal{D}_{11}\Big(\alpha + \gamma^2\kappa^{-2}k_{ab}^2 + \kappa^{-2}k_z^2 + 3\psi_1^2 + \int_{\boldsymbol{p}}(\mathrm{tr}(\hat{m}^{-1}\Phi(\boldsymbol{p})) + 3\mathcal{D}_{11}(\boldsymbol{p}) + \mathcal{D}_{22}(\boldsymbol{p}))\Big), \tag{S25}$$

$$\begin{aligned}\partial_t\mathcal{D}_{22}(\boldsymbol{k},t) = {} & 2\Gamma Tr_0 - 2\Gamma\mathcal{D}_{22}\Big(\alpha + \gamma^2\kappa^{-2}k_{ab}^2 + \kappa^{-2}k_z^2 + \psi_1^2 + \int_{\boldsymbol{p}}(\mathrm{tr}(\hat{m}^{-1}\Phi(\boldsymbol{p})) + \mathcal{D}_{11}(\boldsymbol{p}) + 3\mathcal{D}_{22}(\boldsymbol{p}))\Big) \\ & - 2\Gamma\mathrm{Re}\Big\{-i\tau\chi^{-1}\psi_1\boldsymbol{k}\cdot\boldsymbol{\pi}(\boldsymbol{k}) - i\gamma\kappa^{-1}\psi_1\boldsymbol{k}\,\hat{m}^{-1}\boldsymbol{a}(\boldsymbol{k}) + 2\boldsymbol{a}(\boldsymbol{k})\hat{m}^{-1}\int_{\boldsymbol{p}}\boldsymbol{a}(\boldsymbol{p})\Big\}.\end{aligned} \tag{S26}$$

Cross correlators.

$$\begin{aligned}\partial_t\pi_\alpha(\boldsymbol{k},t) = {} & -\Gamma\Big[i\tau\chi^{-1}\psi_1\Pi_{\alpha\beta}(\boldsymbol{k})k_\beta + i\gamma\kappa^{-1}\psi_1(K^\dagger(\boldsymbol{k}))_{\alpha\gamma}\hat{m}^{-1}_{\gamma\beta}k_\beta + 2(K^\dagger(\boldsymbol{k}))_{\alpha\beta}\hat{m}^{-1}_{\beta\gamma}\int_{\boldsymbol{p}}a_\gamma(\boldsymbol{p}) \\ & + \pi_\alpha(\boldsymbol{k})\Big(\alpha + \gamma^2\kappa^{-2}k_{ab}^2 + \kappa^{-2}k_z^2 + \psi_1^2 + \int_{\boldsymbol{p}}(\mathrm{tr}(\hat{m}^{-1}\Phi(\boldsymbol{p})) + \mathcal{D}_{11}(\boldsymbol{p}) + 3\mathcal{D}_{22}(\boldsymbol{p}))\Big)\Big] \\ & + \Gamma_E\Big[(k^2\delta_{\alpha\beta} - k_\alpha k_\beta)a_\beta(\boldsymbol{k}) - \sigma_{\alpha\beta}(\delta_{\beta\gamma} + \gamma\kappa^{-1}\chi^{-1}k_\beta k_\gamma)\pi_\gamma(\boldsymbol{k}) + \hat{m}^{-1}_{\alpha\beta}a_\beta(\boldsymbol{k})\Big(\psi_1^2 + \int_{\boldsymbol{p}}(\mathcal{D}_{11}(\boldsymbol{p}) + \mathcal{D}_{22}(\boldsymbol{p}))\Big) \\ & + 2\mathcal{D}_{22}(\boldsymbol{k})\hat{m}^{-1}_{\alpha\beta}\int_{\boldsymbol{p}}a_\beta(\boldsymbol{p}) + i\gamma\kappa^{-1}\psi_1\hat{m}^{-1}_{\alpha\beta}k_\beta\mathcal{D}_{22}(\boldsymbol{k})\Big],\end{aligned} \tag{S27}$$

$$\begin{aligned}\partial_t a_\alpha(\boldsymbol{k},t) = {} & -\pi_\alpha(\boldsymbol{k}) - \Gamma\Big[i\tau\chi^{-1}\psi_1K_{\alpha\beta}(\boldsymbol{k})k_\beta + i\gamma\kappa^{-1}\psi_1\Phi_{\alpha\gamma}(\boldsymbol{k})\hat{m}^{-1}_{\gamma\beta}k_\beta + 2\Phi_{\alpha\beta}(\boldsymbol{k})\hat{m}^{-1}_{\beta\gamma}\int_{\boldsymbol{p}}a_\gamma(\boldsymbol{p}) \\ & + a_\alpha(\boldsymbol{k})\Big(\alpha + \gamma^2\kappa^{-2}k_{ab}^2 + \kappa^{-2}k_z^2 + \psi_1^2 + \int_{\boldsymbol{p}}(\mathrm{tr}(\hat{m}^{-1}\Phi(\boldsymbol{p})) + \mathcal{D}_{11}(\boldsymbol{p}) + 3\mathcal{D}_{22}(\boldsymbol{p}))\Big)\Big].\end{aligned} \tag{S28}$$

EM sector.

$$\partial_t\Phi(\boldsymbol{k},t) = -(K(\boldsymbol{k}) + K^\dagger(\boldsymbol{k})), \tag{S29}$$

$$\begin{aligned}\partial_t K_{\alpha\beta}(\boldsymbol{k},t) = {} & -\Pi_{\alpha\beta}(\boldsymbol{k}) + \Gamma_E\Big[\Phi_{\alpha\gamma}(\boldsymbol{k})(k^2\delta_{\gamma\beta} - k_\gamma k_\beta) - K_{\alpha\delta}(\boldsymbol{k})(\delta_{\delta\gamma} + \gamma\kappa^{-1}\chi^{-1}k_\delta k_\gamma)\sigma_{\gamma\beta} \\ & + \Phi_{\alpha\gamma}(\boldsymbol{k})\hat{m}^{-1}_{\gamma\beta}\Big(\psi_1^2 + \int_{\boldsymbol{p}}(\mathcal{D}_{11}(\boldsymbol{p}) + \mathcal{D}_{22}(\boldsymbol{p}))\Big) - i\gamma\kappa^{-1}\psi_1 a_\alpha(\boldsymbol{k})\hat{m}^{-1}_{\beta\gamma}k_\gamma + 2a_\alpha(\boldsymbol{k})\hat{m}^{-1}_{\beta\gamma}\int_{\boldsymbol{p}}a_\gamma(\boldsymbol{p})\Big],\end{aligned} \tag{S30}$$

$$\partial_t\Pi(\boldsymbol{k},t) = 2Tr_0\hat{\sigma}\Gamma_E^2 + Q(\boldsymbol{k}) + Q^\dagger(\boldsymbol{k}), \tag{S31}$$

$$\begin{aligned}Q_{\alpha\beta}(\boldsymbol{k},t) = {} & \Gamma_E\Big[(K^\dagger(\boldsymbol{k}))_{\alpha\gamma}(k^2\delta_{\gamma\beta} - k_\gamma k_\beta) - \Pi_{\alpha\delta}(\boldsymbol{k})(\delta_{\delta\gamma} + \gamma\kappa^{-1}\chi^{-1}k_\delta k_\gamma)\sigma_{\gamma\beta} \\ & + (K^\dagger(\boldsymbol{k}))_{\alpha\gamma}\hat{m}^{-1}_{\gamma\beta}\Big(\psi_1^2 + \int_{\boldsymbol{p}}(\mathcal{D}_{11}(\boldsymbol{p}) + \mathcal{D}_{22}(\boldsymbol{p}))\Big) - i\gamma\kappa^{-1}\psi_1\pi_\alpha(\boldsymbol{k})\hat{m}^{-1}_{\beta\gamma}k_\gamma + 2\pi_\alpha(\boldsymbol{k})\hat{m}^{-1}_{\beta\gamma}\int_{\boldsymbol{p}}a_\gamma(\boldsymbol{p})\Big].\end{aligned} \tag{S32}$$

Equations (S24)-(S32) represent our central technical result. In principle, using these equations, one can directly simulate a photoexcitation event. We note, however, that the total number of independent degrees of freedom is quite large, and one will have to introduce a grid in the three-dimensional momentum space, limiting simulations to relatively small system sizes. One can significantly facilitate simulations of large systems by invoking the cylindrical symmetry of anisotropic superconductors and spherical symmetry of isotropic ones – below we address how to do this in practice. We remark that the thermal state is found self-consistently by putting the right-hand sides of each of the equations to be zero.

$\hat{T}_1\hat{T}_1 = k_{ab}^2\hat{T}_1$	$\hat{T}_2\hat{T}_1 = 0$	$\hat{T}_3\hat{T}_1 = k_{ab}^2\hat{T}_3$	$\hat{T}_4\hat{T}_1 = \hat{T}_1$	$\hat{T}_5\hat{T}_1 = 0$
$\hat{T}_1\hat{T}_2 = k_{ab}^2\hat{T}_2$	$\hat{T}_2\hat{T}_2 = 0$	$\hat{T}_3\hat{T}_2 = k_{ab}^2k_z^2\hat{T}_5$	$\hat{T}_4\hat{T}_2 = \hat{T}_2$	$\hat{T}_5\hat{T}_2 = 0$
$\hat{T}_1\hat{T}_3 = 0$	$\hat{T}_2\hat{T}_3 = k_z^2\hat{T}_1$	$\hat{T}_3\hat{T}_3 = 0$	$\hat{T}_4\hat{T}_3 = 0$	$\hat{T}_5\hat{T}_3 = \hat{T}_3$
$\hat{T}_1\hat{T}_4 = \hat{T}_1$	$\hat{T}_2\hat{T}_4 = 0$	$\hat{T}_3\hat{T}_4 = \hat{T}_3$	$\hat{T}_4\hat{T}_4 = \hat{T}_4$	$\hat{T}_5\hat{T}_4 = 0$
$\hat{T}_1\hat{T}_5 = 0$	$\hat{T}_2\hat{T}_5 = \hat{T}_2$	$\hat{T}_3\hat{T}_5 = 0$	$\hat{T}_4\hat{T}_5 = 0$	$\hat{T}_5\hat{T}_5 = \hat{T}_5$

Table S1. Algebra of operators in the tensor expansion (S34). Note that: $\hat{T}_1^\dagger = \hat{T}_1$, $\hat{T}_2^\dagger = \hat{T}_3$, $\hat{T}_3^\dagger = \hat{T}_2$, $\hat{T}_4^\dagger = \hat{T}_4$, and $\hat{T}_5^\dagger = \hat{T}_5$.

C. Cylindrical symmetry

To take advantage of the cylindrical symmetry we use the following ansatz:

$$\boldsymbol{a}(\boldsymbol{k},t) = ia_{ab}(k_{ab},k_z,t)\boldsymbol{k}_{ab} + ia_z(k_{ab},k_z,t)\boldsymbol{k}_z,\ \boldsymbol{\pi}(\boldsymbol{k},t) = i\pi_{ab}(k_{ab},k_z,t)\boldsymbol{k}_{ab} + i\pi_z(k_{ab},k_z,t)\boldsymbol{k}_z, \tag{S33}$$

$$\Phi_{\alpha\beta}(\boldsymbol{k},t) = \Phi_\parallel(k_{ab},k_z,t)\hat{T}_1 + \Phi_{X,1}(k_{ab},k_z,t)\hat{T}_2 + \Phi_{X,2}(k_{ab},k_z,t)\hat{T}_3 + \Phi_{ab}(k_{ab},k_z,t)\hat{T}_4 + \Phi_z(k_{ab},k_z,t)\hat{T}_5, \tag{S34}$$

and similar expansion holds for the other two electromagnetic tensors. Here $\hat{T}_1 = k_{ab,\alpha}k_{ab,\beta}$, $\hat{T}_2 = k_{ab,\alpha}k_{z,\beta}$, $\hat{T}_3 = k_{z,\alpha}k_{ab,\beta}$, $\hat{T}_4 = \delta^{ab}_{\alpha\beta} = \delta_{\alpha,x}\delta_{\beta,x} + \delta_{\alpha,y}\delta_{\beta,y}$, and $\hat{T}_5 = \delta^z_{\alpha\beta} = \delta_{\alpha,z}\delta_{\beta,z}$ – their algebra is summarized in Table S1. This ansatz allows us to write a closed set of equations solely on the newly introduced quantities, thereby reducing the initial three-dimensional problem to only two-dimensional. Moreover, all of these quantities can be chosen to be real. On symmetry grounds, one has $\Phi^\dagger = \Phi$ and $\Pi^\dagger = \Pi \Rightarrow \Phi_{X,1} = \Phi_{X,2} = \Phi_X$ and $\Pi_{X,1} = \Pi_{X,2} = \Pi_X$, but in general $K_{X,1} \neq K_{X,2}$. Below we summarize the final equations of motion.

SC sector.

$$\begin{aligned}\tau\partial_t\psi_1(t) &= \int_{\boldsymbol{p}} (\tau\chi^{-1}(p_{ab}^2\pi_{ab}(\boldsymbol{p}) + p_z^2\pi_z(\boldsymbol{p})) - \gamma\kappa^{-1}(p_{ab}^2a_{ab}(\boldsymbol{p}) + \gamma^{-2}p_z^2a_z(\boldsymbol{p})))\\ &\quad - \psi_1\Big(\alpha + \psi_1^2 + \int_{\boldsymbol{p}} (p_{ab}^2\Phi_\parallel(\boldsymbol{p}) + 2\Phi_{ab}(\boldsymbol{p}) + \gamma^{-2}\Phi_z(\boldsymbol{p}) + 3\mathcal{D}_{11}(\boldsymbol{p}) + \mathcal{D}_{22}(\boldsymbol{p}))\Big),\end{aligned} \tag{S35}$$

$$\begin{aligned}\partial_t\mathcal{D}_{11}(\boldsymbol{k},t) &= 2\Gamma Tr_0 - 2\Gamma\mathcal{D}_{11}(\boldsymbol{k})\Big(\alpha + \gamma^2\kappa^{-2}k_{ab}^2 + \kappa^{-2}k_z^2 + 3\psi_1^2\\ &\quad + \int_{\boldsymbol{p}} (p_{ab}^2\Phi_\parallel(\boldsymbol{p}) + 2\Phi_{ab}(\boldsymbol{p}) + \gamma^{-2}\Phi_z(\boldsymbol{p}) + 3\mathcal{D}_{11}(\boldsymbol{p}) + \mathcal{D}_{22}(\boldsymbol{p}))\Big),\end{aligned} \tag{S36}$$

$$\begin{aligned}\partial_t\mathcal{D}_{22}(\boldsymbol{k},t) &= 2\Gamma Tr_0 - 2\Gamma\mathcal{D}_{22}(\boldsymbol{k})\Big(\alpha + \gamma^2\kappa^{-2}k_{ab}^2 + \kappa^{-2}k_z^2 + \psi_1^2\\ &\quad + \int_{\boldsymbol{p}} (p_{ab}^2\Phi_\parallel(\boldsymbol{p}) + 2\Phi_{ab}(\boldsymbol{p}) + \gamma^{-2}\Phi_z(\boldsymbol{p}) + \mathcal{D}_{11}(\boldsymbol{p}) + 3\mathcal{D}_{22}(\boldsymbol{p}))\Big)\\ &\quad - 2\Gamma\psi_1\Big[\tau\chi^{-1}(k_{ab}^2\pi_{ab}(\boldsymbol{k}) + k_z^2\pi_z(\boldsymbol{k})) + \gamma\kappa^{-1}(k_{ab}^2a_{ab}(\boldsymbol{k}) + \gamma^{-2}k_z^2a_z(\boldsymbol{k}))\Big].\end{aligned} \tag{S37}$$

Cross correlators.

$$\begin{aligned}\partial_t\pi_{ab}(\boldsymbol{k},t) = -\Gamma\Big[&\tau\chi^{-1}\psi_1(k_{ab}^2\Pi_\parallel(\boldsymbol{k}) + k_z^2\Pi_X(\boldsymbol{k}) + \Pi_{ab}(\boldsymbol{k})) + \gamma\kappa^{-1}\psi_1(k_{ab}^2K_\parallel(\boldsymbol{k}) + \gamma^{-2}k_z^2K_{X,2}(\boldsymbol{k}) + K_{ab}(\boldsymbol{k}))\\ &+ \pi_{ab}(\boldsymbol{k})\Big(\alpha + \gamma^2\kappa^{-2}k_{ab}^2 + \kappa^{-2}k_z^2 + \psi_1^2 + \int_{\boldsymbol{p}} (p_{ab}^2\Phi_\parallel(\boldsymbol{p}) + 2\Phi_{ab}(\boldsymbol{p}) + \gamma^{-2}\Phi_z(\boldsymbol{p}) + \mathcal{D}_{11}(\boldsymbol{p}) + 3\mathcal{D}_{22}(\boldsymbol{p}))\Big)\Big]\\ &+ \Gamma_E\Big[k_z^2(a_{ab}(\boldsymbol{k}) - a_z(\boldsymbol{k})) - \sigma_{ab}(\pi_{ab}(\boldsymbol{k}) + \gamma\kappa^{-1}\chi^{-1}(k_{ab}^2\pi_{ab}(\boldsymbol{k}) + k_z^2\pi_z(\boldsymbol{k})))\\ &+ a_{ab}(\boldsymbol{k})\Big(\psi_1^2 + \int_{\boldsymbol{p}} (\mathcal{D}_{11}(\boldsymbol{p}) + \mathcal{D}_{22}(\boldsymbol{p}))\Big) + \gamma\kappa^{-1}\psi_1\mathcal{D}_{22}(\boldsymbol{k})\Big],\end{aligned} \tag{S38}$$

$$
\begin{aligned}
\partial_t \pi_z(\boldsymbol{k},t) = &-\Gamma\Big[\tau\chi^{-1}\psi_1(k_{ab}^2\Pi_X(\boldsymbol{k})+\Pi_z(\boldsymbol{k}))+\gamma\kappa^{-1}\psi_1(k_{ab}^2K_{X,1}(\boldsymbol{k})+\gamma^{-2}K_z(\boldsymbol{k}))\\
&+\pi_z(\boldsymbol{k})\Big(\alpha+\gamma^2\kappa^{-2}k_{ab}^2+\kappa^{-2}k_z^2+\psi_1^2+\int_{\boldsymbol{p}}(p_{ab}^2\Phi_\parallel(\boldsymbol{p})+2\Phi_{ab}(\boldsymbol{p})+\gamma^{-2}\Phi_z(\boldsymbol{p})+\mathcal{D}_{11}(\boldsymbol{p})+3\mathcal{D}_{22}(\boldsymbol{p}))\Big)\Big]\\
&+\Gamma_E\Big[k_{ab}^2(a_z(\boldsymbol{k})-a_{ab}(\boldsymbol{k}))-\sigma_c(\pi_z(\boldsymbol{k})+\gamma\kappa^{-1}\chi^{-1}(k_{ab}^2\pi_{ab}(\boldsymbol{k})+k_z^2\pi_z(\boldsymbol{k})))\\
&+\gamma^{-2}a_z(\boldsymbol{k})\Big(\psi_1^2+\int_{\boldsymbol{p}}(\mathcal{D}_{11}(\boldsymbol{p})+\mathcal{D}_{22}(\boldsymbol{p}))\Big)+\gamma^{-1}\kappa^{-1}\psi_1\mathcal{D}_{22}(\boldsymbol{k})\Big],
\end{aligned}
\tag{S39}
$$

$$
\begin{aligned}
\partial_t a_{ab}(\boldsymbol{k},t) = &-\pi_{ab}(\boldsymbol{k})-\Gamma\Big[\tau\chi^{-1}\psi_1(k_{ab}^2K_\parallel(\boldsymbol{k})+k_z^2K_{X,1}(\boldsymbol{k})+K_{ab}(\boldsymbol{k}))+\gamma\kappa^{-1}\psi_1(k_{ab}^2\Phi_\parallel(\boldsymbol{k})+\gamma^{-2}k_z^2\Phi_X(\boldsymbol{k})+\Phi_{ab}(\boldsymbol{k}))\\
&+a_{ab}(\boldsymbol{k})\Big(\alpha+\gamma^2\kappa^{-2}k_{ab}^2+\kappa^{-2}k_z^2+\psi_1^2+\int_{\boldsymbol{p}}(p_{ab}^2\Phi_\parallel(\boldsymbol{p})+2\Phi_{ab}(\boldsymbol{p})+\gamma^{-2}\Phi_z(\boldsymbol{p})+\mathcal{D}_{11}(\boldsymbol{p})+3\mathcal{D}_{22}(\boldsymbol{p}))\Big)\Big],
\end{aligned}
\tag{S40}
$$

$$
\begin{aligned}
\partial_t a_z(\boldsymbol{k},t) = &-\pi_z(\boldsymbol{k})-\Gamma\Big[\tau\chi^{-1}\psi_1(k_{ab}^2K_{X,2}(\boldsymbol{k})+K_z(\boldsymbol{k}))+\gamma\kappa^{-1}\psi_1(k_{ab}^2\Phi_X(\boldsymbol{k})+\gamma^{-2}\Phi_z(\boldsymbol{k}))\\
&+a_z(\boldsymbol{k})\Big(\alpha+\gamma^2\kappa^{-2}k_{ab}^2+\kappa^{-2}k_z^2+\psi_1^2+\int_{\boldsymbol{p}}(p_{ab}^2\Phi_\parallel(\boldsymbol{p})+2\Phi_{ab}(\boldsymbol{p})+\gamma^{-2}\Phi_z(\boldsymbol{p})+\mathcal{D}_{11}(\boldsymbol{p})+3\mathcal{D}_{22}(\boldsymbol{p}))\Big)\Big].
\end{aligned}
\tag{S41}
$$

EM sector.

$$
\partial_t\Phi_\parallel(\boldsymbol{k},t)=-2K_\parallel(\boldsymbol{k}),\ \partial_t\Phi_X(\boldsymbol{k},t)=-(K_{X,1}(\boldsymbol{k})+K_{X,2}(\boldsymbol{k})),\ \partial_t\Phi_{ab}(\boldsymbol{k},t)=-2K_{ab}(\boldsymbol{k}),\ \partial_t\Phi_z(\boldsymbol{k},t)=-2K_z(\boldsymbol{k}),
\tag{S42}
$$

$$
\begin{aligned}
\partial_t K_\parallel(\boldsymbol{k},t) = &-\Pi_\parallel(\boldsymbol{k})+\Gamma_E\Big[\Big(k_z^2\Phi_\parallel(\boldsymbol{k})-k_z^2\Phi_X(\boldsymbol{k})-\Phi_{ab}(\boldsymbol{k})\Big)+\Big(\psi_1^2+\int_{\boldsymbol{p}}(\mathcal{D}_{11}(\boldsymbol{p})+\mathcal{D}_{22}(\boldsymbol{p}))\Big)\Phi_\parallel(\boldsymbol{k})\\
&-\sigma_{ab}\Big(K_\parallel(\boldsymbol{k})+\frac{\gamma}{\kappa\chi}(k_{ab}^2K_\parallel(\boldsymbol{k})+k_z^2K_{X,1}(\boldsymbol{k})+K_{ab}(\boldsymbol{k}))\Big)+\gamma\kappa^{-1}\psi_1a_{ab}(\boldsymbol{k})\Big],
\end{aligned}
\tag{S43}
$$

$$
\begin{aligned}
\partial_t K_{X,1}(\boldsymbol{k},t) = &-\Pi_X(\boldsymbol{k})+\Gamma_E\Big[\Big(k_{ab}^2\Phi_X(\boldsymbol{k})-k_{ab}^2\Phi_\parallel(\boldsymbol{k})-\Phi_{ab}(\boldsymbol{k})\Big)+\gamma^{-2}\Big(\psi_1^2+\int_{\boldsymbol{p}}(\mathcal{D}_{11}(\boldsymbol{p})+\mathcal{D}_{22}(\boldsymbol{p}))\Big)\Phi_X(\boldsymbol{k})\\
&-\sigma_c\Big(K_{X,1}(\boldsymbol{k})+\frac{\gamma}{\kappa\chi}(k_{ab}^2K_\parallel(\boldsymbol{k})+k_z^2K_{X,1}(\boldsymbol{k})+K_{ab}(\boldsymbol{k}))\Big)+\gamma^{-1}\kappa^{-1}\psi_1a_{ab}(\boldsymbol{k})\Big],
\end{aligned}
\tag{S44}
$$

$$
\begin{aligned}
\partial_t K_{X,2}(\boldsymbol{k},t) = &-\Pi_X(\boldsymbol{k})+\Gamma_E\Big[\Big(k_z^2\Phi_X(\boldsymbol{k})-\Phi_z(\boldsymbol{k})\Big)+\Big(\psi_1^2+\int_{\boldsymbol{p}}(\mathcal{D}_{11}(\boldsymbol{p})+\mathcal{D}_{22}(\boldsymbol{p}))\Big)\Phi_X(\boldsymbol{k})\\
&-\sigma_{ab}\Big(K_{X,2}(\boldsymbol{k})+\frac{\gamma}{\kappa\chi}(k_{ab}^2K_{X,2}(\boldsymbol{k})+K_z(\boldsymbol{k}))\Big)+\gamma\kappa^{-1}\psi_1a_z(\boldsymbol{k})\Big],
\end{aligned}
\tag{S45}
$$

$$
\partial_t K_{ab}(\boldsymbol{k},t) = -\Pi_{ab}(\boldsymbol{k})+\Gamma_E\Big[(k_{ab}^2+k_z^2)\Phi_{ab}(\boldsymbol{k})-\sigma_{ab}K_{ab}(\boldsymbol{k})+\Big(\psi_1^2+\int_{\boldsymbol{p}}(\mathcal{D}_{11}(\boldsymbol{p})+\mathcal{D}_{22}(\boldsymbol{p}))\Big)\Phi_{ab}(\boldsymbol{k})\Big],
\tag{S46}
$$

$$
\begin{aligned}
\partial_t K_z(\boldsymbol{k},t) = &-\Pi_z(\boldsymbol{k})+\Gamma_E\Big[\Big(k_{ab}^2\Phi_z(\boldsymbol{k})-k_{ab}^2k_z^2\Phi_X(\boldsymbol{k})\Big)+\gamma^{-2}\Big(\psi_1^2+\int_{\boldsymbol{p}}(\mathcal{D}_{11}(\boldsymbol{p})+\mathcal{D}_{22}(\boldsymbol{p}))\Big)\Phi_z(\boldsymbol{k})\\
&-\sigma_c\Big(K_z(\boldsymbol{k})+\frac{\gamma}{\kappa\chi}(k_{ab}^2k_z^2K_{X,2}(\boldsymbol{k})+k_z^2K_z(\boldsymbol{k}))\Big)+\gamma^{-1}\kappa^{-1}\psi_1a_z(\boldsymbol{k})k_z^2\Big],
\end{aligned}
\tag{S47}
$$

$$
\begin{aligned}
\partial_t\Pi_\parallel(\boldsymbol{k},t) = 2\Gamma_E\Big[&\Big(k_z^2K_\parallel(\boldsymbol{k})-k_z^2K_{X,2}(\boldsymbol{k})-K_{ab}(\boldsymbol{k})\Big)+\Big(\psi_1^2+\int_{\boldsymbol{p}}(\mathcal{D}_{11}(\boldsymbol{p})+\mathcal{D}_{22}(\boldsymbol{p}))\Big)K_\parallel(\boldsymbol{k})\\
&-\sigma_{ab}\Big(\Pi_\parallel(\boldsymbol{k})+\frac{\gamma}{\kappa\chi}(k_{ab}^2\Pi_\parallel(\boldsymbol{k})+k_z^2\Pi_X(\boldsymbol{k})+\Pi_{ab}(\boldsymbol{k}))\Big)+\gamma\kappa^{-1}\psi_1\pi_{ab}(\boldsymbol{k})\Big],
\end{aligned}
\tag{S48}
$$

$$\partial_t \Pi_X(\boldsymbol{k},t) = \Gamma_E\Big[\Big((k_{ab}^2+k_z^2)(K_{X,1}(\boldsymbol{k})+K_{X,2}(\boldsymbol{k})) - k_{ab}^2 K_\parallel(\boldsymbol{k}) - k_z^2 K_{X,2}(\boldsymbol{k}) - K_{ab}(\boldsymbol{k}) - K_z(\boldsymbol{k}) - k_{ab}^2 K_{X,1}(\boldsymbol{k})\Big) - \sigma_{ab}\Big(\Pi_X(\boldsymbol{k}) + \frac{\gamma}{\kappa\chi}(k_{ab}^2\Pi_X(\boldsymbol{k}) + \Pi_z(\boldsymbol{k}))\Big) - \sigma_c\Big(\Pi_X(\boldsymbol{k}) + \frac{\gamma}{\kappa\chi}(k_{ab}^2\Pi_\parallel(\boldsymbol{k}) + k_z^2\Pi_X(\boldsymbol{k}) + \Pi_{ab}(\boldsymbol{k}))\Big) + \gamma\kappa^{-1}\psi_1(\pi_z(\boldsymbol{k}) + \gamma^{-2}\pi_{ab}(\boldsymbol{k})) + \Big(\psi_1^2 + \int_{\boldsymbol{p}}(\mathcal{D}_{11}(\boldsymbol{p}) + \mathcal{D}_{22}(\boldsymbol{p}))\Big)(K_{X,1}(\boldsymbol{k}) + \gamma^{-2}K_{X,2}(\boldsymbol{k}))\Big], \tag{S49}$$

$$\partial_t \Pi_{ab}(\boldsymbol{k},t) = 2Tr_0\sigma_{ab}\Gamma_E^2 + 2\Gamma_E\Big[(k_{ab}^2+k_z^2)K_{ab}(\boldsymbol{k}) - \sigma_{ab}\Pi_{ab}(\boldsymbol{k}) + \Big(\psi_1^2 + \int_{\boldsymbol{p}}(\mathcal{D}_{11}(\boldsymbol{p}) + \mathcal{D}_{22}(\boldsymbol{p}))\Big)K_{ab}(\boldsymbol{k})\Big], \tag{S50}$$

$$\partial_t \Pi_z(\boldsymbol{k},t) = 2Tr_0\sigma_c\Gamma_E^2 + 2\Gamma_E\Big[\Big(k_{ab}^2 K_z(\boldsymbol{k}) - k_{ab}^2 k_z^2 K_{X,1}(\boldsymbol{k})\Big) - \sigma_c\Big(\Pi_z(\boldsymbol{k}) + \frac{\gamma}{\kappa\chi}(k_{ab}^2 k_z^2\Pi_X(\boldsymbol{k}) + k_z^2\Pi_z(\boldsymbol{k}))\Big) + \gamma^{-2}\Big(\psi_1^2 + \int_{\boldsymbol{p}}(\mathcal{D}_{11}(\boldsymbol{p}) + \mathcal{D}_{22}(\boldsymbol{p}))\Big)K_z(\boldsymbol{k}) + \gamma^{-1}\kappa^{-1}\psi_1\pi_z(\boldsymbol{k})k_z^2\Big]. \tag{S51}$$

D. Spherical symmetry

For isotropic three-dimensional superconductors, with $\sigma_{ab} = \sigma_c = \sigma$ and $\gamma = 1$, one simplifies the equations of motion using the following ansatz:

$$\boldsymbol{a}(\boldsymbol{k},t) = i a_k(t)\boldsymbol{k},\ \boldsymbol{\pi}(\boldsymbol{k}) = i\pi_k(t)\boldsymbol{k}, \tag{S52}$$

$$\Phi_{\alpha\beta}(\boldsymbol{k},t) = \Phi_k^\parallel(t)\frac{k_\alpha k_\beta}{k^2} + \Phi_k^\perp(t)\Big(\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}\Big), \tag{S53}$$

and similar expansion holds for the other two electromagnetic tensors. With this ansatz, the initial three-dimensional problem reduces to one-dimensional. All of the newly introduced quantities are real.

The final set of equations of motion for isotropic superconductors reads:

$$\tau\partial_t\psi_1 = \int_{\boldsymbol{p}} p^2\left[\tau\chi^{-1}\pi_p - \kappa^{-1}a_p\right] - \psi_1\Big[\alpha + \psi_1^2 + \int_{\boldsymbol{p}}(\Phi_p^\parallel + 2\Phi_p^\perp + 3\mathcal{D}_{11}(p) + \mathcal{D}_{22}(p))\Big], \tag{S54}$$

$$\tau\partial_t\mathcal{D}_{11}(k,t) = 2Tr_0 - 2\mathcal{D}_{11}(k)\Big[\alpha + \kappa^{-2}k^2 + 3\psi_1^2 + \int_{\boldsymbol{p}}(\Phi_p^\parallel + 2\Phi_p^\perp + 3\mathcal{D}_{11}(p) + \mathcal{D}_{22}(p))\Big], \tag{S55}$$

$$\tau\partial_t\mathcal{D}_{22}(k,t) = 2Tr_0 - 2\mathcal{D}_{22}\Big[\alpha + \kappa^{-2}k^2 + \psi_1^2 + \int_{\boldsymbol{p}}(\Phi_p^\parallel + 2\Phi_p^\perp + \mathcal{D}_{11}(p) + 3\mathcal{D}_{22}(p))\Big] - 2\psi_1 k^2[\tau\chi^{-1}\pi_k + \kappa^{-1}a_k], \tag{S56}$$

$$\partial_t\Phi_k^{\parallel(\perp)}(t) = -2K_k^{\parallel(\perp)}, \tag{S57}$$

$$\partial_t K_k^\parallel(t) = -\Pi_k^\parallel + \Gamma_E\Big[-\sigma K_k^\parallel(1 + \kappa^{-1}\chi^{-1}k^2) + \Phi_k^\parallel\Big(\psi_1^2 + \int_{\boldsymbol{p}}(\mathcal{D}_{11}(p) + \mathcal{D}_{22}(p))\Big) + \kappa^{-1}\psi_1 k^2 a_k\Big], \tag{S58}$$

$$\partial_t K_k^\perp(t) = -\Pi_k^\perp + \Gamma_E\Big[-\sigma K_k^\perp + \Phi_k^\perp\Big(k^2 + \psi_1^2 + \int_{\boldsymbol{p}}(\mathcal{D}_{11}(p) + \mathcal{D}_{22}(p))\Big)\Big], \tag{S59}$$

$$\partial_t\Pi_k^\parallel(t) = 2Tr_0\sigma\Gamma_E^2 + 2\Gamma_E\Big[-\sigma\Pi_k^\parallel(1 + \kappa^{-1}\chi^{-1}k^2) + K_k^\parallel\Big(\psi_1^2 + \int_{\boldsymbol{p}}(\mathcal{D}_{11}(p) + \mathcal{D}_{22}(p))\Big) + \kappa^{-1}\psi_1 k^2\pi_k\Big], \tag{S60}$$

$$\partial_t\Pi_k^\perp(t) = 2Tr_0\sigma\Gamma_E^2 + 2\Gamma_E\Big[-\sigma\Pi_k^\perp + K_k^\perp\Big(k^2 + \psi_1^2 + \int_{\boldsymbol{p}}(\mathcal{D}_{11}(p) + \mathcal{D}_{22}(p))\Big)\Big], \tag{S61}$$

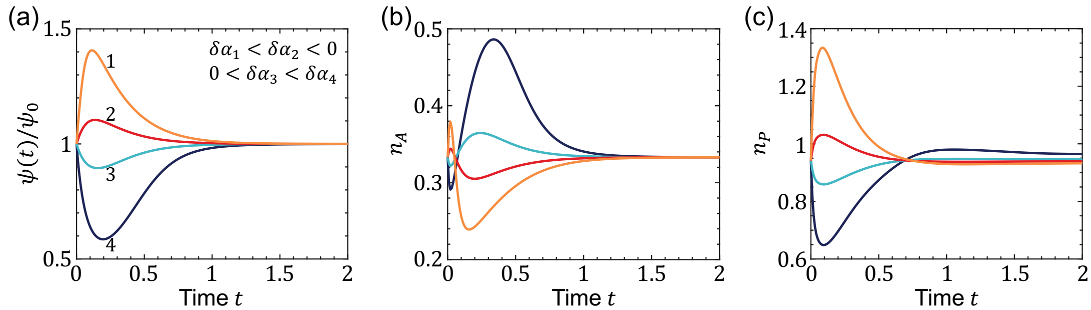

Figure S1. Photoexcitation dynamics in isotropic superconductors below T_c – extension of Fig. 3 (bottom panels) of the main text. (a) Dynamics of the long-range order parameter expectation value $\langle\psi\rangle(t)$: for $\delta\alpha < 0$ ($\delta\alpha > 0$), it becomes transiently enhanced (suppressed) and then exponentially returns to its pre-pulse value ψ_0. We decompose $\langle|\psi|^2\rangle(t) = \psi^2(t)+n_A(t)+n_P(t)$, where $n_A(t)$ represents the longitudinal order parameter fluctuations (b) and $n_P(t)$ describes the transverse fluctuations (c).

$$\partial_t \pi_k(t) = -\Gamma\Big[\tau\chi^{-1}\psi_1\Pi_k^{\parallel} + \kappa^{-1}\psi_1 K_k^{\parallel} + \pi_k\Big(\alpha + \kappa^{-2}k^2 + \psi_1^2 + \int_{\boldsymbol{p}} (\Phi_p^{\parallel} + 2\Phi_p^{\perp} + \mathcal{D}_{11}(p) + 3\mathcal{D}_{22}(p))\Big)\Big]$$
$$+ \Gamma_E\Big[a_k\Big(\psi_1^2 + \int_{\boldsymbol{p}} (\mathcal{D}_{11}(p) + \mathcal{D}_{22}(p))\Big) - \sigma(1+\kappa^{-1}\chi^{-1}k^2)\pi_k + \kappa^{-1}\psi_1\mathcal{D}_{22}(k)\Big], \tag{S62}$$

$$\partial_t a_k(t) = -\pi_k - \Gamma\Big[\tau\chi^{-1}\psi_1 K_k^{\parallel} + \kappa^{-1}\psi_1\Phi_k^{\parallel} + a_k\Big(\alpha + \kappa^{-2}k^2 + \psi_1^2 + \int_{\boldsymbol{p}} (\Phi_p^{\parallel} + 2\Phi_p^{\perp} + \mathcal{D}_{11}(p) + 3\mathcal{D}_{22}(p))\Big)\Big]. \tag{S63}$$

Supplementary Note 4: Dynamics of the order parameter fluctuations after photoexcitation in the symmetry broken phase

When considering quenches in the symmetry broken phase in the main text, we showed only the dynamics of $\langle|\psi|^2\rangle(t)$, which actually contains contributions from both the long-range expectation value $\langle\psi\rangle$ and order parameter fluctuations: $\langle|\psi|^2\rangle = \langle\psi\rangle^2 + n_A + n_P$. Here $n_A(t) = \int_{\boldsymbol{p}} \mathcal{D}_{11}(p,t)$ represents longitudinal order parameter fluctuations and encompasses the order parameter amplitude; $n_P(t) = \int_{\boldsymbol{p}} \mathcal{D}_{22}(p,t)$ describes transverse fluctuations and encodes essentially the order parameter phase. Figure S1 shows the dynamics of each of these quantities.

For concreteness, we stick to quenches with $\delta\alpha < 0$ corresponding to photo-enhancement of superconductivity (yellow and red curves in Fig. S1). The order parameter dynamics is similar to that of $\langle|\psi|^2\rangle(t)$: $\langle\psi\rangle$ is first transiently enhanced and then exponentially restores to its pre-pulse value ψ_0. The stronger the photoexcitation, the stronger the order parameter develops.

The evolution of the longitudinal fluctuations is governed by three stages. During the first quick stage, $n_A(t)$ slightly proliferates because transiently the Ginzburg-Landau free energy becomes steeper. During the second stage, $n_A(t)$ becomes suppressed due to the development of the order parameter expectation value $\langle\psi\rangle(t)$, which renders amplitude fluctuations energetically costly. The final stage is the recovery to the equilibrium state.

The evolution of the transverse fluctuations is different: initially, when the free energy becomes steeper, $n_P(t)$ proliferates and then seemingly recovers to its equilibrium value, following the trend of $\langle\psi\rangle(t)$. However, before actually recovering, $n_P(t)$ transiently becomes slightly suppressed compared to its equilibrium value. This final stage can be understood as follows. In contrast to the amplitude fluctuations, the phase fluctuations are linearly coupled to the electromagnetic field, resulting in the development of a plasma gap. Thus, the initial rise of $n_P(t)$ can be interpreted as a proliferation of plasmons at all length scales. Since at longer times $\langle|\psi|^2\rangle(t)$ exceeds its equilibrium value and since this quantity defines the plasmon frequency at equilibrium, it renders the plasmons to be energetically costly, resulting in their eventual depopulation.

Supplementary Note 5: Dephasing within the Scenario I

In the main text, we primarily studied the situation where the photoexcitation results in a sudden quench of the quadratic coefficient $\alpha(t)$ of the superconducting free energy. Although $\alpha(t)$ shows abrupt dynamics, the evolution of the superconducting order parameter is relatively smooth. As it evolves, it excites the entire plasmon continuum through the generation of momentum conserving plasmon pairs. For smoother order parameter dynamics, the high energy, high momentum modes are less excited. For this reason, the dephasing effect is relatively weak and many cycles of bi-plasmon oscillations are visible before their decay.

In the phenomenological model, if one chooses an extremely large order parameter relaxation time, as encoded in τ, then the resulting bi-plasmon oscillations will be suppressed. This is because even the low momenta plasmons are not being excited in this regime, which is the case of adiabatic order parameter dynamics. On the contrary, if τ is extremely small, then the order parameter displays abrupt evolution. In this case, the amplitude of bi-plasmon oscillations is large, but their lifetime is small due to dephasing. This damping is also stronger for superconductors with small Ginzburg parameter κ and, as such, steeper plasmon dispersion, cf. Supplementary Note 2.

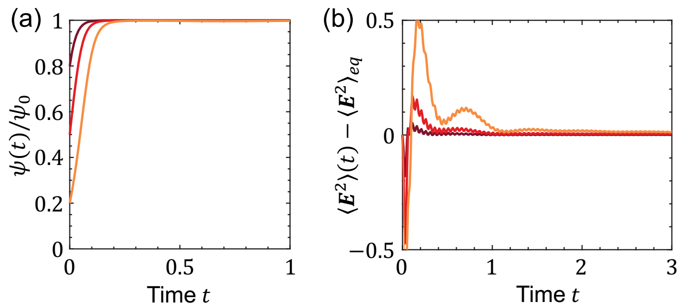

Figure S2. Post-pulse dynamics within the Scenario I in isotropic superconductors below T_c. To mimic the photoexcitation event, we choose the initial state to be thermal, but then we take the order parameter expectation value $\psi(t=0^+)$ to be reduced compared to the pre-pulse value ψ_0. (a) Order parameter $\psi(t)$ displays exponential recovery to the equilibrium value ψ_0. (b) The electric field variance, $\langle \boldsymbol{E}^2\rangle(t) - \langle \boldsymbol{E}^2\rangle_{eq}$, shows periodic dynamics with frequency being twice the plasmon gap. Notably, the lifetime of the oscillations here is smaller compared to the more smooth quenches considered in the main text. Parameters used: $\tau = 1$, $\tau_E = 1$, $\chi^{-1} = 0.1$, $\kappa = 5$, $\sigma = 0.1$, $Tr_0 = 10^{-2}$.

To illustrate the dephasing effect, we consider a situation where the photoexcitation partially evaporates the equilibrium order parameter in a sudden manner, and we choose relatively small κ. Specifically, we prepare the initial state to be thermal below T_c, but then we choose $\langle\psi\rangle(t=0^+)$ to be smaller than the equilibrium value ψ_0. As such, the order parameter dynamics is abrupt [Fig. S2(a)], resulting in a rather strong damping of the bi-plasmon oscillations [Fig. S2(b)]. We remark that the evolution in Fig. S2(b) also displays superficial high-frequency oscillations. Those oscillations arise due to the momentum cutoff chosen in our simulation and indicate that all plasmons up to the highest momentum modes are notably excited by the abrupt change in the order parameter.

Supplementary Note 6: Induced periodic dynamics in regime III

For temperatures $T > T^*$, plasmons are overdamped. However, one can imagine that an impulsive optical quench can induce periodic dynamics. If the order parameter relaxation rate is low, then photoexcitation can result in a transient enhancement of superconducting fluctuations, which only slowly recover to equilibrium. The developed expectation value $\langle|\psi|^2\rangle(t)$ provides the necessary ground to form lasting bi-plasmon oscillations in out-of-equilibrium. As shown in Fig. 3(b) of the main text, we indeed find that such a quench results in oscillatory dynamics of the electromagnetic field. Fourier analysis of those oscillations indicates that the frequency is rather poorly defined. This is because the quasiparticle conductivity is large, so that those out-of-equilibrium oscillations are damped, and also because $\langle|\psi|^2\rangle(t)$ is a time-evolving quantity, which translates as the plasmon frequency is being changed in time.

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