Recomputing Sanskrit Astronomical Tables: The *Amṛtalaharī* of Nityānanda (*c.* 1649/50 CE)

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1. Introduction

Astronomical tables (*koṣṭhakas* or *sāraṇīs*) begin to appear in Sanskrit astral sciences from around the twelfth century CE. These tables described different calendrical quantities (like the division of synodic lunar months or the lunar mansions), a variety of mathematical and trigonometric relations, and the planetary positions and motions. By the early modern period of Indian history, the corpus of Sanskrit astronomical tables had grown to reflect incredible ingenuity in the way complex calendrical and planetary elements were calculated and represented. In Mughal India,[1] as medieval Islamicate astronomy began interacting with Sanskrit mathematical astronomy, the computational practices of Sanskrit astronomers started to reflect this exchange of ideas. It is in this historical context that we find the *Amṛtalaharī* of Nityānanda.

Nityānanda was a seventeenth-century Sanskrit astronomer at the court of the Mughal emperor Shāh Jāhān (r. 1592 to 1666 CE). He was commissioned by Āsaf Khān, the emperor's chief minister (*vazīr*), to translate into Sanskrit the *Zīj-i Shāh-Jahānī*, an enormous compilation of Persian astronomical tables prepared by Mullā Farīd al-Dīn Masʿūd al-Dihlavī in October 1629 CE. Nityānanda dedicated himself to the task and in the early 1630s, he completed his translation the *Siddhāntasindhu* 'Ocean of Siddhāntas'.[2] Around a decade later, in 1639 CE, Nityānanda published his canonical treatise (*siddhānta*)

* Preliminary numerical computations were done with the assistance of Zachary Hynd (Seequent, New Zealand).

[1] Mughal India refers to the cosmopolitan society under the rule of the Mughal emperors (1526 to 1857 CE) where artistic, scientific, and linguistic exchanges between Islamicate (Arabic and Persian) and Sanskrit scholars flourished for over three hundred years, see Truschke, *Culture of Encounters*.

[2] The four complete extant manuscripts of the *Siddhāntasindhu*, one bearing the seal of Emperor Shāh Jāhān himself, are currently held at the City Palace Museum Library in Jaipur, India. These manuscripts are over 450 folia each and contain vast numbers of mathematical, astronomical, and astrological tables of different kinds, see Pingree, *A Descriptive Catalogue*, pp. 138–43.

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the *Sarvasiddhāntarāja* 'King of all Siddhāntas' as an attempt to explain Islamicate (Ptolemaic) astronomical models and parameters in the language of a traditional Sanskrit *siddhānta*.[3] Misra, *The Golādhyāya*, pp. 12–17, discusses the scientific milieu of Mughal India in which Nityānanda lived and worked.

A short paper by David Pingree brought Nityānanda's *Amṛtalaharī* to my attention.[4] The *Amṛtalaharī* is a collection of astronomical tables for computing Indian calendrical elements, planetary positions, and ascensions of zodiacal signs. Pingree made some insightful observations on how the *Amṛtalaharī* was an experiment in bringing elements of Islamicate and Sanskrit astronomy together. The list below summarises some of his main remarks on the tables of the *Amṛtalaharī* (based on MS Sanskrit 19 from the collection of the University of Tokyo).

1. The name *Amṛtalaharī* is reconstructed. As the incomplete incipit on f. 1v indicates, Nityānanda may have called his work *Kheṭakṛti*. However, the manuscript catalogue of the collection of the University of Tokyo identifies this work as the *Amṛtalaharī*, and accordingly, I follow Pingree in referring to this work with its catalogued name.
2. Brief notes (in the paratext surrounding the tables) refer to earlier Sanskrit works, e.g. Makaranda's *Makaranda* (1428 CE) is mentioned in the paratext surrounding the *tithi* tables on f. 2r.[5] There are also certain calendrical elements that, according to Pingree, are Nityānanda's own inventions. For instance, the mean motion tables employ a lunar-solar calendar equivalent to three Metonic cycles of 57 solar years found in Jewish calendars (and explained in Islamicate *zījes*).[6]
3. Pingree conjectures the epoch of the *Amṛtalaharī* as 21 February Julian in 1593 CE. According to him, the epoch year 1593 is the beginning of the 57-year long period within which the work was composed. This puts the *terminus ante quem* of the work around 1649/50 agreeing with Nityānanda's floruit in the early parts of the seventeenth century.
4. Certain features of the lunar and planetary tables of the *Amṛtalaharī* closely resemble those seen in similar tables from Islamicate and Ptolemaic traditions, and mostly absent in Sanskrit astronomical works, e.g. tabulating the mean motions of the anomalies of the Moon, Venus,

[3] See Pingree, 'Indian Reception', pp. 476–80; Pingree, 'The Sarvasiddhāntarāja'; Montelle et al., 'Computation of Sines', and Montelle and Ramasubramanian, 'Determining the Sine'.

[4] Pingree, 'Amṛtalaharī'. In Misra, *The Golādhyāya* only two works are credited to Nityānanda, the *Sarvasiddhāntarāja* and the *Siddhāntasindhu*. The existence of the *Amṛtalaharī* was unknown to me at the time.

[5] See Pingree, 'Amṛtalaharī', footnote 9 on p. 210.

[6] ibid., pp. 211–12.

and Mercury, or the positive norming in the tables of planetary equations.

Pingree concludes his paper with the remark:[7]

> 'It remains unclear why Nityānanda wrote it [the *Amṛtalaharī*]; indeed, it is indeed [*sic*] astonishing that even one copy of this unusual attempt to reform siddhāntic astronomy has survived. It is a curiosity, but perhaps it played some role in history by suggesting to Jayasiṃha's astronomers how they might express de La Hire's Latin tables, which use the Julian and Gregorian calendars, in the form of an adjusted Indian calendar.'

To understand better the implication of Nityānanda's 'attempt to reform siddhāntic astronomy', I recompute and analyse a set of astronomical tables from the *Amṛtalaharī* in this study. My goal is to recompute the *attested* values seen in the manuscript (MS Sanskrit 19) instead of suggesting the *correct* values derived from historically apposite procedures. By identifying the computational methods (including irregularities) and analysing the differences between the attested values and our recomputed results, we can gain an insight into the subtle mathematical practices of *table authors*.[8] The analytical and historical methods applied in this study demonstrate how numerical tables can be seen as mathematical artefacts in the transmission of scientific knowledge between cultures.

Section 2 begins with a description of the source manuscript and a general overview of the tables of the *Amṛtalaharī*. Following this, I describe the set of six tables selected for this study (hereafter referred to as the 'selected corpus') and provide an English translation of the Sanskrit text associated with these tables. Towards the end of the section, I discuss my methodological framework to study the selected corpus, and also describe the mathematical standards adopted in this study. In Section 3, for each table, I first outline my recomputation strategies (including any irregular recomputations that exactly reproduce an attested value), and then analyse the differences between the attested values and my recomputed results individually. These discussions also include a few proposed emendations to the attested values based on my

[7] ibid., p. 213.

[8] In this study, I use the term *table authors* to indicate those historical actors who change certain numerical values based on their own computational decisions as they recopy a table. Other actors, like scribes, who copy the tables without making any computational changes are set apart. This separation is made for expedient reasons; it is not an attempt to divide them into mutually exclusive categories. In fact, both kinds of actors modify a table as they copy it (e.g. through their inadvertent oversights in copying); however, what sets them apart in this study are recomputational interventions. Scribes and table authors may both intentionally intervene to rectify a corrupted/illegible/missing entry, but table authors (often) do so by applying a mathematical algorithm (e.g. interpolating) whereas scribes may simply fill in the numbers by observing a pattern. More on this in Section 2.3.

analysis, particularly, when an inadvertent or intentional scribal effect is evident. Finally, in Section 4, I summarise the main observations of this study, and discuss the methodological questions that arise when recomputing historical tables using modern computational tools.

2. The *Amṛtalaharī* of Nityānanda

2.1. Description of the digitised microfilm

For this study, I have used a digital copy of the only verified manuscript of the *Amṛtalaharī* currently known to be extant.[9] MS Sanskrit 19 (henceforth identified with the siglum 'Tk') is a part of the Sanskrit manuscript collections of the University of Tokyo and contains the tables of the *Amṛtalaharī*.[10] The digitised microfilm of MS Tk[11] contains the (catalogue?) reference numbers 547 (old) and 19 (new) at the very beginning of the reel. The second frame captures an image of the cover page of the manuscript with the word '*Amṛtalaharī*' (in the centre) and number '13' (at the top-left corner) written in Sanskrit. The handwriting on the cover page is notably different from that of the scribe who copied this manuscript. I suspect an earlier cataloguer, or perhaps Prof. Junjirō Takakusu, who brought the manuscript from Nepal to Japan in 1913,[12] wrote this on the cover page of the manuscript. The reel number of the microfilm (MF_13) and the catalogued name *Amṛtalaharī*[13] appear to be based on this writing on the cover page. All remaining frames contain images of two folia of the manuscript, one above the other, with the digital stamp General Library. University of Tokyo at the bottom-right corner of each frame. Plates 8 and 9 show ff. 1v–2r from MS Tk, photographed by Taro Mimura in January 2021, as examples.

2.1.1. *Manuscript description from surrogate*

According to Matsunami's catalogue,[14] MS Tk is a paper manuscript with 51 folia of dimensions 11 × 5 inches that contain a collection of Sanskrit

[9] An unconfirmed manuscript of a work called *Amṛtalaharīsāraṇī* (of unknown authorship) is catalogued in the collection of the Nepalese-German Manuscript Cataloguing Project maintained by the University of Hamburg (https://catalogue.ngmcp.uni-hamburg.de/receive/aaingmcp_ngmcpdocument_00002491). At the time of writing this chapter, I have been unable to independently verify the authenticity or the contents of this manuscript.

[10] MS Tk is referenced in Matsunami, *A Catalogue of the Sanskrit Manuscripts*, pp. 8–9.

[11] This is available at http://picservice.ioc.u-tokyo.ac.jp/03_150219~UT-library_sanskrit_ms/MF13_03_004~MF13_03_004/?pageId=001.

[12] See Pingree, 'Amṛtalaharī', p. 210.

[13] See Matsunami, *A Catalogue of the Sanskrit Manuscripts*, pp. 8–9.

[14] ibid.

astronomical tables in Devanāgarī. I list below some additional features of MS Tk from its digital surrogate.

1. The folio edges of the manuscript are frayed. There are no visible binding marks or string holes suggesting, perhaps, that the stacked folia were merely wrapped in cloth and held together between (wooden?) cover boards (resembling loose-leaf, unbound books called *pothīs*). The handwriting is in black ink, legible, free of any corrections, and produced by a single scribal hand. The tables themselves appear to be written between (faint) double-ruled margins and, in a few instances, the paratext and numbers extend into the margins. The folio numbers are written on verso pages, towards the middle of the page in the right margin.
2. On f. 1v, an incomplete incipit verse is partially visible along the frayed top edge of the folio. It contains the last three quatrains (*pādas*) (of an incomplete verse in the *indravaṃśā* meter):

   yā paṇḍitair indrapurī virājate |
   śrīdevadattasya suto dvijānugaḥ
   tasyāṃ vasan khetakṛtiṃ cikīrṣati ||

   The [city of] Indrapurī that appears beautiful with [the presence] of scholars (*paṇḍita*), the 'twice-born' (*dvija*) [i.e. Brahmin] son of Śrī Devadatta resident in that city desires to complete [this work called] Kheṭakṛti.
3. Towards the top-left corner of f. 1v, the title ⟨*A*⟩*mṛtalaharī* (the initial *a* is lost to the frayed edge of the folio) appears in the left margin. According to Pingree, it is written by a different hand compared to the copyist of the manuscript.[15] The digital surrogate makes it difficult to validate this claim with surety; however, I believe this was written by the same hand as the main copyist. The shape of the remaining letters in the word matches the chirography of the primary scribe.
4. The Sanskrit numerals, along with the paratext, table titles, and row headings in Sanskrit, are written in the Nepālī (Pracalita Lipi, Newar, or Nepāla Lipi) and Devanāgarī scripts, occasionally, conflating the two scripts together. For example, Table 1 shows samples of Sanskrit numbers in MS Tk written in Pracalita Lipi and Devanāgarī.
5. The paratext surrounding the tables also use the two scripts in an intermixed manner. On the top-left corner of f. 1v (below the frayed top edge), the words of the incipit ...यापंडितैरिंद्रपूरी... (...*yā paṃḍitair iṃdrapūrī*...) are in Pracalita Lipi. In other places, identical Sanskrit words are written variously in Pracalita Lipi or Devanāgarī. For example, the compounded words ...धनमृणं... (...*dhanam ṛṇaṃ*...) on lines 3–4

[15] See Pingree, 'Amṛtalaharī', p. 210.

Script	Sample
Hindu-Arabic Numerals	0 1 2 3 4 5 6 7 8 9
Devanāgarī Numerals	० १ २ ३ ४ ५ ६ ७ ८ ९
Pracalita Lipi Numerals	𑑐 𑑑 𑑒 𑑓 𑑔 𑑕 𑑖 𑑗 𑑘 𑑙
Digits ‘0–9’ in Devanāgarī (from f. 14r)	
Digits ‘0–9’ in Pracalita Lipi (from f. 1v)	
Mixed scripts (from f. 1v): Numbers ‘25–29’ in Pracalita Lipi (top line) Numbers ‘35–39’ in Devanāgarī (bottom line)	

Table 1: Samples from MS Tk showing Sanskrit numbers written in different scripts.

of the text block to the right of f. 1v are in Pracalita Lipi, while the same set of words ...धनमृणं... (...*dhanam ṛṇaṃ*...) on line 4 of the text block to the right on f. 2r are in Devanāgarī.

6. When letters in the two scripts are homoglyphic (in handwritten Sanskrit), the scribe appears to write the letters using Pracalita Lipi, e.g. the letter *la*, seen as ल in MS Tk, is closer in appearance to the letter 𑐮 in Pracalita Lipi than the letter ल in Devanāgarī.
7. Finally, the number ‘1’ is written at the beginning of every table title. In Prachalita Lipi, the number 𑑑 stands for the Sanskrit invocation *siddhir astu* ‘may there be success’ as a benedictory supplication.[16]

2.2. Overview of the tables of the *Amṛtalaharī*

Table 2 includes an overview of the types and foliation of the tables of the *Amṛtalaharī* in MS Tk. A more detailed description of these tables, and the different table parameters in each instance, can be found (in the Appendix) in Pingree, ‘Amṛtalaharī’, pp. 214–17.

My study focuses on the collection of tables seen in row VI of Table 2. The selected corpus includes the table of Sines (*kramajyā*);[17] the table of solar declinations (*krānti*); the three tables of shadow lengths for gnomons (*śaṅku-chāyā*) of heights 60 digits, 12 digits, and 7 digits; and the table of lunar latitudes (*śara*).

For each of these six different tables, the arguments range from 1° to 90° in one-degree steps, and their corresponding values are expressed in sexages-

[16] See Sircar, *Indian Epigraphy*, pp. 92–97 for a discussion on auspicious marks in Indian texts and epigraphs.

[17] Throughout this chapter, I use capitalised initials for trigonometric functions ‘Sine’, ’Cosine’, ’Chord’, etc. to indicate a non-unitary radius $\mathcal{R}$ (*sinus totus*). Mathematically, $\mathrm{Sin}\,\theta \equiv \mathcal{R}\sin\theta$, $\mathrm{Cos}\,\theta \equiv \mathcal{R}\cos\theta$, $\mathrm{Crd}\,\theta \equiv \mathcal{R}\,\mathrm{crd}\,\theta$, etc.

Number	Table types	Foliation
I.A	Tables of *tithis*.[18]	Ff. 1v–6v
I.B	Tables of *nakṣatras*.[19]	Ff. 7r–11v
I.C	Tables of *yogas*.[20]	Ff. 11v–17v
II	Tables of *abdapas* and *saṅkrāntīs*.[21]	Ff. 17v–18r
III	Tables of planetary mean motions of the Sun, the Moon, Lunar anomaly, Lunar node, Mars, Mercury's anomaly, Jupiter, Venus' anomaly and Saturn.	Ff. 18v–27r
IV	Tables of planetary equations: (a) *manda* equations for the Sun and the Moon; and (b) the set of first *śīghra*, *manda*, and second *śīghra* equations for the five star-planets.[22]	Ff. 27v–44r
V	Tables of rising times of zodiacal signs (right and oblique ascensions).	Ff. 44v–49r
VI	Tables of (a) Sines (here: Table VI.A); (b) solar declinations (Table VI.B); (c) shadow lengths of gnomons of heights 60 digits (Table VI.C_1), 12 digits (Table VI.C_2), and 7 digits (Table VI.C_3); and (d) lunar latitudes (Table VI.D).	Ff. 49v–50v
VII	Tables of adjustments for the five star-planets.	F. 51r

Table 2: An overview of the tables of the *Amṛtalaharī* in MS Tk.

[18] A *tithi* is the thirtieth part of a synodic lunar month, or the time interval during which the longitudinal difference between the Sun and the Moon increases by 12°.

[19] A *nakṣatra* (or lunar mansion) is the constellation in which the Moon is located. Typically, Sanskrit astronomy lists 27 *nakṣatras* each spanning 13°20′ along the 360° orbit of revolution of the Moon.

[20] A *yoga* (or *nityayoga* 'daily yoga') is the duration in which the combined motions of the Sun and the Moon amount to 1 *nakṣatra* or 13°20′. There are 27 identified *yogas* corresponding to the 27 *nakṣatras*.

[21] The *abdapas* are the weekdays on which particular years commence, and *saṅkrāntīs* refer to the solar ingress (*saṅkramaṇa*) into the 12 zodiacal signs (*rasis*) and 27 lunar mansions (*nakṣatras*).

[22] In Indian astronomy, the *manda-saṃskāras* are the equation-of-centre corrections applied to the mean longitude of the planets (*madhyama-grahas*) to produce the *manda*-corrected longitudes or *manda-sphuṭa-grahas*. In case of the Sun and the Moon, this is the only correction required to obtain their true longitudes (*sphuṭa-grahas*). However, for the other five star-planets—the two interior planets Mercury and Venus and the three exterior planets Mars, Jupiter, and Saturn—an additional *śīghra-saṃskāra* (correction due to the anomaly of conjunction) is applied to their *manda-sphuṭa-grahas* to obtain their true longitudes. For exterior planets, the *manda-sphuṭa-grahas* are their true heliocentric longitudes and the *śīghra-saṃskāra* converts these values to their true geocentric longitudes. For interior planets, the *manda-sphuṭa-graha* is the *manda*-corrected mean Sun that gets *śīghra*-corrected to produce their true geocentric longitudes.

imal numbers (up to a fractional precision of seconds). The six tables are identically arranged over three folia (ff. 49v–50v) of MS Tk. Each folio has thirty arguments in the first row, followed by six successive rows listing the corresponding six function values (i.e. the attested values of each table) in individual rows. Appendix A (pp. 226–31) includes the images of ff. 49v–50v from MS Tk and a diplomatic transcription of the six tables on these folia.

2.2.1. *Translation of the table titles*

The three table titles (seen at the top of ff. 49r–50v of MS Tk respectively) are presented below. The Sanskrit text is transliterated with Latin characters and also translated into English.

> || 1 *atha kramajyā-krānti-ṣaṣṭyaṅgula-śaṅku-dvādaśāṅgula-saptāṅgula-śaṅku-chāyā-candra-śarāṃśāḥ* ||
>
> Now, the Sines (*kramajyā*); the solar declinations (*krānti*); the shadow lengths (*chāyā*) [of] 60-digit gnomon (*ṣaṣṭi-aṅgula-śaṅku*), 12-digit (*dvādaśa-aṅgula*) [gnomon], 7-digit gnomon (*sapta-aṅgula-śaṅku*); [and] the degrees of lunar latitudes (*candra-śara-aṃśa*).
>
> || 1 *pratyaṃśa-kramajyā-krānti-chāyāḥ śarāśca* ||
>
> For every degree (*aṃśa*), the Sines (*kramajyā*), the solar declinations (*krānti*), the shadow lengths (*chāyā*), and the [lunar] latitudes (*śara*).
>
> || 1 *iti pratyaṃśaka-kramajyā-kranti-chāyāḥ śarāśca samāptaḥ* ||
>
> Thus, the Sines (*kramajyā*), the solar declinations (*krānti*), the shadow lengths (*chāyā*), and the [lunar] latitudes (*śara*) for every degree (*aṃśaka*) ends.

At the bottom of f. 50v of MS Tk, we find the following text:

> || *pātonacandro bhujā kārye bhujyaṃśebhyaḥ śaro grāhyāḥ yadi pātonacandraḥ ṣaḍbhonas tadā śaraḥ saumyaḥ yadādhikas tadā yāmyaḥ* ||[23]
>
> In [taking] the longitude (*bhujā*) of the Moon (*candra*) minus the node (*pāta*), the lunar latitude (*śara*) is to be understood from the degrees fulfilling it [i.e. calculated according to the degrees of lunar elongation]. If the [longitude of] the Moon minus the node (*pāta*) is less than six signs (*ṣaḍ-bha*) [i.e. less than 180°] then the lunar latitude (*śara*) is in the northern direction (*saumya*) [i.e. north of the ecliptic plane]; if it is more [i.e. greater than 180°] then [the lunar latitude] is in the southern direction (*yāmya*) [i.e. south of the ecliptic plane].

[23] This sentence is grammatically ill-formed; for example, the attested words *bhujyaṃśebhyaḥ* (instead of *bhujāṃśebhyaḥ*) and *grāhyāḥ* (instead of *grāhyaḥ*) have orthographic defects.

2.3. Methodology of recomputation and analysis

Before describing my general methodology for recomputing and analysing individual tables, I note the following remarks on the selected corpus, and on my mathematical practice of recomputing numerical tables.

1. Ff. 49v–50v of MS Tk do not contain any instructions to compute the attested values of the six functions (Tables VI.A–VI.D). As the translations of the table titles show, the titular text merely identifies the types of tables written on a particular folio.[24] The other table titles throughout the rest of this manuscript (as well as the paratext surrounding those tables) also lack any computational instructions. Hence, the recomputation strategies used in this study are derived from other apposite Sanskrit and Islamicate sources.

   Recent studies on Nityānanda's texts,[25] and more generally, the culture of science that thrived at the Mughal courts of early seventeenth century India,[26] suggest that he was well acquainted with Islamicate (Persianate) theories in addition to Sanskrit siddhāntic astronomy.[27] His *Amṛtalaharī* uses certain parameters that are distinctly Islamicate, e.g. a *sinus totus* of 60, as well as those that are traditionally siddhāntic, e.g. an ecliptic obliquity of 24°. In fact, the *Amṛtalaharī* contains several instances that testify to Nityānanda's familiarity with (and acceptance of) both traditions of knowledge. It is, therefore, reasonable to choose recomputational methods from the Sanskrit texts (e.g. *siddhāntas*, *karaṇas*, or *koṣṭhakas*) or the Islamicate *zījes* that were in circulation in Mughal India during his time.[28]

2. Establishing an absolute agreement between the attested and recomputed values is extremely difficult, if not nearly impossible. While some differences can be explained computationally, there are other unknown factors that lead to differences between the attested and recomputed values.[29] In fact, even at the level of recomputations, the arithmetical practices of table authors (e.g. dividing mixed fractions, rounding/truncating

[24] The text at the bottom of f. 50v of MS Tk explains certain aspects of the lunar latitude (*śara*); however, it does not describe a computational procedure or algorithm.

[25] For example, Misra, *The Golādhyāya*; Montelle et al., 'Computation of Sines', and Montelle and Ramasubramanian, 'Determining the Sine'.

[26] For example, Minkowski, 'Astronomers and Their Reasons' and Truschke, *Culture of Encounters*.

[27] For example, Misra, 'Persian Astronomy in Sanskrit' and Misra, 'Sanskrit Recension of Persian Astronomy'.

[28] See Ansari, 'On the Transmission' and Ansari, 'Survey of Zījes'.

[29] See Appendix A2 *Tabular errors* in van Dalen, 'A Statistical Method', pp. 116–19 for a statistical description of the errors in numerical tables.

the fractions, approximating/interpolating between fractions, etc.) affect our own calculations at every step. The cumulative effect of these decisions create an uncertainty in precisely reproducing the attested value. In this study, all recomputed values are presented up to a level of computational efficacy that retains a residual arithmetical noise.

3. In some instances, the differences between the attested and recomputed values can indicate scribal discrepancies. Typically, these include
   (a) *inadvertent copying oversights* in the digits of an entry (or the whole entry), e.g.
      – permutation or transposition of digits/entries,
      – unwitting alteration of homoglyphic digits (due to misreading);
      – dittography, i.e. copying a sequence of digits/entries twice;
      – haplography, i.e. omitting a sequence of identical digits/entries while copying; or
      – mistranscription, i.e. a general non-purposive mistake in reading and copying an individual digit, a whole entry, or a sequence of digits/entries; and
   (b) *intentional interventions* by historical actors (scribes/table authors) to rectify corrupted/illegible/missing digits of an entry (or the whole entry), e.g.
      – ad hoc substitution, i.e. replacing illegible digits by other digits;
      – assimilation, i.e. merging digits of adjacent entries to create new entries;
      – insertion, i.e. filling missing digits or whole entries by inspecting the sequence; or
      – contamination, i.e. inserting digits/entries from elsewhere (on the folio) to fill missing entries.

It is worth noting that the lists above are neither exhaustive nor mutually exclusive. It is often the case that distinguishing between inadvertent or intentional actions is simply not possible. Moreover, even in the clearest of examples, any emendations to the attested value (that are meant to correct/rectify these actions) remain conjectural. With these caveats, it is nevertheless useful to analyse the differences between the attested and recomputed values. If a difference can be justifiably explained as the result of an inadvertent copying mistake or an intentional (but inaccurate) intervention, the attested value can be emended to a recomputed result as a *proposed emendation*.

For example, in Table VI.A, the attested digits (in the minutes places) for $\mathrm{Sin}\,16^\circ$, $\mathrm{Sin}\,17^\circ$, $\mathrm{Sin}\,18^\circ$, and $\mathrm{Sin}\,19^\circ$ are 32, 33, 32, and 32 respectively. The recomputed Sines for these arguments suggest that digits in the minutes places for each of these arguments should be 32. The abrupt increase of $+1^{\mathrm{m}}$ (for Sin 17°) in an overall monotonic sequence suggests a plausible error in coping '33' instead of '32'. The digits २ and ३ in handwritten Devanāgarī are often homoglyphic, and hence, an unwitting alteration of these digits is not uncommon. Accordingly, I emend the digits in the minutes place of Sin 17° from '33' to the recomputed result '32' in Table VI.A.

4. A particular class of intentional actions, different from the ones listed in the previous remark, are *recomputational interventions*. While scribes may intervene to correct a corrupted/illegible/missing entry following some rudimentary logic, table authors do the same but they recalculate (or estimate) the values using more elaborate mathematical procedures. Sometimes, table authors apply these mathematical procedures to intentionally intervene, but do so inattentively which leads to an erroneous result. By retracing their calculations (using historically apposite procedures instead of modern ones), we can detect the irregularities along the way that lead to the errant result. The goal of this study is to recompute the attested results, and therefore, identifying *recomputational irregularities* is an important part of the process. In my study of the selected corpus, I have identified the following kinds of recomputational irregularities:
   (a) instances where table authors (unwittingly) err in applying a mathematical procedure, e.g. misidentifying an appropriate interval when interpolating;
   (b) instances where table authors perpetuate an erroneous calculation, e.g. using an erroneous Sine to compute the solar declination; and
   (c) instances where table authors round/truncate the sexagesimal digits in a calculation inconsistently.

The six individual tables from the selected corpus are recomputed and analysed following a common methodological routine:

Routine of recomputation

1. Recompute the values of the table for the entire range of arguments using apposite historical procedures.
2. Compare the attested values (in MS Tk) and the results of the first recomputation, and note the differences (between the digits in corresponding sexagesimal places).
3. Inspect all non-zero differences, and where possible, identify any irregular recomputations that reproduce the attested values (and thereby, eliminate these differences).

4. Reassess the revised differences between the attested values and the results of the second recomputation (i.e. recomputations including irregular ones).

Routine of analysis

5. Re-examine the attested values in (the diplomatic transcription and the digital surrogate of) MS Tk for those arguments that still have large (revised) differences.
6. Identify, if possible, any copying oversights or intentional (non-recomputational) interventions in the attested value, and propose emendations or corrections to those values with justifications.

2.3.1. *Mathematical standards*

1. I follow two main mathematical standards to recompute the individual tables in this study:
   (a) recomputed sexagesimal values are reduced to the second fractional place by systematically rounding the digits in the final result instead of truncating them (at the seconds place),[30] and
   (b) recomputed Sines are chosen over attested Sines (in MS Tk) for all calculations.[31]

   Appendices C.1–2 include my statistical justifications for choosing these mathematical standards in this study.
2. When the division of sexagesimal numbers is an intermediate part of a computation, the result of the division is rounded to seconds before proceeding further. Effectively, this implies that,
   - while calculating the solar declination in Section 3.3, $\mathrm{Sin}\,\delta = \mathrm{Sin}\,\lambda \times {}^{\mathrm{Sin}\,24^\circ}/_{60}$ is computed as a sexagesimal number (rounded to seconds) before proceeding to find δ as the inverse arc of $\mathrm{Sin}\,\delta$;
   - while calculating the shadow lengths in Section 3.5, ${}^{\mathrm{Cos}\,a^\circ}/_{\mathrm{Sin}\,a^\circ}$ is computed as a sexagesimal number (rounded to seconds) before multiplying it by the different gnomon heights h to determine the value of their respective shadow lengths; and

[30] For a sexagesimal number $a; b, c, d$ with $a, b, c, d \in [0, 59]$, *systematic rounding* results in either $a; b, c$ for $d < 29$ or $a; b, c+1$ for $d \geq 30$. All calculations in this study follow this standard of systematic rounding. In contrast, *truncation* ignores the final (third) sexagesimal digit d and simply takes the result as $a; b, c$ for any value of d.

[31] Sines are required for recomputing the solar declinations (in Table VI.B), the shadow lengths for gnomons of various heights (in Tables VI.C_1–VI.C_3), and lunar latitudes (in Table VI.D).

– while calculating the lunar latitude in Section 3.7, $\mathrm{Sin}\,\beta = 4;42,25 \times {}^{\mathrm{Sin}\,\omega}/_{60}$ is computed as a sexagesimal number (rounded to seconds) before proceeding to find β as the inverse arc of Sin β.

3. In addition to this:
   (a) the lunar latitudes (in Table VI.D) are recomputed using an exact expression in lieu of an approximate one, and
   (b) the lunar latitude recomputations use $\mathrm{Sin}\,4°30' = 4;45,25$.

   Appendices C.3–4 include my statistical justifications for these choices.

3. Recomputation strategies and analyses of differences for Tables VI.A–D

Following the general methodology described above, my recomputation strategies for each of the six tables from the selected corpus, along with an analyses of the differences between the attested values and my recomputed results, are presented below in separate subsections.

3.1. Table of Sines (*kramajyā*): Recomputation strategy

The Sine table of the *Amṛtalaharī* (in MS Tk) is computed for every degree of arc from 1° to 90° and has a maximum value (*sinus totus* $\mathcal{R}$) of 60;0,0. I recompute the Sines following a sequence of interdependent mathematical operations based on arithmetical, geometrical, and trigonometric arguments. My recomputed table of Sines for the first ninety degrees of arc is presented in Table VI.A on page 233.

The *Amṛtalaharī* (in particular, MS Tk) does not describe any method to compute the Sines; however, Nityānanda's *Sarvasiddhāntarāja* (1639 CE) includes a detailed discussion on Sine computations (sixty verses including several diagrams in six sections) in the *spaṣṭādhikāra* of the *gaṇitādhyāya*, I.3: 19–85. A critical edition, English translation, and technical commentary of the verses from the first five sections can be found in Montelle et al., 'Computation of Sines', and those from the sixth section can be found in Montelle and Ramasubramanian, 'Determining the Sine'. Considering the *Amṛtalaharī* was composed almost contemporaneously with the *Sarvasiddhāntarāja* (i.e. around the first half of the seventeenth century), it is reasonable to assume that Nityānanda used analogous geometrical arguments and trigonometric formulae (including the iterative algorithm for calculating the Sine of 1°) to construct the Sine tables of the *Sarvasiddhāntarāja* and the *Amṛtalaharī*.[32]

[32] See Van Brummelen, *The Mathematics of the Heavens* and Plofker, *Mathematics in India* for a more detailed discussion on the history and development of trigonometry in India.

3.1.1. *Recomputing the Sines of the elementary arcs based on geometrical arguments*

I first compute the Sines of 90°, 72°, 60°, 54°, 45°, 36°, 30°, and 18°. In the *Sarvasiddhāntarāja* I.3: 24–54, Nityānanda computes these Sines using (**a**) geometrical arguments in a circle of radius 60, (**b**) the half-arc and double-arc formulae for Sines, and (**c**) the sum and difference laws for Sines. I list below the different expressions for calculating these Sines. All of these expressions can be derived using simple geometrical arguments; readers may refer to Montelle et al., 'Computation of Sines' where Nityānanda's derivations from the *Sarvasiddhāntarāja* are described in greater detail.

1. $\mathrm{Sin}\,90^\circ$ corresponds to the radius (*vyāsa-khaṇḍa*) of a circle, i.e. we have $\mathrm{Sin}\,90^\circ \equiv \mathcal{R} = 60;0,0,0$ (*Sarvasiddhāntarāja* I.3: 24).[33]
2. $\mathrm{Sin}\,45^\circ$ can be expressed as $\frac{1}{\sqrt{2}}\sqrt{\mathcal{R}^2} \approx 42;25,35,3$ (*Sarvasiddhāntarāja* I.3: 28). This expression is derived using the Pythagorean theorem in an inscribed right triangle at the centre of a circle of radius $\mathcal{R}$.[34]
3. $\mathrm{Sin}\,30^\circ$ can be expressed as $\frac{1}{2}\mathcal{R} = 30;0,0,0$ (*Sarvasiddhāntarāja* I.3: 24). An equilateral triangle subtended at the centre of a circle of radius $\mathcal{R}$ has sides measuring $\mathrm{Crd}\,60^\circ \equiv \mathcal{R}$. The Sine (*jyārdha* 'half the chord') corresponding to an arc (*cāpa*) of 30° is 'half the chord of double the arc', i.e. $\frac{1}{2}\,\mathrm{Crd}\,60^\circ$.[35]
4. $\mathrm{Sin}\,60^\circ$ is approximately 51;57,41,29. This value is computed using Nityānanda's procedure for the Sine of double the arc (*Sarvasiddhāntarāja* I.3: 37) for an arc of 30°. Montelle et al., 'Computation of Sines', pp. 28–29 discuss the two-step procedure for this calculation as well as Nityānanda's Sanskrit expressions of the formula for the Sine of double the arc.
5. $\mathrm{Sin}\,18^\circ$ can be expressed as $\sqrt{\left(\frac{\mathcal{D}}{4}\right)^2 + \frac{1}{4}\left(\frac{\mathcal{D}}{4}\right)^2} - \frac{1}{2}\left(\frac{\mathcal{D}}{4}\right) \approx 18;32,27,40$ (*Sarvasiddhāntarāja* I.3: 24), where the diameter $\mathcal{D} \equiv 2\mathcal{R} = 120$. Nityānanda's geometrical demonstration for this expression (in the *Sarvasiddhāntarāja* I.3: 25–27), and its equivalence to Bhāskara II's expression $\frac{1}{4}(\sqrt{5\mathcal{R}^2} - \mathcal{R})$ stated in terms of the radius $\mathcal{R}$ (in his *Jyotpatti*: 9, 1150 CE) is discussed in Montelle et al., 'Computation of Sines', pp. 18–22.[36]

[33] Montelle et al., 'Computation of Sines', p. 18.

[34] ibid., pp. 22–23.

[35] ibid., p. 18.

[36] Bhāskara II does not derive this equation; Munīśvara (*fl.* 1638 CE), in his commentary *Marīci-ṭīkā* on the *Jyotpatti*, offers a geometrical explanation for it. In fact, Munīśvara proposes the lemma *dasāśra-bhujā-vargo'yaṃ bhuja-trijyā-vadhena yuk trijyāvargo bhavet* 'The square of a side of a regular decagon together with the product of the side and the radius (of the

6. $\text{Sin}\,36^\circ$ is approximately 35;16,1,36. Like $\text{Sin}\,60^\circ$, this value is also computed using Nityānanda's procedure for the Sine of double the arc (*Sarvasiddhāntarāja* I.3: 37) for an arc of 18°.[37]
7. $\text{Sin}\,54^\circ$ is approximately 48;32,27,40. This value is computed using Nityānanda's procedure for the Sine of the difference of two arcs (*Sarvasiddhāntarāja* I.3: 49) for two arcs measuring 90° and 36°, with $\text{Sin}\,90^\circ = 60$ and $\text{Sin}\,36^\circ = 35;16,1,36$. Nityānanda's geometrical demonstration of this expression (in the *Sarvasiddhāntarāja* I.3: 50–54) is discussed in Montelle et al., 'Computation of Sines', pp. 38–41.
8. And finally, $\text{Sin}\,72^\circ$ is approximately 57;3,48,12, also using Nityānanda's procedure for the Sine of the difference of two arcs (*Sarvasiddhāntarāja* I.3: 49) for two arcs measuring 90° and 18°, with $\text{Sin}\,90^\circ = 60$ and $\text{Sin}\,18^\circ = 18;32,27,40$.[38]

3.1.2. *Recomputing the Sines for multiples of 3° of arc*

Next, I compute the Sines of multiples of 3° of arc (in a circle of radius 60). These values are calculated by successively applying the trigonometric formulae for (**a**) the Sine of half the arc and (**b**) the Sine of the sum and differences of arcs.

In his *Sarvasiddhāntarāja* I.3: 31–32 and 36, Nityānanda gives two expressions to determine the Sine of half the arc. The first method calculates the Sine in terms of the Versine (*utkramajyā*), while the second method computes it iteratively. See Montelle et al., 'Computation of Sines', pp. 23–27 for a more detailed description of these methods, including their derivations and equivalence.

As an example, $\text{Sin}\,27^\circ$ is calculated from $\text{Sin}\,54^\circ \approx 48;32,27,40$ (with the first method) as

$$\text{Sin}\,27^\circ = \text{Sin}\left(\frac{54^\circ}{2}\right) = \sqrt{\left(\frac{\text{Vers}\,54^\circ}{2}\right)^2 + \left(\frac{\text{Sin}\,54^\circ}{2}\right)^2}$$
$$\text{(where Vers}\,54^\circ = \mathcal{R} - \text{Cos}\,54^\circ)$$
$$\Rightarrow \text{Sin}\,27^\circ \approx 27;14,21,56.$$

In the *Sarvasiddhāntarāja* I.3: 41 and 49, Nityānanda also gives the expressions for the Sine of the addition of (or the subtraction between) two arcs; see Montelle et al., 'Computation of Sines', pp. 29–46 for Nityānanda's

circumscribing circle) is equal to the square of the radius' to derive an expression for $\text{Sin}\,18^\circ$, see Gupta, 'Sine of Eighteen Degrees'.

[37] Montelle et al., 'Computation of Sines', pp. 28–29.

[38] ibid., pp. 38–41.

geometrical arguments to derive these expressions. Essentially, these formulae help calculate new Sines using previously determined Sines (and corresponding Cosines). For example, $\mathrm{Sin}\,48^\circ$ is calculated from $\mathrm{Sin}\,30^\circ = 30;0,0,0$ and $\mathrm{Sin}\,18^\circ \approx 18;32,27,40$ as

$$\begin{aligned}\mathrm{Sin}\,48^\circ &= \tfrac{1}{60}\,\mathrm{Sin}\,(30^\circ + 18^\circ)\\ &= \tfrac{1}{60}(\mathrm{Sin}\,30^\circ\,\mathrm{Cos}\,18^\circ + \mathrm{Cos}\,30^\circ\,\mathrm{Sin}\,18^\circ) \approx 44;35,19,16.\end{aligned}$$

Similarly, the Sine of the difference between two arcs is calculated using the Sines (and corresponding Cosines) of the two arcs. For example, $\mathrm{Sin}\,6^\circ$ is calculated from $\mathrm{Sin}\,36^\circ = 35;16,1,36$ and $\mathrm{Sin}\,30^\circ = 30;0,0,0$ as

$$\begin{aligned}\mathrm{Sin}\,6^\circ &\equiv \tfrac{1}{60}\,\mathrm{Sin}\,(36^\circ - 30^\circ)\\ &= \tfrac{1}{60}(\mathrm{Sin}\,36^\circ\,\mathrm{Cos}\,30^\circ - \mathrm{Cos}\,36^\circ\,\mathrm{Sin}\,30^\circ) \approx 6;16,18,8.\end{aligned}$$

I calculate the Sines for the thirty arguments that are multiples of 3° of arc by successively applying the formulae for the Sine of half the arc and the Sine of the sums and differences of arcs.

3.1.3. *Recomputing the Sine of 1° of arc*

To calculate the remaining Sines, in particular, the Sines for arguments that are multiples of 2° (distinct from the multiples of 3°), e.g. $\mathrm{Sin}\,4^\circ$ or $\mathrm{Sin}\,56^\circ$, the value of Sine of 1° is essential.

Typically, in the Indian tradition, the Sines were tabulated in 24 blocks of $3^\circ45'$ (or $225'$) for the first 90° (the first quadrant) of a circle of specified radius (identified as the *trijyā* or *sinus totus*).[39] The Sine of a non-tabulated argument was calculated by interpolating between appropriate (successive) values using different interpolation (and iterative) schemes.[40]

[39] Bag, 'Sine Table' describes the different sine tables in the Indian tradition. Also, Subbarayappa and Sarma, *Indian Astronomy*, pp. 62–73 present translations and analyses of the verses (from primary sources) that discuss Sine computations from major Sanskrit texts.

[40] For example, see Hayashi, 'Āryabhaṭa's Rule' for Āryabhaṭa's rule of differences for computing Sines in his *Āryabhaṭīya* (*c.* 499 CE); Gupta, 'Second Order Interpolation', p. 88 for Brahmagupta's second-order finite-difference interpolation scheme for approximating Sines in his *Dhyānagraha* (*c.* early 7th century)—a technique also repeated in his later and more famous work *Khaṇḍakhādyaka* (665 CE); Plofker, 'An Example of the Secant Method' for Parameśvara's fixed-point iterations to compute Sines in his *Siddhāntadīpikā* (*c.* 14th century); Ramasubramanian and Sriram, *Tantrasaṅgraha*, pp. 52–68 for Āryabhaṭa's commentator Nīlakaṇṭha Somayājī's interpolation techniques to compute desired Sines in his *Tantrasaṅgraha* (1501 CE); and Sarma et al., *Gaṇita-yukti-bhāṣā*, pp. 90–102 for Jyeṣṭhadeva's demonstrations of the sine and cosine series approximations—attributed to the famous Kerala astronomer Mādhava of Saṅgamagrāma (fl. *c.* 1380/1420 CE)—in his *Gaṇitayuktibhāṣā* (*c.* 16th century).

In his *Sarvasiddhāntarāja* I.3: 60–66, Nityānanda gives three different iterative algorithms to determine the Sine of one degree as a solution to a cubic equation. Montelle and Ramasubramanian, 'Determining the Sine' discuss, in detail, Nityānanda's algebraic and geometrical rationales in using a cubic equation, his derivation of the Sine of one degree as a recursive solution of a cubic equation, as well as the historical and technical context of this derivation—including its origin in al-Kāshī's method from the 15th century.

I describe below the main steps in calculating Sin 1° following Nityānanda's first iterative method described in his *Sarvasiddhāntarāja* I.3: 60–63.[41]

1. Calculate $\mathrm{Sin}\,3^\circ$. In the *Sarvasiddhāntarāja* I.3: 66, Nityānanda expressly mentions the value of $\mathrm{Sin}\,3^\circ$ as 3;8,24,33,59,34,28,14,50; however, for the present purpose, a recomputed value (to thirds) provides an identical estimate of $\mathrm{Sin}\,1^\circ$ up to the fourth fractional place in this algorithm. The formula for the Sine of half the arc for an arc of 6° (with $\mathrm{Sin}\,6^\circ \approx$ 6;16,18,8) gives $\mathrm{Sin}\,3^\circ \approx$ 3;8,24,33.

2. Solve a cubic equation in X (with $X \equiv 2 \times \mathrm{Sin}\,1^\circ$) of the form (in modern notation)

$$X = \frac{2 \times \mathrm{Sin}\,3^\circ}{3} + \frac{X^3}{3R^2} \quad \text{or} \quad \mathrm{Sin}\,1^\circ = \frac{\mathrm{Sin}\,3^\circ}{3} + \frac{(\mathrm{Sin}\,1^\circ)^3}{3R^2}.$$

   By treating the number X as a sequence of successive sexagesimal digits $p_0, p_1, p_2, \ldots, p_n$ (up to the n^{th} level of precision), Nityānanda's first iterative method (*Sarvasiddhāntarāja* I.3: 60–63) generates the individual digits p_i for $i \in \mathbb{N}_n$ recursively in n iterations. Essentially, this method uses successive divisions of remainders to determine a progressively more accurate root of the cubic equation in $\mathrm{Sin}\,1^\circ$. According to Montelle and Ramasubramanian, 'Determining the Sine', pp. 15–16, this algorithm gives $\mathrm{Sin}\,1^\circ =$ 1;2;49,43,11 (calculated up to the fourth fractional place).

3.1.4. *Recomputing the Sines of the remaining arcs*

The Sines for all remaining integer-valued arcs between 1° and 90° can be easily recomputed with $\mathrm{Sin}\,1^\circ$ and the formulae for Sines of half the arc and the sums and differences of arcs. For example, $\mathrm{Sin}\,2^\circ$ is calculated from $\mathrm{Sin}\,3^\circ \approx$ 3;8,24,33 and $\mathrm{Sin}\,1^\circ \approx$ 1;2;49,43 as

$$\mathrm{Sin}\,2^\circ \equiv \mathrm{Sin}\,(3^\circ - 1^\circ) = \tfrac{1}{60}\,(\mathrm{Sin}\,3^\circ\,\mathrm{Cos}\,1^\circ - \mathrm{Cos}\,3^\circ\,\mathrm{Sin}\,1^\circ) \approx 2;5,38,17.$$

[41] See Montelle and Ramasubramanian, 'Determining the Sine', pp. 13–14.

3.2. Table of Sines (*kramajyā*): Analysis of differences

List of proposed emendations to the attested Sines in MS Tk:[42]

Based on inadvertent copying oversights

1. Sin 1°_{m}: $5 \to 2$ and Sin 2°_{m}: $0 \to 5$. The value of Sin 1° is an important part of the recomputation of Sines, and hence, an error in the minutes place of Sin 1° suggests an unintentional copying mistake rather than an irregular recomputation. The digits '5' and '0' in Sin 1°_{m} and Sin 2°_{m} could have been mistakenly transposed during copying; however, Sin $1^{\circ}_{\mathrm{m}} = 0$ is still a significant error.
2. Sin 17°_{m}: $33 \to 32$. Suspected alteration of homoglyphic digits '2' and '3' in handwritten Devanāgarī. Also, Sin 16°_{m}, Sin 17°_{m}, Sin 18°_{m}, and Sin 19°_{m} appear in the sequence '32', '<u>33</u>', '32', and '32' respectively so a mistranscription is just as likely.
3. Sin 37°_{m}: $16 \to 6$. Suspected dittography. Sin 36°_{m} and Sin 37°_{m} appear in the sequence '16' and '<u>16</u>' respectively.
4. Sin 50°_{u}: $46 \to 45$. Suspected mistranscription. Sin 49°_{u}, Sin 50°_{u}, and Sin 51°_{u} appear in the sequence '45', '<u>46</u>', and '46' respectively.
5. Sin 75°_{s}: $30 \to 20$. Suspected alteration of homoglyphic digits '2' and '3' in handwritten Devanāgarī.

Based on intentional interventions

6. Sin $88^{\circ}_{\mathrm{m,s}}$: $59{,}27 \to 57{,}48$. Suspected contamination. Adjacent entries Sin 88° and Sin 89° are both 59;59,27. This could also suggest a dittography; however, the entries for *all* six functions corresponding to the 88th and 89th arguments are identical in MS Tk. (Pages 230–31 show the printed reproduction and a diplomatic transcription of f. 50v from MS Tk.) I suspect a table author intentionally copied the entire column of (correct) entries corresponding to the 89th argument (from a parent manuscript) to replace a corrupted/illegible/missing column of entries for the 88th argument.

Remarks on Table VI.A

1. On f. 49v of MS Tk, the digits '2' and '0' (of the number 20) in Sin 57°_{m} have overhead marks: , $\check{2}\ddot{0}$. This could suggest a correction

[42] The subscripts 'u', 'd', 'm', and 's' are used to indicate digits in the units, degrees, minutes, and seconds place respectively. I use '→' to represent a change between digits, in other words, the digits to the left of '→' are emended to the ones on its right. I follow these conventions to indicate my proposed emendations for the rest of this chapter.

to the (digits in the) number 20; however, there are no marginal corrections visible on the folio and hence I simply record this entry as 20 in my transcription.

2. The attested and recomputed $\text{Sin}\,46°$, $\text{Sin}\,49°$, $\text{Sin}\,50°$, $\text{Sin}\,52°$, $\text{Sin}\,53°$, $\text{Sin}\,54°$, and $\text{Sin}\,57°$ differ as $\pm 1'$. My recomputations (including irregular ones) have been unsuccessful in removing this difference, and there are no discernible copying mistakes or scribal corrections in any of these instances. Therefore, I present the attested digits (in the minutes place) of these Sines in Table VI.A without suggesting any emendations.

   However, looking at Nityānanda's Sine table from his *Sarvasiddhāntarāja*,[43] we find:[44]

$$\text{Sin}\,46° = 43;\underline{9},37,23,49$$

$$\begin{array}{ll} \text{Sin}\,49° = 45;\underline{16},57,16,10 & \text{Sin}\,50° = 45;\underline{57},45,35,59 \\ \text{Sin}\,52° = 47;\underline{16},50,19,22 & \text{Sin}\,53° = 47;\underline{55},\ 5,16,13 \\ \text{Sin}\,54° = 48;\underline{32},27,40,15 & \text{Sin}\,57° = 50;\underline{19},12,50,34 \end{array}$$

   The underlined digits (in the minutes place) of these values are identical to the corresponding digits of my recomputed Sines in Table VI.A. The similarity between these Sines in the *Sarvasiddhāntarāja* and the *Amṛtalaharī* alludes to a common computational nuance, or perhaps a common textual ancestor.

3. $\text{Sin}\,14°_s$, $\text{Sin}\,21°_s$, $\text{Sin}\,45°_s$, and $\text{Sin}\,65°_s$ have a difference of $+1$ between the attested values (from MS Tk) and the recomputed results. This difference appears to be the result of an unknown (and possibly, irregular) arithmetical calculation by a table author. I leave the digits (in the seconds place) of these Sines unchanged in Table VI.A.

3.3. Table of solar declinations (*krānti*): Recomputation strategy

The table of solar declination (*krānti*) of the *Amṛtalaharī* (in MS Tk) is computed for every degree of celestial (tropical or *sāyana*) longitude λ from $1°$ to $90°$ and has a maximum value (equal to the obliquity of the ecliptic ε) of $24°0'0''$. The solar declination δ is related to the celestial longitude λ with the expression

$$\text{Sin}\,\delta = \text{Sin}\,\lambda \times \frac{\text{Sin}\,\varepsilon}{\mathcal{R}} \equiv \text{Sin}\,\lambda \times \frac{\text{Sin}\,24°}{60} \quad \because\ \text{Sin}\,90° = \mathcal{R} = 60.$$

[43] MS Sans γ550, f. 19r, from the Wellcome Institute for the History of Medicine and MS Reel No. B 354/15, f. 15r, from the National Archive Kathmandu.

[44] The fifth digit '49' of $\text{Sin}\,46°$ is illegible in MS B 354/15. Also, $\text{Sin}\,54°_s$ resembles '20' in MS B 354/15.

This expression is commonly found in most Sanskrit *siddhāntas* from very early times, e.g. Brahmagupta's *Brāhmasphuṭasiddhānta* (628 CE): II.55. See Plofker, 'An Example of the Secant Method', pp. 91–92 for a simple geometric derivation of this expression applying the 'rule of three' to similar right triangles inscribed between the ecliptic and the equator. Having calculated the Sine of the declination, the method to find the arc of declination corresponding to it involves estimating the inverse arc of Sine. Several Sanskrit texts describe the method to find the inverse Sine, i.e. the arc measure (*cāpa* or *dhanus*, 'bow') corresponding to a particular Sine (*kramajyā*) value.[45]

Typically, the unknown arc θ for a given $\operatorname{Sin}\theta$ is linearly interpolated using localised Sine differences. The general algorithm of this method (in modern notation) is as follows:

1. Identify the interval $\operatorname{Sin}\theta_i < \operatorname{Sin}\theta < \operatorname{Sin}\theta_{i+1}$ for $i \in \mathbb{Z}^+_{90}$ in the table of Sines. The Sine function is a monotonic function that increases from 0 to $\mathcal{R}$ in the interval $[0°, 90°]$, and therefore, the corresponding interval of the argument θ can be identified as $\theta_i < \theta < \theta_{i+1}$ for $i \in \mathbb{Z}^+_{90}$.
2. Compute $\delta\theta$, where $\delta\theta \overset{\text{def}}{=} \theta - \theta_i$ and hence $\theta = \theta_i + \delta\theta$. The increment $\delta\theta$ can be computed from a linear incremental ratio in the unit interval $[\theta_i, \theta_{i+1}]$ as
$$\frac{\operatorname{Sin}\theta - \operatorname{Sin}\theta_i}{\theta - \theta_i} = \frac{\operatorname{Sin}\theta_{i+1} - \operatorname{Sin}\theta_i}{1}$$
$$\Rightarrow \theta - \theta_i = \frac{\operatorname{Sin}\theta - \operatorname{Sin}\theta_i}{\operatorname{Sin}\theta_{i+1} - \operatorname{Sin}\theta_i} \Rightarrow \delta\theta = \frac{\delta\operatorname{Sin}\theta}{\Delta_i\operatorname{Sin}\theta}.$$
3. Calculate θ from θ_i and $\delta\theta$ with $\theta = \theta_i + \delta\theta$.

It is worth noting that table authors are not as systematic in linearly interpolating between successive values as described above. Sometimes, certain (re)computational irregularities are easy to identify, e.g. choosing $\operatorname{Sin}\theta_{i+2} - \operatorname{Sin}\theta_{i+1}$ instead of $\operatorname{Sin}\theta_{i+1} - \operatorname{Sin}\theta_i$ in calculating $\delta\theta$. However, in other instances, table authors make intuitive choices like approximating the argument instead of interpolating it (for smaller values), making it difficult to explain an anomalous entry. My recomputations of the solar declinations attested in MS Tk admit to this level of uncertainty in a few instances.

3.3.1. *Worked example*

Calculating the solar declination δ corresponding to a celestial longitude λ of $52°$:

[45] For example, see Bhāskara II's *Karaṇakutūhala* (1183 CE): II.8 (Rao and Uma, *Karaṇakutūhalam*, p. S19) or Nilakaṇṭha's *Tantrasaṅgraha* (1501 CE): II.7 (Ramasubramanian and Sriram, *Tantrasaṅgraha*, pp. 68–70).

1. For the celestial longitude $\lambda = 52^\circ$, using the recomputed results $\mathrm{Sin}\,52^\circ = 47;16,50$ and $\mathrm{Sin}\,24^\circ = 24;24,15$ from Table VI.A,
$$\mathrm{Sin}\,\delta(52^\circ) = \mathrm{Sin}\,52^\circ \times \frac{\mathrm{Sin}\,24^\circ}{60} \approx 19;13,50,33.$$
2. To determine the arc $\delta(52^\circ)$ corresponding to a Sine of 19;13,51 (rounded to seconds), observe from Table VI.A that $\mathrm{Sin}\,18^\circ \equiv 18;32,28 < \mathrm{Sin}\,\delta(52^\circ) < \mathrm{Sin}\,19^\circ \equiv 19;32,3$. Therefore,
$$\begin{aligned}\delta(52^\circ) &= 18^\circ + \frac{\mathrm{Sin}\,\delta(52^\circ) - \mathrm{Sin}\,18^\circ}{\mathrm{Sin}\,19^\circ - \mathrm{Sin}\,18^\circ}\\ &= 18^\circ + \left[\frac{19;13,51 - 18;32,28}{19;32,3 - 18;32,28}\right]_{\text{in degrees}} = 18^\circ + \left[\frac{0;41,23}{0;59,35}\right]_{\text{in degrees}}\\ &= 18^\circ + 0^\circ 41' 40'' \text{ (rounded to seconds)} \approx 18^\circ 41' 40''\end{aligned}$$

   The recomputed solar declination corresponding to a celestial longitude of 52° is $18^\circ 41' 40''$.

Table VI.B on page 234 presents the recomputed solar declinations for every degree of celestial longitude from 1° to 90°. Most of these recomputations follow the algorithm described above; however, a few entries are calculated irregularly as described below.

3.3.2. *Recomputational irregularities in solar declination calculations*

1. Recomputing the solar declination for a celestial longitude of 28°. For $\lambda = 28^\circ$, $\mathrm{Sin}\,\delta(28^\circ) = \mathrm{Sin}\,28^\circ \times {}^{\mathrm{Sin}\,24^\circ}/_{60}$. With $\mathrm{Sin}\,28^\circ = 28;10,6$ and $\mathrm{Sin}\,24^\circ = 24;24,15$, $\mathrm{Sin}\,\delta(28^\circ) = 11.45707836 \approx 11;27,25$. A regular interval to determine the inverse arc of this Sine (by interpolation) is
$$\begin{gathered}\mathrm{Sin}\,11^\circ < \mathrm{Sin}\,\delta(28^\circ) < \mathrm{Sin}\,12^\circ\\ \Rightarrow 11;26,55 < \mathrm{Sin}\,\delta(28^\circ) \approx 11;27,25 < 12;28,29,\end{gathered}$$
   which gives $\delta \approx 11;0,30$ (rounded to seconds). However, the irregular interval
$$\begin{gathered}\mathrm{Sin}\,10^\circ < \mathrm{Sin}\,\delta(28^\circ) < \mathrm{Sin}\,12^\circ\\ \Rightarrow 10;25,8 < \mathrm{Sin}\,\delta(28^\circ) \approx 11;27,25 < 12;28,29\end{gathered}$$
   gives
$$\begin{aligned}\delta(28^\circ) &= 10^\circ + \frac{\mathrm{Sin}\,\delta(28^\circ) - \mathrm{Sin}\,10^\circ}{\mathrm{Sin}\,12^\circ - \mathrm{Sin}\,11^\circ}\\ &\approx 10^\circ + 1^\circ 0' 41'' 54''' \approx 11^\circ 0' 41'' 54'''.\end{aligned}$$
   The truncated value $\delta(28^\circ) = 11^\circ 0' 41''$ is identical to the attested value in MS Tk.

2. Recomputing the solar declinations for the celestial longitudes 2°, 7°, 12°, 15°, 18°, 37°, 43°, 45°, 49°, 53°, 55°, 61°, 64°, 72°, 74°, 78°, 80°, and 82°. The recomputed values match the attested values in MS Tk when the final results are truncated to seconds (instead of systematically rounding them to seconds), e.g.

$$\begin{array}{rcl} \delta \text{ (recomputed, up to thirds)} & & \delta \text{ (attested in MS Tk)} \\ \delta(2^\circ) = 0^\circ 48' 47'' 44''' & \longleftrightarrow & \delta(2^\circ) = 0^\circ 48' 47'' \\ \delta(18^\circ) = 7^\circ 13' 14'' 45''' & \longleftrightarrow & \delta(18^\circ) = 7^\circ 13' 14'' \end{array}$$

The truncated versions of these recomputed values are not suggested as emendations; instead, the third row of differences between the attested and recomputed values (corresponding to these arguments) in Table VI.B registers them as a difference of '-1'.

3.4. Table of solar declinations (*krānti*): Analysis of differences

List of proposed emendations to the attested solar declinations in MS Tk:

Based on inadvertent copying oversights

1. $\delta(23^\circ)_d$: $8 \rightarrow 9$. Suspected mistranscription. $\delta(22^\circ)_d$, $\delta(23^\circ)_d$, and $\delta(24^\circ)_d$ appear in the sequence '8', '$\underline{8}$', and '9' respectively.
2. $\delta(67^\circ)_s$: $25 \rightarrow 15$. Suspected alteration of homoglyphic digits '1' and '2' in handwritten Devanāgarī.
3. $\delta(70^\circ)_m$: $29 \rightarrow 28$. Suspected mistranscription. $\delta(68^\circ)_m$, $\delta(69^\circ)_m$, and $\delta(70^\circ)_m$ appear in the sequence '9', '19', and '$\underline{29}$' respectively.
4. $\delta(77^\circ)_s$: $44 \rightarrow 55$. Suspected alteration of homoglyphic digits '4' and '5' in handwritten Devanāgarī.

Based on intentional interventions

5. $\delta(88^\circ)_s$: $46 \rightarrow 4$. Suspected contamination. Adjacent entries $\delta(88^\circ)$ and $\delta(89^\circ)$ are both $23^\circ 59' 46''$. All six functions corresponding to the 88th and 89th arguments are identical in MS Tk. See note 6 on page 204.

Remarks on Table VI.B

The digits in the seconds place of the attested and recomputed solar declinations for several degrees of celestial longitudes vary by ± 1. A few entries differ by up to $\pm 4''$, with one instance of a $+5''$ variation. I suspect these differences are a result of irregular arithmetic calculations, or selecting incorrect interpolation intervals. However, I have not been able to explain these differences mathematically (or justify them as interventions/oversights), and therefore, I do not emend the attested digits (in the seconds places) of the solar declinations corresponding to these longitudes in Table VI.B.

3.5. Tables of shadow lengths (*śaṅkuchāyā*): Recomputation strategy

The tables of lengths of shadows (*chāyā*) of gnomons (*śaṅku*) of the *Amṛtalaharī* (in MS Tk) are computed for every degree of solar altitude (*lambaka*)[46] from 1° to 90° for gnomons of heights 60 digits, 12 digits, and 7 digits.[47] The shadow length (*śaṅkuchāyā*) of a gnomon of height h digits, hereafter abbreviated as $Ch\bar{a}y\bar{a}_h$, is related to the solar altitude a with the expression

$$Ch\bar{a}y\bar{a}_h\, a = h \times \frac{\operatorname{Cos} a}{\operatorname{Sin} a},$$

where $a \equiv$ solar altitude and $h \equiv$ gnomon height (in digits). A simple geometric derivation for this expression (for a 12 digit gnomon, a typical measure in Indian astronomy) is described in Ramasubramanian and Sriram, *Tantrasaṅgraha*, p. 135. Another way to interpret the shadow length is to consider the argument as the terrestrial latitude φ of an observer. The tabulated shadow lengths then represent the length of the equinoctial noon shadow cast by the gnomon (of a particular height h). On the day of the equinox, the declination of the Sun is zero and hence the diurnal path of the Sun (almost) traces the celestial equator in the sky. At midday on this day, the local zenith crossing of the Sun corresponds to the local terrestrial latitude (measured from the local zenith). Thus, the equinoctial noon shadow of the gnomon (*viṣuvatchāyā*) can be expressed as a function of the local terrestrial latitude, i.e. $h \times {}^{\operatorname{Sin}\varphi}/_{\operatorname{Cos}\varphi}$.[48] Several Sanskrit texts, beginning from very early times, describe how the shadow lengths of gnomons (for known heights) are computed, e.g. Kauṭilīya's *Arthaśāstra* (2nd century–3rd century CE) or Āryabhaṭa's *Āryabhaṭīya* (*c.* 499 CE).[49]

3.5.1. *Worked example*

Calculating the shadow length $Ch\bar{a}y\bar{a}_h$ corresponding to a solar altitude a of 52° for gnomons of heights $h = 60$ digits, 12 digits, and 7 digits:

1. For the solar altitude $a = 52°$ and gnomon height $h = 60$, $\operatorname{Sin} a \equiv \operatorname{Sin} 52° \approx 47;16,50$ and $\operatorname{Cos} a \equiv \operatorname{Cos} 52° = \operatorname{Sin}(38°) \approx 36;56,23$ (using recomputed Sines from Table VI.A). Thus,

$$Ch\bar{a}y\bar{a}_{60}\, 52° = h \times \frac{\operatorname{Cos} 52°}{\operatorname{Sin} 52°} \equiv 60 \times \frac{36;56,23}{47;16,50} \approx 46;52,38$$

[46] The solar altitude (*lambaka*) (above the horizon) is the complement of the zenith distance (*natāṃśa*) of the Sun.

[47] A digit or *aṅgula* is a unit of linear measure of a finger breadth, approximately, $(^1/_{24})^{\text{th}}$ part of a cubit (*hasta*).

[48] Ramasubramanian and Sriram, *Tantrasaṅgraha*, p. 140.

[49] See, respectively, Abraham, 'The Gnomon' and Shukla and Sarma, *Āryabhaṭīya*.

(rounded to seconds). The recomputed shadow length for a gnomon of height 60 digits and corresponding to a solar altitude of 52° is 46;52,38.

2. Similarly, for $h = 12$ and $h = 7$: *Chāyā*$_{12}$ $52° \equiv 12 \times \frac{36;56,23}{47;16,50} \approx 9;22,32$ and *Chāyā*$_{7}$ $52° \equiv 7 \times \frac{36;56,23}{47;16,50} \approx 5;28,8$. The recomputed shadow lengths for gnomons of height 12 digits and 7 digits corresponding to a solar altitude of 52° are 9;22,32 and 5;28,8 respectively. Both values are rounded to the second fractional place.

Tables VI.C$_1$ (page 235), VI.C$_2$ (page 236), and VI.C$_3$ (page 237) present the recomputed shadow lengths for every degree of solar altitude from 1° to 90° for gnomon lengths 60 digits, 12 digits, and 7 digits respectively. Most of these recomputations follow the algorithm described above; however, a few entries are calculated irregularly as described below.

3.5.2. *Recomputational irregularities in shadow-length calculations: 60-digit gnomon (Table VI.C$_1$)*

1. Recomputing the shadow length for a solar altitude of 14°. Using the attested value $\text{Sin}\,14° = 14;30,56$ from MS Tk (see Table VI.A) gives *Chāyā*$_{60}$ $(14°) = 60 \times \frac{\text{Cos}\,14°}{\text{Sin}\,14°} = 60 \times \frac{58;13,4}{14;30,56} \approx 60 \times 4;0,38,34,45 \approx$ 240;38,34,45 (truncated to seconds). This value is identical to the attested value in MS Tk. The shadow length value with the recomputed Sin 14° as 14;30,55 is 240;38,51 (rounded to seconds).
2. The recomputations of the shadow lengths for the following arguments agree with their attested values in MS Tk if irregular Sine (or Cosine) values are considered. These recomputational scenarios are seemingly random; nevertheless, I list them below for completeness. The attested or recomputed Sines stated below can be found in Table VI.A.
   (a) With the recomputed $\text{Sin}\,11° = 11;26,55$ and an *arbitrary* $\text{Cos}\,11° = \text{Sin}\,79° = 58;53,53$, *Chāyā*$_{60}$ $(11°) \approx 308;40,25$ (rounded to seconds), which agrees with the attested value in MS Tk. The shadow length with the attested/recomputed Cos 11° as 58;53,51 is 308;40,14 (rounded to seconds).
   (b) With the *attested* $\text{Sin}\,21° = 21;30,8$ and the recomputed $\text{Cos}\,21° = \text{Sin}\,69° = 56;0,53$, *Chāyā*$_{60}$ $(21°) \approx 156;18,14$ (rounded to seconds), which agrees with the attested value in MS Tk. The shadow length with the recomputed Sin 21° as 21;30,6,59 is 156;18,22 (rounded to seconds).
   (c) With an *arbitrary* $\text{Sin}\,42° = 40;8,50$ and the recomputed $\text{Cos}\,42° = \text{Sin}\,48° = 44;35,19$, *Chāyā*$_{60}$ $(42°) \approx 66;38,16$ (rounded to sec-

onds), which agrees with the attested value in MS Tk. The shadow length with the attested/recomputed Sin 42° as 40;8,52 is 66;38,12 (rounded to seconds).

(d) With an *arbitrary* Sin 43° = 40;55,16 and the recomputed Cos 43° = Sin 47° = 43;52,52, $Ch\bar{a}y\bar{a}_{60}(43^\circ) \approx 64;20,24$ (rounded to seconds), which agrees with the attested value in MS Tk. The shadow length with the attested/recomputed Sin 43° as 40;55,12 is 64;20,30 (rounded to seconds).

(e) With an *arbitrary* Sin 50° = 45;57,47 and the recomputed Cos 50° = Sin 40° = 38;34 (rounded to minutes), $Ch\bar{a}y\bar{a}_{60}(50^\circ) \approx 50;20,41$ (rounded to seconds), which agrees with the attested value in MS Tk. The shadow length with the attested Sin 50° as 46;56,46 and the recomputed Cos 50° as 38;34,2 (up to the seconds) is 49;17,29 (rounded to seconds), whereas the shadow length with the recomputed Sin 50° as 45;57,46,0 and the recomputed Cos 50° as 38;34,2 (up to the seconds) is 50;20,45 (rounded to seconds).

3. Recomputing the shadow lengths of a 60-digit gnomon for the solar altitudes 3°, 5°, 23°, 27°, 40°, 41°, 49°, 51°, 59°, 61°, 64°, 68°, 80°, 83°, 86°, and 87°. The recomputed values match the attested values in MS Tk when the final results are truncated to seconds instead of systematically rounding them to seconds), e.g.

$$\begin{array}{rcl} Ch\bar{a}y\bar{a}_{60}\,(\text{recomputed to thirds}) & & Ch\bar{a}y\bar{a}_{60}\,(\text{attested value}) \\ Ch\bar{a}y\bar{a}_{60}(3^\circ) = 1114;49,27,53 & \longleftrightarrow & Ch\bar{a}y\bar{a}_{60}(3^\circ) = 1114;49,27 \\ Ch\bar{a}y\bar{a}_{60}(49^\circ) = 52;9,26,35 & \longleftrightarrow & Ch\bar{a}y\bar{a}_{60}(49^\circ) = 52;9,26 \end{array}$$

The truncated versions of these recomputed values are not suggested as emendations; instead, the third row of differences between the attested and recomputed values (corresponding to these arguments) in Table VI.C_1 registers them as a difference of '−1'.

3.5.3. *Recomputational irregularities in shadow length calculations: 12-digit gnomon (Table VI.C_2)*

1. Recomputing the attested shadow length for a solar altitude of 4°. The shadow-lengths of gnomons of heights 60 and 12 digits are related by $Ch\bar{a}y\bar{a}_{12} = \frac{1}{5} Ch\bar{a}y\bar{a}_{60}$. For a solar altitude of 4°, using the attested value of $Ch\bar{a}y\bar{a}_{60}(4^\circ) = 859;3,48$ from MS Tk (see Table VI.A) gives $Ch\bar{a}y\bar{a}_{12}(4^\circ) = \frac{859;3,48}{5} \approx 171;48,45,36 \approx 171;48,46$ (rounded to seconds). This value agrees with the attested value 172; 48,46 in MS Tk

if the digits '172' in the units place are considered a copying oversight for '171'. (The digits '1' and '2' are homoglyphic in handwritten Devanāgarī.) Using the recomputed value $\textit{Chāyā}_{60}(4^\circ)$ as 858;3,48, $\textit{Chāyā}_{12}(4^\circ) \approx 171;36,46$ (rounded to seconds).

2. Recomputing the shadow length for a solar altitude of 24°. With the *arbitrary* shadow length $\textit{Chāyā}_{60}(24^\circ) = 134;45,5$, $\textit{Chāyā}_{12}(24^\circ) = \frac{1}{5} \times 134;45,5 = 26;57,1$, which agrees with the attested value in MS Tk. The shadow length $\textit{Chāyā}_{12}(24^\circ)$ with the attested/recomputed $\textit{Chāyā}_{60}(24^\circ)$ as 134;45,45 is 26;57,9.
3. Recomputing the shadow length for a solar altitude of 87°. The regular expression for $\textit{Chāyā}_{12}\, 87^\circ$ is $12 \times \frac{\text{Cos}\, 87^\circ}{\text{Sin}\, 87^\circ}$. However, using $\text{Cos}\, 88^\circ = \text{Sin}\, 2^\circ = 2;0,38$ (the attested value in MS Tk) instead of $\text{Cos}\,(87^\circ)$ gives $\textit{Chāyā}_{12}(87^\circ) = 12 \times \frac{\text{Cos}\, 88^\circ}{\text{Sin}\, 87^\circ} = 12 \times \frac{2;0,38}{59;55,4} = 0;24,9,35 \approx 0;24,9$ (truncated to seconds), which agrees with the attested value in MS Tk. A regular recomputation of $\textit{Chāyā}_{12}(87^\circ)$ (using recomputed $\text{Sin}\, 87^\circ$ and $\text{Cos}\, 87^\circ = \text{Sin}\, 3^\circ$) gives 0;37,44 (rounded to seconds).
4. Recomputing the shadow lengths of a 12-digit gnomon for the solar altitudes 3°, 17°, 20°, 39°, 46°, 47°, 56°, 58°, 65°, and 86°. The recomputed values match the attested values in MS Tk when final results are truncated to seconds (instead of systematically rounding them to seconds), e.g.

$$\begin{array}{rcl} \textit{Chāyā}_{12}\,(\text{recomputed to thirds}) & & \textit{Chāyā}_{12}\,(\text{attested value}) \\ \textit{Chāyā}_{12}(3^\circ) = 228;57,53,34 & \longleftrightarrow & \textit{Chāyā}_{12}(3^\circ) = 228;57,53 \\ \textit{Chāyā}_{12}(39^\circ) = 14;49,7,47 & \longleftrightarrow & \textit{Chāyā}_{12}(39^\circ) = 14;49,7 \end{array}$$

The truncated versions of these recomputed values are not suggested as emendations; instead, the third row of differences between the attested and recomputed values (corresponding to these arguments) in Table VI.C_2 registers them as a difference of '-1'.

3.5.4. *Recomputational irregularities in shadow length calculations: 7 digits (Table VI.C_3)*

1. Recomputing the shadow length for a solar altitude of 4°. The shadow-lengths of gnomons of heights 60 and 7 digits are related by $\textit{Chāyā}_7 = \frac{7}{60}\textit{Chāyā}_{60}$. For a solar altitude of 4°, using the attested value $\textit{Chāyā}_{60}(4^\circ) = 859;3,48$ from MS Tk (see Table VI.A) gives $\textit{Chāyā}_7(4^\circ) = \frac{7 \times 859;3,48}{60} \approx 100;13,26,36 \approx 100;13,27$ (rounded to seconds). This value agrees with the attested value in MS Tk. Using the re-

computed value $Ch\bar{a}y\bar{a}_{60}(4°)$ as 858;3,48 gives $Ch\bar{a}y\bar{a}_{7}(4°) \approx$ 100;6,27 (rounded to seconds).

2. Recomputing the shadow length for a solar altitude of 12°. For an *arbitrary* value of Sin 12° = 12;28,30, $Ch\bar{a}y\bar{a}_{7}(12°)$ = 32;55,54, which agrees with the attested value in MS Tk. The recomputed Sin 12° as 12;28,28,55 gives 32;55,57 (rounded to seconds).
3. Recomputing the shadow length for a solar altitude of 37°. With the recomputed Sin 37° = 36;6,32 and an *arbitrary* Cos 37° = Sin 53° = 47;56,50, $Ch\bar{a}y\bar{a}_{7}(37°) \approx$ 9;17,42 (rounded to seconds), which agrees with the attested value in MS Tk. The shadow length with the recomputed Sin 37° as 36;6,32 and the attested Cos 37° as 47;56,5 is 9;17,33 (rounded to seconds), whereas the shadow length with the recomputed Sin 37° as 36;6,32 and the recomputed Cos 37° as 47;55,5 is 9;17,21 (rounded to seconds).
4. Recomputing the shadow lengths of a 7-digit gnomon for the solar altitudes 39°, 42°, and 59°. The recomputed values match the attested values in MS Tk when final results are truncated to seconds (instead of systematically rounding them to seconds), e.g.

$$\begin{array}{rcl} Ch\bar{a}y\bar{a}_{7}\,(\text{recomputed to thirds}) & & Ch\bar{a}y\bar{a}_{7}\,(\text{attested value}) \\ Ch\bar{a}y\bar{a}_{7}(39°) = 8;38,39,40 & \longleftrightarrow & Ch\bar{a}y\bar{a}_{7}(39°) = 8;38,39 \\ Ch\bar{a}y\bar{a}_{7}(42°) = 7;46,27,48 & \longleftrightarrow & Ch\bar{a}y\bar{a}_{7}(42°) = 7;46,27 \end{array}$$

The truncated versions of these recomputed values are not suggested as emendations; instead, the third row of differences between the attested and recomputed values (corresponding to these arguments) in Table VI.C_3 registers them as a difference of '−1'.

3.6. Table of shadow lengths (*śaṅkuchāyā*): Analysis of differences

In the following subsections, I present a list of proposed emendations to the attested values of shadow lengths for gnomons of heights 60 digits, 12 digits, and 7 digits respectively.

3.6.1. *Shadow length for gnomon of height 60 digits (Table VI.C_1)*

Based on inadvertent copying oversights

1. $Ch\bar{a}y\bar{a}_{60}(10°)_s$: 24 → 34. Suspected alteration of homoglyphic digits '2' and '3' in handwritten Devanāgarī.
2. $Ch\bar{a}y\bar{a}_{60}(12°)_s$: 19 → 39. Suspected alteration of homoglyphic digits '1' and '3' in handwritten Devanāgarī.

3. *Chāyā*$_{60}(67^\circ)_{\mathrm{s}}$: $0 \rightarrow 7$. Suspected alteration of homoglyphic digits ‘0’ and ‘7’ in handwritten Devanāgarī.

Based on intentional interventions

5. *Chāyā*$_{60}(88^\circ)$: $1;2,50 \rightarrow 2;5,43$. Suspected contamination. Adjacent entries *Chāyā*$_{60}(88^\circ)$ and *Chāyā*$_{60}(89^\circ)$ are both 1;2,50. All six functions corresponding to the 88$^{\mathrm{th}}$ and 89$^{\mathrm{th}}$ arguments are identical in MS Tk. See note 6 on page 204.

Remarks on Table VI.C$_1$

1. The attested entry ‘859’ for *Chāyā*$_{60}(4^\circ)_{\mathrm{u}}$ could be emended to ‘858’ as a suspected mistranscription by a table author (or scribe). This emendation would agree with the recomputed result, and also avoid the difference of 1 integer unit between the attested and recomputed entries (a significant statistical anomaly). However, the attested shadow lengths of the 60-digit and 12-digit gnomons corresponding to 4° of solar altitude in MS Tk are computationally interrelated. The irregular recomputation of *Chāyā*$_{12}(4^\circ)$ uses 859;3,48 as the attested value of *Chāyā*$_{60}(4^\circ)$ (see note 1 in Section 3.5.3).
2. On f. 49v of MS Tk, the digit ‘0’ (of the number 30) in *Chāyā*$_{60}(23^\circ)_{\mathrm{s}}$ had a dot under it: ३०̣, 30̣. An underdot is sometimes used as a *signe de renvoi* (cancellation mark) in Sanskrit, and the recomputational evidence also suggests *Chāyā*$(23^\circ)_{\mathrm{s}} = 3$. Hence, I record the value of *Chāyā*$_{60}(23^\circ)_{\mathrm{s}}$ as 3 in my transcription.
3. The digits in the seconds place of the attested and recomputed shadow lengths of a 60-digit gnomon for certain degrees of solar altitudes (e.g. 38°, 56°, or 65°) vary by up to ± 3. I have not been able to justify these differences mathematically (or as obvious interventions/oversights), and therefore, I do not propose any emendations in Table VI.C$_1$ to change the attested digits (in the seconds place) of the shadow lengths corresponding to these arguments.

3.6.2. *Shadow length for gnomon of height 12 digits (Table VI.C$_2$)*

Based on inadvertent copying oversights

1. *Chāyā*$_{12}(4^\circ)_{\mathrm{u}}$: $172 \rightarrow 171$. Suspected alteration of homoglyphic digits ‘1’ and ‘2’ in handwritten Devanāgarī.
2. *Chāyā*$_{12}(19^\circ)_{\mathrm{m}}$: $1 \rightarrow 51$. Suspected mistranscription. *Chāyā*$_{12}(18^\circ)_{\mathrm{m}}$, *Chāyā*$_{12}(19^\circ)_{\mathrm{m}}$, and *Chāyā*$_{12}(20^\circ)_{\mathrm{m}}$ appear in the sequence ‘55’, ‘1’, and ‘58’ respectively.

3. *Chāyā* $_{12}$ $(23°)_s$: 23 → 13. Suspected alteration of homoglyphic digits '1' and '2' in handwritten Devanāgarī.

4. *Chāyā* $_{12}$ $(25°)_s$: 12 → 2. Suspected mistranscription. *Chāyā* $_{12}$ $(24°)_s$, *Chāyā* $_{12}$ $(25°)_s$ and *Chāyā* $_{12}$ $(26°)_s$ appear in the sequence '1', '<u>12</u>', and '14' respectively.

5. *Chāyā* $_{12}$ $(33°)_m$: 48 → 28. Suspected mistranscription. *Chāyā* $_{12}$ $(32°)_m$, *Chāyā* $_{12}$ $(33°)_m$ and *Chāyā* $_{12}$ $(34°)_m$ appear in the sequence '12', '<u>48</u>', and '47' respectively.

6. *Chāyā* $_{12}$ $(43°)_m$: 55 → 52. Suspected mistranscription. *Chāyā* $_{12}$ $(42°)_m$, *Chāyā* $_{12}$ 43°, and *Chāyā* $(44°)_m$ appear in the sequence '19', '<u>55</u>', and '25' respectively.

7. *Chāyā* $_{12}$ $(53°)_s$: 34 → 14. Suspected alteration of homoglyphic digits '1' and '3' in handwritten Devanāgarī.

8. *Chāyā* $_{12}$ $(68°)_s$: 4 → 54. Suspected mistranscription (perhaps, an inadvertent omission of the digit '5' in '54').

9. *Chāyā* $_{12}$ $(86°)_s$: 29 → 21. Suspected alteration of homoglyphic digits '1' and '9' in handwritten Devanāgarī.

10. *Chāyā* $_{12}$ $89°_m$: 32 → 12 and *Chāyā* $_{12}$ $89°_s$: 24 → 34. Suspected alteration of homoglyphic digits '1', '2', and '3' in handwritten Devanāgarī.

Based on intentional interventions

11. *Chāyā* $_{12}$ (88°) : 0;32,24 → 0;25,9. Suspected contamination. Adjacent entries *Chāyā* $_{12}$ (88°) and *Chāyā* $_{12}$ (89°) are both 0;32,24. All six functions corresponding to the 88th and 89th arguments are identical in MS Tk. See note 6 on page 204.

Remarks on Table VI.C$_2$

The digits in the seconds place of the attested and recomputed shadow lengths of a 12-digit gnomon for several degrees of solar altitudes vary by ± 1. For a few entries, the values differ by up to $+3''$ or $-4''$. I suspect these differences are a result of irregular sexagesimal divisions. However, I have not been able to justify these differences mathematically (or observe inadvertent or intentional scribal discrepancies). Therefore, I present the attested digits (in the seconds place) of the shadow lengths corresponding to these arguments in Table VI.C$_2$ without suggesting any emendations.

3.6.3. *Shadow length for gnomon of height 7 digits (Table VI.C$_3$)*

Based on inadvertent copying oversights

1. *Chāyā*$_7$ $(1°)_{\mathrm{u}}$: 410 → 400. Suspected mistranscription. Also, the digits '0' and '1 can (sometimes) appear homoglyphic in handwritten Devanāgarī suggesting a possible unwitting alteration.
2. *Chāyā*$_7$ $(67°)_{\mathrm{m}}$: 59 → 58. Suspected mistranscription. *Chāyā*$_7$ $(67°)_{\mathrm{m}}$ and *Chāyā*$_7$ $(68°)_{\mathrm{m}}$ appear in the sequence '59' and '49' respectively. Also, the digits '8' and '9' can (sometimes) appear homoglyphic in handwritten Devanāgarī suggesting a possible alteration.
3. *Chāyā*$_7$ $(81°)_{\mathrm{u}}$: 16 → 6. Suspected mistranscription. *Chāyā*$_7$ $(80°)_{\mathrm{u}}$ and *Chāyā*$_7$ $(81°)_{\mathrm{u}}$ appear in the sequence '14' and '16' respectively.
4. *Chāyā*$_7$ $(82°)_{\mathrm{u}}$: 34 → 2. Suspected mistranscription. *Chāyā*$_7$ (82°), *Chāyā*$_7$ (83°), and *Chāyā*$_7$ (84°) appear in the sequence '34', '34', and '34' respectively.
5. *Chāyā*$_7$ $(84°)_{\mathrm{u}}$: 34 → 9. Suspected mistranscription. *Chāyā*$_7$ (82°), *Chāyā*$_7$ (83°), and *Chāyā*$_7$ (84°) appear in the sequence '34', '34', and '34' respectively.
6. *Chāyā*$_7$ $(89°)_{\mathrm{s}}$: 10 → 20. Suspected alteration of homoglyphic digits '1' and '2' in handwritten Devanāgarī.

Based on intentional interventions

7. *Chāyā*$_7$ (87°): 0;14,40 → 0;22,1. Suspected contamination. The recomputed value of *Chāyā*$_7$ (88°) is 0;14,40; this value appears under the 87$^{\text{th}}$ argument as a dislocated or displaced entry (perhaps, to replace a corrupted/illegible/missing entry; however, this could also be an unintentional mistranscription).
8. *Chāyā*$_7$ (88°) : 0;7,20 → 0;14,40. Suspected contamination. Adjacent entries *Chāyā*$_7$ (88°) and *Chāyā*$_7$ (89°) are both 0;7,20. All six functions corresponding to the 88$^{\text{th}}$ and 89$^{\text{th}}$ arguments are identical in MS Tk. See note 6 on page 204.

Remarks on Table VI.C$_3$

The digits in the seconds place of the attested and recomputed shadow lengths of a 7-digit gnomon for the solar altitudes of 18°, 25°, 34°, 38°, 43°, 51°, and 77° vary by +1. Without any mathematical justification for these differences (or any evidence to suggest scribal interventions/oversights), I leave digits (in the seconds place) of these shadow lengths in Table VI.C$_3$ unemended.

3.7. Table of lunar latitudes (*śara*): Recomputation strategy

The table of lunar latitude (*śara*) of the *Amṛtalaharī* (in MS Tk) is computed for every degree of the lunar-nodal elongation[50] from 1° to 90° and has a maximum value (equal to the inclination i of the lunar orbit) of 4°30′. The lunar latitude β is related to the lunar-nodal elongation (also known as the argument of lunar latitude) ω with the expression

$$\operatorname{Sin}\beta = \operatorname{Sin} i \times \frac{\operatorname{Sin}\omega}{\mathcal{R}} \equiv \operatorname{Sin} 4^\circ 30' \times \frac{\operatorname{Sin}\omega}{60} \quad \because\ i = 4^\circ 30' \text{ and } \mathcal{R} = 60.$$

1. Most Sanskrit *siddhāntas* approximate the lunar latitude β as $4;30 \times {}^{\operatorname{Sin}\omega}/_{\mathcal{R}}$ (in degrees), e.g. Lalla's *Śiṣyadhīvṛddhidatantra* (*c.* early 9th century): V.11.[51] However, MS Tk uses the exact form of the expression to calculate the lunar latitude.[52] Appendix C.3 includes a statistical analysis of the differences between the attested lunar latitudes (from MS Tk) and the recomputed results when the approximate expression ($4;30 \times \operatorname{Sin}\omega/\mathcal{R}$) or the exact equation ($\operatorname{Sin} 4^\circ 30' \times \operatorname{Sin}\omega/\mathcal{R}$) are used separately.
2. The value of the parameter Sin 4°30′ can be calculated in two different ways:
   (a) by linear interpolation using the recomputed values of Sin 4° and Sin 5° as 4;11,17 and 5;13,46 (from Table VI.A) respectively, or
   (b) by using the formula for the Sine of half the arc for an arc of 9° and the recomputed value Cos 9° = Sin 81° = 59;15,41 (from Table VI.A).

   The method of linear interpolation gives 4;42,26,29,59 (with all subsequent fractions greater than 30), or 4;42,27 (successively rounded to seconds). Using the trigonometric formula gives 4;42,26,8,59, or approximately 4;42,26 (rounded to seconds). My recomputations, however, indicate that the lunar latitude calculations in MS Tk use Sin 4°30′ = 4;42,25. I select the value 4;42,25 by statistically testing the differences between the attested values in MS Tk and my recomputed results (using all three values of the parameter Sin 4°30′ separately) to find the parametric estimate that minimises these differences, see Appendix C.4.

[50] The lunar-nodal elongation is the difference between the celestial longitude of the orbital lunar node (☊ or ☋) and the orb of the Moon, i.e. $\omega = \lambda_{\text{Moon}} - \lambda_{☊ \text{ or } ☋}$. The lunar-nodal elongation ranges from 0° to ±180° depending on the position of the Moon (along its orbit) and the lunar node.

[51] Chatterjee, *Śiṣyadhīvṛddhida Tantra*, pp. 113–14, includes a derivation of Lalla's method to compute the lunar latitude using the approximate expression.

[52] The maximum value of β (at ω = 90°) is equal to the inclination of the lunar orbit, i.e. 4°30′. As $\operatorname{Sin} 4^\circ 30' \approx 4;30$, most Sanskrit texts take $\operatorname{Sin}\beta \approx \beta$ for all $0^\circ \leq \beta \leq 4^\circ 30'$.

The method of determining the lunar latitude (from its Sine) is similar to that of the solar declination. Having calculated the Sine of the lunar latitude, the corresponding latitude (in degrees) is determined by finding the inverse arc of Sine. See Section 3.3 for the algorithm to inversely interpolate the measure of arc corresponding to a particular Sine.

3.7.1. *Worked example*

Calculating the lunar latitude β corresponding to a lunar-nodal elongation ω of 52°:

1. For a lunar-nodal elongation $\omega = 52^\circ$, using the recomputed $\mathrm{Sin}\,52^\circ = 47;16,50$ from Table VI.A, $\mathrm{Sin}\,\beta(52^\circ) = 4;42,25 \times {}^{\mathrm{Sin}\,52^\circ}/_{60} \approx 3;42,33$ (rounded to seconds).
2. To determine the lunar latitude $\beta(52^\circ)$ corresponding to a Sine of 3;42,33, observe from Table VI.A that $\mathrm{Sin}\,3^\circ \equiv 3;8,25 < \mathrm{Sin}\,\beta(52^\circ) < \mathrm{Sin}\,4^\circ \equiv 4;11,7$. Therefore,

$$\begin{aligned}\beta(52^\circ) &= 3^\circ + \frac{\mathrm{Sin}\,\beta(52^\circ) - \mathrm{Sin}\,3^\circ}{\mathrm{Sin}\,4^\circ - \mathrm{Sin}\,3^\circ}\\ &= 3^\circ + \left[\frac{3;42,33 - 3;8,25}{4;11,7 - 3;8,25}\right]_{\text{in degrees}} = 3^\circ + \left[\frac{0;34,9}{1;2,42}\right]_{\text{in degrees}}\\ &= 3^\circ + 0^\circ32'40'' \text{ (rounded to seconds)} \approx 3^\circ32'40''.\end{aligned}$$

   The recomputed lunar latitude corresponding to a lunar-nodal elongation of 52° is $3^\circ32'40''$.

Table VI.D on page 238 presents the recomputed lunar latitudes for every degree of lunar-nodal elongation from 1° to 90°. Most of these recomputations follow the algorithm described above; however, a few entries are calculated irregularly as described below.

3.7.2. *Recomputational irregularities in lunar latitude calculations*

1. Recomputing the lunar latitude β for a lunar-nodal elongation of $\omega = 90^\circ$. For $\omega = 90^\circ$, $\mathrm{Sin}\,\beta(90^\circ) = \mathrm{Sin}\,4^\circ30'$ as $\mathrm{Sin}\,\omega = \mathrm{Sin}\,90^\circ = \mathcal{R}$. Hence, $\beta(90^\circ)$ is simply $4^\circ30'$ (the inclination of the lunar orbit). Alternatively, with $\mathrm{Sin}\,(4^\circ30') \approx 4;42,25$, $\beta(90^\circ) \equiv \mathrm{arcSin}\,(4;42,25) \approx 4^\circ29'59''$ (rounded to seconds). This value is inversely interpolated using the recomputed values $\mathrm{Sin}\,4^\circ = 4;11,7$ and $\mathrm{Sin}\,5^\circ = 5;13,46$ from Table VI.A. The attested value of 4;30′ in MS Tk agrees with this interpolated value (rounded to minutes).
2. Recomputing the lunar latitudes for the lunar-nodal elongations 4°, 7°, 24°, 25°, 26°, 42°, 48°, 50°, 62°, and 79°. The recomputed values

match the attested values in MS Tk when the final results are truncated to seconds (instead of systematically rounding them to seconds), e.g.

$$\begin{array}{rcl} \beta\ \text{(recomputed, up to thirds)} & & \beta\ \text{(attested in MS Tk)} \\ \beta(4^\circ) = 0^\circ 18' 48'' 42''' & \longleftrightarrow & \beta(4^\circ) = 0^\circ 18' 48'' \\ \beta(24^\circ) = 1^\circ 49' 42'' 48''' & \longleftrightarrow & \beta(24^\circ) = 1^\circ 49' 42'' \end{array}$$

The truncated versions of these recomputed values are not suggested as emendations; instead, the third row of differences between the attested and recomputed values (corresponding to these arguments) in Table VI.D registers them as a difference of '-1'.

3.8. Table of lunar latitudes (*śara*): Analysis of differences

List of proposed emendations to the attested lunar latitudes in MS Tk:

Based on inadvertent copying oversights

1. $\beta(12^\circ)_d$: $1 \rightarrow 0$. Suspected mistranscription. $\beta(10^\circ)_d$, $\beta(11^\circ)_d$, $\beta(12^\circ)_d$, and $\beta(13^\circ)_d$ appear in the sequence '0', '0', '$\underline{1}$', and '1' respectively.
2. $\beta(44^\circ)_s$: $36 \rightarrow 26$. Suspected alteration of homoglyphic digits '2' and '3' in handwritten Devanāgarī.

Based on intentional interventions

3. $\beta(85^\circ)_s$: $20 \rightarrow 57$. Suspected contamination. The recomputed value of $\beta(86^\circ)_s$ is '19'; the number '20' ($\sim$ '19' at the level of arithmetical noise) appears under the 85$^{\text{th}}$ argument as a dislocated or displaced entry, perhaps, to replace a corrupted/illegible/missing entry. However, this could also be an unintentional mistranscription by a scribe/table author.
4. $\beta(86^\circ)_s$: $37 \rightarrow 20$. Suspected contamination. The recomputed value of $\beta(87^\circ)_s$ is '37'; this value appears under the 86$^{\text{th}}$ argument as a dislocated or displaced entry (perhaps, to replace a corrupted/illegible/missing entry or a perpetuated mistranscription).
5. $\beta(87^\circ)_s$: $50 \rightarrow 37$ Suspected contamination. The recomputed value of $\beta(88^\circ)_s$ is '50'; the number '49' ($\sim$ '50' at the level of arithmetical noise) appears under the 87$^{\text{th}}$ argument as a dislocated or displaced entry (perhaps, to replace a corrupted/illegible/missing entry or a perpetuated mistranscription).
6. $\beta(88^\circ)_s$: $57 \rightarrow 50$. Suspected contamination. Adjacent entries $\beta(88^\circ)_s$ and $\beta(89^\circ)_s$ are '57'. All six functions corresponding to the 88$^{\text{th}}$ and 89$^{\text{th}}$ arguments are identical in MS Tk. See note 6 on page 204.

Remarks on Table VI.D

1. The lunar latitudes for 57° to 60° of lunar-nodal elongation are illegible in the minutes and seconds places in MS Tk. I represent these illegible entries, the differences between the corresponding sexagesimal digits, and their proposed emendations as '[--]' in Table VI.D.
2. The attested and recomputed lunar latitudes for 70°, 71°, 74°, 75°, 76°, 77°, 81°, and 82° of lunar-nodal elongation differ by $\pm 1'$. My recomputations (including irregular ones) have been unsuccessful in removing this difference, and there are no discernible copying mistakes or scribal corrections in any of these instances. Therefore, I present the attested digits (in the minutes place) of these lunar latitudes in Table VI.D without suggesting any emendations.
3. Also, the digits in the seconds place of the attested and recomputed lunar latitudes for several degrees of lunar-nodal elongations vary by up to ± 3. I suspect these differences are a result of irregular arithmetic calculations or selecting incorrect interpolation intervals. However, I have not been able to explain these differences mathematically (or justify them as interventions/oversights), and therefore, I do not emend the attested digits (in the seconds place) for these arguments in Table VI.D.

4. Conclusion and Discussion

In this study, I recomputed a selection of six tables from Nityānanda's *Amṛtalaharī* to understand the algorithms, the irregularities, and the interdependencies that capture the mathematics of these tables. I also analysed the differences between the attested values (in a single witness MS Tk) and my recomputed results to identify plausible scribal discrepancies (inadvertent copying oversights or intentional interventions), which then allowed to propose a few emendations to the attested values. The process of recomputing attested tables not only reveals the subtle mathematical decisions that table authors make as they recalculate or rectify entries, but also indicates patterns of errors and oversights that get transmitted as the tables are recopied over time. This study brings to light the challenges in applying this process when working with a single manuscript witness. I summarise below the main observations of my study, and the ensuing questions they pose as we begin to build modern digital tools to understand better the historical process of computing astronomical tables.

1. The attested values corresponding to the 88th and 89th arguments are identical for all six functions tabulated on the manuscript. The digital surrogates (of ff. 49v–50v) of MS Tk show faint vertical rules separating thirty columns of arguments on each folio, with corresponding six sets

of functions vertically stacked below them and mutually separated by horizontal rules. This formatted (grid-like) presentation of the six tables on MS Tk suggests that a professional scribe could have copied the entries from a parent manuscript, column by column, and while doing so, inadvertently duplicated all six sets of values for the 88th and 89th arguments as they populated the grid.

However, there are other instances where individual digits (in the sexagesimal places of the value of a function) appear to be shifted horizontally into adjacent cells, e.g. the leftwards displacement of the digits (in the seconds place) for lunar latitudes corresponding to the 86th, 87th, and 88th arguments. These horizontal shifts suggest that the tables (or certain parts of the tables) were perhaps copied cell by cell along each row. Certain mathematical aspects of a function (e.g. monotonicity) become evident when copying the values progressively, and hence, table authors may have found it intuitive to copy the sexagesimal digits (of the value of a function) row-wise. The various patterns of computational irregularities or scribal discrepancies noted in this study suggest different directions in which the tables were possibly copied. The extent to which anomalous entries can expose the direction of copying, and perhaps, the intention of the copyist themselves, is a challenging question that requires more advanced methods of analysis applied to larger selections of tables from a manuscript.

2. While the identical sets of values for the 88th and 89th arguments on f. 50v of MS Tk could be the result of an inadvertent copying oversight, it is just as likely the result of an intentional change. At some point in the transmission of the tables, a diligent scribe (or a table author) may have simply copied the six sets of values for the 89th argument into the column of the 88th argument to rectify a corrupted, illegible, or missing column in a parent manuscript (perhaps, treating the small differences between these values to be mathematically insignificant). These speculations indicate how inadvertent or intentional choices of successive historical actors (scribes or table authors) modify a particular table, and separate each subsequent copy from the previous one (and the original) by an added degree of uncertainty.

3. In this study, there are some cases where irregular recomputations eliminate the differences between the attested values (in MS Tk) and my recomputed results. In other instances, inadvertent or intentional scribal changes are evident enough to justify emending the attested values, and by doing so, reduce or remove the differences. Nevertheless, there are still several (small) differences between the attested and recomputed val-

ues in every table that cannot be justified as anomalous calculations or scribal discrepancies. Perhaps, in some measure, these differences are the result of historical actors making tacit decisions *ad libitum*. Most historical recomputations of astronomical tables, including those presented here, admit to this level of residual noise.

4. In my study of the selected corpus, I found a single instance where an attested Sine from MS Tk (different from my recomputed Sine) reproduces an attested value (of another function) identically and exclusively. With $\mathrm{Cos}\,88° = \mathrm{Sin}\,2° = 2;0,38$, the recomputed shadow length of a 12-digit gnomon for a solar altitude of 87° is identical to its attested value in MS Tk. Mathematically, this recomputation is highly irregular as it not only enters a wrong Cosine in the algorithm ($\mathrm{Cos}\,88°$ instead of the regular $\mathrm{Cos}\,87° = \mathrm{Sin}\,3°$), but also uses an inaccurate Sine ($\mathrm{Sin}\,2°$ should be 2;5,38) in the calculation that follows. Accordingly, this attested (or irregularly recomputed) shadow length for the 87th argument makes the sequence $\mathit{Ch\bar{a}y\bar{a}}_{12}(86°) = 0;50,20$, $\mathit{Ch\bar{a}y\bar{a}}_{12}(87°) = 0;24,9$, and $\mathit{Ch\bar{a}y\bar{a}}_{12}(88°) = 0;32,24$ in MS Tk mathematically inconsistent. (The shadow length is a monotonically decreasing function for the first ninety degrees of the argument.) The recomputational irregularities that involve interdependencies between attested values from different tables are a strong indication of secondary interventions. In this case, it is very likely that a (later) table author (mis)calculated the shadow length for a corrupted/illegible/missing entry corresponding to the 87th argument by simply using the attested value of Sine (in the parent manuscript).
5. The three tables of shadow lengths in MS Tk reveal further interdependencies between their entries, e.g. the shadow lengths $\mathit{Ch\bar{a}y\bar{a}}_{7}(4°)$ and $\mathit{Ch\bar{a}y\bar{a}}_{60}(4°)$, $\mathit{Ch\bar{a}y\bar{a}}_{12}(4°)$ and $\mathit{Ch\bar{a}y\bar{a}}_{60}(4°)$, or $\mathit{Ch\bar{a}y\bar{a}}_{12}(24°)$ and $\mathit{Ch\bar{a}y\bar{a}}_{60}(24°)$. These computational interdependencies also indicate that historical actors (presumably, different from the original author) regularly modified tables by recomputing certain entries using attested values from a parent manuscript.

The observations of this study show how historical actors carelessly or consciously modify a table as they copy it. Their modifications increasingly distance earlier versions of the table from what is attested in a present witness. Essentially, each witness is a mathematical artefact from a particular time that contains an aggregated picture of the changes made (and unmade) by previous actors. Our modern recomputations simulate historical procedures, identify computational irregularities, and analyse scribal discrepancies to help us trace the mathematical practices of these actors. As more advanced tools from data sciences (in particular, knowledge discovery processes and machine learning) are adapted to analyse and predict patterns in these table entries, methodological questions become important for designing meaningful algorithms. For instance, how do table authors modify theoretical (canonical) formulae for practical computations? What combinations of arithmetical operations reproduce the anomalous values attested in a table? Do residual differences follow a behavioural trend for a selected corpus? What is a sensible taxonomy of recomputational irregularities and scribal discrepancies? How can competing recomputational strategies be statistically chosen? This study addresses some of these questions by examining a few selected tables of the *Amṛtalaharī*.

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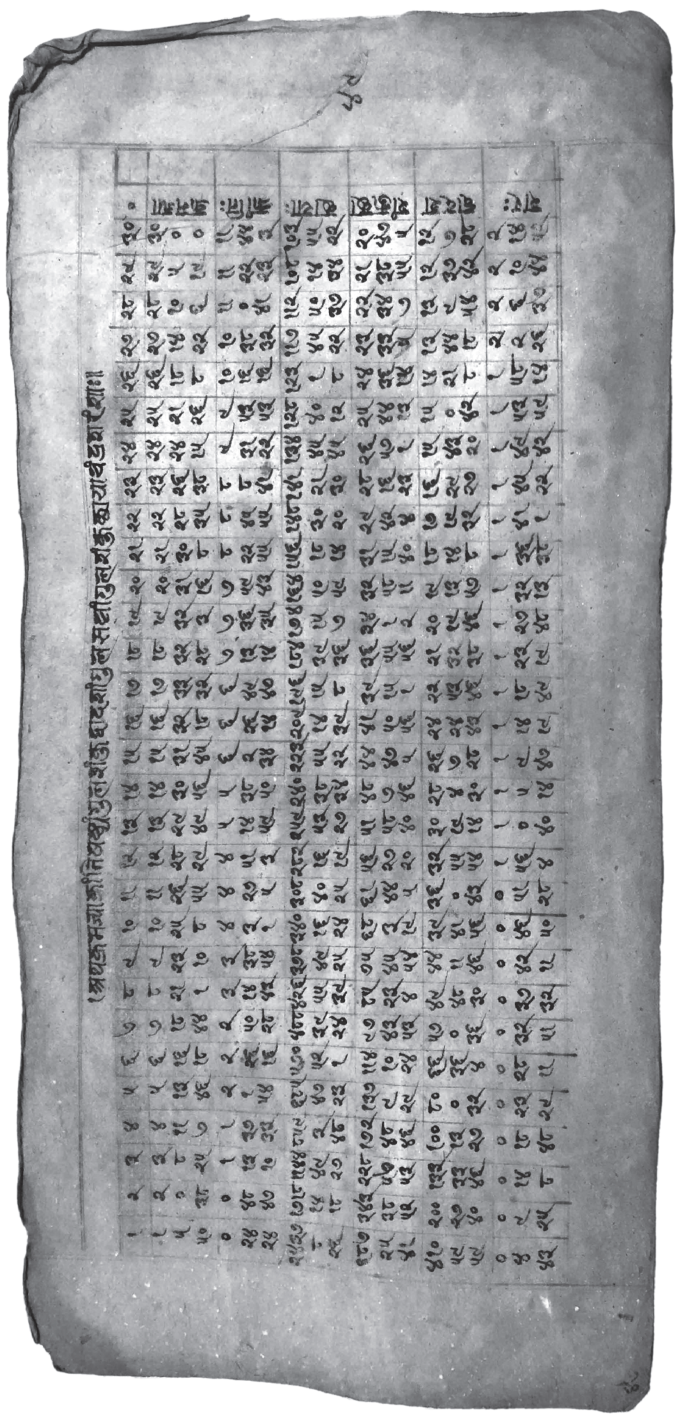
Appendix A: Diplomatic transcription of the table on ff. 49v–50v of MS Tk

Pages 226–31 include the printed reproductions and diplomatic transcriptions of the six tables on ff. 49v–50v of MS Tk. The diplomatic transcriptions preserve the attested (landscape) layout of the tables. The orthography of the attested text is transcribed without modifying any erroneous or missing letters. Illegible letters are indicated as '[-*x*-]' (where each '*x*' indicates an individual letter) or '[-*x*?-]' (where '*x*?' indicates an unknown number of missing letters). The paratext and table titles are transliterated into the Latin script. Sanskrit euphonic rules (*saṃdhi*) are silently applied to make the transliterated words morphologically regular, e.g. the solar declination or *krānti* is correctly transliterated with internal /*n*/ instead of *krāṃti* (with the allophonic /*ṃ*/). The numbers in the tables are copied as they appear in the manuscript. Any missing or illegible numbers are indicated as '[--]' with the number of dashes (within the square brackets) indicating, tentatively, the maximum size of the missing number. Any unclear or dysmorphic numbers are disambiguated by inspecting the handwriting for consistency, or by identifying local or global trends in the sequence in which these numbers appear.

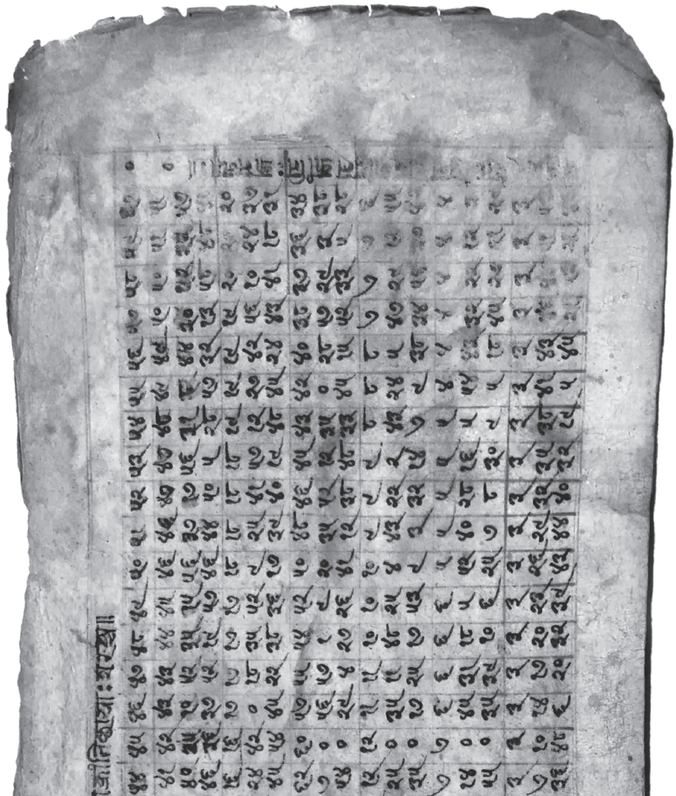


F. 49v of MS Tk

Diplomatic transcription of the tables on f. 49v of MS Tk

|| atha kramajyākrāntiṣaṣṭyaṅgulaśaṅkudvādaśāṅgulasaptāṅgulaśaṅkuchāyācandraśaraṃśāḥ ||

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	o		
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	*kramajyā*		VI.A
5	0	8	11	13	16	18	21	23	25	26	28	29	30	31	32	33	32	32	31	30	28	26	24	21	18	14	10	5	0			
50	38	25	7	46	18	44	1	10	8	55	29	49	56	45	18	32	28	3	16	8	35	38	15	26	8	22	6	19	0			
0	0	1	1	2	2	2	3	3	4	4	4	5	5	6	6	6	7	7	7	8	8	8	9	9	10	10	11	11	11	*krāntiḥ*		VI.B
24	48	13	37	1	26	50	14	38	3	27	51	14	38	2	26	49	13	36	59	22	42	8	31	53	16	38	0	22	44			
24	47	10	33	54	16	28	43	54	1	1	3	59	50	34	14	47	14	35	43	52	52	41	22	53	16	32	41	23	3			
3437	1718	1144	859	685	570	488	426	378	340	308	282	259	240	223	209	196	184	174	164	156	148	141	134	128	123	117	112	108	103	*chāyāḥ*		
8	14	49	2	47	52	39	55	49	16	40	16	53	38	55	14	15	39	15	50	18	30	21	45	40	1	45	50	14	55			VI.C_1
26	18	27	48	23	1	24	39	25	34	25	19	27	34	22	39	8	36	7	59	14	20	30	45	12	8	22	37	34	22		49	
687	343	228	172	137	114	97	85	75	68	61	56	51	48	44	41	39	36	34	32	31	29	28	26	25	24	23	22	21	20	*śaṅkuchā*		
25	38	57	48	9	10	43	23	45	3	44	27	58	7	47	50	15	55	1	58	15	42	16	57	44	36	33	34	38	47			VI.C_2
41	52	53	46	29	24	52	4	54	19	5	20	40	46	5	56	1	56	2	11	40	4	23	1	2	14	5	7	55	5			
410	200	133	100	80	66	57	49	44	39	36	32	30	28	26	24	22	21	20	19	18	17	16	15	15	14	13	13	12	12	*dvādaśa*		
59	27	33	12	0	36	0	48	11	41	0	55	19	4	7	24	53	32	19	13	14	19	29	43	0	21	44	9	37	7			VI.C_3
59	40	46	27	32	4	36	30	46	56	43	54	14	30	28	43	46	38	46	57	8	32	27	20	42	8	18	54	42	28			
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	*śaraḥ*		
4	9	14	18	23	28	32	37	42	46	51	56	0	5	9	14	18	23	27	32	36	41	45	49	53	58	2	6	10	14			VI.D
43	25	8	48	29	11	51	32	11	50	28	4	40	14	47	19	49	19	46	13	38	1	22	42	59	14	26	30	44	52			

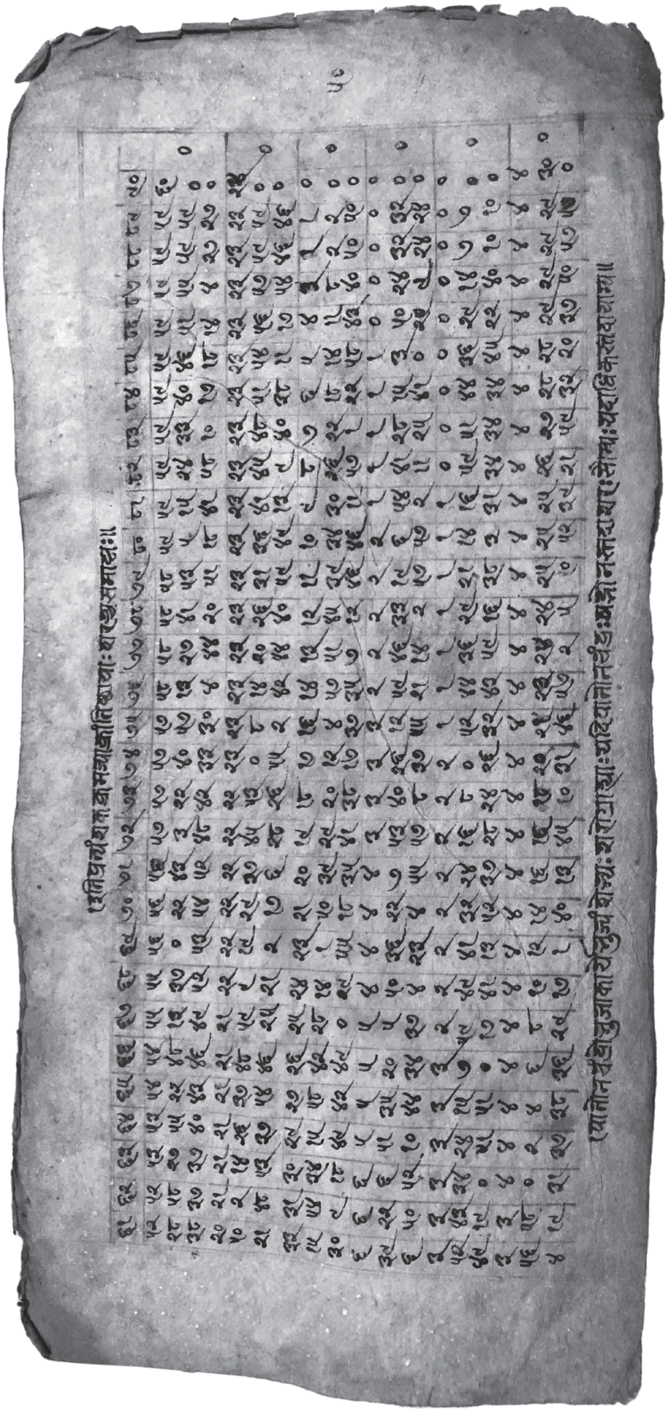


F. 50r of MS Tk

Diplomatic transcription of the tables on f. 50r of MS Tk

|| pratyaṃśakramajyākrāntichāyāḥ śaraśca ||

31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	o	
30	31	32	33	34	35	36	36	37	38	39	40	40	41	42	43	43	44	45	46	46	47	47	48	49	49	50	50	51	51	o	VI.A
54	47	40	33	24	16	6	56	45	34	21	8	55	40	25	10	52	35	15	56	37	17	56	31	8	44	20	52	22	57		
8	43	42	6	53	2	32	23	33	2	49	52	12	46	36	37	52	19	57	46	44	50	5	28	57	32	13	58	48	41		
12	12	12	13	13	13	14	14	14	15	15	15	16	16	16	17	17	17	17	18	18	18	18	19	19	19	19	20	20	20	*kramajyā*	VI.B
5	26	47	8	29	49	10	30	49	9	28	47	6	24	42	0	18	35	52	9	25	41	57	12	27	42	56	10	24	37		
30	51	56	49	29	55	8	10	52	22	37	35	17	45	54	45	22	38	36	17	35	40	19	48	45	24	43	41	18	31		
99	96	92	88	85	82	79	76	74	71	69	66	64	62	60	57	55	54	52	50	48	46	45	43	42	40	38	37	36	34	[-x-]*āṅgula krāntiḥ*	$VI.C_1$
51	1	23	57	41	34	37	47	5	30	1	38	20	7	0	56	57	1	9	20	35	52	12	35	0	28	57	29	3	38		
25	10	31	9	18	59	21	48	37	19	18	16	24	54	0	29	4	27	26	41	12	38	48	33	45	15	52	33	5	28		
19	19	18	17	17	16	15	15	14	14	13	12	13	12	12	11	11	10	10	10	9	9	9	8	8	8	7	7	7	6	[-xxx-]	$VI.C_2$
58	12	48	47	8	31	55	21	49	18	48	19	55	25	0	35	11	48	25	4	43	22	2	43	24	5	47	29	12	55		
17	14	42	27	16	0	28	33	7	4	16	38	6	35	0	17	25	17	53	9	3	32	14	7	9	38	34	54	37	42		
11	11	10	10	9	9	9	8	8	8	8	7	7	7	7	6	6	6	6	5	5	5	5	5	4	4	4	4	4	4	[-x?-]	$VI.C_3$
39	12	46	22	59	38	17	57	38	20	3	46	30	14	0	45	31	18	5	52	40	28	16	5	54	43	32	22	12	2		
0	8	40	41	49	5	42	35	39	32	9	27	24	55	0	35	39	10	6	25	7	8	30	8	5	18	45	27	21	29		
2	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	[-x?-]	VI.D
18	22	26	30	34	38	42	46	49	53	57	0	4	7	10	14	17	20	23	26	29	32	35	38	41	43	[--]	[--]	[--]	[--]		
56	57	55	50	44	32	22	6	47	25	0	31	0	36	48	6	25	32	39	42	44	40	32	59	5	45	[--]	[--]	[--]	[--]		



F. 50v of MS Tk

Diplomatic transcription of the tables on f. 50v of MS Tk

|| iti pratyaṃśakakramajyākrantichāyāḥ śaraścasamāptaḥ ||

61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	o		
52	52	53	53	54	54	55	55	56	56	56	57	57	57	57	58	58	58	58	59	59	59	59	59	59	59	59	59	59	60			
28	58	27	55	22	48	13	37	0	22	43	3	22	40	57	13	27	41	53	5	15	24	33	40	46	51	55	59	59	0	o		VI.A
38	37	37	40	42	46	49	52	53	54	52	48	42	33	30	4	44	20	51	18	41	58	10	17	18	14	4	27	27	0			
20	21	21	21	21	21	21	22	22	22	22	22	22	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	24			
50	2	14	26	37	48	59	9	19	29	37	45	53	0	8	14	20	26	31	36	41	45	48	51	54	56	57	59	59	0	o		VI.B
21	48	53	37	54	46	25	21	2	17	6	28	26	55	2	42	54	40	59	49	13	9	40	38	11	17	54	46	46	0			
33	31	30	29	27	26	25	24	23	21	20	19	18	17	16	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0			
15	54	34	15	58	42	28	14	1	50	39	29	20	12	4	57	51	45	39	34	30	26	22	18	14	11	8	2	2	0	o		VI.C$_1$
30	8	18	49	42	49	0	29	55	18	35	41	38	17	37	35	7	12	46	46	11	57	1	22	58	43	40	50	50	0		50	
6	6	6	5	5	5	5	4	4	4	4	3	3	3	3	2	2	2	2	2	1	1	1	1	1	0	0	0	0	0			
39	22	6	51	35	20	5	50	36	22	7	53	40	26	12	59	46	33	19	6	54	41	28	15	3	50	24	32	32	0	o		VI.C$_2$
6	50	52	10	44	34	37	4	23	4	55	57	8	27	55	31	14	2	57	57	2	11	25	41	0	20	9	24	24	0			
3	3	3	3	3	3	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0			
52	43	34	24	15	7	59	49	41	32	24	16	8	0	52	44	36	29	21	14	16	59	51	44	36	29	14	7	7	0	o		VI.C$_3$
49	19	0	51	51	0	17	41	13	52	37	28	24	26	32	42	59	16	38	3	31	34	34	34	45	22	40	10	10	0			
3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4			
56	58	0	2	4	6	8	10	12	14	16	16	18	22	24	22	24	24	25	25	25	26	27	28	28	29	29	29	29	30	o		VI.D
4	19	31	27	38	36	29	17	1	40	13	45	10	31	46	57	2	5	0	52	39	21	59	32	20	37	50	57	57	0			

pātonacandro bhujā kārye bhujyaṃśedbhyaḥ śaro grāhyāḥ yadi pāto nacandraḥ ṣadbhonas tadā śaraḥ saumyaḥ yadādhikas tadā yāmya ||

Appendix B: Recomputation and analysis of Tables VI.A–D

Conventions for representing the tables

The six tables (Tables VI.A, VI.B, VI.C_{1-3}, and VI.D) from the selected corpus are presented on pp. 233–38.

1. Each table has four separate rows of (sexagesimal) entries, placed one below the other, in three argument blocks 1° to 30°, 31° to 60°, and 61° to 90°. The arguments (in degrees) represent the following different quantities for the respective tables:
   (a) Table of Sines (VI.A): measure of arc;
   (b) Table of solar declinations (VI.B): celestial longitude;
   (c) Table of shadow lengths of gnomons of 60-digit (VI.C_1), 12-digit (VI.C_2), and 7-digit heights (VI.C_3): solar altitude; and
   (d) Table of lunar latitudes (VI.D): lunar-nodal elongation.
2. The sexagesimal values of the table entries are written vertically. The digits at the top of a vertical stack represent the integer part (i.e. units or degrees) of the number, those in the middle indicate the first fractional part (i.e. minutes), and the digits at the bottom of a stack signify the second fractional part (i.e. seconds).
3. In each argument block of thirty degrees,
   (a) the first row lists the attested values from MS Tk;
   (b) the second row presents the recomputed values with
      – the digits (in individual sexagesimal places) that result from irregular recomputations enclosed in a rectangular box;
   (c) the third row shows the difference in digits between corresponding sexagesimal places of the attested and recomputed values (from the previous two rows) with
      – all non-zero differences enclosed in shaded grey boxes; and
   (d) the fourth row lists the proposed emendation to the attested values where
      – any modified entries (in individual sexagesimal places) are enclosed in circles.

These conventions allow (**a**) recomputational irregularities (digits in rectangular boxes), (**b**) non-zero revised differences (digits in grey cells), and (**c**) proposed emendations (encircled digits) to be clearly identified. For a collection of tables from a single manuscript, this visual representation allows the recomputational and the text-critical versions of individual tables to be seen concurrently.

Table VI.A: Recomputations and analyses of the Table of Sines (*kramajyā*) discussed in Sections 3.1–3.2

Measure of arc	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
Attested Sine in MS Tk	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	UNITS
	5	0	8	11	13	16	18	21	23	25	26	28	29	30	31	32	33	32	32	31	30	28	26	24	21	18	14	10	5	0	MINUTES
	50	38	25	7	46	18	44	1	10	8	55	29	49	56	45	18	32	28	3	16	8	35	38	15	26	8	22	6	19	0	SECONDS
Recomputed Sine (rounded to seconds)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
	2	5	8	11	13	16	18	21	23	25	26	28	29	30	31	32	32	32	32	31	30	28	26	24	21	18	14	10	5	0	
	50	38	25	7	46	18	44	1	10	8	55	29	49	55	45	18	32	28	3	16	7	35	38	15	26	8	22	6	19	0	
Difference between the digits in corresponding sexagesimal places	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	+3	−5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	
Proposed emendation to the attested value in MS Tk	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
	2	5	8	11	13	16	18	21	23	25	26	28	29	30	31	32	32	32	32	31	30	28	26	24	21	18	14	10	5	0	
	50	38	25	7	46	18	44	1	10	8	55	29	49	56	45	18	32	28	3	16	8	35	38	15	26	8	22	6	19	0	

Measure of arc	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	
Attested Sine in MS Tk	30	31	32	33	34	35	36	36	37	38	39	40	40	41	42	43	43	44	45	46	46	47	47	48	49	49	50	50	51	51	UNITS
	54	47	40	33	24	16	16	56	45	34	21	8	55	40	25	10	52	35	15	56	37	17	56	31	8	44	20	52	25	57	MINUTES
	8	43	42	6	53	2	32	23	33	2	49	52	12	46	36	37	52	19	57	46	44	50	5	28	57	32	13	58	48	41	SECONDS
Recomputed Sine (rounded to seconds)	30	31	32	33	34	35	36	36	37	38	39	40	40	41	42	43	43	44	45	45	46	47	47	48	49	49	50	50	51	51	
	54	47	40	33	24	16	6	56	45	34	21	8	55	40	25	9	52	35	16	57	37	16	55	32	8	44	19	52	25	57	
	8	43	42	6	53	2	32	23	33	2	49	52	12	46	35	37	52	19	57	46	44	50	5	28	57	32	13	58	48	41	
Difference between the digits in corresponding sexagesimal places	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	+10	0	0	0	0	0	0	0	0	+1	0	0	−1	−1	0	+1	+1	−1	0	0	+1	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Proposed emendation to the attested value in MS Tk	30	31	32	33	34	35	36	36	37	38	39	40	40	41	42	43	43	44	45	45	46	47	47	48	49	49	50	50	51	51	
	54	47	40	33	24	16	6	56	45	34	21	8	55	40	25	10	52	35	15	56	37	17	56	31	8	44	20	52	25	57	
	8	43	42	6	53	2	32	23	33	2	49	52	12	46	36	37	52	19	57	46	44	50	5	28	57	32	13	58	48	41	

Measure of arc	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	
Attested Sine in MS Tk	52	52	53	53	54	54	55	55	56	56	56	57	57	57	57	58	58	58	58	59	59	59	59	59	59	59	59	59	59	60	UNITS
	28	58	27	55	22	48	13	37	0	22	43	3	22	40	57	13	27	41	53	5	15	24	33	40	46	51	55	59	59	0	MINUTES
	38	37	37	40	43	46	49	52	53	54	52	48	42	33	30	4	44	20	51	18	41	58	10	17	18	14	4	27	27	0	SECONDS
Recomputed Sine (rounded to seconds)	52	52	53	53	54	54	55	55	56	56	56	57	57	57	57	58	58	58	58	59	59	59	59	59	59	59	59	59	59	60	
	28	58	27	55	22	48	13	37	0	22	43	3	22	40	57	13	27	41	53	5	15	24	33	40	46	51	55	57	59	0	
	38	37	37	40	42	46	49	52	53	54	52	48	42	33	20	4	44	20	51	18	41	58	10	17	18	14	4	48	27	0	
Difference between the digits in corresponding sexagesimal places	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+2	0	0	
	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	+10	0	0	0	0	0	0	0	0	0	0	0	0	−21	0	0	
Proposed emendation to the attested value in MS Tk	52	52	53	53	54	54	55	55	56	56	56	57	57	57	57	58	58	58	58	59	59	59	59	59	59	59	59	59	59	60	
	28	58	27	55	22	48	13	37	0	22	43	3	22	40	57	13	27	41	53	5	15	24	33	40	46	51	55	57	59	0	
	38	37	37	40	43	46	49	52	53	54	52	48	42	33	20	4	44	20	51	18	41	58	10	17	18	14	4	48	27	0	

Table VI.B: Recomputations and analyses of the Table of solar declinations (*krānti*) discussed in Sections 3.3–3.4

Celestial longitude	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	
Attested solar declination in MS Tk	0	0	1	1	2	2	2	3	3	4	4	4	5	5	6	6	6	7	7	7	8	8	8	9	9	10	10	11	11	11	DEGREES
	24	48	13	37	1	26	50	14	38	3	27	51	14	38	2	26	49	13	36	59	22	45	8	31	53	16	38	0	22	44	MINUTES
	24	47	10	33	54	16	28	43	54	1	5	3	59	50	34	14	47	14	35	43	55	55	41	22	53	16	32	41	23	3	SECONDS
Recomputed solar declination (rounded to seconds)	0	0	1	1	2	2	2	3	3	4	4	4	5	5	6	6	6	7	7	7	8	8	9	9	9	10	10	11	11	11	
	24	48	13	37	1	26	50	14	38	3	27	51	14	38	2	26	49	13	36	59	22	45	8	31	53	16	38	0	22	44	
	24	48	11	33	54	12	29	42	54	1	5	4	59	50	35	14	47	15	35	47	54	51	41	22	53	16	28	$\boxed{41}$	23	3	
Difference between the digits in corresponding sexagesimal places	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	−1	−1	0	0	+4	−1	+1	0	0	0	−1	0	0	−1	0	0	−1	0	−4	+1	+4	0	0	0	0	+4	0	0	0	
Proposed emendation to the attested value in MS Tk	0	0	1	1	2	2	2	3	3	4	4	4	5	5	6	6	6	7	7	7	8	8	9	9	9	10	10	11	11	11	
	24	48	13	37	1	26	50	14	38	3	27	51	14	38	2	26	49	13	36	59	22	45	8	31	53	16	38	0	22	44	
	24	47	10	33	54	16	28	43	54	1	5	3	59	50	34	14	47	14	35	43	52	52	41	22	53	16	32	41	23	3	

Celestial longitude	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	
Attested solar declination in MS Tk	12	12	12	13	13	13	14	14	14	15	15	15	16	16	16	17	17	17	17	18	18	18	18	19	19	19	19	20	20	20	DEGREES
	5	26	47	8	29	49	10	30	49	9	28	47	6	24	42	0	18	35	52	9	25	41	57	12	27	42	56	10	24	37	MINUTES
	30	51	56	49	29	55	8	10	52	22	37	35	17	45	54	45	22	38	36	17	39	40	19	48	45	24	43	41	18	31	SECONDS
Recomputed solar declination (rounded to seconds)	12	12	12	13	13	13	14	14	14	15	15	15	16	16	16	17	17	17	17	18	18	18	18	19	19	19	19	20	20	20	
	5	26	47	8	29	49	10	30	49	9	28	47	6	24	42	0	18	35	52	9	25	41	57	12	27	42	56	10	24	37	
	33	51	56	49	31	56	9	9	52	22	37	35	18	45	55	45	22	39	37	17	39	40	20	43	46	25	42	41	18	31	
Difference between the digits in corresponding sexagesimal places	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	−3	0	0	0	−2	−1	−1	+1	0	0	0	0	−1	0	−1	0	0	−1	−1	0	0	0	−1	+5	−1	−1	+1	0	0	0	
Proposed emendation to the attested value in MS Tk	12	12	12	13	13	13	14	14	14	15	15	15	16	16	16	17	17	17	17	18	18	18	18	19	19	19	19	20	20	20	
	5	26	47	8	29	49	10	30	49	9	28	47	6	24	42	0	18	35	52	9	25	41	57	12	27	42	56	10	24	37	
	30	51	56	49	29	55	8	10	52	22	37	35	17	45	54	45	22	38	36	17	39	40	19	48	45	24	43	41	18	31	

Celestial longitude	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	
Attested solar declination in MS Tk	20	21	21	21	21	21	21	22	22	22	22	22	22	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	24	DEGREES
	50	2	14	26	37	48	59	9	19	29	37	45	53	0	8	14	20	26	31	36	41	45	48	51	54	56	57	59	59	0	MINUTES
	21	48	53	37	54	46	25	21	2	17	6	28	26	55	2	42	44	40	59	49	13	9	40	38	11	17	54	46	46	0	SECONDS
Recomputed solar declination (rounded to seconds)	20	21	21	21	21	21	21	22	22	22	22	22	22	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	24	
	50	2	14	26	37	48	59	9	19	28	37	45	53	0	8	14	20	26	31	36	41	45	48	51	54	56	57	59	59	0	
	22	48	55	38	54	48	15	21	2	17	6	29	25	56	2	42	55	41	59	50	13	10	38	39	12	17	55	4	46	0	
Difference between the digits in corresponding sexagesimal places	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	−1	0	−2	−1	0	−2	+10	0	0	0	0	−1	+1	−1	0	0	−11	−1	0	−1	0	−1	+2	−1	−1	0	−1	+42	0	0	
Proposed emendation to the attested value in MS Tk	0	21	21	21	21	21	21	22	22	22	22	22	22	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	24	
	50	2	14	26	37	48	59	9	19	28	37	45	53	0	8	14	20	26	31	36	41	45	48	51	54	56	57	59	59	0	
	21	48	53	37	54	46	15	21	2	17	6	28	26	55	2	42	55	40	59	49	13	9	40	38	11	17	54	4	46	0	

Table VI.C$_1$: Recomputations and analyses of the Table of shadow lengths (*śaṅkuchāyā*) of a 60-digit gnomon discussed in Sections 3.5–3.6

Solar altitude	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
Attested shadow length of a 60-digit gnomon in MS Tk	3437	1718	1144	859	685	570	488	426	378	340	308	282	259	240	223	209	196	184	174	164	156	148	141	134	128	123	117	112	108	103	UNITS
	8	14	49	3	47	52	39	55	49	16	40	16	53	38	55	14	15	39	15	50	18	30	21	45	40	1	45	50	14	55	MINUTES
	26	18	27	48	23	1	24	39	25	24	25	19	27	34	22	39	8	36	7	59	14	20	3	45	12	8	22	37	34	22	SECONDS
Recomputed shadow length of a 60-digit gnomon (rounded to seconds)	3437	1718	1144	858	685	570	488	426	378	340	308	282	259	240	223	209	196	184	174	164	156	148	141	134	128	123	117	112	108	103	
	8	14	49	3	47	51	39	55	49	16	40	16	53	38	55	14	15	39	15	50	18	30	21	45	40	1	45	50	14	55	
	26	18	28	48	24	59	24	40	26	34	$\boxed{25}$	39	27	$\boxed{34}$	22	39	8	36	7	59	$\boxed{14}$	20	4	45	10	8	23	37	34	22	
Difference between the digits in corresponding sexagesimal places	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	−1	0	−1	−58	0	−1	−1	−10	0	−20	0	0	0	0	0	0	0	0	0	0	−1	0	+2	0	−1	0	0	0	
Proposed emendation to the attested value in MS Tk	3437	1718	1144	859	685	570	488	426	378	340	308	282	259	240	223	209	196	184	174	164	156	148	141	134	128	123	117	112	108	103	
	8	14	49	3	47	52	39	55	49	16	40	16	53	38	55	14	15	39	15	50	18	30	21	45	40	1	45	50	14	55	
	26	18	27	48	23	1	24	39	25	34	25	39	27	34	22	39	8	36	7	59	0	20	3	45	12	8	22	37	34	22	

Solar altitude	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	
Attested shadow length of a 60-digit gnomon in MS Tk	99	96	92	88	85	82	79	76	74	71	69	66	64	62	60	57	55	54	52	50	48	46	45	43	42	40	38	37	36	34	UNITS
	51	1	23	57	41	34	37	47	5	30	1	38	20	7	0	56	57	1	9	20	35	52	12	35	0	28	57	29	3	38	MINUTES
	25	10	31	9	18	59	21	48	39	19	18	16	24	54	0	29	4	27	26	41	12	38	48	33	45	15	52	33	5	28	SECONDS
Recomputed shadow length of a 60-digit gnomon (rounded to seconds)	99	96	92	88	85	82	79	76	74	71	69	66	64	62	60	57	55	54	52	50	48	46	45	43	42	40	38	37	36	34	
	51	1	23	57	41	34	37	47	5	30	1	38	20	7	0	56	57	1	9	20	35	52	12	35	0	28	57	29	3	38	
	25	10	31	12	17	56	21	47	39	20	19	$\boxed{16}$	$\boxed{24}$	54	0	29	4	27	27	$\boxed{41}$	13	38	48	33	45	14	52	33	6	28	
Difference between the digits in corresponding sexagesimal places	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	−3	+1	+3	0	+1	0	−1	−1	0	0	0	0	0	0	0	−1	0	−1	0	0	0	0	+1	0	0	−1	0	
Proposed emendation to the attested value in MS Tk	99	96	92	88	85	82	79	76	74	71	69	66	64	62	60	57	55	54	52	50	48	46	45	43	42	40	38	37	36	34	
	51	1	23	57	41	34	37	47	5	30	1	38	20	7	0	56	57	1	9	20	35	52	12	35	0	28	57	29	3	38	
	25	10	31	9	18	59	21	48	37	19	18	16	24	54	0	29	4	27	26	41	12	38	48	33	45	15	52	33	5	28	

Solar altitude	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	
Attested shadow length of a 60-digit gnomon in MS Tk	33	31	30	29	27	26	25	24	23	21	20	19	18	17	16	14	13	12	11	10	9	8	7	6	5	4	3	1	1	0	UNITS
	15	54	34	15	58	42	28	14	1	50	39	29	20	12	4	57	51	45	39	34	30	26	22	18	14	11	8	2	2	0	MINUTES
	30	9	18	49	42	49	0	29	55	18	35	41	38	17	37	35	7	12	46	46	11	57	1	22	58	43	40	50	50	0	SECONDS
Recomputed shadow length of a 60-digit gnomon (rounded to seconds)	33	31	30	29	27	26	25	24	23	21	20	19	18	17	16	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	
	15	54	34	15	58	42	28	14	1	50	39	29	20	12	4	57	51	45	39	34	30	25	22	18	14	11	8	5	2	0	
	31	9	18	50	43	49	7	30	54	17	35	43	37	17	37	35	7	12	46	47	11	56	2	22	58	44	41	43	50	0	
Difference between the digits in corresponding sexagesimal places	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	−3	0	0	
	−1	0	0	−1	−1	0	−7	−1	+1	+1	0	−2	+1	0	0	0	0	0	0	−1	0	+1	−1	0	0	−1	−1	−7	0	0	
Proposed emendation to the attested value in MS Tk	33	31	30	29	27	26	25	24	23	21	20	19	18	17	16	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	
	15	54	34	15	58	42	28	14	1	50	39	29	20	12	4	57	51	45	39	34	30	26	22	18	14	11	8	5	2	0	
	30	8	18	49	42	49	7	29	55	18	35	41	38	17	37	35	7	12	46	47	11	57	1	22	58	43	40	43	50	0	

Table VI.C$_2$: Recomputations and analyses of the Table of shadow lengths (*śaṅkuchāyā*) of a 12-digit gnomon discussed in Sections 3.5–3.6

Solar altitude		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Attested shadow length of a 60-digit gnomon in MS Tk	UNITS	687	343	228	172	137	114	97	85	75	68	61	56	51	48	44	41	39	36	34	32	31	29	28	26	25	24	23	22	21	20
	MINUTES	25	38	57	48	9	10	43	23	45	3	44	27	58	7	47	50	15	55	1	58	15	42	16	57	44	36	33	34	38	47
	SECONDS	41	52	53	46	29	24	53	4	54	19	5	20	40	46	5	56	1	56	2	11	40	4	23	1	12	14	5	7	55	5
Recomputed shadow length of a 12-digit gnomon (rounded to seconds)		687	343	228	171	137	114	97	85	75	68	61	56	51	48	44	41	39	36	34	32	31	29	28	26	25	24	23	22	21	20
		25	38	57	48	9	10	43	23	45	3	44	27	58	7	47	50	15	55	51	58	15	42	16	57	44	36	33	34	38	47
		41	52	54	46	29	24	53	8	53	19	3	20	41	43	4	56	2	55	1	12	39	4	13	1	2	14	5	7	55	4
Difference between the digits in corresponding sexagesimal places		0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−50	0	0	0	0	0	0	0	0	0	0	0
		0	0	−1	0	0	0	0	−4	+1	0	+2	0	−1	+3	+1	0	−1	+1	+1	−1	+1	0	+10	0	+10	0	0	0	0	+1
Proposed emendation to the attested value in MS Tk		687	343	228	171	137	114	97	85	75	68	61	56	51	48	44	41	39	36	34	32	31	29	28	26	25	24	23	22	21	20
		25	38	57	48	9	10	43	23	45	3	44	27	58	7	47	50	15	55	51	58	15	42	16	57	44	36	33	34	38	47
		41	52	53	46	29	24	53	4	54	19	5	20	40	46	5	56	1	56	2	11	40	4	13	1	2	14	5	7	55	5

Solar altitude		**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Attested shadow length of a 12-digit gnomon in MS Tk	UNITS	19	19	18	17	17	16	15	15	14	14	13	13	12	12	12	11	11	10	10	10	9	9	9	8	8	8	7	7	7	6
	MINUTES	58	12	48	47	8	31	55	21	49	18	48	19	55	25	0	35	11	48	25	4	43	22	2	43	24	5	47	29	12	55
	SECONDS	17	14	42	27	16	0	28	33	7	4	16	38	6	35	0	17	25	17	53	9	3	32	14	7	9	38	34	54	37	42
Recomputed shadow length of a 12-digit gnomon (rounded to seconds)		19	19	18	17	17	16	15	15	14	14	13	13	12	12	12	11	11	10	10	10	9	9	9	8	8	8	7	7	7	6
		58	12	28	47	8	31	55	21	49	18	48	19	52	25	0	35	11	48	25	4	43	22	2	43	24	5	47	29	12	55
		17	14	42	26	16	0	28	33	8	4	16	39	5	35	0	18	26	17	53	9	3	32	34	7	9	39	34	55	37	42
Difference between the digits in corresponding sexagesimal places		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	+20	0	0	0	0	0	0	0	0	0	+3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	+1	0	0	0	0	−1	0	0	−1	+1	0	0	−1	−1	0	0	0	0	0	−20	0	0	−1	0	−1	0	0
Proposed emendation to the attested value in MS Tk		19	19	18	17	17	16	15	15	14	14	13	13	12	12	12	11	11	10	10	10	9	9	9	8	8	8	7	7	7	6
		58	12	28	47	8	31	55	21	49	18	48	19	52	25	0	35	11	48	25	4	43	22	2	43	24	5	47	29	12	55
		17	14	42	27	16	0	28	33	7	4	16	38	6	35	0	17	25	17	53	9	3	32	34	7	9	38	34	54	37	42

Solar altitude		**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
Attested shadow length of a 12-digit gnomon in MS Tk	UNITS	6	6	6	5	5	5	5	4	4	4	4	3	3	3	3	2	2	2	2	2	1	1	1	1	1	0	0	0	0	0
	MINUTES	39	22	6	51	35	20	5	50	36	22	7	53	40	26	12	59	46	33	19	6	54	41	28	15	3	50	24	32	32	0
	SECONDS	6	50	52	10	44	34	37	4	23	4	55	57	8	27	55	31	14	2	57	57	2	11	25	41	0	29	9	24	24	0
Recomputed shadow length of a 12-digit gnomon (rounded to seconds)		6	6	6	5	5	5	5	4	4	4	4	3	3	3	3	2	2	2	2	2	1	1	1	1	1	0	0	0	0	0
		39	22	6	51	35	20	5	50	36	22	7	53	40	26	12	59	46	33	19	6	54	41	28	15	3	50	24	25	12	0
		6	50	52	10	45	34	37	54	23	3	55	57	7	27	55	31	13	2	57	57	2	11	24	40	0	21	9	9	34	0
Difference between the digits in corresponding sexagesimal places		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+7	+20	0
		0	0	0	0	−1	0	0	−50	0	+1	0	0	+1	0	0	0	+1	0	0	0	0	0	+1	+1	0	+8	0	+15	−10	0
Proposed emendation to the attested value in MS Tk		6	6	6	5	5	5	5	4	4	4	4	3	3	3	3	2	2	2	2	2	1	1	1	1	1	0	0	0	0	0
		39	22	6	51	35	20	5	50	36	22	7	53	40	26	12	59	46	33	19	6	54	41	28	15	3	50	24	25	12	0
		6	50	52	10	44	34	37	54	23	4	55	57	8	27	55	31	14	2	57	57	2	11	25	41	0	21	9	9	34	0

Table VI.C$_3$: Recomputations and analyses of the Table of shadow lengths (*śaṅkuchāyā*) of a 7-digit gnomon discussed in Sections 3.5–3.6

Solar altitude	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
Attested shadow length of a 7-digit gnomon in MS Tk	410	200	133	100	80	66	57	49	44	39	36	32	30	28	26	24	22	21	20	19	18	17	16	15	15	14	13	13	12	12	UNITS
	59	27	33	13	0	36	0	48	11	41	0	55	19	4	7	24	53	32	19	13	14	19	29	43	0	21	44	9	37	7	MINUTES
	59	40	46	27	32	4	36	30	46	56	43	54	14	30	28	43	46	38	46	57	8	32	27	20	42	8	18	54	42	28	SECONDS
Recomputed shadow length of a 7-digit gnomon (rounded to seconds)	400	200	133	100	80	66	57	49	44	39	36	32	30	28	26	24	22	21	20	19	18	17	16	15	15	14	13	13	12	12	
	59	27	33	13	0	36	0	48	11	41	0	55	19	4	7	24	53	32	19	13	14	19	29	43	0	21	44	9	37	7	
	59	40	46	27	32	4	36	30	46	56	43	54	14	30	28	43	46	37	46	57	8	32	27	20	41	8	18	54	42	28	
Difference between the digits in corresponding sexagesimal places	+10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	+1	0	0	0	0	0	
Proposed emendation to the attested value in MS Tk	400	200	133	100	80	66	57	49	44	39	36	32	30	28	26	24	22	21	20	19	18	17	16	15	15	14	13	13	12	12	
	59	27	33	13	0	36	0	48	11	41	0	55	19	4	7	24	53	32	19	13	14	19	29	43	0	21	44	9	37	7	
	59	40	46	27	32	4	36	30	46	56	43	54	14	30	28	43	46	38	46	57	8	32	27	20	42	8	18	54	42	28	

Solar altitude	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	
Attested shadow length of a 7-digit gnomon in MS Tk	11	11	10	10	9	9	9	8	8	8	8	7	7	7	7	6	6	6	6	5	5	5	5	5	4	4	4	4	4	4	UNITS
	39	12	46	22	59	38	17	57	38	20	3	46	30	14	0	45	31	18	5	52	40	28	16	5	54	43	32	22	12	2	MINUTES
	0	8	45	41	49	5	42	35	39	32	9	27	24	55	0	35	39	10	6	25	7	8	30	9	5	18	45	27	21	29	SECONDS
Recomputed shadow length of a 7-digit gnomon (rounded to seconds)	11	11	10	10	9	9	9	8	8	8	8	7	7	7	7	6	6	6	6	5	5	5	5	5	4	4	4	4	4	4	
	39	12	46	22	59	38	17	57	38	20	3	46	30	14	0	45	31	18	5	52	40	28	16	5	54	43	32	22	12	2	
	0	8	45	40	49	5	42	34	40	32	9	28	23	55	0	35	39	10	6	25	6	8	30	9	5	18	45	27	22	29	
Difference between the digits in corresponding sexagesimal places	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	+1	0	0	0	+1	−1	0	0	−1	+1	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	−1	0	
Proposed emendation to the attested value in MS Tk	11	11	10	10	9	9	9	8	8	8	8	7	7	7	7	6	6	6	6	5	5	5	5	5	4	4	4	4	4	4	
	39	12	46	22	59	38	17	57	38	20	3	46	30	14	0	45	31	18	5	52	40	28	16	5	54	43	32	22	12	2	
	0	8	45	41	49	5	42	35	39	32	9	27	24	55	0	35	39	10	6	25	7	8	30	9	5	18	45	27	21	29	

Solar altitude	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	
Attested shadow length of a 7-digit gnomon in MS Tk	3	3	3	3	3	3	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	UNITS
	52	43	34	24	15	7	59	49	41	32	24	16	8	0	52	44	36	29	21	14	16	59	51	44	36	29	14	7	7	0	MINUTES
	49	19	0	51	51	0	17	41	13	52	37	28	24	26	32	43	59	16	38	3	31	34	34	34	45	22	40	10	10	0	SECONDS
Recomputed shadow length of a 7-digit gnomon (rounded to seconds)	3	3	3	3	3	3	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	
	52	43	34	24	15	7	58	49	41	32	24	16	8	0	52	44	36	29	21	14	6	59	51	44	36	29	22	14	7	0	
	49	19	0	51	51	0	17	41	13	52	37	28	24	26	32	43	58	16	38	3	31	2	34	9	45	22	1	40	20	0	
Difference between the digits in corresponding sexagesimal places	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	+10	0	0	0	0	0	−8	−7	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	+32	0	+25	0	0	+39	−30	−10	0	
Proposed emendation to the attested value in MS Tk	3	3	3	3	3	3	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	
	52	43	34	24	15	7	58	49	41	32	24	16	8	0	52	44	36	29	21	14	6	59	51	44	36	29	22	14	7	0	
	49	19	0	51	51	0	17	41	13	52	37	28	24	26	32	43	59	16	38	3	31	2	34	9	45	22	1	40	20	0	

Table VI.D: Recomputations and analyses of the Table of lunar latitudes (*śara*) discussed in Sections 3.7–3.8

Lunar-nodal elongation	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	
Attested lunar latitude in MS Tk	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	DEGREES
	4	9	14	18	23	28	32	37	42	46	51	56	0	5	9	14	18	23	27	32	36	41	45	49	53	58	2	6	10	14	MINUTES
	43	25	8	48	29	11	51	32	11	50	28	4	40	14	47	19	49	19	48	13	38	1	22	42	59	14	26	37	44	52	SECONDS
Recomputed lunar latitude (rounded to seconds)	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	
	4	9	14	18	23	28	32	37	42	46	51	56	0	5	9	14	18	23	27	32	36	41	45	49	54	58	2	6	10	14	
	43	24	7	49	30	11	52	32	11	49	27	4	40	14	49	21	51	21	49	16	40	3	24	43	0	15	28	39	47	54	
Difference between the digits in corresponding sexagesimal places	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	
	0	+1	+1	−1	−1	0	−1	0	0	+1	+1	0	0	0	−2	−2	−2	−2	−1	−3	−2	−2	−2	−1	+59	−1	−2	−2	−3	−2	
Proposed emendation to the attested value in MS Tk	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	
	4	9	14	18	23	28	32	37	42	46	51	56	0	5	9	14	18	23	27	32	36	41	45	49	53	58	2	6	10	14	
	43	25	8	48	29	11	51	32	11	50	28	4	40	14	47	19	49	19	48	13	38	1	22	42	59	14	26	37	44	52	

Lunar-nodal elongation	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	
Attested lunar latitude in MS Tk	2	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	DEGREES
	18	22	26	30	34	38	42	46	49	53	57	0	4	7	10	14	17	20	23	26	29	32	35	38	41	43	[--]	[--]	[--]	[--]	MINUTES
	56	57	55	50	44	35	22	6	47	25	0	31	0	36	48	6	20	32	39	43	44	40	32	19	5	45	[--]	[--]	[--]	[--]	SECONDS
Recomputed lunar latitude (rounded to seconds)	2	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
	18	22	26	30	34	38	42	46	49	53	57	0	4	7	10	14	17	20	23	26	29	32	35	38	41	43	46	48	51	53	
	56	58	56	52	44	35	22	6	47	25	0	32	0	26	48	6	21	33	40	44	44	40	32	20	5	45	21	53	21	45	
Difference between the digits in corresponding sexagesimal places	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	[--]	[--]	[--]	[--]	
	0	−1	−1	−2	0	0	0	0	0	0	0	−1	0	+10	0	0	−1	−1	−1	−1	0	0	0	−1	0	0	[--]	[--]	[--]	[--]	
Proposed emendation to the attested value in MS Tk	2	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
	18	22	26	30	34	38	42	46	49	53	57	0	4	7	10	14	17	20	23	26	29	32	35	38	41	43	[--]	[--]	[--]	[--]	
	56	57	55	50	44	35	22	6	47	25	0	31	0	26	48	6	20	32	39	43	44	40	32	19	5	45	[--]	[--]	[--]	[--]	

Lunar-nodal elongation	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	
Attested lunar latitude in MS Tk	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	DEGREES
	56	58	0	2	4	6	8	10	12	14	16	16	18	20	21	22	24	24	25	25	25	26	27	28	28	29	29	29	29	30	MINUTES
	4	19	31	37	38	36	29	17	1	40	13	45	10	31	46	57	2	5	0	52	39	21	59	32	20	37	50	57	57	0	SECONDS
Recomputed lunar latitude (rounded to seconds)	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	56	58	0	2	4	6	8	10	12	13	15	16	18	19	20	21	23	24	25	25	26	27	27	28	28	29	29	29	29	$\boxed{30}$	
	4	20	30	36	38	36	29	17	0	40	15	45	10	30	46	57	3	4	1	52	38	21	58	29	57	19	37	49	56	$\boxed{0}$	
Difference between the digits in corresponding sexagesimal places	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	+1	+1	0	0	+1	+1	+1	+1	0	0	0	−1	−1	0	0	0	0	0	0	0	0	
	0	−1	+1	+1	0	0	0	0	+1	0	−2	0	0	+1	0	0	−1	+1	−1	0	+1	0	+1	+3	−37	+18	+13	+8	+1	0	
Proposed emendation to the attested value in MS Tk	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
	56	58	0	2	4	6	8	10	12	14	16	16	18	20	21	22	24	24	25	25	25	26	27	28	28	29	29	29	29	30	
	4	19	31	37	38	36	29	17	1	40	13	45	10	31	46	57	2	5	0	52	39	21	59	32	57	20	37	50	57	0	

Appendix C: Statistical Analysis

C.1. Choosing systematic rounding over truncation

All regular recomputations in this study express sexagesimal numbers up to the second fractional place. To reduce a sexagesimal number in the final result of a recomputation, I chose to *systematically round* the number to the seconds place instead of *truncating* it; in other words, for a number of the form $a; b, c, d$, I round the number to $a; b, c$ when $d < 30$ or $a; b, c + 1$ when $d \geq 30$ (instead of truncating it to $a; b, c$ for any value of d). To validate this choice, I statistically test the proportion of differences between the attested and recomputed values when two mutually independent strategies are used to reduce the final result, namely, systematic rounding and truncation. In both reduction strategies, computing the differences between the attested values and the recomputed results are considered as binary events, i.e. they generate zero (0-state) or non-zero values (1-state) of the differences. The z-test for two population proportions is then used to test the efficacy of these two strategies in minimising the proportion of the differences for every table from the selected corpus. The parameters, hypotheses, and test statistic in implementing this test are described below.

1. The ninety reduced entries (i.e. the final results) using systematic rounding and those using truncation are considered as two independent populations with a common size. The total number of determinate events n_{det} is selected as the common sample size from both populations. The determinate events are those instances where a clear distinction can be made between the choice of sexagesimal reduction.[53] For every table, the reduced sample size n_{det} is large enough (i.e. greater than thirty) to assume normality, and the individual events (in the 0-state or 1-state) in the sample are mutually independent.

2. In the two samples of size n_{det}, $x_{\text{det}}^{\text{sys.rnd}}$ indicates the number of 1-state events (i.e. those producing non-zero differences between the attested and recomputed values) generated by the first population (systematic rounding) and $x_{\text{det}}^{\text{trunc}}$ indicates the 1-state events generated by the second population (truncation). With these values, the sample proportions for the two populations are computed as

[53] For a sexagesimal result $a; b, c, d$ with $d \leq 29$, systematic rounding or truncation reduce the number to $a; b, c$ identically. Such instances are called *indeterminate* events n_{indet} as the two reduction strategies are indistinguishable. The present analysis only includes *determinate* events n_{det} where the reduction strategies can be clearly identified from one another; in other words, cases where the recomputed results are $a; b, c, d$ with $d \geq 30$ (and hence reduced to $a; b, c + 1$ by systematic rounding or $a; b, c$ by truncation). For every table, $n_{\text{det}} + n_{\text{indet}} = 90$.

$$\hat{p}^{\text{sys.rnd}}_{\text{det}} = \frac{x^{\text{sys.rnd}}_{\text{det}}}{n_{\text{det}}} \quad \text{and} \quad \hat{p}^{\text{trunc}}_{\text{det}} = \frac{x^{\text{trunc}}_{\text{det}}}{n_{\text{det}}}.$$

3. To statistically test:

   the null hypothesis H_0: $\hat{p}^{\text{sys.rnd}}_{\text{det}} \leq \hat{p}^{\text{trunc}}_{\text{det}}$ against
   the alternative hypothesis H_a: $\hat{p}^{\text{sys.rnd}}_{\text{det}} > \hat{p}^{\text{trunc}}_{\text{det}}$.

   The null hypothesis maintains that the proportion of 1-state events in the first population is lower or equal to those in the second population, whereas the alternative hypothesis claims the converse. In other words, the null hypothesis expresses the belief that systematic rounding is statistically better (or at least, equivalent to) truncating the digits when the two reduction strategies are compared. The alternative hypothesis, if true, shows that truncating the digits, instead of systematically rounding them, is significantly better at minimising the non-zero differences between the attested and recomputed results.

4. The test statistic based on the pooled sample proportion is:

$$z\text{-statistic: } z = \frac{\hat{p}^{\text{sys.rnd}}_{\text{det}} - \hat{p}^{\text{trunc}}_{\text{det}}}{\sqrt{\hat{p}_{\text{det}} \times (1 - \hat{p}_{\text{det}}) \times \left(\frac{2}{n_{\text{det}}}\right)}},$$

   where

$$\hat{p}_{\text{det}} \equiv \frac{\hat{p}^{\text{sys.rnd}}_{\text{det}} \times n_{\text{det}} + \hat{p}^{\text{trunc}}_{\text{det}} \times n_{\text{det}}}{n_{\text{det}} + n_{\text{det}}} = \frac{x^{\text{sys.rnd}}_{\text{det}} + x^{\text{trunc}}_{\text{det}}}{2n_{\text{det}}}$$

   is the pooled proportion. For every table, n_{det} is large enough to ensure $\hat{p}_{\text{det}} \times n_{\text{det}} \geq 5$ and $(1 - \hat{p}_{\text{det}}) \times n_{\text{det}} \geq 5$. This allows the z-statistic to be validly applied.

5. The hypothesis is tested at a 5% level of significance α using the right-tailed z-test for two population proportions. For $\alpha = 0.05$, the decision rule is:

   Reject H_0 $\forall$ $z \in R$, where the rejection region $R := \{z : z > 1.645\}$.

   The critical value of the right-tailed z-test is taken as $z_c \equiv z_\alpha = 1.645$.

As Table 3 shows, the calculated z-statistic lies outside the rejection region for all six tables of the selected corpus, and therefore, the null hypothesis H_0 is retained and the alternative H_a is rejected. At a 5% level of significance, the proportion of non-zero differences between the attested and recomputed values using systematic rounding is lower (or at the very least, equal to) the proportion when truncation is used. The recomputations in this study, in particular, the final results of a calculation, are reduced to seconds by systematically rounding the digits based on this statistical inference.

Type of recomputation	Sexagesimal reduction strategies	
	Systematic rounding	Truncation
Sines Table VI.A	$n_{\text{det}} = 46$ and $n_{\text{indet}} = 44$	
	$x_{\text{det}}^{\text{sys.rnd}} = 3,\quad \hat{p}_{\text{det}}^{\text{sys.rnd}} \approx 0.065$	$x_{\text{det}}^{\text{trunc}} = 46,\quad \hat{p}_{\text{det}}^{\text{trunc}} = 1$
	$\hat{p}_{\text{det}} = {}^{49}/_{92} \approx 0.533$ and $z \approx {}^{-0.935}/_{0.104} \approx -8.985 < z_c = 1.645$ $\because z \notin R \Rightarrow$ Accept H_0 and reject H_a	
	statistically preferred	statistically rejected
Solar declinations Table VI.B	$n_{\text{det}} = 45$ and $n_{\text{indet}} = 45$	
	$x_{\text{det}}^{\text{sys.rnd}} = 26,\quad \hat{p}_{\text{det}}^{\text{sys.rnd}} = 0.578$	$x_{\text{det}}^{\text{trunc}} = 26,\quad \hat{p}_{\text{det}}^{\text{trunc}} \approx 0.578$
	$\hat{p}_{\text{det}} = {}^{52}/_{90} \approx 0.578$ and $z \approx {}^{0}/_{0.104} = 0 < z_c = 1.645$ $\because z \notin R \Rightarrow$ Accept H_0 and reject H_a	
	statistically preferred	statistically rejected
Shadow lengths: 60-digit gnomon Table VI.C_1	$n_{\text{det}} = 42$ and $n_{\text{indet}} = 48$	
	$x_{\text{det}}^{\text{sys.rnd}} = 19,\quad \hat{p}_{\text{det}}^{\text{sys.rnd}} \approx 0.452$	$x_{\text{det}}^{\text{trunc}} = 23,\quad \hat{p}_{\text{det}}^{\text{trunc}} \approx 0.548$
	$\hat{p}_{\text{det}} = {}^{42}/_{84} = 0.5$ and $z \approx {}^{-0.096}/_{0.109} \approx -0.881 < z_c = 1.645$ $\because z \notin R \Rightarrow$ Accept H_0 and reject H_a	
	statistically preferred	statistically rejected
Shadow lengths: 12-digit gnomon Table VI.C_2	$n_{\text{det}} = 48$ and $n_{\text{indet}} = 42$	
	$x_{\text{det}}^{\text{sys.rnd}} = 13,\quad \hat{p}_{\text{det}}^{\text{sys.rnd}} \approx 0.271$	$x_{\text{det}}^{\text{trunc}} = 37,\quad \hat{p}_{\text{det}}^{\text{trunc}} = 0.771$
	$\hat{p}_{\text{det}} = {}^{50}/_{96} \approx 0.521$ and $z \approx {}^{-0.5}/_{0.102} \approx -4.903 < z_c = 1.645$ $\because z \notin R \Rightarrow$ Accept H_0 and reject H_a	
	statistically preferred	statistically rejected
Shadow lengths: 7-digit gnomon Table VI.C_3	$n_{\text{det}} = 41$ and $n_{\text{indet}} = 49$	
	$x_{\text{det}}^{\text{sys.rnd}} = 5,\quad \hat{p}_{\text{det}}^{\text{sys.rnd}} = 0.122$	$x_{\text{det}}^{\text{trunc}} = 38,\quad \hat{p}_{\text{det}}^{\text{trunc}} = 0.927$
	$\hat{p}_{\text{det}} = {}^{43}/_{82} \approx 0.524$ and $z \approx {}^{-0.805}/_{0.110} \approx -7.297 < z_c = 1.645$ $\because z \notin R \Rightarrow$ Accept H_0 and reject H_a	
	statistically preferred	statistically rejected
Lunar latitudes Table VI.D	$n_{\text{det}} = 49$ and $n_{\text{indet}} = 41$	
	$x_{\text{det}}^{\text{sys.rnd}} = 29,\quad \hat{p}_{\text{det}}^{\text{sys.rnd}} \approx 0.592$	$x_{\text{det}}^{\text{trunc}} = 36,\quad \hat{p}_{\text{det}}^{\text{trunc}} \approx 0.735$
	$\hat{p}_{\text{det}} = {}^{65}/_{98} \approx 0.663$ and $z \approx {}^{-0.143}/_{0.095} \approx -1.496 < z_c = 1.645$ $\because z \notin R \Rightarrow$ Accept H_0 and reject H_a	
	statistically preferred	statistically rejected

Table 3: Statistical test (right-tailed z-test for two population proportions) to select between systematic rounding or truncation (two mutually independent reduction strategies) to reduce the final results of the recomputations to the second fractional place for the six tables from MS Tk.

C.2. Choosing recomputed Sines over the attested Sines in MS Tk

The solar declinations, shadow lengths of gnomons of various heights, and lunar latitudes in this study are calculated using the recomputed Sines (Sin_r) instead of the attested Sines (Sin_a) in MS Tk.[54] I justify this choice on the basis of the following two statistical measures:

1. The first measure compares the differences d_i s between the attested values of these functions (from MS Tk) and their recomputed values using Sin_r and Sin_a separately, i.e.

$$d_i^{\mathrm{Sin}\,a} = |\mathrm{Value}_i^{\mathrm{recomp}}[\mathrm{Sin}_a] - \mathrm{Value}_i^{\mathrm{attest}}| \qquad \text{and}$$
$$d_i^{\mathrm{Sin}\,r} = |\mathrm{Value}_i^{\mathrm{recomp}}[\mathrm{Sin}_r] - \mathrm{Value}_i^{\mathrm{attest}}| \forall\ i \in \mathbb{N}_{90}$$

   Similar to the 0-state and 1-state described in Appendix C.1, these differences (i.e. $d_i^{\mathrm{Sin}\,a}$ and $d_i^{\mathrm{Sin}\,r}$) are considered as binary events. Accordingly, I consider

   - $x_{\mathrm{sim}}^{\mathrm{Sin}\,a}$ and $x_{\mathrm{sim}}^{\mathrm{Sin}\,r}$ as the number of 0-states (i.e. instances when the differences d_i s are similar or zero) using the attested and recomputed Sines respectively, and
   - $x_{\mathrm{diss}}^{\mathrm{Sin}\,a}$ and $x_{\mathrm{diss}}^{\mathrm{Sin}\,r}$ as the number of 1-states (i.e. instances when the differences d_i s are dissimilar or non-zero) using the attested and recomputed Sines respectively.

   For a total of $n = 90$ entries for each function, the proportion of 0 and 1 states using Sin_r and Sin_a separately can be expressed as

$$p_{\mathrm{sim}}^{Sin_a} = \frac{x_{\mathrm{sim}}^{Sin_a}}{n}, \quad p_{\mathrm{sim}}^{Sin_r} = \frac{x_{\mathrm{sim}}^{Sin_r}}{n}, \quad p_{\mathrm{diss}}^{Sin_a} = \frac{x_{\mathrm{diss}}^{Sin_a}}{n}, \quad \text{and} \quad p_{\mathrm{diss}}^{Sin_r} = \frac{x_{\mathrm{diss}}^{Sin_r}}{n}.$$

   Table 4 presents these four proportions (in percentages) for the recomputations of the solar declinations, shadow lengths of gnomons of various heights, and lunar latitudes in 2×2 contingency tables. For lunar latitudes, the attested values for 57°, 58°, 59 °, and 60° are illegible in MS Tk, and accordingly, $n = 86$ for calculating these proportions. The percentage proportion of dissimilar (non-zero) differences between the attested and recomputed function values are typically lower (or, at the very least, comparably equal) when recomputed Sines are used instead of the attested Sines from MS Tk. Equivalently, the percentage

[54] I use the attested Sine values with my proposed emendations (to correct for scribal discrepancies) in this analysis. For example, the attested $\mathrm{Sin}_a 1°$ is taken as 1;2,50 instead of 1;5,50 (seen in MS Tk). Without these emendation, the recomputed function values based on the attested Sines become highly irregular and statistically superfluous. Also, for all calculations in this analysis, the final sexagesimal results are systematically rounded to the second fractional place.

Solar declinations		
Differences / Sines	0-state (similar or zero)	1-state (dissimilar or non-zero)
Attested	$p_{\text{sim}}^{Sin_a} = {}^{50}/_{90} \approx 55.56\%$	$p_{\text{diss}}^{Sin_a} = {}^{40}/_{90} \approx 44.44\%$
Recomputed	$p_{\text{sim}}^{Sin_r} = {}^{50}/_{90} \approx 55.56\%$	$p_{\text{diss}}^{Sin_r} = {}^{40}/_{90} \approx 44.44\%$

Shadow lengths: 60-digit gnomon		
Differences / Sines	0-state (similar or zero)	1-state (dissimilar or non-zero)
Attested	$p_{\text{sim}}^{Sin_a} = {}^{49}/_{90} \approx 54.44\%$	$p_{\text{diss}}^{Sin_a} = {}^{41}/_{90} \approx 45.56\%$
Recomputed	$p_{\text{sim}}^{Sin_r} = {}^{58}/_{90} \approx 64.44\%$	$p_{\text{diss}}^{Sin_r} = {}^{32}/_{90} \approx 35.56\%$

Shadow lengths: 12-digit gnomon		
Differences / Sines	0-state (similar or zero)	1-state (dissimilar or non-zero)
Attested	$p_{\text{sim}}^{Sin_a} = {}^{52}/_{90} \approx 57.78\%$	$p_{\text{diss}}^{Sin_a} = {}^{38}/_{90} \approx 42.22\%$
Recomputed	$p_{\text{sim}}^{Sin_r} = {}^{63}/_{90} = 70\%$	$p_{\text{diss}}^{Sin_r} = {}^{27}/_{90} = 30\%$

Shadow lengths: 7-digit gnomon		
Differences / Sines	0-state (similar or zero)	1-state (dissimilar or non-zero)
Attested	$p_{\text{sim}}^{Sin_a} = {}^{67}/_{90} \approx 74.44\%$	$p_{\text{diss}}^{Sin_a} = {}^{23}/_{90} \approx 25.56\%$
Recomputed	$p_{\text{sim}}^{Sin_r} = {}^{80}/_{90} \approx 88.89\%$	$p_{\text{diss}}^{Sin_r} = {}^{10}/_{90} \approx 11.11\%$

Lunar latitudes		
Differences / Sines	0-state (similar or zero)	1-state (dissimilar or non-zero)
Attested	$p_{\text{sim}}^{Sin_a} = {}^{14}/_{86} \approx 16.28\%$	$p_{\text{diss}}^{Sin_a} = {}^{72}/_{86} \approx 83.72\%$
Recomputed	$p_{\text{sim}}^{Sin_r} = {}^{34}/_{86} \approx 39.53\%$	$p_{\text{diss}}^{Sin_r} = {}^{52}/_{86} \approx 60.47\%$

Table 4: 2×2 contingency tables showing the proportions of differences (in percentages) between the attested and recomputed values of solar declinations, shadow lengths for gnomons of various heights, and lunar latitudes calculated using the attested Sines (in MS Tk) and the recomputed Sines separately.

Type of Recomputation	Sines	
	Attested	Recomputed
Solar declinations	RMSD $\approx 7.258^s$ AAD $\approx 2.779^s$	RMSD $\approx 1.265^s$ AAD $\approx 0.689^s$
Shadow lengths: 60-digit gnomon	RMSD $\approx 380.840^s$ AAD $= 52.7^s$	RMSD $\approx 379.528^s$ AAD $\approx 41.067^s$
Shadow lengths: 12-digit gnomon	RMSD $\approx 6.536^s$ AAD $\approx 2.811^s$	RMSD $\approx 0.767^s$ AAD $= 0.367^s$
Shadow lengths: 7-digit gnomon	RMSD $\approx 3.485^s$ AAD $\approx 1.367^s$	RMSD $\approx 0.333^s$ AAD $\approx 0.111^s$
Lunar latitudes	RMSD $\approx 18.161^s$ AAD $\approx 6.779^s$	RMSD $\approx 18.219^s$ AAD $\approx 6.256^s$

Table 5: Table comparing the Root Mean Square Deviation (RMSD) and Average Absolute Deviation (AAD) (both measures in seconds) in recomputing the solar declinations, shadow lengths of gnomons of various heights, and lunar latitudes using the attested Sines (in MS Tk) and the recomputed Sines separately.

of similar (zero) differences between the attested and recomputed function values are typically higher (or comparably equal) when recomputed Sines are used. This provides the first measure of validation for using the recomputed Sines in calculating the other functions in this study.

2. In addition to the percentage proportions of differences, I calculate the Root Mean Square Deviation (RMSD) and the Average Absolute Deviation (AAD) for an Ordinary Least Squares (OLS) regression model as a second statistical measure to validate my choice. For each recomputed function, treating the attested $\text{Value}_i^{\text{attest}}$ as the predicted value y_i and the recomputed $\text{Value}_i^{\text{recomp}}[\text{Sin}_a]$ or $\text{Value}_i^{\text{recomp}}[\text{Sin}_r]$ as the observed value $\hat{y}_i^\alpha$ (α being Sin_a or Sin_r, and $i \in \mathbb{N}_{90}$), the i^{th} residual is $e_i^\alpha = \hat{y}_i^\alpha - y_i$ (among the total $n = 90$ residuals). With this

$$\text{RMSD} = \sqrt{\frac{\sum_i^n \left(e_i^\alpha\right)^2}{n}} \equiv \sqrt{\frac{\sum_i^n \left(\hat{y}_i^\alpha - y_i\right)^2}{n}} \qquad \text{and}$$

$$\text{AAD} = \sqrt{\frac{\sum_i^n \left|e_i^\alpha\right|}{n}} \equiv \sqrt{\frac{\sum_i^n \left|\hat{y}_i^\alpha - y_i\right|}{n}}.$$

The RMSD measures the square root of the variance of the residual; in other words, it indicates the standard deviation of the unexplained variance between the prediction and the observation. The AAD indicates the absolute average value of the residual, i.e. the average difference between the attested and recomputed values of the functions. Both measures of fit are absolute measures (in the units of the entries themselves) with lower values indicating a better fit. In OLS regression models, RMSD and AAD are used to indicate how accurately a model predicts the response. Table 5 lists the RMSD and AAD values (in seconds) for my recomputations of the solar declinations, shadow lengths of gnomons of various heights, and lunar latitudes using the attested and recomputed Sines. (The lunar latitude calculations use $n = 86$ as four attested entries in MS Tk are illegible.) The RMSD and AAD values are lower in most recomputations when recomputed Sines are used (instead of the attested Sines in MS Tk), and thus, provide a second reason to choose recomputed Sines to calculate the other functions in this study.[55]

C.3. Choosing the exact expression of lunar latitude over the approximate one

In this study, the lunar latitude β is recomputed for each degree of lunar-nodal elongation ω using the exact expression $\mathrm{Sin}\,\beta = \mathrm{Sin}\,4°30' \times {}^{\mathrm{Sin}\,\omega}/_{60}$ instead of the approximate expression $\beta \approx 4°30' \times {}^{\mathrm{Sin}\,\omega}/_{60}$. I justify this choice based on the following two statistical measures:

1. The first measure compares the proportion of differences between the attested and recomputed lunar latitudes when the two expressions are used separately. Similar to the first statistical measure in Appendix C.2 (note 1), the proportions of the 0-state (similar or zero) and 1-state (dissimilar or non-zero) differences using the exact and approximate expressions of lunar latitudes separately can be calculated as

$$p^{\mathrm{exact}}_{\mathrm{sim}} = \frac{x^{\mathrm{exact}}_{\mathrm{sim}}}{n},\ p^{\mathrm{approx}}_{\mathrm{sim}} = \frac{x^{\mathrm{approx}}_{\mathrm{sim}}}{n},\ p^{\mathrm{exact}}_{\mathrm{diss}} = \frac{x^{\mathrm{exact}}_{\mathrm{diss}}}{n},\ \text{and } p^{\mathrm{approx}}_{\mathrm{diss}} = \frac{x^{\mathrm{approx}}_{\mathrm{diss}}}{n}.$$

where $x^{\mathrm{exact}}_{\mathrm{sim}}$ and $x^{\mathrm{approx}}_{\mathrm{sim}}$ are the number of 0-states using the respective expressions; $x^{\mathrm{exact}}_{\mathrm{diss}}$ and $x^{\mathrm{approx}}_{\mathrm{diss}}$ are the number of 1-states using the respective expressions; and $n = 86$ (since four entries corresponding to the arguments 57° to 60° are illegible in MS Tk). Table 6 presents these four proportions (in percentages) for the lunar latitude recomputations in a 2×2 contingency table. Following previous calculations, the final sexagesimal results are systematically rounded to the second fractional place, and recomputed Sines (instead of the attested Sines in MS Tk) are used. The percentage proportion of dissimilar (non-zero) differences between the attested and recomputed lunar latitudes is lower when the exact expression is used instead of the approximate one. Or equivalently,

[55] The RMSD is sensitive to outliers as the effect of each residual is proportional to the size of its squared value. On account of this, the RMSD value for the lunar latitude recomputations using recomputed Sines is slightly larger than the corresponding value using attested Sines in Table 5.

Expressions \ Differences	0 state (similar or zero)	1 state (dissimilar or non-zero)
Exact	$p^{\text{exact}}_{\text{sim}} = {}^{34}/_{86} \approx 39.53\%$	$p^{\text{exact}}_{\text{diss}} = {}^{52}/_{86} \approx 60.47\%$
Approximate	$p^{\text{approx}}_{\text{sim}} = {}^{10}/_{86} \approx 11.63\%$	$p^{\text{approx}}_{\text{diss}} = {}^{80}/_{86} \approx 93.02\%$

Table 6: 2×2 contingency table showing the proportions of differences (in percentages) between the attested and recomputed values of lunar latitudes calculated using the exact and approximate expressions separately.

the percentage of similar (zero) differences between the attested and recomputed lunar latitudes is higher when the exact expression is used. This provides the first measure of validation for using the exact expression to recompute lunar latitudes.

2. I calculate the Median Absolute Deviation (MAD) of the differences between the attested and recomputed lunar latitudes using the exact and approximate expressions separately to establish the second statistical measure. With the i^{th} difference $d_i = \text{Value}_i^{\text{recomp}}[\tilde{\alpha}] - \text{Value}_i^{\text{attest}}$ where $\tilde{\alpha}$ is the exact or approximate expression and $i \in \mathbb{N}_{86}$,
$$\text{MAD} = \text{Median}\left(|d_i - \text{Median}(d_i)|\right).$$
MAD provides a robust measure of the variability of the differences with non-normal distributions.[56] With it, a median-centred interval $[\nu - 2\,\text{MAD}, \nu + 2\,\text{MAD}]$ can be constructed to identify statistical outliers that lie outside the limits. Table 7 provides the descriptive statistics for 86 entries of d_i s using the exact and approximate expressions of lunar latitude. When the exact expression is used,
   - 75 entries (out of 86) are within $\pm 2\,\text{MAD}$ of the median, in other words, a set of 75 differences $d_i^{\text{corrected}} \in [-2, 2]$ are statistically relevant; while
   - 78 entries (out of 86) are within $\pm 2\,\text{MAD}$ of the median when the appropriate expression is used, i.e. 78 differences $d_i^{\text{corrected}} \in [-7, 17]$ are statistically relevant.

   Among these outlier-corrected differences $d_i^{\text{corrected}}$,
   - there are 41 dissimilar (non-zero) differences out of 75, i.e. around 54.67%, when the exact expression is used, and
   - there are 72 dissimilar (non-zero) differences out of 78, i.e. around 92.03%, when the approximate expression is used.

[56] Typically, a normal distribution has skewness $\varsigma \sim 0$ and kurtosis $\kappa \sim 3$, with the mean $\mu \sim$ median ν. As Table 7 shows, the differences between the attested and recomputed lunar latitudes using the exact and approximate expressions are not normally distributed.

Type of Recomputation	Expressions of lunar latitude	
	Exact	Approximate
Median Absolute Deviation (MAD)	1	6
Median $\nu \equiv \text{Median}(d_i)$	0	5
Mean μ	≈ -2.442	≈ 1.953
Standard Deviation σ	≈ 18.055	≈ 18.578
Skewness $\varsigma(d_i)$	≈ -1.279	≈ -1.509
Kurtosis $\varkappa(d_i)$	≈ 7.275	≈ 6.789

Table 7: Table showing the descriptive statistics, including the Median Absolute Deviation (MAD) of the differences between the attested and recomputed lunar latitudes calculated using the exact and approximate expressions separately.

The lower percentage of outlier-corrected dissimilar (non-zero) differences between the attested and recomputed lunar latitudes using the exact expression (compared to the approximate one) validates its choice in this study.

C.4. Choosing the parameter Sin $4^\circ 30' = 4;45,25$ for lunar latitude recomputations

In this study, the lunar latitude β is recomputed for each degree of lunar-nodal elongation ω using the exact expression with the parameter Sin $4^\circ 30' = 4;42,25$ instead of 4;42,26 or 4;42,27.[57] I justify this choice based on the following two statistical measures:

1. The first measure compares the proportion of differences between the attested and recomputed values when the three estimates of the parameter Sin $4^\circ 30'$ are used separately. Similar to the first statistical measures in Appendices C.2–3 (note 1), the proportion of the 0-state (similar or zero) and 1-state (dissimilar or non-zero) differences can be separately calculated using 4;42,25, 4;42,26, and 4;42,27 as

$$p^{25^s}_{\text{sim}} = \frac{x^{25^s}_{\text{sim}}}{n}, \; p^{26^s}_{\text{sim}} = \frac{x^{26^s}_{\text{sim}}}{n}, \; p^{26^s}_{\text{sim}} = \frac{x^{27^s}_{\text{sim}}}{n},$$

$$p^{25^s}_{\text{diss}} = \frac{x^{25^s}_{\text{diss}}}{n}, \; p^{26^s}_{\text{diss}} = \frac{x^{26^s}_{\text{diss}}}{n}, \; \text{and } p^{26^s}_{\text{diss}} = \frac{x^{27^s}_{\text{diss}}}{n},$$

where $x^{25^s}_{\text{sim}}$, $x^{26^s}_{\text{sim}}$, and $x^{27^s}_{\text{sim}}$ are the 0-states using 4;42,25, 4;42,26, and 4;42,27 respectively; $x^{25^s}_{\text{diss}}$, $x^{26^s}_{\text{diss}}$, and $x^{27^s}_{\text{diss}}$ are the 1-states using the same

[57] The different estimates of the parameter Sin $4^\circ 30'$ are derived using different methods, see Section 3.7 (note 2).

Differences / Sin 4°30′	0 state (similar or zero)	1 state (dissimilar or non-zero)
4;42,25	$p^{25^s}_{\text{sim}} = {}^{38}/_{86} \approx 39.53\%$	$p^{25^s}_{\text{diss}} = {}^{52}/_{86} \approx 60.47\%$
4;42,26	$p^{26^s}_{\text{sim}} = {}^{30}/_{86} \approx 34.88\%$	$p^{26^s}_{\text{diss}} = {}^{60}/_{86} \approx 69.77\%$
4;42,27	$p^{27^s}_{\text{sim}} = {}^{14}/_{86} \approx 16.28\%$	$p^{27^s}_{\text{diss}} = {}^{76}/_{86} \approx 88.37\%$

Table 8: 3×2 contingency table showing the proportions of differences (in percentages) between the attested and recomputed values of lunar latitudes calculated with the parametric estimates 4;42,25, 4;42,26, and 4;42,27 separately.

parametric estimates respectively; and $n = 86$ (since four entries corresponding to the arguments 57° to 60° are illegible in MS Tk). Table 8 presents these six proportions (in percentages) for the lunar latitude recomputations in a 3×2 contingency table. Like the previous calculations, the final sexagesimal results are systematically rounded to the second fractional place, and recomputed Sines (instead of those attested in MS Tk) are used. The percentage proportions of dissimilar (non-zero) differences between the attested and recomputed lunar latitudes is lower with the parametric estimate 4;42,25 instead of 4;42,26 or 4;42,27. Or equivalently, the percentage of similar (zero) differences between the attested and recomputed lunar latitudes is higher when 4;42,25 is used. This provides the first measure to statistically validate using $\text{Sin}\,4°30' = 4;42,25$ to recompute the lunar latitudes.

2. The second statistical measure uses the Median Absolute Deviation (MAD) calculated for the three parametric estimates separately. As described in note 2 of Appendix C.3, the MAD values determines a median-centred interval $[\nu - 2\,\text{MAD}, \nu + 2\,\text{MAD}]$ of differences d_i s between the attested and recomputed lunar latitudes for each of the three parametric estimates. Table 9 provides the descriptive statistics for 86 entries of d_i s calculated with the parametric estimates 4;42,25, 4;42,26, and 4;42,27 separately.
   - Using 4;42,25, 75 entries (out of 86) are within $\pm 2\,\text{MAD}$ of the median, i.e. 75 differences $d_i^{\text{corrected}} \in [-2, 2]$ are statistically relevant;
   - using 4;42,26, 77 entries (out of 86) are within $\pm 2\,\text{MAD}$ of the median, i.e. 77 differences $d_i^{\text{corrected}} \in [-1, 3]$ are statistically relevant; and
   - using 4;42,27, 78 entries (out of 86) are within $\pm 2\,\text{MAD}$ of the median, i.e. 78 differences $d_i^{\text{corrected}} \in [-2, 4]$ are statistically relevant.

Type of Recomputation	$\text{Sin}\,4°30'$		
	4;42,25	4;42,26	4;42,27
Median Absolute Deviation (MAD)	1	1	1.5
Median $\nu \equiv \text{Median}(d_i)$	0	1	1
Mean μ	≈ -2.442	≈ -1.930	≈ -1.291
Standard Deviation σ	≈ 18.055	≈ 18.086	≈ 18.072
Skewness $\varsigma(d_i)$	≈ -1.279	≈ -1.741	≈ -1.638
Kurtosis $\kappa(d_i)$	≈ 7.275	≈ 7.64	≈ 7.690

Table 9: Table showing the descriptive statistics, including the Median Absolute Deviation (MAD) of the differences between the attested and recomputed lunar latitudes calculated with the parametric estimates 4;42,25, 4;42,26, and 4;42,27 separately.

Among these outlier-corrected differences $d_i^{\text{corrected}}$,

- there are 41 dissimilar (non-zero) difference out of 75, i.e. around 54.67%, when $\text{Sin}\,4°30' = 4;42,25$;
- there are 50 dissimilar (non-zero) difference out of 77, i.e. around 64.94%, when $\text{Sin}\,4°30' = 4;42,26$; and
- there are 70 dissimilar (non-zero) difference out of 78, i.e. around 89.74%, when $\text{Sin}\,4°30' = 4;42,27$.

The lower percentage of outlier-corrected dissimilar (non-zero) differences between the attested and recomputed lunar latitudes calculated with the parameter $\text{Sin}\,4°30' = 4;42,25$ (compared to the estimates 4;42,26 and 4;42,27) validates its choice in this study.