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Spectral Dissipation of Finite-depth Gravity Waves Due to Turbulent Bottom Friction[1]

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ABSTRACT

The spectral dissipation of finite-depth random gravity waves is evaluated for a given empirical relationship between the local bottom stress and the wave field. The dissipation is quasilinear. Computations for a quadratic friction law yield satisfactory agreement with wave measurements at two stations in the Gulf of Mexico.

1. *Introduction.* The traditional parametrical approach to wave prediction is being widely subseded by more fundamental methods based on the numerical integration of the radiative transfer equation (cf. Gelci and Cazalé 1962, Fons 1966, Pierson et al. 1966, Barnett 1966). The central prediction problem has thereby become the determination of the source functions in the transfer equation. Although many transfer processes are now understood theoretically, many gaps remain, particularly in our knowledge of the dissi-

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pative processes, which at present are bridged by partly empirical, largely intuitive, assumptions.

As a particular case, we consider here the dissipation due to turbulent bottom friction, which is known to be an important factor in wave prediction at coasts bounding extensive shallow-water areas, such as a continental shelf.

A rigorous treatment of the full problem, involving a nonstationary turbulent boundary layer at the ocean bottom, appears to be impossible at present. As a starting point, we assume therefore that a 'friction law' expressing the bottom wall stress, $\boldsymbol{\tau}$, as a function of the wave field has been established empirically. The problem then is to evaluate the spectral distribution of the associated wave-energy dissipation. Clearly this will not contribute to our understanding of the basic wave-turbulence interaction process; we are concerned, rather, with the practical question of determining a spectral source (sink) function for a random wave field from a given deterministic force-field relationship.

As an application, we compute the local dissipation and decay of the wave spectrum for the quadratic friction law

$$\boldsymbol{\tau} = -\varrho c_f \boldsymbol{u}^b |\boldsymbol{u}^b|, \tag{1.1}$$

where ϱ is the density of water, $\boldsymbol{u}^b$ is the bottom velocity in the absence of a boundary layer (including a mean current component), and c_f is a friction coefficient; c_f is in general a slowly varying function of the characteristic flow parameters, but it is regarded as constant for a given wave field.

The computed cases for shallow water over a gently sloping beach agree well with wave observations at two stations in the Gulf of Mexico. Further examples illustrate the significance of bottom friction effects in larger and somewhat deeper areas typical of a continental shelf.

2. *The Dissipation Function.* To a first approximation, a homogeneous random gravity-wave field in water of constant depth, H, is given by the linear irrotational solution

$$\zeta = \sum \zeta_{\boldsymbol{k}}, \tag{2.1}$$

where ζ is the wave height,

$$\begin{aligned} \zeta_{\boldsymbol{k}} &= \tilde{\zeta}_{\boldsymbol{k}} + \tilde{\zeta}_{\boldsymbol{k}}^*, \\ \tilde{\zeta}_{\boldsymbol{k}} &= Z_{\boldsymbol{k}} e^{i(\boldsymbol{k}\cdot\boldsymbol{x} - \sigma t)}, \\ Z_{\boldsymbol{k}} &= \text{const}, \\ \sigma &= (gk \tanh kH)^{1/2}, \end{aligned} \tag{2.2}$$

and $\boldsymbol{x}$, $\boldsymbol{k}$ are horizontal coordinate and wave-number vectors, respectively.

The first and second moments of the random amplitudes, $Z_{\boldsymbol{k}}$, are given by

$$\langle Z_{\boldsymbol{k}} \rangle = 0, \qquad \langle Z_{\boldsymbol{k}} Z_{\boldsymbol{k}'} \rangle = 0, \tag{2.3}$$

$$\langle Z_{\boldsymbol{k}} Z_{\boldsymbol{k}'}^{*} \rangle = \frac{1}{2} \delta_{\boldsymbol{k}\boldsymbol{k}'} F(\boldsymbol{k}) \Delta \boldsymbol{k}, \tag{2.4}$$

where $\langle \ldots . \rangle$ denotes the ensemble mean, $\Delta \boldsymbol{k}$ is the wave-number increment of the Fourier sum (2.1), used here as a notational convenience in place of a Fourier-Stieltjes integral, and $F(\boldsymbol{k})$ is the wave spectrum, normalized so that

$$\langle \zeta^2 \rangle = \int F(\boldsymbol{k}) d\boldsymbol{k}. \tag{2.5}$$

The velocity field of the wave motion may be similarly written

$$\boldsymbol{u} = \sum u_{\boldsymbol{k}},$$

where $\boldsymbol{u}_{\boldsymbol{k}} = \tilde{\boldsymbol{u}}_{\boldsymbol{k}} + \tilde{\boldsymbol{u}}_{\boldsymbol{k}}^{*}$, and $\tilde{\boldsymbol{u}}_{\boldsymbol{k}}$ is proportional to $\tilde{\zeta}_{\boldsymbol{k}}$. In particular,

$$\tilde{\boldsymbol{u}}_{\boldsymbol{k}} = \frac{g\boldsymbol{k}}{\sigma \cosh kH} \tilde{\zeta}_{\boldsymbol{k}} \quad \text{at the bottom,} \quad z = -H, \tag{2.6}$$

where z is the vertical coordinate, measured positive upward from the equilibrium free surface.

In the linear approximation, the wave field is Gaussian (cf. Hasselmann 1967). It is therefore completely specified statistically by the spectrum $F(\boldsymbol{k})$.

Consider now the modification of the free-wave solution due to the turbulent boundary layer at the ocean bottom. Interactions with a nonuniform current, $\boldsymbol{c} = (c_1(z), c_2(z), 0)$, and with the atmosphere also affect the wave field, but we restrict our investigation here to the boundary-layer interactions. The mean current enters the problem only in so far as it affects the boundary layer. In this case, the wave motion outside the boundary layer remains a potential flow, $\boldsymbol{u}$, which is uniquely determined by the surface displacement. The total velocity field is then $\boldsymbol{w} = \boldsymbol{u} + \boldsymbol{c} + \boldsymbol{v}$, where $\boldsymbol{v}$ is a turbulent velocity field that is nonzero only in the bottom boundary layer, $-H \leqq z < -H + \delta$. We assume $\delta \ll H$.

Since the wave field remains homogeneous, the representation (2.1) and the definition (2.4) of the spectrum remain valid. However, the amplitudes $Z_{\boldsymbol{k}}$, and therefore the spectrum $F(\boldsymbol{k})$, become slowly varying functions of time.

The rate of change of the spectrum may be determined from energy considerations. If we multiply the equations of motion,

$$\varrho \frac{\partial w_i}{\partial t} = -\frac{\partial p}{\partial x_i} - \varrho \frac{\partial}{\partial x_j}(w_i w_j) + \nu \varrho \nabla^2 w_i$$

$$(\nu = \text{kinematic viscosity}),$$

by u_{ik}, integrate from $-H$ to ζ, and take expectation values, we obtain, on allowing for the usual free-surface boundary conditions at $z = \zeta$ and the orthogonality relationship (2.4),

$$\Delta k \varrho g \frac{\partial F(\boldsymbol{k})}{\partial t} = \langle \tau_i u_{i\boldsymbol{k}} \rangle_{z=-H} - \int_{-H}^{-H+\delta} \langle T_{ij} \frac{\partial u_{i\boldsymbol{k}}}{\partial x_j} \rangle \, dz - \varrho \frac{\partial}{\partial t} \int_{-H}^{-H+\delta} \langle (v_i + c_i) u_{i\boldsymbol{k}} \rangle \, dz,$$

where

$$T_{ij} = -\varrho w_i w_j + \varrho \nu \left(\frac{\partial w_i}{\partial x_j} + \frac{\partial w_j}{\partial x_i} \right)$$

and $\tau_i = T_{3i}$.

For $\delta \ll H$, the last two terms on the right side can be neglected, so that

$$\Delta \boldsymbol{k} \varrho g \frac{\partial F(\boldsymbol{k})}{\partial t} = \langle \tau_i u_{i\boldsymbol{k}} \rangle_{z=-H} = -\Theta(\boldsymbol{k}) \Delta \boldsymbol{k}. \quad (2.7)$$

Hence, the energy loss, $\Theta(\boldsymbol{k})\Delta \boldsymbol{k}$, of the wave component, $\zeta_{\boldsymbol{k}}$, is given by the work done by the total bottom stress, $\boldsymbol{\tau}$, against the bottom velocity of the component $\boldsymbol{u_k}(z = -H)$, as may have been anticipated.

We assume now that $\boldsymbol{\tau}$ is given empirically as a function of the wave field and the mean current, $\boldsymbol{\tau} = \boldsymbol{\tau}(\zeta, \boldsymbol{c})$. If the dissipation is weak, i. e., if $\partial F/\partial t \ll \omega_c F$, where ω_c is a characteristic frequency of the spectrum, $\Theta(\boldsymbol{k})$ can be evaluated to the first order by substituting the free-wave solution in the right side of (2.7).

To express the dissipation in terms of the free-wave spectrum, we expand the functional $\boldsymbol{\tau}$ about the wave field

$$\zeta' = \zeta - \zeta_{\boldsymbol{k}},$$

which differs from the wave field ζ by an infinitesimal component. According to (2.4), all Fourier components of ζ' are statistically orthogonal to $\zeta_{\boldsymbol{k}}$ (since the only-correlated $\boldsymbol{k}$'th Fourier component is missing in ζ'). Furthermore, since the fields ζ' and $\zeta_{\boldsymbol{k}}$ are jointly Gaussian, it follows that ζ' and $\zeta_{\boldsymbol{k}}$ are in fact statistically independent.

Hence

$$\begin{aligned} \Theta(\boldsymbol{k}) \Delta \boldsymbol{k} &= -\langle \tau_i u_{i\boldsymbol{k}} \rangle, \\ &= -\langle (\tau_i(\zeta') + \frac{\delta \tau_i}{\delta \zeta}(\zeta') \zeta_{\boldsymbol{k}} + \ldots) u_{i\boldsymbol{k}} \rangle, \quad (2.8) \\ &= -\langle \frac{\delta \tau_i}{\delta \zeta}(\zeta') \rangle \langle \zeta_{\boldsymbol{k}} u_{i\boldsymbol{k}} \rangle + \ldots, \end{aligned}$$

where $\delta \tau_i / \delta \zeta$ is the functional derivative. (Velocities here and in the following refer to bottom velocities).

Explicitly,

$$\tau_i(\zeta + d\zeta;\, \boldsymbol{x}, t) = \tau_i(\zeta;\, \boldsymbol{x}, t) + \int_{-\infty}^{+\infty} K_i(\zeta;\, \boldsymbol{x}, \boldsymbol{x}', t, t')\,\delta\zeta(\boldsymbol{x}', t')\,d\boldsymbol{x}'dt' + \ldots, \quad (2.9)$$

where

$$K_i \equiv \frac{\delta\tau_i}{\delta\zeta}$$

and (2.8) should be read

$$\Theta(\boldsymbol{k})\Delta\boldsymbol{k} = -\int \langle K_i(\zeta';\, \boldsymbol{x}, \boldsymbol{x}', t, t')\rangle \langle \zeta_{\boldsymbol{k}}(\boldsymbol{x}', t')\, u_{i\boldsymbol{k}}(\boldsymbol{x}, t)\rangle\, d\boldsymbol{x}'dt' \quad (2.10)$$

(cf. Collatz 1967: 473).

For a homogeneous time-independent physical system, the functional $\boldsymbol{\tau}$ is independent of space and time translations. It follows that the kernel, K_i, depends only on the difference variables, $\boldsymbol{\varrho} = \boldsymbol{x} - \boldsymbol{x}'$ and $\lambda = t - t'$. Allowing for (2.4) and (2.6), eq. (2.10) then becomes

$$\Theta(\boldsymbol{k}) = -\frac{gk_i}{\sigma\cosh kH} F(\boldsymbol{k}) \int K_i(\zeta, \boldsymbol{\varrho}, \lambda)\cos(\boldsymbol{k}.\boldsymbol{\varrho} - \sigma\lambda)\,d\boldsymbol{\varrho}\,d\lambda, \quad (2.11)$$

where the field ζ' in K_i has been replaced again by ζ.

Alternatively, (2.9) may be written in the equivalent Fourier form

$$\tau_i(\zeta + d\zeta;\, \boldsymbol{x}, t) = \tau_i(\zeta;\, \boldsymbol{x}, t) + 2\,\mathrm{Re}\left\{\sum_{\boldsymbol{k}} \int_{-\infty}^{+\infty} \tilde{K}_{i\boldsymbol{k}}(\zeta;\, t - t')\,\delta\tilde{\zeta}_{\boldsymbol{k}}(\boldsymbol{x}, t')\,dt'\right\}, \quad (2.12)$$

where

$$\tilde{K}_{i\boldsymbol{k}} \equiv \frac{\delta\tau_i}{\delta\tilde{\zeta}_{\boldsymbol{k}}}.$$

This yields

$$\Theta(\boldsymbol{k}) = -\frac{gk_i}{\sigma\cosh kH} F(\boldsymbol{k})\, 2\,\mathrm{Re} \int_{-\infty}^{+\infty} \tilde{K}_{i\boldsymbol{k}}(\zeta, \lambda)\, e^{i\sigma\lambda}\, d\lambda. \quad (2.13)$$

We note that the dissipation, $\Theta(\boldsymbol{k})$, is quasilinear; it is proportional to the spectrum at $\boldsymbol{k}$, as in the case of laminar friction, but the proportionality factor depends now on the entire wave field.

3. *Application to Specific Friction Laws.* We apply (2.13) first to the quadratic friction law (1.1), which is known to be an acceptable though rather crude approximation for a wide variety of turbulent flows, including periodic boundary layers (cf. Savage 1953, Iwagaki et al. 1965, Jonsson 1965, Putnam and Johnson 1949, Bretschneider and Reid 1954, Kajiura 1964).

Since

$$\frac{\delta\tau_i}{\delta\zeta}\delta\zeta = \frac{\partial\tau_i}{\partial u_j^b}\frac{\delta u_j^b}{\delta\zeta}\delta\zeta,$$

$$= -\sum_{\boldsymbol{k}} \frac{\varrho g c_f}{\sigma \cosh kH}\left\{u^b k_i + \frac{u_i^b u_j^b}{u^b} k_j\right\}\delta\zeta_{\boldsymbol{k}},$$

we have

$$\tilde{K}_{i\boldsymbol{k}}(\lambda) = -\delta(\lambda)\frac{\varrho g c_f}{2\,\sigma \cosh kH}\left(u^b k_i + \frac{u_i^b u_j^b k_j}{u^b}\right) \quad (3.1)$$

and

$$\Theta(\boldsymbol{k}) = \nu_{ij} k_i k_j \varrho g F(\boldsymbol{k}), \quad (3.2)$$

with the anisotropic "viscosity" tensor

$$\nu_{ij} = \frac{g c_f}{\sigma^2 \cosh^2 kH}\left\{\delta_{ij}\langle u^b\rangle + \langle\frac{u_i^b u_j^b}{u^b}\rangle\right\}. \quad (3.3)$$

The mean quantities $\langle u^b\rangle$ and $\langle u_i^b u_j^b/u^b\rangle$ can be readily evaluated, since the joint distribution of the variables is Gaussian. The mean of the distribution is $\langle u_i^b\rangle = c_i$ and the covariance matrix is given by the spectrum

$$\langle(u_i^b - c_i)(u_j^b - c_j)\rangle = \langle u_i u_j\rangle = g^2\int\frac{k_i k_j}{\sigma^2\cosh^2 kH}F(\boldsymbol{k})d\boldsymbol{k}. \quad (3.4)$$

In the zero-current case, $\boldsymbol{u}^b = \boldsymbol{u}$, we obtain the closed expressions

$$\langle u\rangle = \alpha\boldsymbol{E},$$

$$\langle\frac{u_1^2}{u}\rangle = \alpha\left\{\frac{\boldsymbol{E}}{\varkappa^2} - \frac{\boldsymbol{K}}{\varkappa^2}(1-\varkappa^2)\right\},$$

$$\langle\frac{u_2^2}{u}\rangle = \alpha\frac{(1-\varkappa^2)}{\varkappa^2}(\boldsymbol{K}-\boldsymbol{E}),$$

where

$$\alpha = \left(\frac{2\langle u_1^2\rangle}{\pi}\right)^{1/2}$$

$$\varkappa = (1 - \langle u_2^2\rangle/\langle u_1^2\rangle)^{1/2}$$

and

$$\boldsymbol{K} = \boldsymbol{K}(\varkappa), \quad \boldsymbol{E} = \boldsymbol{E}(\varkappa)$$

are complete elliptic integrals of the first and second kind, respectively. The coordinate system is chosen such that

$$\langle u_1 u_2\rangle = 0, \quad \langle u_2^2\rangle < \langle u_1^2\rangle.$$

The damping is a maximum in the mean propagation direction x_1. In the extreme case of a unidirectional wave beam parallel to x_1, $\nu_{11} = 2\nu_{22}$. Generally, $\nu_{22} < \nu_{11} < 2\nu_{22}$.

The friction law (1.1) can be modified in several ways. Boundary-layer measurements in stationary flows indicate that c_f is in general a slowly varying function of the boundary-layer parameters, which depend on the history of the boundary layer. Experiments in sinusoidal oscillating flows show a variation of c_f with the period and amplitude of the external flow; they also show a small phase shift in the relation (1.1) (cf. Jonsson 1965).

The experimental data for periodic flows could be generalized to random wave fields by defining a mean period and amplitude in terms of the wave spectrum and then regarding c_f in (3.3) as a function of these variables. Perhaps a more satisfactory approach is to modify the basic force-field relationship (1.1) to include a dependence on the past history of the flow. For example, a simple generalization of (1.1) that allows for memory and phase-shift effects is

$$\tau(t) = -\varrho u^b \left\{ c_f^{(0)} \boldsymbol{u}^b + \beta c_f^{(1)} \int_{-\infty}^{t} e^{\beta(t-t')} \boldsymbol{u}(t')\, dt', \right. \tag{3.5}$$

where $c_f^{(0)}$, $c_f^{(1)}$, and β are empirical constants that depend on the average flow parameters.

In this case,

$$\delta\tau_i = \delta\tau_i^{(0)} - \varrho g c_f^{(1)} \int_{-\infty}^{t} e^{\beta(t-t')} \sum_{\boldsymbol{k}} (\sigma \cosh kH)^{-1} \left\{ k_i u^b(t)\, \delta\zeta_{\boldsymbol{k}}(t') + \right.$$

$$\left. + \frac{u_i(t') u_j^b(t) k_j}{u^b(t)}\, \delta\zeta_{\boldsymbol{k}}(t) \right\} dt',$$

so that

$$\langle \tilde{K}_{i\boldsymbol{k}}(\lambda) \rangle = \langle \tilde{K}_{i\boldsymbol{k}}^{(0)}(\lambda) \rangle - \frac{\varrho g c_f^{(1)} S(\lambda)}{2\sigma \cosh kH} \left\{ k_i \langle u^b \rangle e^{-\beta\lambda} + \delta(\lambda) k_j \langle \frac{\hat{u}_i u_j^b}{u^b} \rangle \right\},$$

where

$$S(\lambda) = \begin{cases} 0 \text{ for } \lambda < 0 \\ 1 \text{ for } \lambda \geqq 0, \end{cases}$$

$$\hat{u}_i(t) = \int_0^{\infty} u_i(t-\lambda) e^{-\beta\lambda} \beta\, d\lambda = \sum_{\boldsymbol{k}} 2\,\mathrm{Re} \left(\frac{\beta}{\beta - i\sigma}\, \tilde{u}_{i\boldsymbol{k}} \right).$$

The superscript $^{(0)}$ refers to the $c_f^{(0)}$ term.

Applying (2.13), the dissipation function is again found to be of the form (3.2), with a modified viscosity tensor

$$\nu_{ij} = \nu_{ij}^{(0)} + \frac{g c_f^{(1)}}{\sigma^2 \cosh^2 kH} \left\{ \delta_{ij} \langle u^b \rangle \frac{\beta^2}{\beta^2 + \sigma^2} + \langle \frac{\hat{u}_i u_j^b}{u^b} \rangle. \right. \tag{3.6}$$

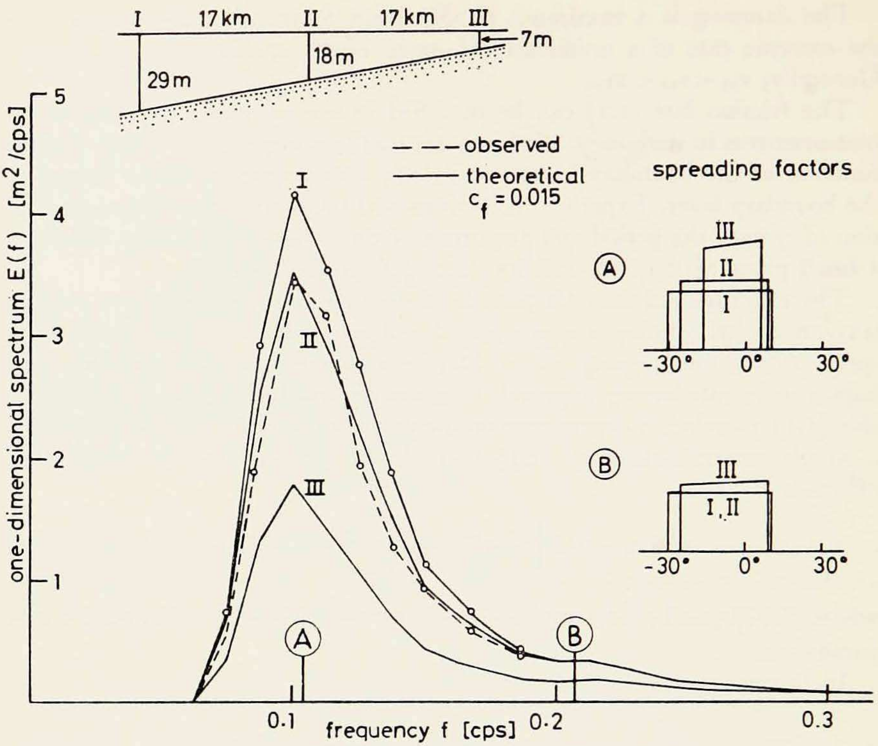

Figure 1. Decay of a wave spectrum due to bottom friction for near-normal incidence on a constant-slope beach. (Hurricane Hilda, 1964).

The mean quantities $\langle u^b \rangle$ and $\langle \hat{u}_i u_j^b / u^b \rangle$ can be evaluated as before by noting that the joint distribution of the variables $\hat{u}_i$, u_j^b is Gaussian with a covariance matrix that is determined by the spectrum.

4. *Computations.* The spectral decay was computed for some typical cases by numerical integration of the radiative transfer equation

$$\frac{\partial F}{\partial t}(\boldsymbol{k}, \boldsymbol{x}) + \dot{x}_i \frac{\partial F}{\partial x_i} + \dot{k}_i \frac{\partial F}{\partial k_i} = S, \tag{4.1}$$

where

$$\dot{x}_i = \frac{\partial \sigma}{\partial k_i}(\boldsymbol{x}, \boldsymbol{k}),$$

$$\dot{k}_i = -\frac{\partial \sigma}{\partial x_i}(\boldsymbol{x}, \boldsymbol{k})$$

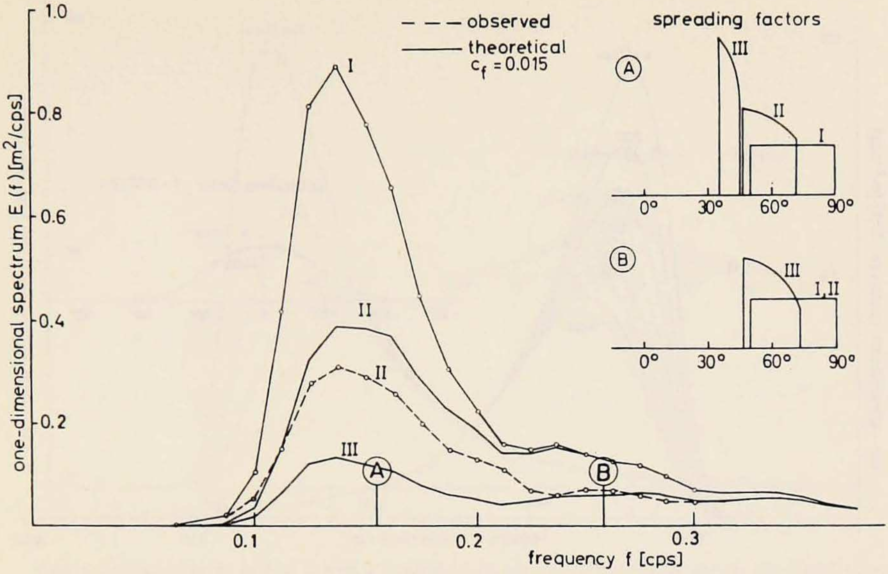

Figure 2. Decay of a wave spectrum due to bottom friction for near-glancing incidence on a constant-slope beach. The observed spectra for $f > 0.2$ *cps* are probably locally generated by offshore winds. A comparison with theory is meaningful only for $f < 0.2$ *cps*. (Hurricane Hilda, 1964).

represent the equations of a wave-group path in $\boldsymbol{x}-\boldsymbol{k}$ phase space. The second term on the left side represents the convection of energy, the third term the effect of refraction. The source function was assumed to consist of only the bottom-friction dissipation, $S = -\Theta$. In all cases a quadratic friction law (1.1) was assumed.

The spectra are presented as energy densities with respect to frequency, f, and direction, φ,

$$F(\boldsymbol{k})d\boldsymbol{k} = \tilde{F}(f,\varphi)df\,d\varphi = E(f)\,S(f,\varphi)df\,d\varphi, \tag{4.2}$$

where $E(f)$ is the one-dimensional frequency spectrum and $S(f,\varphi)$ is the spreading factor, with

$$\int_{-\pi}^{+\pi} S(f,\varphi)d\varphi = 1\,.$$

Fig. 1 shows the computed decay of a wave spectrum incident almost normally on a beach of constant slope, 6.5×10^{-4}. The topography and the positions of Sts. I and II correspond to wave-measuring installations in the Gulf of Mexico near Panama City, Florida. St. III is a hypothetical extrapolation; the beach actually shallowed more rapidly shoreward of St. II, the shore lying between Sts. II and III; see Kirst and Gaul (1964).

The initial frequency distribution at St. I corresponds to the observed

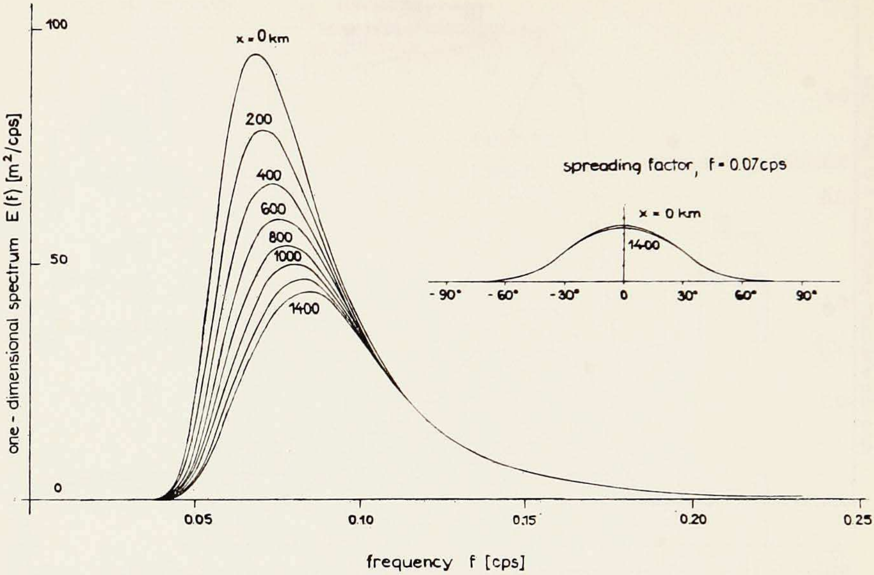

Figure 3. Decay of a wave spectrum due to bottom friction in a constant-depth continental-shelf region. The initial distribution at $x = 0$ corresponds to a 40-knot Pierson-Moskowitz (1964) spectrum with a spreading factor proportional to $\cos^4 \varphi$. The water depth is 100 m.

spectrum. The associated spreading factor was estimated from the known position and extent of the wave source. The wave records were obtained during Hurricane Hilda in 1964. The three hourly North American weather maps were used to estimate the initial spreading factors by observing the subtended angle of the wave-generating area. The predicted spectrum at St. II agrees well with the observed spectrum for a value of $c_f = 0.015$, which is consistent with laboratory measurements (cf. Jonsson 1965) and previous field estimates (Bretschneider and Reid 1954).

The frictional dissipation exceeds the increase in the frequency spectrum that would have resulted from refraction alone. However, the peaking of the spreading factor is almost entirely refractional. Some broadening may have been expected from the anisotropy of the viscosity tensor, but this is practically cancelled by the longer travel path of waves at larger angles of incidence; this applies also to the spreading factor in Fig. 3.

The variations in travel path result in a strong dependence of the normal-to-shore decay rate on the angle of incidence, particularly at large angles. Fig. 2 shows the decay of a spectrum for the same topography as in Fig. 1, but with a more tangential initial angular distribution,

$$S(f, \varphi) = \begin{cases} \text{const} & \text{for} \quad 50^\circ < \varphi \leqslant 90^\circ \\ 0 & \text{for} \quad -90^\circ < \varphi \leqslant 50^\circ. \end{cases}$$

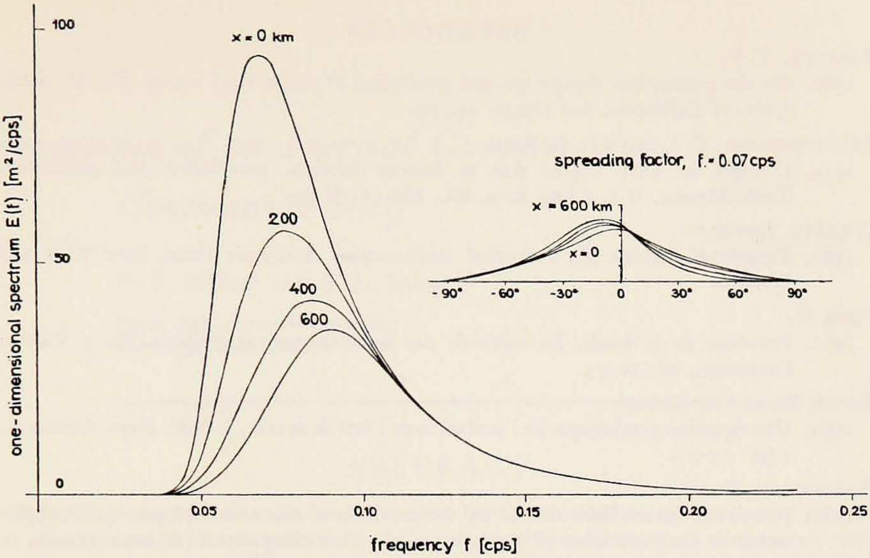

Figure 4. The same as in Fig. 3 with a superimposed current of 0.7 m/s at 45° to the initial mean wavepropagation direction, $\varphi = 0$.

The initial frequency spectrum is again the observed distribution. The predicted spectrum at St. II agrees reasonably well with the observed spectrum. The fit could clearly have been improved by choosing a slightly more tangential angular distribution initially (or by increasing c_f). However, the agreement has only qualitative significance in this case. The weather maps indicated approximately tangential angles of incidence, but the beam width could not be estimated accurately. Computations of the bottom-friction dissipation on a continental-shelf scale are shown in Figs. 3 and 4. The dimensions are characteristic of the North Sea. In Fig. 4, a constant mean current of 0.7 m/s is superimposed on the wave field at an angle of 45° to the mean wave propagation direction.

The value of c_f used in the previous examples was retained for the sake of comparison. In the case of Fig. 4, this is probably too high; the increase in dissipation due to the mean current may be offset by a smaller friction coefficient. However, the angular asymmetry induced by the current is not strongly affected by the value of c_f.

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