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GENERATION OF WAVES BY TURBULENT WIND

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ABSTRACT

Interactions between random gravity waves and the turbulent atmosphere boundary layer can be treated by an extension of wave-wave interaction theory. The energy transfer resulting from various interaction combinations can be characterized by transfer diagrams corresponding to the Feynman diagrams of wave-wave interactions. The complete set of lowest order transfer diagrams is shown to include Miles' and Phillips' theories of wave generation and a further set of wave-turbulence interactions which have not been considered previously. The predicted wave growth for the various interactions is compared with existing measurements. But a conclusive answer to the question of wave generation must await more detailed experiments correlating wave measurements with turbulence measurements in the atmosphere boundary layer.

INTRODUCTION

The fundamental papers of Miles (1) and Phillips (2) marked an important advance in our understanding of the basic processes by which waves can be generated by wind. It has been widely hoped that the growth of waves could be largely accounted for by a superposition of these two processes. However, it is known that although the theories describe independent aspects of the wave-atmosphere interaction, they are not entirely complementary; there remain interactions which are included in neither theory. Miles considers the coupling of the wave field with the mean atmospheric boundary-layer flow, but ignores the wave-turbulence interactions. These are similarly neglected in Phillips' treatment, which is concerned only with the external excitation of the waves by the undisturbed turbulent field.

Apart from the general theoretical interest, a complete theory of wave-atmosphere interactions has now become particularly desirable through the field

study of Snyder and Cox (3), which indicates that the combined Miles-Phillips mechanisms fail to account for the observed wave growth by almost an order of magnitude.

We shall attempt here to outline a complete theory based on a systematic expansion of the coupled fields. Besides the Miles and Phillips processes, the theory yields three further processes at lowest order: a nonlinear interaction with the mean boundary-layer flow and two forms of wave-turbulence interaction.

The problem may be divided into two parts: the analysis of the coupling between the wave field and the turbulent boundary-layer flow, and the determination of the energy transfer due to the coupling. The first part concerns the details of the interaction expansion. The second part may be regarded as a particular application of a general transfer theory for random wave fields in weakly coupled systems.

WAVE-ATMOSPHERE INTERACTIONS

We present here only the general structure of the interaction analysis; a detailed derivation is given in Ref. 4. Let ζ be the surface displacement, $\mathbf{u} = \mathbf{U} + \mathbf{u}'$ be the turbulent velocity field in the atmosphere, consisting of a mean flow \mathbf{U} and a superimposed fluctuating component \mathbf{u}' of zero mean, and $\delta\mathbf{u}$ be the wave-induced perturbation of the turbulent velocity field. We assume that all fields are statistically homogeneous, so that they may be represented as a superposition of mutually statistically orthogonal Fourier components of amplitude $\zeta_{\mathbf{k}}$, $\mathbf{u}'_{\mathbf{k}}$, $\delta\mathbf{u}_{\mathbf{k}}$, where \mathbf{k} is the two-dimensional, horizontal wavenumber vector.

The equations of motion of the coupled wave-atmosphere system may then be expressed in the form

$$L(\delta\mathbf{u}_{\mathbf{k}}) = Q(\mathbf{u}'_{\mathbf{k}}, \delta\mathbf{u}_{\mathbf{k}}) \quad (z > 0), \quad (1)$$

$$\delta\mathbf{u}_{\mathbf{k}} = R[\mathbf{u}'_{\mathbf{k}}, \zeta_{\mathbf{k}}] \quad (z = 0), \quad (2)$$

$$\ddot{\zeta}_{\mathbf{k}} + \sigma^2 \zeta_{\mathbf{k}} = S[\zeta_{\mathbf{k}}, \mathbf{u}'_{\mathbf{k}}, \delta\mathbf{u}_{\mathbf{k}}] \quad (z = 0), \quad (3)$$

where $\sigma = (gk \tanh kH)^{1/2}$, g is the gravitational acceleration, H is the water depth, z is the vertical coordinate, measured positive upward, L represents a linear (essentially the Orr-Sommerfeld) operator, and Q , R , and S are nonlinear functionals of the coupled fields.

The wave-atmosphere interactions are proportional to the air-to-water density ratio and are therefore weak. This suggests a solution by iteration. To first order, the forcing function S in the harmonic-oscillator equation (Eq. (3)) can be neglected, yielding a stationary wave field of free, sinusoidal waves. The free-wave field can then be substituted in the boundary condition (Eq. (2)), which together with Eq. (1) determines the wave-induced velocity field $\delta\mathbf{u}$. Substitution of the solution $\delta\mathbf{u}$ in the forcing function S then determines a second-order solution for the wave field, and so forth.

Miles and Phillips introduce additional simplifications. Phillips ignores the dependence of s on the fields ζ_k and δu_k , so that Eq. (3) reduces to a purely external excitation by the turbulence field u' (physically, s corresponds in this case to the unmodified turbulent surface pressure). Miles ignores the nonlinear term Q in Eq. (1) (i.e., the wave-induced perturbation of the turbulent Reynolds stress) and the u' -dependence of R in Eq. (2). This reduces Eqs. (1) and (2) to linear, constant-coefficient equations, and the boundary-value problem of determining δu becomes tractable.

If the Miles approximation is regarded as an acceptable first-order solution, the general case can be approached by introducing a second iteration loop in which Q and the u' dependence of R are treated as further perturbations. The n th iteration is obtained by solving Eqs. (1) and (2) with the $(n-1)$ th iteration substituted in the right-hand sides. In this manner, the wave-induced velocity field is obtained as a power series in the components ζ_k and u'_k , and the forced-harmonic-oscillator equation (Eq. (3)) takes the form

$$\bar{\zeta}_k + \sigma^2 \zeta_k = P_k + B_k \zeta_k + \sum_{k_1+k_2=k} C_{k_1 k_2} \zeta_{k_1} u'_{k_2} + \dots \quad (4)$$

The first two terms on the right correspond to the Phillips and Miles approximations, respectively; P_k denotes the external forcing term due to random turbulent pressure fluctuations; and $B_k, C_{k_1 k_2}, \dots$ are coupling coefficients, which are determined by solving the Orr-Sommerfeld equation, Eq. (1), under boundary condition (2). In general, this is possible only by numerical methods or by restriction to simple boundary-layer models. (It is known that a simple constant-velocity or constant-slope profile is inadequate in the Miles approximation, in which the energy transfer is determined by the local profile curvature at the critical layer. However, the detailed properties of the velocity profile are probably less important for the higher order processes.)

Correlation measurements of wave height and surface pressure by Longuet-Higgins et al. (5) indicate that the Miles approximation does indeed yield a reasonable first-order description of the wave-induced fluctuations in the atmosphere. It should be noted, however, that this does not necessarily apply to the Miles transfer expression. In Miles' approximation, the wave-induced pressure fluctuations are almost 90 degrees out of phase with the wave height over most of the wave spectrum, so that only a small fraction of the pressure field is effective in generating waves. It is therefore conceivable that the higher order pressure fluctuations, although smaller in absolute magnitude, lead to a larger energy transfer.

THE ENERGY TRANSFER

After determining the coefficients of the interaction expansion, the problem remains of evaluating the energy transfer resulting from the coupled equations (Eqs. (4)). The analysis is basically straightforward but involved algebraically. We summarize here only the results, referring to Refs. 4 and 6 for details. The problem may be regarded as a generalization of the theory of wave-wave interactions, first considered by Peierls (7) in his classic study on the heat conduction

in solids and now developed to a standard scattering formalism in various fields of physics. The theory has recently also found a number of geophysical applications (8).

The energy transfer arises from interactions between combinations of Fourier components whose wave numbers k_1, \dots, k_n and frequencies $\omega_1, \dots, \omega_n$ satisfy the transfer conditions

$$\sum_{j=1}^{n-1} s_j k_j = k_n \quad (5)$$

and

$$\sum_{j=1}^{n-1} s_j \omega_j = \omega_n, \quad (6)$$

where $s_j = \pm 1$. Equation (5) follows from the homogeneity of the physical system and applies to all interactions. An energy transfer between the interacting components occurs only if the additional *resonance* condition of Eq. (6) is also satisfied.

The net energy transfer is found by summing the contributions from all combinations of resonant interactions. The final expression consists of a number of integrals containing various spectral products, which can conveniently be divided into two classes. Integrals in which the transfer conditions of Eqs. (5) and (6) occur as δ factors are associated with *scattering* processes, the remaining integrals are associated with *parametric* processes.

To distinguish between the various transfer terms, it is further convenient to introduce a notation based on *transfer* diagrams. The transfer diagram for a scattering process consists of a number of wavenumber vectors k_1, \dots, k_{n-1} entering a vertex and a single wave component k_n leaving the vertex. The components satisfy the transfer conditions of Eqs. (5) and (6). Components associated with a negative sign $s_j = -1$ are indicated by a cross stroke.

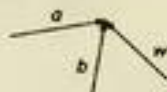
The transfer diagram for a parametric process consists only of ingoing components. There are no side conditions on the wavenumbers or frequencies. Parametric processes occur only in interacting systems in which the total energy and momentum of the wave fields are not conserved. They have no counterpart in the theory of wave-wave interactions, but there is a close analogy with the interactions occurring in nonlinear parametric amplifiers.

The structure of the various transfer expressions can be deduced from the transfer diagrams with the aid of a single transfer rule: the rate of change of the energy spectrum of any wave component in a transfer diagram is proportional to the product of the spectral densities of the ingoing components. Thus for any interacting system, the set of all transfer expressions for a particular wave field w is obtained by applying the transfer rule to all wave components w in all possible transfer diagrams (Fig. 1).



$$\frac{\partial F_n}{\partial t} = \int T_n F_a F_b F_c \delta(k_a - k_b - k_1 - k_2) \delta(n_b - n_1 - n_2) dk_a dk_b dk_c \quad (n=1,2; \bar{n}=2,1)$$

(i)



$$\frac{\partial F_w}{\partial t} = \int T_w F_a F_b F_w dk_a dk_b$$

(ii)

Fig. 1 - Examples of transfer diagrams and transfer expression for (i) a third-order scattering process and (ii) a third-order parametric process. a and b represent arbitrary field components; w_1 , w_2 , and w represent wave components; T_n , T_w are transfer functions; and the indices n , \bar{n} refer to the wave components w_1 , w_2 .

It is important to note that the transfer diagrams reflect only the structure of the transfer expressions. They are normally not directly related to the basic component-interactions responsible for the energy transfer. Thus although all transfer expressions are due entirely to resonant interactions, the resonant interaction conditions of Eqs. (5) and (6) occur only in the scattering, not the parametric transfer diagrams. The structure of the interaction analysis can be summarized independently in terms of *interaction* diagrams (6). (However, for conservative wave-wave interactions, the interaction and transfer diagrams are very simply interrelated (8).)

APPLICATION TO WAVE-ATMOSPHERE INTERACTIONS

The complete set of lowest-order transfer diagrams in the case of wave-atmosphere interactions are shown in Fig. 2. The linear interaction with the mean boundary-layer flow according to Miles appears as the degenerate parametric diagram (i). Phillips' external excitation by the atmospheric turbulence field is represented by the diagrams (iii). (If the external field is expressed in terms of the turbulent pressure p^t instead of the turbulent velocity components u , these reduce to a single linear diagram.) The remaining processes represent a nonlinear interaction with the mean boundary-layer flow,

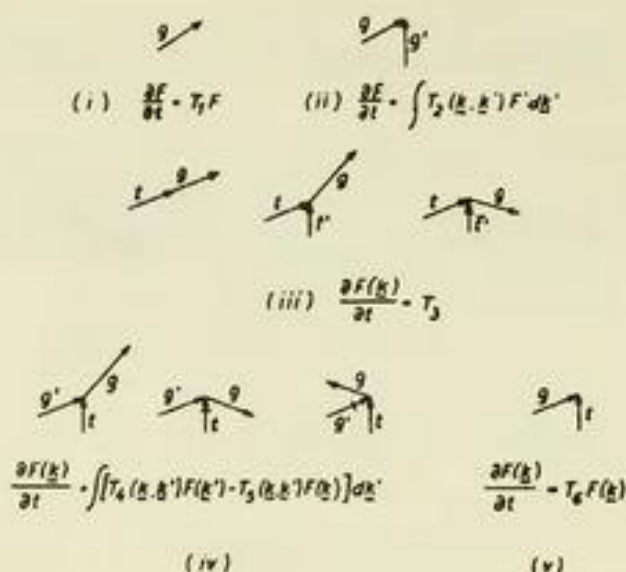


Fig. 2 - Lowest order transfer diagrams and transfer expressions for wave-atmosphere interactions: (i) Miles, (ii) nonlinear interaction with mean wind, (iii) Phillips, (iv) wave-turbulence scattering processes, and (v) wave-turbulence parametric process. The components g , t , and g'' represent gravity-wave, turbulent-velocity and turbulent-pressure components, respectively.

diagram (ii), and wave-turbulence interactions, diagrams (iv) and (v). The transfer expressions derived from the transfer rule are also shown. Only the dependence on the wave spectrum is given explicitly. The transfer functions T_1, \dots, T_6 depend on the coupling coefficients, and in the case of diagrams (iii), (iv), and (v), on the atmospheric turbulence spectra. The expressions are given in full in Ref. 4.

THE PRESSURE SPECTRA

Figure 3 shows schematically a two-dimensional k_1 - ω section of the three-dimensional surface pressure spectra $F_p(k, \omega)$ associated with the various transfer processes. The scattering processes (iii) and (iv) of Fig. 2 correspond to three-dimensional pressure distributions, whereas the parametric processes (i), (ii), and (v) yield two-dimensional distributions concentrated on the gravity-wave dispersion surfaces $\omega = \pm \sigma(k) = \pm (gk \tanh kh)^{1/2}$. Only the pressure fluctuations in resonance with free gravity waves, i.e., on the dispersion surface, transfer energy to the wave field.

The three-dimensional turbulent pressure distribution is concentrated about the "convection surface" $\omega + k_1 U_m = 0$, where U_m is the mean

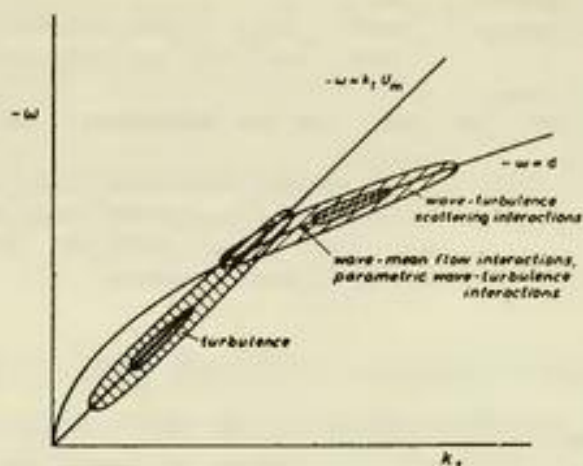


Fig. 3 - Two-dimensional $k_1 - \omega$ section of the three-dimensional surface pressure spectra $F_p(k, \omega)$. (The mean wind U_m is parallel to the x_1 axis.)

("anemometer") wind speed and k_1 is the wavenumber component parallel to the wind. The distribution follows from Taylor's hypothesis, according to which the wavenumber and frequency spectra of a turbulence field are approximately related as though the turbulence were a frozen special pattern convected bodily downstream with the mean flow velocity. It follows that the energy transfer due to Phillips' mechanism is appreciable only where the convection surface and dispersion surface intersect, i.e., for gravity waves whose phase velocities in wind direction are approximately equal to the wind speed (2) (see also Ref. 9 for the present interpretation of Phillips' result).

Linear wave interactions with the mean wind lead to pressure fluctuations of the same wavenumber and frequency as the wave components. Miles' pressure spectrum is therefore represented by a two-dimensional distribution on the dispersion surface.

The parametric processes (ii) and (v) also corresponds to two-dimensional pressure distributions. The process (ii) is due to a cubic wave-wind interaction involving a wave component (k, σ) and a complex conjugate pair of wave components (k', σ') , $(-k', -\sigma')$. This leads to a pressure fluctuation with the wavenumber and frequency of the first wave component. Process (iv) is due to a similar cubic interaction between a wave component (k, σ) and a complex conjugate pair of turbulence components (k', ω') , $(-k', -\omega')$, again producing a pressure fluctuation of wavenumber k and frequency σ .

The scattering processes (iv) are associated with quadratic interactions between wave components (k, σ) and turbulence components (k', ω') . In this case the induced pressure fluctuations can have arbitrary wavenumbers $k + k'$ and frequencies $\sigma + \omega'$, and the spectrum is a three-dimensional continuum.

Since atmospheric turbulence spectra are normally peaked at much lower wave-numbers and frequencies than wave spectra, it may be expected that for most interactions $k' \ll k$, $\omega' \ll \sigma$, so that the resultant pressure distribution lies rather close to the dispersion curve. Furthermore, the maximum of the distribution will lie close to the wave spectral maximum. The strongest wave generation may therefore be expected for frequencies close to the wave spectral peak, in accordance with the observed sequential development of the wave spectrum from high to low frequencies, the waves growing only in a narrow frequency band about the momentary wave peak. (However, other explanations of the sequential wave growth have also been suggested.)

CONCLUSIONS

The recent field study of Snyder and Cox (3) indicate that both Miles' and Phillips' theories are incapable of explaining the wave growth observed in the ocean, strongly suggesting that one or more of the remaining lowest-order processes, in particular the wave-turbulence interactions, are the principal source of wave energy. However, the question of wave generation must be regarded as open until further measurements and transfer computations have been made. Although a complete theory of expansible interactions has been developed, the expansions are valid only for weak spacially uniform interactions. Strong, local effects, such as flow separation at the wave crests, are therefore not included in the theory.

REFERENCES

1. Miles, J.W., *J. Fluid Mech.* 3:185-204 (1957)
2. Phillips, O.M., *J. Fluid Mech.* 2:417-445 (1957)
3. Snyder, R.L., and Cox, C.S., *J. Marine Res.* 24:141-178 (1966)
4. Hasselmann, K., in "Basic Developments in Fluid Mechanics," Vol. II, M. Holt, editor, Academic Press, 1968
5. Longuet-Higgins, M.S., Cartwright, D.E., and Smith, N.D., pp. 111-136 in "Ocean Wave Spectra, Proceedings of a Conference," Easton, Maryland, 1961, Prentice-Hall, 1963
6. Hasselmann, K., *Proc. Roy. Soc. A*299:77-100 (1967)
7. Peierls, R., *Ann. Phys.* 3:1055-1101 (1929)
8. Hasselmann, K., *Rev. Geophys.* 4:1-32 (1966)
9. Hasselmann, K., *Schiffstechnik* 1:191-195 (1960)

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