

COMPARISON OF ASSIMILATION RESULTS  
FROM AN OPTIMAL INTERPOLATION AND THE GREEN'S FUNCTION METHOD  
USING ERS-1 SAR WAVE MODE SPECTRA

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### ABSTRACT

The optimal interpolation and the Green's function method have been applied successfully to the assimilation of ocean wave spectra retrieved from ERS-1 SAR wave mode spectra into the WAModel. Both assimilation methods determine wind corrections with respect to initial winds. Spectra are partitioned into windsea and swell components and wind corrections are computed from windsea corrections. For both methods the wind corrections are of compatible magnitude. The Green's function determines also wind corrections from swell partitionings, by following the swell component back to its origin. Wind corrections are, therefore, scattered in space and time. It is shown that both assimilation schemes agree quite well and that the assimilation of two dimensional wave spectra is useful to improve modeling of ocean waves and produce wind field corrections as data for meteorological data assimilation.

### 1. INTRODUCTION

The global measurements of two-dimensional wave spectra retrieved from SAR wave mode in quasi-real time give fresh impetus to the analysis of wave fields and also of wind fields by means of wave data assimilation. In the beginning, wave data assimilation schemes used only significant wave heights ( $H_s$ ) observed by satellite altimeters. To correct the entire wave spectrum consistently in energy and frequency range fetch laws were used for a wind sea spectrum. For swell spectra the correction was carried out under the assumption that the wave steepness is constant. Furthermore, the model winds were corrected if the spectrum could be identified as wind sea. (e.g. Thomas, 1988; Bauer et al., 1992; Lionello et al., 1992, 1995).

However, two-dimensional spectra in the open ocean usually consist of several wave systems. Therefore, the retrieval of 2d wave spectra from ERS

SAR wave mode image spectra caused an essential progress in wave data assimilation.

Error in the wave field are to the larger extent due to errors in the forcing wind field (Cardone et al. (1996)). Therefore, wave data assimilation has to include the correction of the forcing wind field for an assimilation time window. Otherwise, the corrected windsea part of a spectrum will fall back into its incorrect state immediately.

Two wave data assimilation schemes are presented which assimilate spectral wave data. The first scheme is an optimal interpolation scheme (Hasselmann et al., 1997) and the second is a dynamical scheme called Green's function method (Bauer et al., 1996). The Green's function method determines from the history of the wave evolution the spectral wave age and the wind corrections at the time and position of origin of a wave system. Both assimilation schemes have been applied using the same wave spectra retrieved from ERS-1 SAR wave mode spectra for the Atlantic in November 1992. It is shown that the wind corrections obtained from the two assimilation schemes agree quite well.

We briefly describe the SAR retrieval algorithm in section 2. The optimal interpolation scheme and the Green's function method are described in sections 3 and 4, respectively. The wind and wave conditions of the assimilation case are shown in section 5 and results are presented in section 6. Results are discussed in the concluding section 7.

### 2. SPECTRA RETRIEVED FROM ERS-1 SAR WAVE MODE

Ocean wave spectra can be retrieved from ERS SAR wave mode spectra in quasi-real time by the inversion method of Brüning et al. (1994) and Hasselmann et al. (1996) based on Hasselmann and Hasselmann (1991). The method inverts the nonlinear spectral integral transform mapping an ocean wave spectrum into a SAR image spectrum. The method

requires a first guess spectrum, usually taken from the wave model WAM, to resolve the directional ambiguity of the SAR image spectrum and to augment the spectrum beyond the azimuthal cut-off. The azimuthal cut-off which depends on the azimuthal displacement induced by the velocity bunching effect can be quite large. But studies have shown that the information taken from the first guess diminishes rather rapidly in the course of the iterations of the inversion method (Heimbach et al., 1997; Bauer and Heimbach, 1997).

The assimilation study for the Atlantic in November 1992 is based on more than 5000 SAR-retrieved spectra.

### 3. OPTIMAL INTERPOLATION ASSIMILATION

The optimal interpolation scheme applied to wave height spectra is described in detail in Hasselmann et al. (1997). A correction of the forcing wind at a grid point is determined from all wave observations located within a certain correlation length scale (in our case 200km) from the observational point and within a 6-hour interval centered at the analysis time. Local wind corrections are computed from windsea corrections by a growth law relation (Hasselmann et al., 1976):

$$U = (Ag^{B2}E^{-1}f^B)^{B4} \quad (1)$$

where  $U$  denotes the wind speed,  $E$  and  $f$  are the energy and mean frequency of the windsea partitioning, respectively, and  $g$  is the acceleration of gravity. The constants  $A$  and  $B$  were determined empirically and  $B2 = -(B + 2)$  and  $B4 = -(B + 4)^{-1}$ .

The simpler optimal interpolation scheme is implemented operationally at ECMWF using the significant wave height of ERS altimeter. The extension of this scheme described above will be implemented shortly to use the spectral data from ERS SAR wave mode spectra.

### 4. GREEN'S FUNCTION ASSIMILATION METHOD

The Green's function method computes wind vector corrections from 2d spectral wave data. The method is based on the energy transport equation of waves (Komen et al., 1994; Bauer et al., 1996). The linearized equation describes the changes  $\delta F$  of the energy spectrum  $F$  caused by perturbations  $u$  of the initial wind  $U$ .

$$\frac{D\delta F}{Dt} = \frac{\partial S_{tot}}{\partial F} \delta F + \frac{\partial S_{tot}}{\partial U} \cdot u \quad (2)$$

$D/Dt$  denotes the derivate with time  $t$  and the horizontal coordinates  $x$ , and  $S_{tot}$  is the total source function defined as the sum of the wind input  $S_{in}$ , the dissipation  $S_{ds}$  and the nonlinear transfer function  $S_{nl}$ .

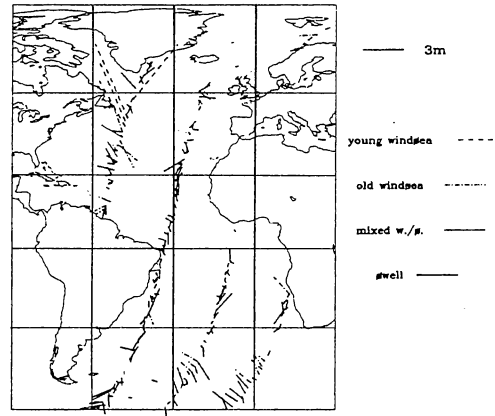


Figure 1: Wave height correction vectors [m] determined from wave height vectors of SAR-retrieved wave systems minus WAM first guess wave systems obtained along the ERS-1 satellite swath on November 3 in the 6-hour interval centered at 12Z. The wave height correction vectors are shown for young and old wind sea, mixed windsea/swell and swell separately through different line styles.

The response of the spectrum due to a perturbation of the wind is expressed by the Green's function of the system. Based on the dynamics of the waves the Green's function can be approximated by a  $\delta$ -function leading to a linear relationship

$$\delta F(\mathbf{k}; \mathbf{x}, t) = W^u u(\mathbf{k}, \mathbf{x}_p, t_p) + W^v v(\mathbf{k}, \mathbf{x}_p, t_p) \quad (3)$$

where the impact functions  $W^u$  and  $W^v$  describe the maximal sensitivity of the spectrum with respect to perturbations of the  $u$  and the  $v$  wind component, respectively. The coordinates  $(\mathbf{x}, t)$  refer to the observation position and  $(\mathbf{x}_p, t_p)$  to the position of origin of the wave component with wavenumber  $\mathbf{k}$ . The origin of a wave component is inferred from the wave age which is the time passed since the wave component received its last forcing input, and from the corresponding group velocity.

For each observed 2d spectrum a cost function is minimized. Then a mean wind correction and a mean wave age for each spectral partitioning is determined using the partitioning method of Hasselmann et al. (1996). Applications of the Green's function method are presented in Bauer (1994) and Bauer et al. (1995).

### 5. ASSIMILATION STUDY

The wave data assimilation study was carried out for the Atlantic, November, 1992 because several strong low pressure systems evolved in the N-Atlantic. The wave model WAM was driven by

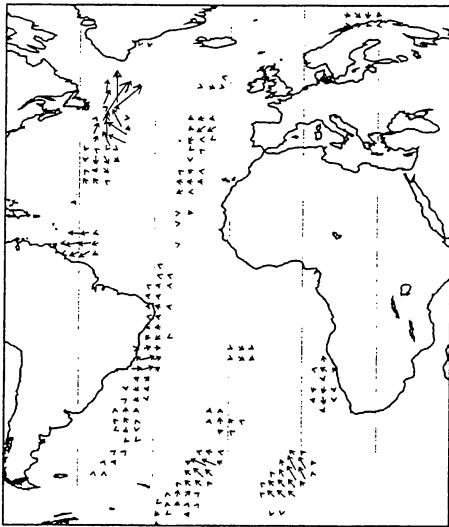


Figure 2: Wind correction vectors [m/s] computed from windsea corrections with the optimal interpolation method using SAR-retrieved spectra from November 3, 1992 in the 6-hour interval centered at 12Z. Only every second wind correction is shown.

ECMWF winds. The monthly mean  $H_s$  retrieved from SAR wave mode spectra was 2.50 m and slightly larger than the mean of 2.45 m obtained from the collocated WAM spectra. The scatter was rather large expressed by a rather small correlation coefficient of 0.78.

In cases in which the largest deviations are seen, however,  $H_s$  of WAM was larger than  $H_s$  of SAR. The case with deviations exceeding even 5 m is displayed in Figure 1. In Figure 1 the differences of  $H_s$  of the partitionings from WAM and SAR are shown along the ERS-1 satellite paths on November 3, 1992 from a 6-hour interval centered at 12Z. The large differences in the NW-Atlantic are associated with a strong storm. Although the satellite did not pass over the western flank of the storm where the strongest winds and maybe the largest errors occurred, the waves generated by those winds were clearly detected by the SAR.

## 6. WIND CORRECTIONS FROM ASSIMILATION

### 6.1 STORM ON NOVEMBER 3, 1992

Wind corrections obtained with the optimal interpolation scheme can directly be compared with local wind corrections for windsea from the Green's function scheme. This is demonstrated for the wind corrections obtained from the optimal interpolation (Figure 2) and from the Green's function (Figure 3)

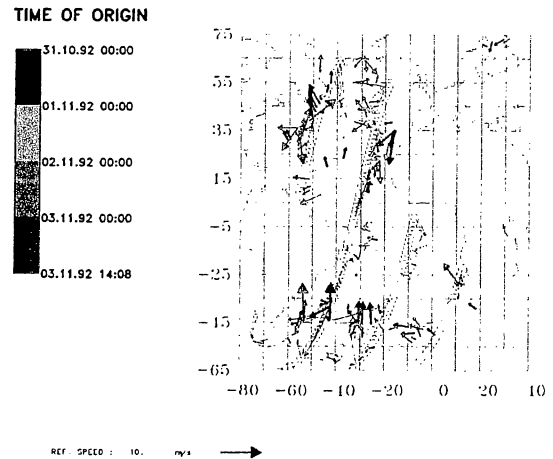


Figure 3: Wind correction vectors [m/s] computed with the Green's function method using the SAR-retrieved spectra as in Figure 2. For each wave system one wind correction vector is shown at the position of origin of the wave system which is determined from the corresponding wave age and the group velocity. The red wind correction vectors refer to windsea corrections.

assimilation scheme using the SAR-retrieved spectra from November 3, of the 6-hour interval centered at 12Z. Both schemes inferred the same large wind corrections for the storm on November 3 in the NW-Atlantic. The SSE winds of the cyclone were found to be overestimated by the ECMWF model up to 10 m/s.

Repeated confirmations for the overestimated model winds in this case are received from the Green's function method. During the following 12 days the swell which propagates from the storm to the S-Atlantic is permanently seen to be too large. The assimilation of the SAR-retrieved spectra by the Green's function method thus yields repeatedly the same wind vector corrections at the same storm in the N-Atlantic (Figure 4). This may be seen as a clear confirmation of the overpredicted winds at the western flank of the storm.

Good agreement between the wind corrections from the optimal interpolation and the Green's function method is further evident in the western tropical Atlantic and in the strong west-wind regions of the S-Atlantic. In the tropical region the corrections show that the initial easterly winds need to be increased whereas in the south the strong westerly winds need to be decreased.

### 6.2 MEAN WIND CORRECTION NOV. 1992

As the differences between modeled and observed wave spectra varied considerably the wind vector cor-

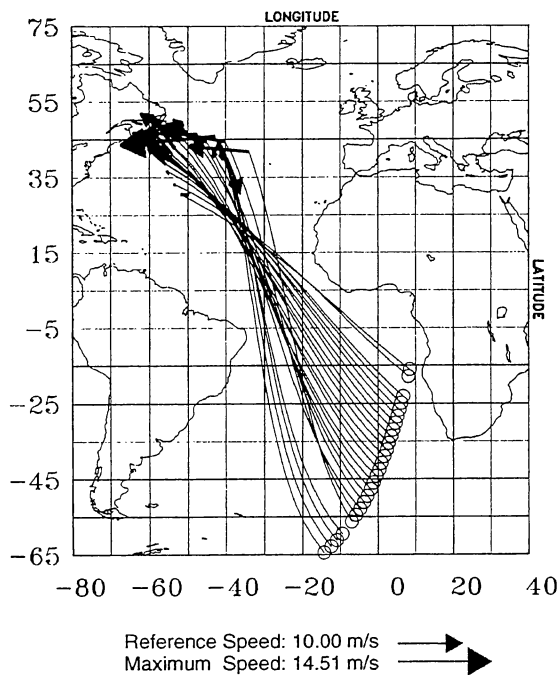


Figure 4: Wind correction vectors [m/s] computed with the Green's function method at times of origin between November 3 and 5, 1992 using the SAR-retrieved spectra from November 15, between 9.36Z and 9.50Z in the S-Atlantic. Observation positions and wind correction positions are connected by great circle lines.

rections are expected to be unevenly distributed in space and time. An estimate of the monthly mean correction from the optimal interpolation and the Green's function method is shown in Figure 5 and 6, respectively.

The comparison of the mean wind corrections from both assimilation schemes with the mean ECMWF wind field for November 1992 (Figure 7) suggests that the monthly mean wind field should slightly be reduced in the strong wind regions of the Extra-Tropics and slightly be enhanced in the western Tropics.

The wind corrections from the Green's function method are most frequently determined in strong wind regions (Figure 8). This is consistent because if a strong wind is incorrect then the error is visible through the high waves which can be traced back to their origin for longer times and longer distances. A much smaller number of wind corrections is obtained by the Green's function method in the Tropics. This can be explained by the fact that the waves in the Tropics are dominated by swell having their origin in the Extra-Tropics.

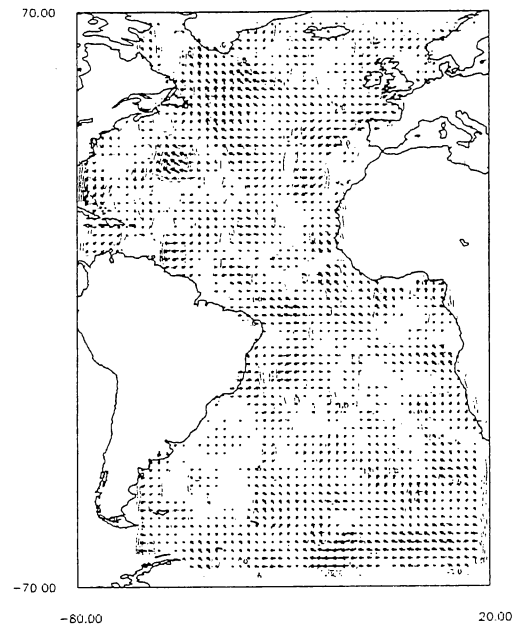


Figure 5: Mean wind corrections [m/s] averaged for November 1992 as computed with the optimal interpolation method. Isolines of the wind corrections are given in steps of 0.5 m/s starting from 0.5 m/s.

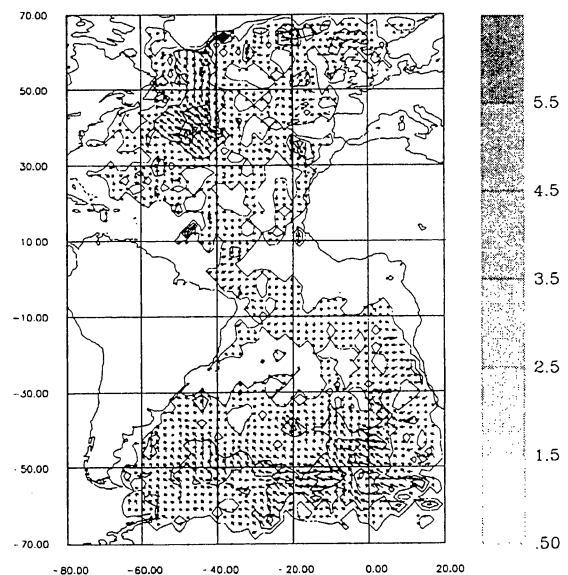


Figure 6: Mean wind corrections [m/s] averaged for November 1992 as computed with the Green's function method. Isolines of the wind corrections are given in steps of 1 m/s starting from 0.5 m/s.



## 7. DISCUSSION

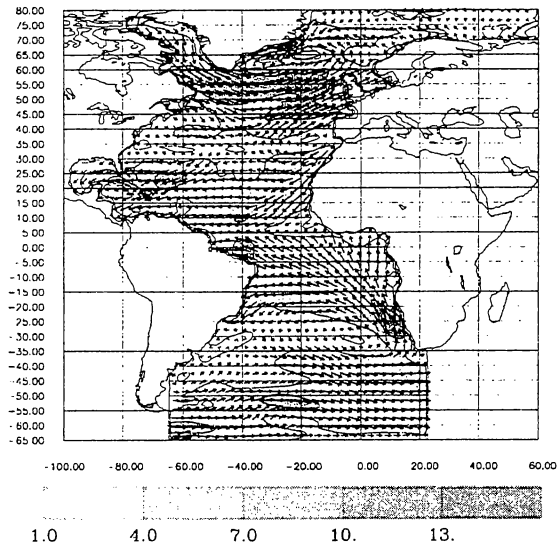


Figure 7: Mean  $u_{10}$  wind field from the ECMWF model for November 1992 which were used as initial winds for the WAM model.

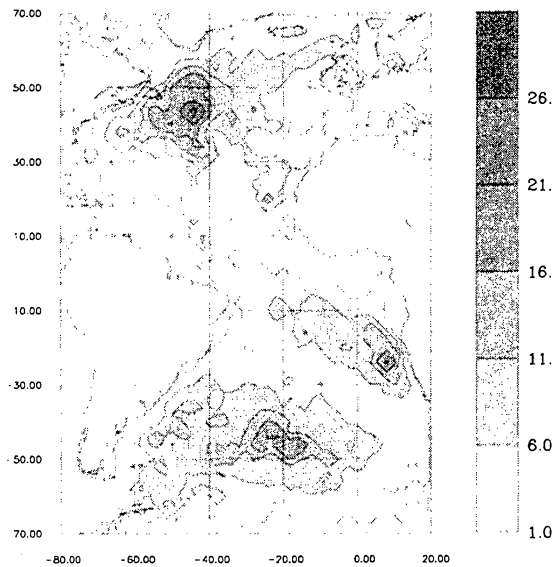


Figure 8: Map of the frequency of wind corrections computed with the Green's function method for November 1992. The frequencies are determined on a  $2^\circ \times 2^\circ$  grid and the isolines are in steps of 5 starting from 1.

Wind corrections obtained from the assimilation of SAR-retrieved spectra with the optimal interpolation and the Green's function scheme agree quite well in magnitude and direction for windsea systems. Although the wind corrections are unevenly distributed in space and time this study for November 1992 reveals that in the northern and southern strong wind regions of the Atlantic the initial winds from ECMWF are occasionally too strong. On the other hand the initial winds in the Tropics appear to be sometimes too weak.

Now, an attempt is made to discuss the results of the present assimilation study with respect to the mutual consistency of wind and waves. Various studies on the assessment of the data quality are currently performed. For instance, a comparison study of  $H_s$  retrieved from SAR wave mode spectra against ERS-1 and TOPEX altimeter measurements shows that the SAR-retrieved spectra are highly reliable (Bauer and Staabs, 1996; Bauer and Heimbach, 1997). Furthermore, the analysis of Heimbach et al. (1997) shows that on average WAM overestimates windsea and underestimates swell in comparison to SAR-retrieved  $H_s$ . As swell is present in the major part of the global oceans the total significant wave height is seen here to be underestimated by WAM.

The assimilation study has shown that the windsea of WAM is too large, because at least occasionally, the model winds are too strong. But it appears still open whether we may attribute the underestimated swell of WAM to an error in the winds in general. The errors can also be caused by an incorrect swell dissipation term in the source terms of the wave model (Heimbach et al., 1997). A continuation of assimilation experiments will help to identify certain conditions in which initial winds are too strong and other conditions in which initial winds are too weak. More examinations are needed if the windsea is overestimated by WAM in general. Even if we have clear indications that the initial winds of a storm have been too strong comparisons of  $H_s$  from WAM against  $H_s$  from TOPEX altimeter show that in pure windsea conditions WAM often underestimates the wave height. If this is true than it could mean that the generation of windsea in WAM has to be enhanced which would consequently increase the swell in WAM. Even if this study can not give a conclusive answer yet on the windsea-swell differences dynamical wave data assimilation schemes will be a useful tool for finding answers.

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