# Controlling Spiral Waves in Confined Geometries by Global Feedback

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The evolution of spiral waves on a circular domain and on a spherical surface is studied by numerical integration of a reaction-diffusion system with a global feedback. It is shown that depending on intensity, sign, and/or time delay in the feedback loop a global coupling can be effectively used either to stabilize the rigid rotation of a spiral wave or to completely destroy spiral waves and to suppress self-sustained activity in a confined domain of an excitable medium. An explanation of the numerically observed effects is produced by a kinematical model of spiral wave propagation. [S0031-9007(97)03067-6]

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Controlling the dynamics of spiral waves is important for many excitable media including biological systems like cardiac tissue [1] or aggregating slime-mold cells [2] and physical systems like the CO oxidation on platinum surfaces [3]. Well developed spiral structures with a very long front describing many whorls can be easily observed in chemical systems like the Belousov-Zhabotinsky (BZ) reaction [4–6]. On the other hand, very often the rotation of spiral waves occurs in restricted domains like chicken retina [7] or single cardiac cells [8,9], where the length of the spiral wave front is relatively small.

A confined geometry introduces new specific properties to the common features of spiral waves in unrestricted media. Particularly, in the case of a circular domain, the rotation frequency of a spiral wave strongly depends on its radius [10,11]. Moreover, an interaction with the nonflux boundary imposes a new two-periodic regime [12,13], in addition to the rigid rotation which is common for an unrestricted medium. Two similar dynamic regimes have been observed numerically on a sphere [14,15], which is an example of a boundaryless confined geometry.

Spiral waves can be effectively controlled by application of an external forcing [16–19]. This forcing can be given *a priori* (i.e., as a periodic modulation of excitability [17,18]) or computed on-line using the data of the momentary state of the medium by closing a feedback loop [19,20]. In many respects feedback control is preferable to *a priori* given forcing, because it adjusts to the present state of the medium. Especially effective is a global feedback when the intensity of external forcing is proportional to the integral of the activity taken over the whole medium with a confined geometry [21,22]. Such a global coupling is naturally observed in surface reactions where the partial pressure in the gas phase is determined by the integral of absorption and desorption rates over the whole surface [3]. The properties of a global feedback can be studied experimentally by using the photosensitive BZ solution where the absorption of transmitted light depends on the concentration of chemical species [20].

In this Letter we investigate the possibility to control the evolution of spiral waves on a disk and on a sphere by introducing such a global feedback based on an integral information about the distribution of the excitation within these confined regions.

We use in our computations a general mathematical model describing excitable media in terms of "reaction-diffusion" equations [23],

$$\begin{aligned} \frac{\partial u}{\partial t} &= D_u \nabla^2 u + F(u,v), \\ \frac{\partial v}{\partial t} &= D_v \nabla^2 v + \epsilon G(u,v), \end{aligned} \tag{1}$$

where variables $u$ and $v$ (sometimes called activator and inhibitor) represent the concentrations of the reagents or the temperature, the electric potential, etc.

For the functions $F(u,v)$ and $G(u,v)$ we use the form suggested in Ref. [10], which is suitable for both numerical and analytical studies,

$$\begin{aligned} F(u,v) &= f(u) - v, \\ f(u) &= -k_1 u, \qquad u < \sigma, \\ &= k_f(u-a), \qquad \sigma < u < 1-\sigma, \\ &= k_2(1-u), \qquad 1-\sigma < u, \\ G(u,v) &= k_g u - v, \qquad k_g u - v \geq 0, \\ &= k_\epsilon(k_g u - v), \qquad k_g u - v < 0, \end{aligned} \tag{2}$$

with the following parameter values: $k_f = 1.7$, $k_g = 2$, $k_\epsilon = 6.0$, $a = 0.1$, $\sigma = 0.01$, $\epsilon = 0.3$, $D_u = 1$, and $D_v = 0$. The parameters $k_1$ and $k_2$ are chosen in such a way that the function $f(u)$ is continuous at $u = \sigma$ and $u = 1 - \sigma$.

This model has a steady state which is stable with respect to a small perturbation. However, a superthreshold perturbation, once locally applied, gives rise to a wave propagating through the medium.

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The system (1) was integrated for a circular domain or a spherical surface with radius $R$. Because of the rotational symmetry of the problem it is natural to use a polar coordinate system. The Laplacian was calculated by using a quasiuniform computational grid proposed earlier [13,15] with the time step $\Delta t = 0.01$ and the space step $\Delta r = 0.5$.

To introduce a global coupling the first equation in (2) was modified [14],

$$F(u, v, t) = f(u) - v + k_{\mathrm{fb}}[B(t - \tau) - B_0], \quad (3)$$

where

$$B(t) = \frac{1}{S} \int_S v \, ds \,. \quad (4)$$

Thus, the intensity of the feedback signal is controlled by the coefficient $k_{\mathrm{fb}}$ and is proportional to the integral value $B$ of the second variable over the simulated domain $S$. The constant $B_0$ is the value of this integral for the case of a stationary rotation. Hence, the application of such a feedback does not change the parameters of the stationary rotation, since the feedback signal vanishes for this case. A positive value of the coefficient $k_{\mathrm{fb}}$ provides a "positive feedback"; that is, an increase of $B$ leads to a higher excitability of the medium due to a decrease of the excitation threshold. In turn, this change in excitability results in a further increase of the integral activity $B$.

The dynamics of spiral waves in a disk and on a sphere is studied below under different signs and absolute values of the coefficient $k_{\mathrm{fb}}$. We will also use the time delay $\tau$ as another control parameter in the feedback loop that usually exists in real systems or can be introduced artificially to achieve a desired dynamic behavior.

To create a spiral wave on a disk a special initial condition is chosen. In our computations we start from a nonuniform distribution of the variables [15]. A superthreshold value $u = 1$ is given within a narrow sector of the disk to induce a propagating wave. The second variable $v$ increases in clockwise direction within this sector from $v = v_f$ (here $du/dt > 0$) up to $v = v_b$ (here $du/dt < 0$). As a result the boundary of the sector consists of two parts which are the front (with $du/dt > 0$) and the back (with $du/dt < 0$) of a wave circulating in counterclockwise direction. The tip of the evolving spiral wave corresponds to the point at which $du/dt = 0$. The initial location of the tip with respect to the center of the disk depends on the internal radius of the sector $r_0$.

If the tip is initially located near the center of the disk a centrosymmetrical rotation of a spiral wave can be created. A two-periodic regime corresponding, in fact, to a drift of a spiral wave core along the nonflux boundary can be induced, if the tip is initially placed near the boundary of the disk.

In the first series of our computations a transformation of the two-periodic regime due to a global feedback was studied as illustrated in Fig. 1. Four trajectories of the spiral wave tip are shown as computed for different values of the intensity $k_{\mathrm{fb}}$ in the feedback loop. All computations started from the same initial conditions resulting in a drift along the boundary. The global feedback was switched on at the moment indicated by a cross.

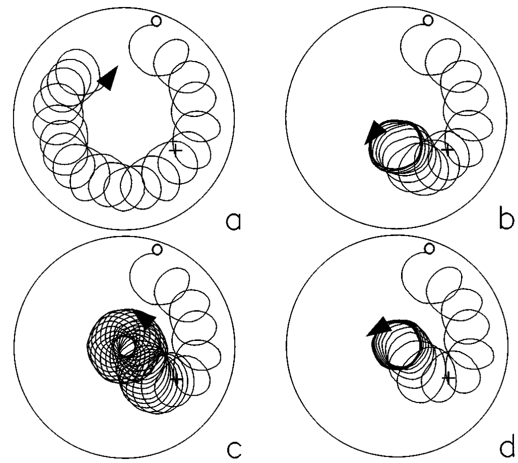


FIG. 1. Trajectories of a spiral wave tip on a circular domain of radius $R = 26$ computed for different intensities of the negative global feedback: (a) $k_{\mathrm{fb}} = -0.02$, (b) $k_{\mathrm{fb}} = -0.04$, (c) $k_{\mathrm{fb}} = -0.06$, and (d) $k_{\mathrm{fb}} = -0.1$. Time delay $\tau = 0$. The initial position of the spiral wave tip and the time instance of feedback application are indicated by the symbols "○" and "+," correspondingly.

A relatively low intensity of the global coupling decreases the drift velocity [Fig. 1(a)]. Moreover the drift can be completely stopped if the intensity of the feedback becomes sufficiently strong [Fig. 1(b)]. A further increase of the intensity results in a stabilization of the tip motion near the center of the disk. At first the tip describes a cycloidal trajectory around the center [Fig. 1(c)] which indicates that some oscillations in the angular velocity take place. But these oscillations are suppressed with growing intensity until complete synchronization is reached [Fig. 1(d)]. Thus a negative global feedback of sufficiently high intensity results in a stabilization of the rigid rotation of a spiral wave in the center of the disk.

A quite different scenario of spiral wave evolution is observed for opposite (positive) sign in the feedback loop (Fig. 2). Here we start from initial conditions which, in

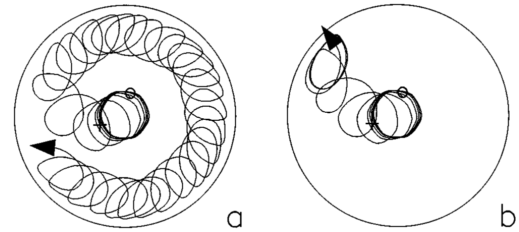


FIG. 2. Trajectories of a spiral wave tip on a circular domain of radius $R = 26$ computed for two different intensities of the positive global feedback: (a) $k_{\mathrm{fb}} = 0.1$, (b) $k_{\mathrm{fb}} = 0.15$. Time delay $\tau = 0$. Symbols "○" and "+" as in Fig. 1.

the absence of the feedback, result in a regime of centrosymmetrical rotation. Switching on the feedback destabilizes the rigid rotation: the tip trajectory approaches the boundary and then drifts along it [Fig. 2(a)]. In some respects the finally observed two-periodic process looks similar to the drift without feedback [cf. Fig. 1(a)]. However, the shape of a single loop of the trajectory is strongly deformed as compared to the unperturbed one. A further increase of the intensity speeds up the process of destabilization. When the intensity of the positive feedback exceeds a certain value, the spiral wave dies after collision with the boundary [Fig. 2(b)].

This scenario can be changed by introducing a time delay into the feedback loop. For a time delay short with respect to the rotation period $T = 50$ its effect is negligible, and switching on the feedback leads to an annihilation of the spiral wave on the boundary. In the example shown in Fig. 3(a) the feedback is applied after a two-periodic regime has been established. In this case the spiral waves died very quickly after switching on the feedback. A larger delay leads to a special drift behavior along the boundary [Fig. 3(b)]. Note that the direction of this drift is counterclockwise and thus opposite to the drift direction in the absence of the feedback. With a time delay of about one-half of $T$ the tip trajectory is stabilized around the center of the disk [Fig. 3(c)]. Further increase leads again to a destabilization of the centrosymmetrical tip motion up to annihilation of the spiral wave due to its collision with the boundary [Fig. 3(d)].

On a boundary-less surface like that of a sphere the contour curve of a wave is always closed. For one part of this contour $du/dt > 0$ (this is the front of the wave); for the other one $du/dt < 0$ (the back of the wave). Thus, on a wave contour at least two points should exist at which the front coincides with the back of the wave. A double spiral (a wave with two spiral tips) fulfills this requirement and is the simplest wave object on a sphere.

Such a double spiral can be created if the initial value of $u$ is taken equal to 1 within a thin stripe located along a meridian, with two edges located near the north and the south poles. The second variable $v$ increases in the east direction from $v_f$ to a critical value $v_b$, which prevents the propagation towards the east. Consequently, the wave starts to curl around its open north and south ends. This process creates two spiral waves that rotate in the northern and southern hemispheres and collide with each other on the equator [14,15].

In the presence of a negative feedback a stationary shape of the wave is established rather quickly and the double spiral rotates as a rigid body around the vertical axis as shown in Fig. 4(a). Introducing a positive feedback changes this regime drastically. The open ends are not attracted any more by the poles, and their trajectories resemble a cycloid placed on the sphere [Fig. 4(b)]. This is typical for a two-periodic regime where the open end

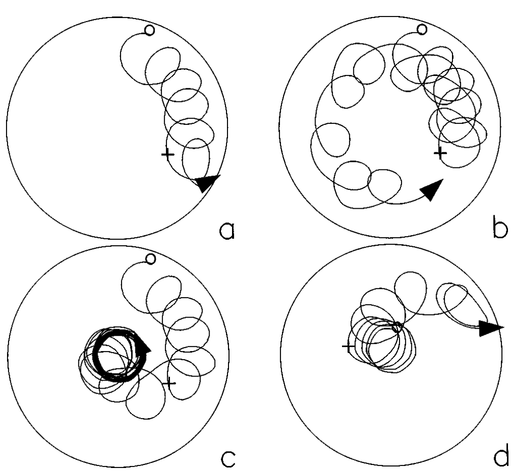


FIG. 3. Trajectories of a spiral wave tip on a circular domain of radius $R = 26$ computed for $k_{\mathrm{fb}} = 0.15$ and different values of the time delay in a feedback loop (a) $\tau = 5$, (b) $\tau = 15$, (c) $\tau = 25$, and (d) $\tau = 40$. Symbols "○" and "+" as in Fig. 1.

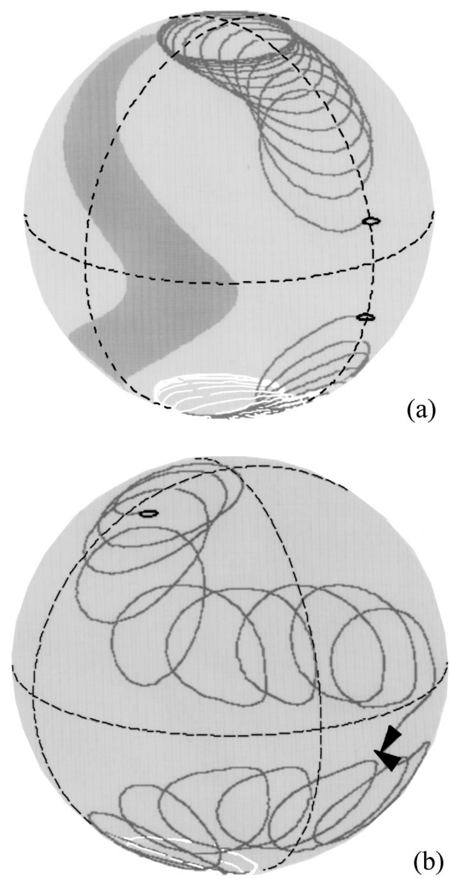


FIG. 4. Trajectories of a spiral wave tip on a sphere of radius $R = 25$ computed (a) for negative global feedback with $k_{\mathrm{fb}} = -0.05$ and (b) for positive feedback with $k_{\mathrm{fb}} = 0.1$. Time delay $\tau = 0$. Initial position of the spiral wave tip is indicated by the symbol "○". Dark shaded area in (a) corresponds to the location of the excited region at the end of the computational interval.

rotates around some core, and the core moves with a constant speed along a specific latitude [14,15]. When the positive feedback has a sufficiently large intensity, two arms of the double spiral mutually annihilate each other [as in Fig. 4(b)]. As a consequence, self-sustained activity on the sphere is completely suppressed.

There is a close similarity between controlling spiral waves on a disk and on a sphere. For both cases a negative global feedback results in the stabilization of a rigid rotation, whereas a positive feedback leads to the annihilation of spiral waves and suppression of self-sustained activity. This similarity is not surprising because, if a wave on a sphere is symmetric with respect to the equator plane, no flux exists between the northern and the southern hemispheres. Topologically a hemisphere with the equator as a nonflux boundary is very similar to a disk.

The open question is why a global coupling is so extremely efficient to control spiral wave motion in a confined geometry. A qualitative answer to this question can be given within the frameworks of a kinematical description of a propagating wave [10,23,24]. Let us consider the case of a weakly excitable medium for which a propagating wave looks like a thin curve moving across a surface. It was shown that in this limit and in a medium without feedback the angular velocity of a spiral wave decreases with the arclength $L$ of the wave rotating on a disk [12]. This dependency predetermines the instability of a centrosymmetrical rotation on a disk and any perturbation results in a transition to a two-periodic regime. In a weakly excitable medium the integral of the second variable $v$ and, hence, the intensity of the feedback computed in accordance with (4) should be proportional to $L$.

In the case of a positive value of the coefficient $k_{\mathrm{fb}}$ the decrease of the rotation velocity with the arclength $L$ is stronger. Because of this the instability of the centrosymmetrical rotation becomes more pronounced than in the absence of the feedback. It means that the spiral wave is pushed towards the boundary more intensively until, for a sufficiently large intensity of the feedback, complete annihilation occurs. The global feedback with a negative value of $k_{\mathrm{fb}}$ results in an increase of the angular velocity with arclength $L$. Thus it works against the destabilizing effects typical for a medium without feedback and tends to suppress any oscillations of the front length. Naturally, such a feedback stabilizes a centrosymmetrical rigid rotation. On the other hand, it can destabilize a two-periodic regime and induce a transition to rigid rotation.

Our computational results demonstrate a very high efficiency of controlling spiral wave by application of a global feedback. Corresponding qualitative kinematical considerations corroborate that this powerful tool can be used either to stabilize or destroy spiral waves not only in this particular model case but also in different excitable media with confined geometries.

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