

# Quantum Floquet engineering with an exactly solvable tight-binding chain in a cavity

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## Abstract

Recent experimental advances enable the manipulation of quantum matter by exploiting the quantum nature of light. However, paradigmatic exactly solvable models, such as the Dicke, Rabi or Jaynes-Cummings models for quantum-optical systems, are scarce in the corresponding solid-state, quantum materials context. Focusing on the long-wavelength limit for the light, here, we provide such an exactly solvable model given by a tight-binding chain coupled to a single cavity mode via a quantized version of the Peierls substitution. We show that perturbative expansions in the light-matter coupling have to be taken with care and can easily lead to a false superradiant phase. Furthermore, we provide an analytical expression for the groundstate in the thermodynamic limit, in which the cavity photons are squeezed by the light-matter coupling. In addition, we derive analytical expressions for the electronic single-particle spectral function and optical conductivity. We unveil quantum Floquet engineering signatures in these dynamical response functions, such as analogs to dynamical localization and replica side bands, complementing paradigmatic classical Floquet engineering results. Strikingly, the Drude weight in the optical conductivity of the electrons is partially suppressed by the presence of a single cavity mode through an induced electron-electron interaction.

## I. INTRODUCTION

The control of matter through light, or more generally electromagnetic (EM) radiation, is a research direction that has gained tremendous attention recently.<sup>1</sup> It connects to many topical fields including information processing and steering chemical reactions.<sup>2-9</sup> In recent years, some exciting progress has been made towards this goal by periodically driving materials with light in a regime where the quantum nature of the light field can be disregarded.<sup>10,11</sup> In this classical-light regime the physics of materials under continuous-wave irradiation is efficiently described by Floquet theory.<sup>12-14</sup> Within Floquet theory, a time-periodic Hamiltonian is replaced by a quasi-static, effective so-called Floquet Hamiltonian, which can include renormalized effective model parameters, new synthetically generated terms, as well as Floquet sidebands, i.e., shakeoff features separated by the driving frequency from the main resonances, in frequency-dependent spectra. The search for driving protocols that realize certain effective Hamiltonians with specific desired properties has become known as Floquet engineering.<sup>14,15</sup> Along these lines several ways to control matter with light have been proposed, for example, the manipulation of topologically non-trivial states,<sup>10,11,16-22</sup> strongly correlated materials<sup>23-27</sup> and superconductors.<sup>28-33</sup> However, a fundamental problem for driving materials with classical light is heating,<sup>31,34,35</sup> which in many realistic setups prohibits

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versatile control.

To circumvent detrimental heating, control of materials through quantum light has recently been proposed.<sup>6,9,36-38</sup> The basic idea is to place a material into an optical cavity by which the light-matter coupling can be enhanced<sup>9,39</sup> since the coupling is inversely proportional to the square-root of the effective mode volume<sup>39,40</sup>. One can therefore bolster the coupling by manufacturing smaller devices, or by employing near-field enhancement effects.<sup>41</sup> Through this enhancement of the coupling, vacuum fluctuations or few photon states of the cavity can already have a sizeable effect on the matter degrees of freedom, alleviating the need of strong classical driving fields. In the emerging field of cavity engineering, ultra-strongly coupled light-matter systems have been realized based on different implementation schemes, starting from the first results obtained with microwave and optical cavities.<sup>42,43</sup> More recently, sizeable light-matter coupling (LMC) has been implemented in superconducting circuits,<sup>44</sup> and it is nowadays possible to couple few electrons to EM fields in split-ring resonators.<sup>45-47</sup> These technological advances have led to the observation of LMC-controlled phenomena such as transport properties being tuned by polaritonic excitations<sup>48</sup> and Bose-Einstein condensation of exciton-polaritons.<sup>49-51</sup> Another route to control matter by quantum light is to influence chemical reactions<sup>52,53</sup> through the selective enhancement of desired reactive paths and blocking of others. In addition, there have been several proposals to influence superconductivity in a cavity, either by coupling cavity modes to the phonons involved in electronic pairing,<sup>54</sup> to magnons that are believed to form the pairing glue in cuprates,<sup>55</sup> or by directly coupling to the electronic degrees of freedom.<sup>56-60</sup> Concurrently, experimental evidence of cavity-enhanced superconductivity was recently reported, whose origin and interpretation are still under debate.<sup>61</sup>

To turn the question around and to add another facet to the problem of LMC, one can inversely ask: How can one engineer the light field of a cavity using matter? One prominent and widely discussed route is the realization of a superradiant phase in thermal equilibrium.<sup>62-69</sup> Generally, systems that require a quantum-mechanical treatment of both light and matter will host hybrid states that mix light and matter degrees of freedom.<sup>70</sup> Describing such light-matter systems is a formidable challenge and often relies on using few-body simplifications. For instance, describing matter through effective few-level systems has led to paradigmatic models such as the Dicke, Rabi or Jaynes-Cummings models. These simplified models capture certain aspects of the underlying physics well.<sup>39,71-74</sup> However, in order to capture collective phenomena of solid-state systems, a many-body description of the material is needed. Efforts in this direction include first-principles approaches, such as the density functional reformulation of QED,<sup>75-77</sup> generalized coupled cluster

theory<sup>78</sup> or hybrid-orbital approaches.<sup>79,80</sup> In addition, a recent work presents the analytic solution of the free 2D electron gas coupled to a cavity.<sup>81</sup>

In this work, we introduce and study a paradigmatic, exactly solvable quantum lattice model for a solid coupled to the quantized light field of a cavity. At the same time, we aim at connecting quantum-photon phenomena to previous results of Floquet engineering by investigating the quantum-to-classical crossover. To this end, we focus on a tight-binding chain coupled to a single mode of a cavity, through a quantized version of the Peierls substitution introduced in Refs. 82–85. As we aim to describe solid-state systems, we are mainly interested in the thermodynamic (TD) limit of this model, but we also connect to prior finite system size studies. First, we determine the groundstate (GS) of the system. By exact numerical means, we exclude the existence of an equilibrium superradiant phase, consistent with existing no-go theorems.<sup>62,64</sup> We show explicitly that gauge invariance must be taken into account carefully to prohibit false signatures of a superradiant phase upon expanding the Peierls substitution in orders of the LMC. We then concentrate on the thermodynamic limit where the electronic groundstate is found to remain the Fermi sea of the uncoupled system centered at quasi-momentum  $k = 0$  consistent with the findings of Ref. 81. Using this insight, we analytically determine the photonic GS of the system to be a squeezed state. Additionally, an analytical expression for the electronic spectral function is given. With this we establish the quantum analogues to paradigmatic Floquet results, such as dynamical localization or the emergence of replica bands, and pinpoint the differences between the classical and quantum cases. To make the connection to Floquet results explicit, we analyze the quantum-to-classical crossover and show that the nonequilibrium spectral function of the system approaches that of a classically driven system in the limit of strong driving. Finally, the current response to a spatially uniform external field, i.e., the optical conductivity, is calculated and a f-sum rule for cavity-coupled systems is identified. The presence of the single cavity mode induces a non-complete suppression of the Drude peak that remains even in the TD limit. This result is consistent with that previously found in Ref. 81 for the 2D electron gas. We attribute this feature to the effective electron-electron interaction mediated by the cavity.

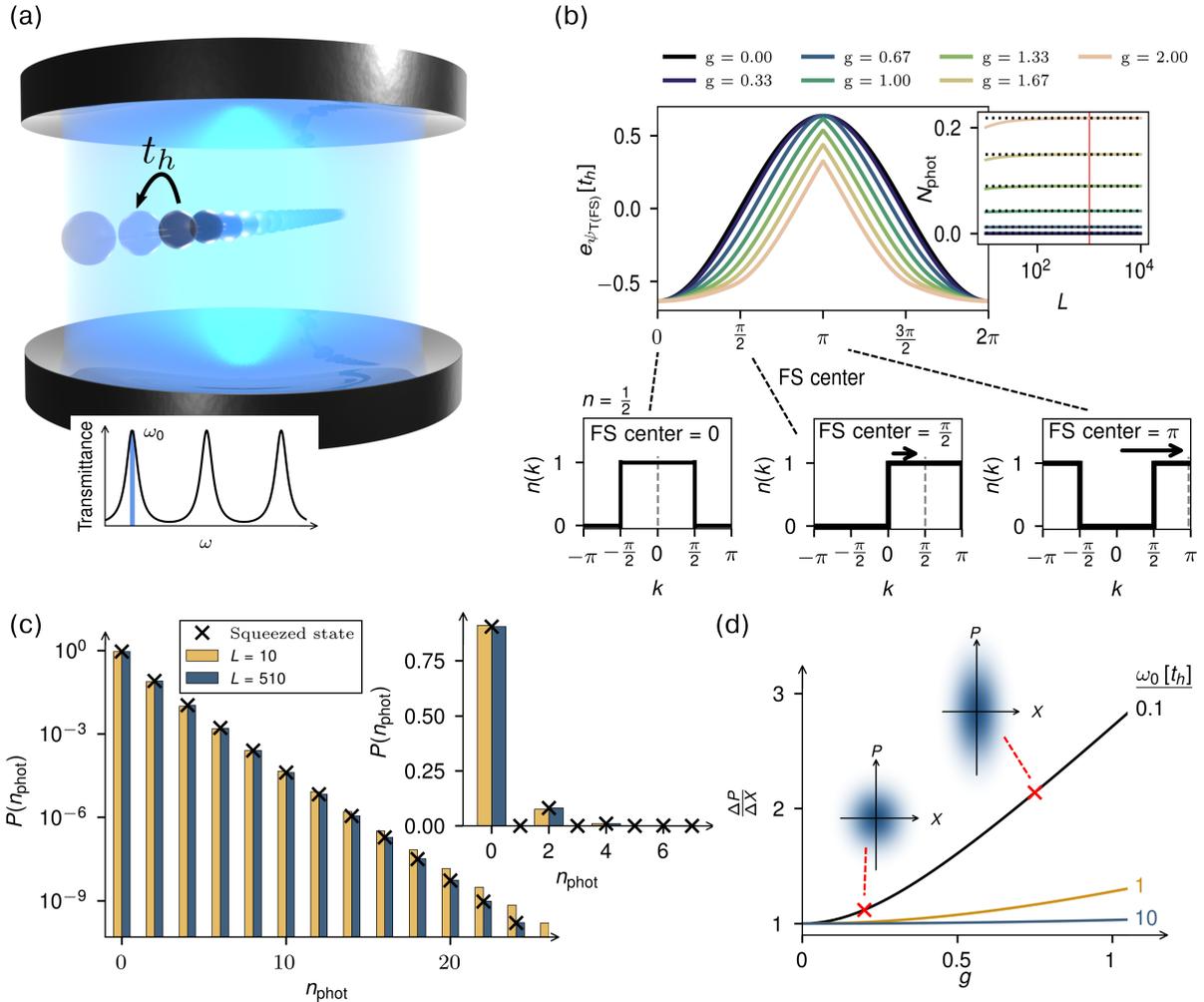


FIG. 1. **Model and groundstate.** (a) Illustration of the studied model: A one dimensional tight-binding chain with nearest neighbour hopping  $t_h$  is coupled to a single cavity mode (first transmittance resonance) with frequency  $\omega_0$  (see inset). (b) Minimum energy density  $e_{\psi_{\text{T(FS)}}}$  according to Eq. (5), with the electronic part of the wavefunction  $|\psi\rangle_f$  chosen as a single connected quasi-momentum region being occupied (Fermi sea, FS). We vary the FS center quasi-momentum and find that the FS centered around  $k = 0$  leads to the lowest energy. A fully unbiased variation of the electronic state has confirmed that this indeed yields the true groundstate. Inset: Average photon number  $N_{\text{phot}} := \langle a^\dagger a \rangle$  for varying coupling strength  $g$ , as function of the system size  $L$ . For all  $g$  values shown, the number of bosons in the cavity converges at large  $L$  to a finite value (black dashed lines). The red line corresponds to the system size used in the main plot ( $L = 1010$ ). (c) The exact probability distribution  $P(n_{\text{phot}})$  in logarithmic scale of the photon number is compared to the one given by a squeezed state (black crosses) for the groundstate of a chain of length  $L = 510$  (blue bars) and  $L = 10$  (yellow bars). Here the coupling constant is set to  $g = 2$ . In the inset, the same quantity is plotted on a linear scale. (d)  $\Delta P / \Delta X$  as function of  $g$  for three different values of  $\omega_0$  and two representative squeezing ellipses for  $g = 0.2$  and  $g = 0.75$ , respectively.

## II. RESULTS

### A. Model

We consider a non-interacting tight-binding chain with nearest-neighbour hopping, as illustrated in Fig. 1(a). We couple the chain to a single cavity mode lying in the first transmittance resonance and consider the dipole approximation, thus neglecting the spatial dependence of the electromagnetic vector potential, which is a good approximation for the wavelengths of typical optical cavities. The Hamiltonian of this model reads<sup>82</sup>

$$H = \omega_0 \left( a^\dagger a + \frac{1}{2} \right) - \sum_{j=1}^L \left[ t_h e^{-i \frac{g}{\sqrt{L}} (a^\dagger + a)} c_{j+1}^\dagger c_j + \text{h.c.} \right], \quad (1)$$

where  $\omega_0$  is the resonant frequency of the uncoupled cavity (at  $g = 0$ ),  $c_j(c_j^\dagger)$  is the fermionic annihilation (creation) operator at lattice cite  $j$ , and  $a(a^\dagger)$  is the bosonic annihilation (creation) operator of the cavity mode. The latter are related to the quantized electromagnetic vector potential via  $A = \frac{g}{\sqrt{L}}(a + a^\dagger)$ , with the convention  $e = \hbar = c = 1$  and  $L$  the number of lattice sites. We use periodic boundary conditions and set the lattice constant to 1. One can show that, within a few-band truncation, inclusion of the relevant effects of the LMC as well as gauge invariance are guaranteed by the quantized form of the Peierls substitution employed to set up the Hamiltonian given in Eq. (1).<sup>82-84</sup> The coupling constant  $g$  depends on the specifics of the system, such as the geometry and material composition of the cavity. We keep the explicit dependence  $1/\sqrt{L}$ , instead of including it in the dimensionless coupling parameter  $g$ , in order to simplify the analysis of the thermodynamic limit. In quasi-momentum space, the model takes the form

$$H = \cos \left( \frac{g}{\sqrt{L}} (a + a^\dagger) \right) \mathcal{T} + \sin \left( \frac{g}{\sqrt{L}} (a + a^\dagger) \right) \mathcal{J} + \omega_0 \left( a^\dagger a + \frac{1}{2} \right), \quad (2)$$

where we have introduced the kinetic energy and current operators

$$\begin{aligned} \mathcal{T} &:= \sum_k -2t_h \cos(k) c_k^\dagger c_k =: \sum_k \varepsilon_k c_k^\dagger c_k \\ \mathcal{J} &:= \sum_k 2t_h \sin(k) c_k^\dagger c_k =: \sum_k v_k c_k^\dagger c_k, \end{aligned} \quad (3)$$

and  $\varepsilon_k, v_k$  are the band dispersion and band velocity at quasi-momentum  $k$ , respectively.  $c_k^{(\dagger)}$  annihilates (creates) and electron at quasi-momentum  $k$ . These expressions highlight the extensive number of constants of motion of the model, namely  $\rho_k = c_k^\dagger c_k$  with  $[\rho_k, H] = 0$  for all  $k \in \text{BZ}$

(Brillouin Zone), which is a consequence of the spatially constant vector potential not breaking the lattice periodicity and preserving fermionic quasi-momentum in any electron-photon scattering process.<sup>81</sup> As a consequence, the eigenstates of the Hamiltonian can be factorized as

$$H|\Psi\rangle = E_{\Psi}|\Psi\rangle ; \quad |\Psi\rangle = |\phi\rangle_b \otimes |\psi\rangle_f, \quad (4)$$

where  $|\phi\rangle_b$  is the photonic part of the wavefunction, and  $|\psi\rangle_f$  is an eigenstate of the electronic density operator  $\rho = \frac{1}{L} \sum_k c_k^\dagger c_k$ .

## B. Groundstate

We determine the GS of the system  $|\Psi_{\text{GS}}\rangle = |\phi_{\text{GS}}\rangle_b \otimes |\psi_{\text{GS}}\rangle_f$  in two different ways: (i) by a variational scheme that exploits the extensive number of constants of motion varying the electronic occupation and using exact diagonalization for the remaining non-harmonic bosonic system (see Sec. IV A) and (ii) by full exact diagonalization of the combined electronic and bosonic system (ED). The variational scheme can be performed for hundreds of lattice sites. We employ a cutoff of the maximum boson number in the Fock space such that all shown results are converged with respect to this cutoff. The ED calculations serve to verify the variational results for small system sizes. We consider a half-filled electronic system with  $n := \langle \rho \rangle = \frac{1}{2}$ , and choose the cavity frequency  $\omega_0 = t_h$ , unless explicitly denoted otherwise.

Within the variational scheme, we find that the electronic part of the GS wavefunction  $|\psi_{\text{GS}}\rangle_f$  is the Fermi sea (FS) around  $k = 0$  even at non-zero  $g$ . In Fig. 1(b) we illustrate this for a subset of possible electronic configurations. Here, following the procedure explained in Sec. IV A, we take as fermionic trial wavefunctions  $|\psi_{\text{T(FS)}}\rangle_f$  only connected regions in  $k$ -space centered at different positions (FS center). Then we numerically determine the GS energy  $E_{\psi_{\text{T(FS)}}}$  of the resulting bosonic hamiltonian  $H_{\psi_{\text{T(FS)}}} = \int \langle \psi_{\text{T(FS)}} | H | \psi_{\text{T(FS)}} \rangle_f$ . In Fig. 1(b) we show the energy density

$$e_{\psi_{\text{T(FS)}}} = \frac{E_{\psi_{\text{T(FS)}}}}{L} \quad (5)$$

as a function of the center of the connected region (FS center). The energetic minimum always remains at the FS centered around  $k = 0$  for all considered coupling values. This shows that the fermionic part of the GS wavefunction remains unchanged upon turning on a coupling to the bosonic mode, a result that is consistent with the two-dimensional electron gas considered in Ref. 81. The unbiased variational scheme (see Sec. IV A) is not limited to connected regions in  $k$ -space, and a full variation in electronic state space confirms the unshifted Fermi sea as the true ground state.

We now discuss the bosonic part of the wavefunction,  $|\phi_{\text{GS}}\rangle_b$ . To this end, we define the photon number eigenstates as  $a^\dagger a |n_{\text{phot}}\rangle = n_{\text{phot}} |n_{\text{phot}}\rangle$  and introduce the probability distribution  $P(n_{\text{phot}}) := |\langle n_{\text{phot}} | \phi_{\text{GS}} \rangle|^2$  of finding  $n_{\text{phot}}$  photons in the GS.

$P(n_{\text{phot}})$  for  $g = 2$  (Fig. 1(c)) shows that only even number states contribute, implying that the bosonic wavefunction has a probability distribution that is incompatible with a coherent state. Instead,  $P(n_{\text{phot}})$  agrees perfectly with a squeezed state with the same average photon number, indicated by the black crosses in Fig. 1(c). This finding does not change qualitatively for different values of  $g$ . In the inset of Fig. 1(b) we show the scaling of the average photon number in the GS,  $N_{\text{phot}} = \langle a^\dagger a \rangle$ .  $N_{\text{phot}}$  is found not to grow extensively with the system size, which excludes the existence of a superradiant phase.

Put differently, the absence of a superradiant phase implies that the expectation value of the bosonic operators in the GS does not scale with the system size. This allows us to perform a scaling analysis of contributions to the GS energy

$$\begin{aligned} \langle \Psi_{\text{GS}} | H | \Psi_{\text{GS}} \rangle &= \underbrace{\langle \Psi_{\text{GS}} | \omega_0 \left( a^\dagger a + \frac{1}{2} \right) | \Psi_{\text{GS}} \rangle}_{\sim 1} + \underbrace{\langle \Psi_{\text{GS}} | \mathcal{T} | \Psi_{\text{GS}} \rangle}_{\sim L} + \underbrace{\langle \Psi_{\text{GS}} | \frac{g}{\sqrt{L}} (a^\dagger + a) \mathcal{J} | \Psi_{\text{GS}} \rangle}_{\sim \sqrt{L}} \\ &\quad - \underbrace{\langle \Psi_{\text{GS}} | \frac{1}{2} \frac{g^2}{L} (a^\dagger + a)^2 \mathcal{T} | \Psi_{\text{GS}} \rangle}_{\sim 1} + \mathcal{O} \left( \frac{1}{\sqrt{L}} \right). \end{aligned} \quad (6)$$

In the TD limit, the GS energy is entirely composed of terms that are at most quadratic in the photon field amplitude  $A = \frac{g}{\sqrt{L}}(a^\dagger + a)$ . In order to simplify the following discussion, we diagonalize the Hamiltonian up to quadratic ( $A^2$ ) order by a combined squeezing and displacement transformation yielding (see Supplementary Material (SM), Sec. I)

$$H^{\text{D}} = \mathcal{W}[\mathcal{T}] \left( \beta^\dagger \beta + \frac{1}{2} \right) + \mathcal{T} - \frac{g^2 \omega_0}{L \mathcal{W}[\mathcal{T}]^2} \mathcal{J}^2 ; \quad \mathcal{W}[\mathcal{T}] = \omega_0 \sqrt{1 - 2 \frac{g^2}{L \omega_0} \mathcal{T}}. \quad (7)$$

where  $\beta^{(\dagger)}$  annihilates (creates) a coherent squeezed state.<sup>30</sup> In terms of the original creation and annihilation operators of the unsqueezed cavity photons, the corresponding squeezed-state operators are given as

$$\begin{aligned} \beta^\dagger &= \cosh \left( \frac{1}{2} \ln \left( \frac{\mathcal{W}[\mathcal{T}]}{\omega_0} \right) \right) \left( a^\dagger + \frac{g \omega_0}{L \mathcal{W}[\mathcal{T}]^2} \mathcal{J} \right) + \sinh \left( \frac{1}{2} \ln \left( \frac{\mathcal{W}[\mathcal{T}]}{\omega_0} \right) \right) \left( a + \frac{g \omega_0}{L \mathcal{W}[\mathcal{T}]^2} \mathcal{J} \right), \\ \beta &= \cosh \left( \frac{1}{2} \ln \left( \frac{\mathcal{W}[\mathcal{T}]}{\omega_0} \right) \right) \left( a + \frac{g \omega_0}{L \mathcal{W}[\mathcal{T}]^2} \mathcal{J} \right) + \sinh \left( \frac{1}{2} \ln \left( \frac{\mathcal{W}[\mathcal{T}]}{\omega_0} \right) \right) \left( a^\dagger + \frac{g \omega_0}{L \mathcal{W}[\mathcal{T}]^2} \mathcal{J} \right). \end{aligned} \quad (8)$$

The last term in  $H^{\text{D}}$  of Eq. (7) highlights that the cavity induces an effective electron-electron interaction.

Knowing that the electronic part of the GS wavefunction is the unshifted FS, we define the expectation value of the electronic kinetic energy and current in the GS as

$$\begin{aligned} T_{\text{GS}} &= {}_f\langle\psi_{\text{GS}}|\mathcal{T}|\psi_{\text{GS}}\rangle_f < 0, \\ J_{\text{GS}} &= {}_f\langle\psi_{\text{GS}}|\mathcal{J}|\psi_{\text{GS}}\rangle_f = 0, \end{aligned} \quad (9)$$

and the dressed cavity frequency as

$$\tilde{\omega} = \mathcal{W}[T_{\text{GS}}] = \omega_0 \sqrt{1 - 2\frac{g^2}{\omega_0 L} T_{\text{GS}}}. \quad (10)$$

The bosonic part of the GS wavefunction is then given by the GS of the electronically renormalized bosonic Hamiltonian

$$H_b^{\text{D}} = {}_f\langle\psi_{\text{GS}}|H^{\text{D}}|\psi_{\text{GS}}\rangle_f = \tilde{\omega} \left( \beta^\dagger \beta + \frac{1}{2} \right) + T_{\text{GS}} \quad (11)$$

which is a squeezed vacuum state  $|\phi_{\text{GS}}\rangle_b$ <sup>73,86–88</sup> that is connected to the bare cavity vacuum  $|0\rangle$  through a squeezing transformation,

$$|\phi_{\text{GS}}\rangle_b = e^{\frac{1}{2}(\zeta^* a^2 - \zeta (a^\dagger)^2)} |0\rangle. \quad (12)$$

The squeeze factor  $\zeta$ <sup>89</sup> is given by (see SM, Sec. I)

$$\zeta = \frac{1}{2} \ln \left( \frac{\tilde{\omega}}{\omega_0} \right). \quad (13)$$

The squeezed state that was numerically observed to match the exact  $P(n_{\text{phot}})$  for the GS Fig. 1(c) corresponds precisely to the squeeze factor  $\zeta$  defined in Eq. (13). In Fig. 1(d) we show how the amount of squeezing depends on the cavity coupling strength  $g$ . Defining  $X := (a^\dagger + a)$  and  $P := i(a^\dagger - a)$ , and  $\Delta\mathcal{O} = \sqrt{\langle\mathcal{O}^2\rangle - \langle\mathcal{O}\rangle^2}$  for a generic operator  $\mathcal{O}$ , a squeezed state minimizes the Heisenberg uncertainty  $\Delta P \Delta X = 1$ . The ratio

$$\frac{\Delta P}{\Delta X} = e^{2\zeta} = \frac{\tilde{\omega}}{\omega_0} \quad (14)$$

characterizes the degree of squeezing.<sup>73,89</sup> The squeezing of the vacuum is reminiscent of the finding presented in Ref. 90, which was obtained for a different light-matter model. It has recently become possible to directly measure the vacuum fluctuations inside a cavity,<sup>91,92</sup> which enables experimental tests of our prediction.

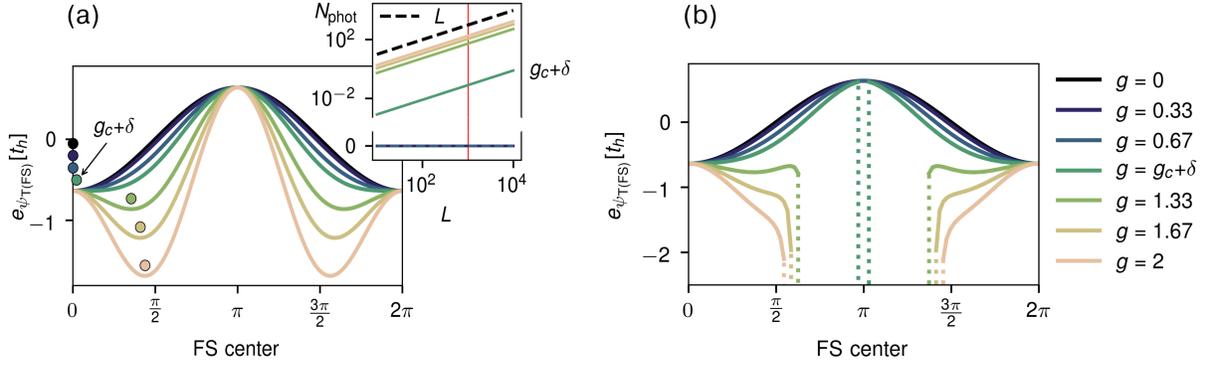


FIG. 2. **False superradiance and instability for the truncated Hamiltonian.** (a) Minimum energy density  $e_{\psi_{T(\text{FS})}}$  (Eq. (5)) of the Hamiltonian truncated at first order for an electronic wavefunction being a single connected occupied region in  $k$ -space, as function of the shift of this Fermi sea. The position of one minimum of the curves is indicated by a circle. At  $g_c = \sqrt{\frac{\pi\omega_0}{4t_h}}$  the center of the Fermi sea realizing the minimal energy moves to a finite  $k$ -value which is illustrated by the small shift of the minimum of the curve corresponding to  $g = g_c + \delta$  where  $\delta = 0.001$ . Inset: Average photon number  $\langle a^\dagger a \rangle$  for varying coupling strengths as function of the system size  $L$ . Above the critical value  $g_c$ , superradiant scaling of the photonic occupancy sets in. The red line denotes the system size used in the main plot ( $L = 1010$ ). (b) Minimum energy density of the second-order truncated Hamiltonian. When the shift is sufficiently large such that the kinetic energy of the electrons is positive, it is possible to obtain a spectrum of the electronically renormalized bosonic Hamiltonian that is not bounded from below anymore, rendering the system unstable. The instability is indicated by the dotted line. Here  $L = 1010$ .

### C. False superradiant phase transition in the approximate model

Next, we analyze the effect of truncating the Hamiltonian at first and second order in  $A = \frac{g}{\sqrt{L}}(a^\dagger + a)$  on the GS at finite  $L$

$$\begin{aligned}
 H^{1\text{st}} &= \omega_0 \left( a^\dagger a + \frac{1}{2} \right) + \mathcal{T} + \frac{g}{\sqrt{L}} (a^\dagger + a) \mathcal{J} \\
 H^{2\text{nd}} &= \omega_0 \left( a^\dagger a + \frac{1}{2} \right) + \mathcal{T} + \frac{g}{\sqrt{L}} (a^\dagger + a) \mathcal{J} - \frac{1}{2} \frac{g^2}{L} (a^\dagger + a)^2 \mathcal{T}.
 \end{aligned} \tag{15}$$

For the first-order truncated Hamiltonian  $H^{1\text{st}}$  we again determine the GS by the unbiased variational scheme (Sec. IV A). The GS is given by a connected region in  $k$ -space that is, however, not always centered at  $k = 0$ . This is shown in Fig. 2(a), where the energy density  $e_{\psi_{T(\text{FS})}}$  (Eq. (5)) for  $H^{1\text{st}}$  is evaluated as function of the FS shift, in analogy to our analysis in Sec. II B. We find that at a critical coupling strength  $g_c$  there is a phase transition to a GS hosting a finite current signified by the shift of the FS, Fig. 2(a). This is complemented by an occupation of the cavity

mode that scales linearly with  $L$  as shown in the inset of Fig. 2(a). The critical coupling is given by  $g_c = \sqrt{\frac{\pi\omega_0}{4t_h}}$ . A symmetric or anti-symmetric combination of the degenerate GS wavefunctions (FS shifted either to the left or the right) would yield a net zero current restoring the inversion symmetry of the system but still result in a macroscopic occupation of the cavity mode. This transition is reminiscent of the one in the Dicke model, for which neglecting the diamagnetic ( $A^2$ ) coupling yields a superradiant phase with a macroscopically occupied photon mode,<sup>71,93</sup> which is absent for the full gauge-invariant coupling.<sup>94</sup>

In the lattice case, only the inclusion of coupling terms to all orders in  $A$  of the the Peierls substitution guarantees gauge invariance. If one instead includes only terms up to second order ( $A^2$ ), a large coupling strength  $g$  results in a spectrum of the Hamiltonian that is not bounded from below. Fig. 2(b) is obtained in an analogous way to Fig. 1(b) illustrating the absence of a GS above a critical coupling strength as follows: Fixing the electronic part of the wavefunction to be a shifted FS, an increased shift will yield a corresponding bosonic problem with a decreased frequency. At some point the effective frequency vanishes, leading to the absence of a GS of the remaining bosonic problem beyond that point. We indicate this point by a dotted line in Fig. 2(b). This instability can be cured by including an arbitrarily small  $A^4$  term, signalling the breakdown of the truncation.

#### D. Momentum-resolved spectral function in the TD limit

The effects of the cavity on electrons could be investigated via ARPES measurements. For this reason, but also to point out analogs to Floquet results, we calculate the electronic spectral-function defined as

$$A(k, \omega) = -\frac{1}{\pi} \text{Im} G^{\text{R}}(k, \omega), \quad (16)$$

with

$$G^{\text{R}}(k, \omega) = -\int_0^{\infty} dt i \langle [c_k(t), c_k^{\dagger}]_+ \rangle e^{i\omega t} \quad (17)$$

where  $[\cdot]_+$  is the anti-commutator. We perform the calculation for a finite chain of  $L = 170$  including all orders of the Peierls coupling. The result is given in Fig. 3(a). In the TD limit, we can use similar arguments to the ones previously utilized in Sec. II B to give an analytic expression for the electronic spectral function. No operator in the expectation value Eq. (17) creates a macroscopic number of photons. We can thus conclude by a similar scaling analysis as in Eq.(6) that in the

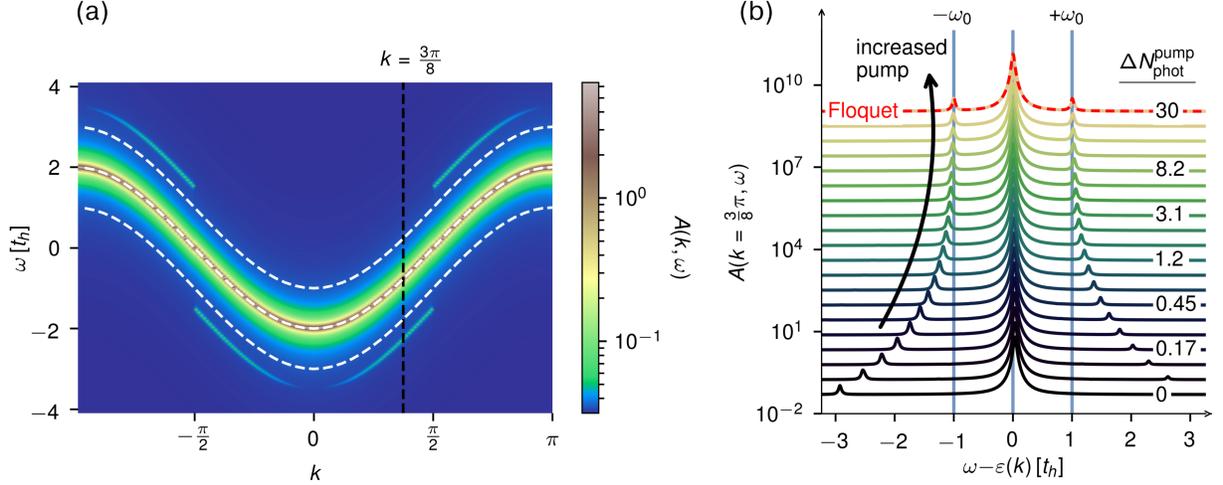


FIG. 3. **Momentum-resolved spectral function in equilibrium and for a driven cavity.** (a) False-color plot of the momentum-resolved spectral function  $A(k, \omega)$  Eq. (18) at  $T = 0$ . The central white dashed curve shows the bare electronic band. Replicas of the bare band offset by  $\pm\omega_0$  are shown by white dashed curves. The quantum replica bands seen in the false-color spectra are at an increased distance from the main band, which is set by the dressed cavity frequency  $\tilde{\omega} > \omega_0$ . The replica bands are below (above) the main band in the occupied (unoccupied) quasi-momentum regions, reflecting the overall particle-hole symmetry of the half-filled system. The dashed line at  $k = \frac{3\pi}{8}$  denotes the  $k$ -space position of the plot in panel (b). Here we consider  $L = 170$  and  $g = 1$ , the delta functions of Eq. (18) are represented by Lorentzians with broadening  $\eta = 0.025$ . (b) Nonequilibrium time- and momentum-resolved spectral function according to Eq.(22) evaluated at  $k = \frac{3\pi}{8}$  for several cavity pumping strengths, characterized by the displacement parameter  $\alpha$  with  $|\alpha|^2 = \Delta N_{\text{phot}}^{\text{pump}}$  (colored lines).  $g^2 \Delta N_{\text{phot}}^{\text{pump}}$  is kept constant, implying that  $g \rightarrow 0$  as the pumping  $\Delta N_{\text{phot}}^{\text{pump}} \rightarrow \infty$ . The black line corresponds to the ground state for  $g = 2.5$  for which the y-axis reports the amplitude, while the coloured lines are vertically shifted for clarity and follow the progressive occupation  $\Delta N_{\text{phot}}^{\text{pump}}$  indicated on the right. For increasing pump strength the side-bands become more symmetric and their position approaches  $\omega_0$  as  $\tilde{\omega} \xrightarrow{g \rightarrow 0} \omega_0$ . For the largest pump  $\Delta N_{\text{phot}}^{\text{pump}}$  the curve is overlaid with the Floquet result (red dashed line), that matches the pumped-cavity result. Here  $L = 90$ , and a Lorentzian broadening  $\eta = 0.025$  has been included in the delta functions.

TD limit the time evolution can be written with the diagonal Hamiltonian Eq. (7). The spectral function keeping leading  $1/L$  corrections is analytically found to be

$$\begin{aligned}
 A(k, \omega) = & (1 - n_k) e^{-\frac{g^2 v_k^2 \omega_0}{L \tilde{\omega}^3}} \sum_{\ell} \frac{\left(\frac{g^2 v_k^2 \omega_0}{L \tilde{\omega}^3}\right)^{\ell}}{\ell!} \delta\left(\omega - \varepsilon_k \left(1 - \frac{g^2 \omega_0}{2L \tilde{\omega}}\right) - \Sigma_k - \tilde{\omega} \ell\right) \\
 & + n_k e^{-\frac{g^2 v_k^2 \omega_0}{L \tilde{\omega}^3}} \sum_{\ell} \frac{\left(\frac{g^2 v_k^2 \omega_0}{L \tilde{\omega}^3}\right)^{\ell}}{\ell!} \delta\left(\omega - \varepsilon_k \left(1 - \frac{g^2 \omega_0}{2L \tilde{\omega}}\right) + \Sigma_k + \tilde{\omega} \ell\right).
 \end{aligned} \tag{18}$$

Here  $n_k = \langle \rho_k \rangle$  and the self-energy  $\Sigma_k$  is given by

$$\Sigma_k = \frac{g^2 \omega_0}{\tilde{\omega}^2 L} v_k^2. \quad (19)$$

The details of the calculation are presented in Sec. II of the SM. From Eq. (18) the spectral function of the unperturbed electrons,

$$A(k, \omega) \xrightarrow{L \rightarrow \infty} A_0(k, \omega) = \delta(\omega - \varepsilon_k), \quad (20)$$

is recovered in the limit  $L \rightarrow \infty$ . This can be intuitively understood by noticing that a single cavity mode with a vanishing energy density is not able to influence the energies of macroscopically many electrons with a finite energy density. We include explicitly the leading order finite-size correction ( $1/L$ ) terms in Eq. (18), since they contain interesting insights to quantum analogs to Floquet results. Another way to render analogous terms relevant is by extending our model to include a macroscopic number of cavity modes, which would yield a finite energy density for the photon modes even in the thermodynamic limit.

Most prominently, the spectral function Eq. (18) contains a sum over  $\delta$  functions with distance  $\tilde{\omega}$  between each other, given by the dressed instead of bare cavity frequency, which is a direct consequence of the quantum nature of the photons. This is the quantum analog to the Floquet replica bands visible in Fig. 3(a). Contrary to the Floquet replica bands, the quantum replica bands lie either above or below the main band, but only on one side for fixed quasi-momentum  $k$  at zero temperature, depending on whether the respective momentum state is filled or empty. This reflects the particle-hole symmetry of the half-filled system, in which a combined  $\omega \rightarrow -\omega$  and  $k \rightarrow k + \pi$  sublattice particle-hole transformation leaves the spectral function invariant.

Importantly, despite the fact that the cavity induces an effective all-to-all electron-electron interaction, there is no broadening of the  $\delta$ -peaks. This is related to the vanishing momentum transfer of the interaction and the resulting fact that the Bloch states remain exact electronic eigenstates. As a consequence, the interaction results in a purely real electronic self-energy  $\Sigma_k$ , leading to band renormalizations without broadening.

The presence of the cavity squeezes the band dispersion  $\varepsilon_k$  by a factor  $\left(1 - \frac{g^2 \omega_0}{2L\tilde{\omega}}\right) < 1$ . This is the quantum analog to the dynamical localization that leads to a suppression of the band width. The band renormalization factor  $1 - \frac{g^2 \omega_0}{2L\tilde{\omega}}$  is consistent to leading order in  $\frac{1}{L}$  with the expectation value of the bosonic operator  $\langle \cos\left(\frac{g}{\sqrt{L}}(a^\dagger + a)\right) \rangle$  as a multiplicative factor to the kinetic energy of the electrons. The electrons are thus effectively localized by coupling to the vacuum fluctuations of the electromagnetic field.

### E. Quantum to Floquet crossover

In the following, we analyze the quantum to classical crossover and recover known Floquet physics in the regime of  $N_{\text{phot}} \rightarrow \infty$  and  $g \rightarrow 0$ , keeping  $g^2 \Delta N_{\text{phot}}^{\text{pump}} = \text{const.}$ <sup>83</sup>

We employ a protocol where the cavity mode is coherently displaced with respect to the GS with displacement parameter  $\alpha$

$$|\alpha\rangle = e^{\alpha(a^\dagger - a)} |\phi_{\text{GS}}\rangle_b. \quad (21)$$

The photon number is thereby increased relative to the one in the GS by  $|\alpha|^2 = \Delta N_{\text{phot}}^{\text{pump}}$ . The coherent displacement considered here models the application of a laser pumping the cavity on time scales too short for the coupled system to follow. Thus, the laser is assumed to place the cavity into a squeezed coherent state in the limit of large system size. The subsequent time evolution of the light-matter coupled system is considered from starting time  $t = 0$ . While for the equilibrium spectral function only the first two orders in  $g$  of the Hamiltonian had to be taken into account, the time evolution is now affected by all orders of the Peierls coupling due to the occupation of the photonic mode that is macroscopic in the classical limit.

We calculate the nonequilibrium spectral function, defined via the full double-time retarded Green's function,<sup>95</sup>

$$A_{\text{non-eq.}}(k, \omega) = \frac{1}{\pi} \text{Im} \frac{1}{\tilde{\tau}} \int_{\Delta T - \frac{\tilde{\tau}}{2}}^{\Delta T + \frac{\tilde{\tau}}{2}} \left[ \int_0^\infty i e^{i\omega_0(t-t')} {}_f\langle \psi_{\text{GS}} | \otimes \langle \alpha | \left[ c_k(t), c_k^\dagger(t') \right]_+ | \alpha \rangle \otimes | \psi_{\text{GS}} \rangle_f d(t-t') \right] d\left(\frac{t+t'}{2}\right) \quad (22)$$

where  $\tilde{\tau} = \frac{2\pi}{\omega}$  is the period corresponding to the dressed cavity frequency. The form is chosen in analogy to the diagonal elements of the Floquet representation of the GF.<sup>96</sup> Here we include a waiting time  $\Delta T$  after the start of the real-time evolution, set to a large value with respect to the intrinsic timescale,  $\Delta T = 200\tilde{\tau}$ , in the numerical simulation. For comparison, we also consider the nonequilibrium spectral function of a classically driven system where the time evolution is governed by the Hamiltonian

$$H^c(t) = - \sum_j t_h e^{-iA(t)} c_{j+1}^\dagger c_j + h.c. \quad (23)$$

In this case, we couple the chain to the classical field  $A(t) = A_0 \sin(\omega_0 t)$ , that oscillates with the eigenfrequency of the unperturbed cavity  $\omega_0$ . Similar to the quantum case, we calculate the

nonequilibrium spectral function according to

$$A_{\text{Floquet}}(k, \omega) = \frac{1}{\pi} \text{Im} \frac{1}{\tau} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \left[ \int_0^{\infty} i e^{i\omega(t-t')} \langle \psi_{\text{GS}} | [c_k(t)_{H^c(t)}, c_k^\dagger(t')_{H^c(t)}]_+ | \psi_{\text{GS}} \rangle_f d(t-t') \right] d\left(\frac{t+t'}{2}\right) \quad (24)$$

where  $\tau = \frac{2\pi}{\omega_0}$ . Here  $(\cdot)(t)_{H^c}$  denotes the time dependence governed by the semi-classical Hamiltonian Eq. (23). The spectral function fulfills

$$A_{\text{Floquet}}(k, \omega + m\omega_0)|_{\omega \in (-\frac{\omega_0}{2}, \frac{\omega_0}{2})} = -\frac{1}{\pi} \text{Im} G_{mm}(\omega) \quad (25)$$

with  $G_{mm}(\omega)$  the diagonal part of the *Floquet representation* of the GF.<sup>96</sup>

We show the evolution from quantum to Floquet spectra for a representative quasi-momentum  $k = \frac{3\pi}{8}$  inside the FS in Fig. 3(b). In the extreme quantum case (GS) the replica band only appears below the main band. Furthermore, it is not located at the bare cavity frequency  $\omega_0$  but at the eigenfrequency of the coupled light-matter system  $\tilde{\omega}$ . By contrast, as the classical limit is approached, the symmetry of the replica bands is restored and their position moves to  $\omega_0$ . For the largest displacement ( $\Delta N_{\text{phot}}^{\text{pump}} = 30$ ) the spectrum matches precisely the Floquet spectrum.

## F. Optical conductivity

In order to discuss the impact of the light-matter coupling on a paradigmatic electronic two-particle response function, we compute the optical conductivity using the standard Kubo formalism. To this end the cavity-chain system is coupled to a spatially uniform external field  $A_{\text{ext}}(t)$ , in addition to the quantized cavity field. The resulting optical conductivity in the long-wavelength limit is obtained in the standard form<sup>97</sup>

$$\sigma(\omega) = -\frac{\langle e_{\text{kin}} \rangle - \Lambda(q=0, \omega)}{i(\omega + i0^+)}, \quad (26)$$

where

$$e_{\text{kin}} = \frac{1}{L} \cos\left(\frac{g}{\sqrt{L}}(a^\dagger + a)\right) \mathcal{T} \quad (27)$$

is the effective kinetic energy density of the electrons in the cavity-modified GS, and  $\Lambda$  is the current-current correlator

$$\Lambda(q=0, \omega) = -\frac{i}{L} \int_0^{\infty} dt e^{i\omega t} \langle [j_{q=0}^p(t), j_{q=0}^p] \rangle, \quad (28)$$

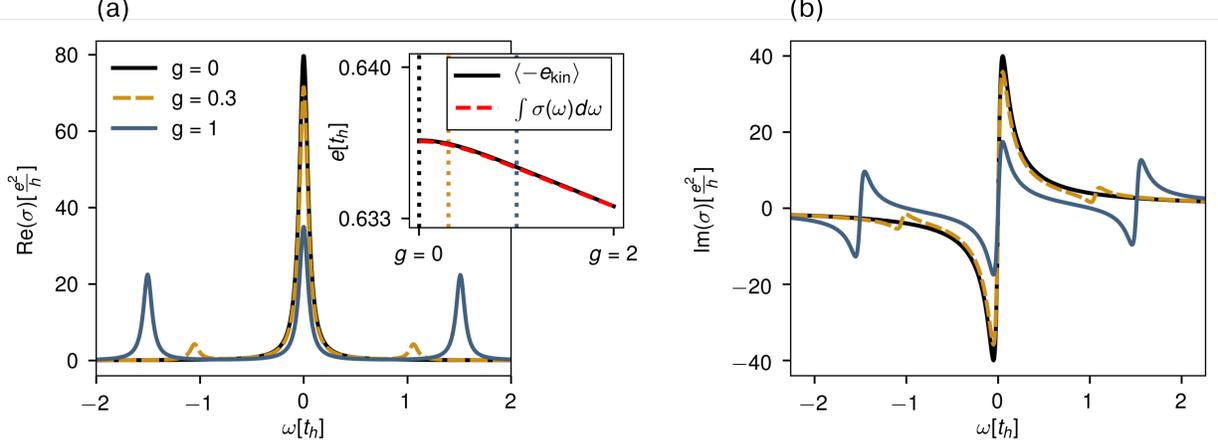


FIG. 4. **Optical conductivity** (a) Real part of the conductivity, Eq. (30), for strong ( $g = 1$ , dark blue line) and intermediate ( $g = 0.3$ , dashed yellow line) couplings. The result for  $g = 0$  is shown for comparison (black line). The Drude peak is suppressed with increasing  $g$ , and two side peaks appear at the same time. The inset shows the negative effective kinetic energy and the integrated conductivity. The dashed lines indicate the coupling strengths from the main plot. They match fulfilling the f-sum rule Eq. (35), here we set  $L = 170$  and a Lorentzian broadening  $\eta = 0.05$ . (b) Corresponding imaginary parts of the conductivity (Eq. (36)). Again the central  $\frac{1}{\omega}$  feature is suppressed and two side features appear at  $\omega = \pm\tilde{\omega}$ .

with  $j_{q=0}^p$  the paramagnetic current density operator at  $q = 0$ . The latter is obtained from the charge continuity equation as

$$j_{q=0}^p = -\cos\left(\frac{g}{\sqrt{L}}(a^\dagger + a)\right) \sum_k 2t_h \sin(k) c_k^\dagger c_k - \sin\left(\frac{g}{\sqrt{L}}(a^\dagger + a)\right) \sum_k 2t_h \cos(k) c_k^\dagger c_k. \quad (29)$$

We evaluate Eq. (26) numerically for  $L = 170$  and finite broadening  $0^+ \rightarrow 0.05$ . The result is shown in Fig. 4(a)-(b).

One can gain additional insight into the properties of the optical conductivity by evaluating it analytically in the TD limit. For the real part of the conductivity we find

$$\text{Re } \sigma(\omega) = D\delta(\omega) + \sigma_{\text{reg}}(\omega), \quad (30)$$

where the Drude weight  $D$  is given as

$$\frac{D}{\pi} = -\frac{T_{\text{GS}}}{L} \left( 1 - \frac{g^2 \omega_0}{2L \tilde{\omega}} + 2 \frac{g^2 \omega_0 T_{\text{GS}}}{\tilde{\omega}^2 L} \right). \quad (31)$$

The second term in the brackets in Eq. (31) derives from the squeezing of the band, previously coined quantum dynamical localization, Sec. IID, and vanishes in the TD limit. The last term originates from the current-current correlator and remains finite even in the TD limit, resulting in

a partial suppression of the Drude weight (note  $T_{\text{GS}} < 0$ ). Writing

$$\gamma = \frac{\omega_p^2}{\omega_0^2 + \omega_p^2}; \quad \omega_p^2 = -2 \frac{g^2 \omega_0}{L} T_{\text{GS}} \quad (32)$$

we find for  $D$  in the TD limit

$$D = D_0(1 - \gamma); \quad 0 \leq \gamma \leq 1 \quad (33)$$

where  $D_0$  is the Drude weight of the uncoupled chain. This is consistent with the findings of Ref. 81 for an electron gas. For the second contribution  $\sigma_{\text{reg}}$  in Eq. (30) one finds

$$\frac{\sigma_{\text{reg}}(\omega)}{\pi} = \frac{g^2 \omega_0}{\tilde{\omega}^2} \frac{T_{\text{GS}}^2}{L^2} (\delta(\omega + \tilde{\omega}) + \delta(\omega - \tilde{\omega})). \quad (34)$$

Two side-peaks at  $\omega = \pm \tilde{\omega}$  appear that balance the suppression of the Dude weight. These effects are illustrated in Fig. 4(a).

The inset of Fig. 4(a) shows that the real part of the conductivity satisfies the f-sum rule, similar to other electron-boson models<sup>98</sup>,

$$\frac{D}{\pi} + \int_{-\infty}^{\infty} \sigma_{\text{reg}}(\omega) d\omega = -\langle e_{\text{kin}} \rangle, \quad (35)$$

which is also evident from the corresponding analytical expression.

For completeness, we also state the imaginary part of the conductivity

$$\text{Im} \sigma(\omega) = -\frac{T_{\text{GS}}}{L} \frac{1}{\omega} \left( 1 - \frac{g^2 \omega_0}{2L \tilde{\omega}} \right) + \frac{g^2 \omega_0}{\tilde{\omega}} \frac{T_{\text{GS}}^2}{L^2} \frac{1}{\omega} \left( \frac{1}{\omega - \tilde{\omega}} - \frac{1}{\omega + \tilde{\omega}} \right). \quad (36)$$

which fulfills the usual Kramers-Kronig relation  $\text{Im} \sigma(\omega) = -\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{\text{Re} \sigma(\omega')}{\omega' - \omega} d\omega'$  and is shown in Fig. 4(b). Similar to the real part we find a suppression at  $\omega = 0$  and shakeoff features at  $\omega = \pm \tilde{\omega}$ .

### III. DISCUSSION

In this work, we have discussed a tight-binding chain coupled to a single spatially constant cavity mode. The exact solution of this model is enabled by the macroscopic number of constants of motion that results from the absence of momentum transfer between photons and electrons in the long-wavelength limit. Consequently, the GS of the system is a product state of electrons and photons (Sec. II).

Removing these constants of motion, either through relaxing the dipole approximation or including an electron-electron interaction, is expected to lead to interesting new results, as the GS of the coupled system is then expected to be an entangled state of electrons and photons. It is

well known that a one-dimensional system with local interactions is susceptible to form a charge density wave at zero temperature.<sup>99</sup> The effective interaction induced by the cavity considered in this work does not lead to such a symmetry-broken GS, since it is featureless. Including local interactions, it would therefore be interesting to study the effect of the cavity on charge-ordered phases. Relaxing the dipole approximation would lead to a finite-ranged but nonlocal effective electron-electron interaction, which opens new opportunities for inducing or modifying materials properties.<sup>100</sup> Including either Coulomb interactions or abandoning the dipole approximation (or both) also circumvents existing no-go theorems to superradiance.<sup>63–65</sup> It is therefore worthwhile to revisit the question whether an equilibrium photon condensate can exist in the GS.

Another assumption that enabled the exact solution of the model is the single-mode approximation. In the presence of multiple modes, cavity-matter system would lead to coupled oscillators, since the higher order terms arising from the Peierls substitution effectively couple the different modes. Furthermore, all orders of the Peierls phase would then give a relevant contribution to the energy. In our model, relaxing the single-mode approximation while keeping the long-wavelength approximation would still entail a macroscopic number of constants of motion, and again the GS would be a product state of an electronic state and a photonic state with no entanglement.

We have furthermore calculated the single-particle Green’s function analytically (Sec. IID). This analytical expression might provide the basis for future studies by building a many-body perturbation theory around this solution to investigate many-body instabilities diagrammatically, such as superconductivity. Using insights from the squeezing transformation, it might also be possible to treat systems with two modes analytically. One interesting prospect is to include an optically active phonon into the model that couples quadratically to the electrons.<sup>28,30,101</sup> Extending the here-presented analytical methods to a bimodal squeezing, it might be possible to analytically obtain GS properties and signatures in electronic spectra of the coupled bosonic modes. This could open up a pathway to realize multi-mode squeezed states, with important applications to quantum information.<sup>102</sup> In a similar spirit, one could also study two distinct photonic cavity modes and search for signatures of the matter-induced photon-photon interaction on the basis of the exactly solvable model put forward in the present work.

Concerning the connection to experiments, a temperature lower than the eigenfrequency of the cavity is needed in order for our zero-temperature calculations to hold qualitatively. For a resonance at  $\omega_0 = 0.41\text{THz}$  as used in Ref. 103 this would correspond to temperatures well below 3.1K. The electronic spectra calculated here (Fig. 3(a)) should in principle be observable in ARPES measurements. A quality factor that ensures a linewidth that is smaller than the cavity frequency

is required to observe the side bands, which appears within experimental reach.<sup>103</sup> The fact that deviations from the spectral function of uncoupled electrons are vanishing in the TD limit, is a consequence of the single-mode approximation. In an experimental setup one naturally has a continuum of modes (even for high-quality cavities), rendering the corrections finite. Qualitative effects, such as the asymmetry of the shake-off bands in the quantum limit, remain present also in the many-mode case. The experimental observation of such asymmetric shake-off bands would complement the successful demonstration of classical Floquet replica bands.<sup>10</sup>

Another prediction of the present work is the squeezing of the vacuum fluctuations in the GS consistent with predictions for other models.<sup>88,90</sup> Recently progress in probing the vacuum fluctuations of light<sup>91,92</sup> puts an experimental confirmation of our prediction within reach.

Finally, a suppression of the Drude peak (Fig. 4(a)) has already been observed experimentally.<sup>48</sup> It has previously been explained by Rokaj et al.<sup>81</sup> via an analogous result to the one presented by us but for an electron gas instead of a tight-binding chain. It is an interesting question why the single cavity mode can influence the macroscopically many electrons in this particular case. From our point of view, the reason lies in the induced electron-electron interaction that does not vanish in the TD limit and is probed indirectly through the optical conductivity.

## IV. METHODS

### A. Variational scheme

Here, we describe the variational scheme that we use to determine the exact GS. As discussed before, the Bloch states are fermionic eigenstates of the system. Thus the input to the procedure is a vector of length  $L$  specifying the occupations of each Bloch-state at quasi-momentum  $k$ . This determines the electronic part  $|\psi_T\rangle_f$  of the trial wavefunction  $|\Psi_T\rangle = |\phi_T\rangle_b \otimes |\psi_T\rangle_f$ , with which we calculate the eigenvalues of the operators  $\mathcal{T}$  and  $\mathcal{J}$

$$T_{\psi_T} = {}_f \langle \psi_T | \mathcal{T} | \psi_T \rangle_f ; \quad J_{\psi_T} = {}_f \langle \psi_T | \mathcal{J} | \psi_T \rangle_f . \quad (37)$$

Evaluating the electronic part of the expectation value for the GS energy one is left with the purely photonic Hamiltonian

$$H_{\psi_T} = \omega_0 \left( a^\dagger a + \frac{1}{2} \right) + \cos \left( \frac{g}{\sqrt{L}} (a^\dagger + a) \right) T_{\psi_T} + \sin \left( \frac{g}{\sqrt{L}} (a^\dagger + a) \right) J_{\psi_T} . \quad (38)$$

The problem reduces to that of an anharmonic oscillator, that can be solved by numerical diagonalization introducing a cutoff in the Fock space usually chosen as  $N_{\text{cut}} = 100$ . All results are

converged with respect to this cutoff. The scheme then varies over trial wavefunctions optimizing for the smallest GS energy of the remaining bosonic problem Eq. (38). It thus only compares eigenenergies of exact eigenstates making it possible to find the true GS.

We verified our results against an exact diagonalization of the full Hamiltonian for small system sizes.

## CODE AVAILABILITY

The code is openly available via github <https://github.com/ce335805/comeChainComeShine.git>.

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#### AUTHOR CONTRIBUTIONS

C.J.E. carried out the simulations with the variational code, G.P. and M.O. performed the ED simulations. Analytical calculations were done by C.J.E. All authors analyzed the data and discussed the results. C.J.E., G.P., M.A.S. and D.M.K. wrote the manuscript with input from all authors. The project was conceived by D.M.K. and M.A.S.

#### SUPPLEMENTARY INFORMATION

### VI. DIAGONALIZATION OF THE HAMILTONIAN IN THE TD LIMIT

In this part we show how to diagonalize the Hamiltonian expanded to second order in the field that gave the only non-vanishing contribution in the TD limit to the GS energy in Eq. (6). It reads

$$H^{2\text{nd}} = \omega_0 \left( a^\dagger a + \frac{1}{2} \right) + \mathcal{T} + \frac{g}{\sqrt{L}} \left( a^\dagger + a \right) \mathcal{J} - \frac{g^2}{2L} (a^\dagger + a)^2 \mathcal{T} \quad (\text{S1})$$

and can be diagonalized using a combined squeezing and displacement transformation<sup>30,86</sup>

$$H^D = e^{S^d[\mathcal{T}, \mathcal{J}]} e^{S^{\text{sq}}[\mathcal{T}]} H^{A, A^2} e^{-S^d[\mathcal{T}, \mathcal{J}]} e^{-S^{\text{sq}}[\mathcal{T}]}$$

$$S^d[\mathcal{T}, \mathcal{J}] = \frac{g}{\sqrt{L}\omega_0} \left( \frac{\mathcal{W}[\mathcal{T}]}{\omega_0} \right)^{-\frac{3}{2}} \left( a^\dagger - a \right) \mathcal{J}, \quad (\text{S2})$$

$$S^{\text{sq}}[\mathcal{T}] = \frac{1}{4} \ln \left( \frac{\mathcal{W}[\mathcal{T}]}{\omega_0} \right) \left( a^2 - (a^\dagger)^2 \right).$$

The diagonal Hamiltonian  $H^D$  is given in the main text Eq. (7) together with the definition of  $\mathcal{W}[\mathcal{T}]$ . Both displacement and squeezing transformations depend on fermionic operators namely

the kinetic energy  $\mathcal{T}$  and the current  $\mathcal{J}$ . Since  $\mathcal{T}$  and  $\mathcal{J}$  are diagonal in  $k$ -space the GS of the whole system is given as (see also Eq. (11) of the main text and below)

$$\begin{aligned} |\Phi_{\text{GS}}\rangle &= |\psi_{\text{GS}}\rangle_f \otimes |0_\beta\rangle \\ &= |\psi_{\text{GS}}\rangle_f \otimes e^{S^{\text{d}}[T_{\text{GS}}, J_{\text{GS}}]} e^{S^{\text{sq}}[T_{\text{GS}}]} |0\rangle. \end{aligned} \quad (\text{S3})$$

where  $|\psi_{\text{GS}}\rangle_f$  is the unshifted FS and  $|0_\beta\rangle$  is the vacuum state of the annihilators(creators)  $\beta^{(\dagger)}$  of the coherent squeezed states, defined in the main text Eq. (8).  $|0\rangle$  is the vacuum state of the non-squeezed bosonic operators  $a^\dagger$  and  $a$ . Since we found  $J_{\text{GS}} = 0$  due to the vanishing shift of the FS we have  $e^{S^{\text{d}}[T_{\text{GS}}, J_{\text{GS}}]} = I_b$  where  $I_b$  is the identity on the bosonic part of the Hilbertspace. The photon part of the GS wavefunction is thus given by Eq. (12) of the main part.

## VII. MOMENTUM-RESOLVED SPECTRAL FUNCTION IN THE TD LIMIT

In this part we show how to analytically calculate the spectral function  $A(k, \omega)$  of the electrons in the TD limit. Since we do this at temperature  $T = 0$  the expectation values appearing in the definition of the spectral function (Eq. (16) of the main text) are taken just with respect to the GS. None of the operators in the expectation value creates a macroscopic occupation of the photonic mode. Therefore, the scaling analysis of Eq. (6) of the main text can be applied in this case allowing us to diagonalize the problem by the combined squeezing and displacement transformation Eq. (S2). To evaluate the expectation values we also need the behaviour of the fermionic creation (annihilation) operators under these transformations which read

$$\begin{aligned} e^{S^{\text{d}}} e^{S^{\text{sq}}} c_k e^{-S^{\text{sq}}} e^{-S^{\text{d}}} &= c_k X Y, \\ e^{S^{\text{d}}} e^{S^{\text{sq}}} c_k^\dagger e^{-S^{\text{sq}}} e^{-S^{\text{d}}} &= c_k^\dagger X^\dagger Y^\dagger \end{aligned} \quad (\text{S6})$$

with

$$\begin{aligned} \ln(X) &= -\frac{g\omega_0}{\sqrt{L}\mathcal{W}^2} v_k (a^\dagger - a) + \mathcal{O}\left(\frac{1}{L^{\frac{3}{2}}}\right), \\ \ln(Y) &= \frac{1}{2} \frac{\frac{g^2}{\omega_0 L} \varepsilon_k}{1 - 2\frac{g^2}{\omega_0 L} \mathcal{T}} (a^2 - (a^\dagger)^2) + \mathcal{O}\left(\frac{1}{L^{\frac{3}{2}}}\right). \end{aligned} \quad (\text{S7})$$

Considering the first expectation value from the spectral function, Eq. (16) of the main text, we find

$$\begin{aligned}
\langle c_k(t)c_k^\dagger \rangle &= {}_f \langle \psi_{\text{GS}} | \otimes {}_b \langle \phi_{\text{GS}} | \overbrace{1}^{e^{iHt} c_k e^{-iHt} c_k^\dagger} \underbrace{1}_{e^{-S^{\text{sq}}[\mathcal{T}] e^{-S^{\text{d}}[\mathcal{T}, \mathcal{J}] e^{S^{\text{d}}[\mathcal{T}, \mathcal{J}] e^{S^{\text{sq}}[\mathcal{T}]}}} | \phi_{\text{GS}} \rangle_b \otimes | \psi_{\text{GS}} \rangle_f \\
&= {}_f \langle \psi_{\text{GS}} | \otimes \langle 0 | e^{iH^{\text{D}}t} c_k X Y e^{-iH^{\text{D}}t} c_k^\dagger X^\dagger Y^\dagger | 0 \rangle \otimes | \psi_{\text{GS}} \rangle_f + \mathcal{O}\left(\frac{1}{L^{\frac{3}{2}}}\right) \\
&= \langle \psi_{\text{GS}} |_f \otimes \langle 0 | c_k(t)_{H^{\text{D}}} X(t)_{H^{\text{D}}} Y(t)_{H^{\text{D}}} c_k^\dagger X^\dagger Y^\dagger | 0 \rangle \otimes | \psi_{\text{GS}} \rangle_f + \mathcal{O}\left(\frac{1}{L^{\frac{3}{2}}}\right).
\end{aligned} \tag{S8}$$

With the subscript  $(\cdot)(t)_{H^{\text{D}}}$  we signify that the time dependence is determined by the diagonal Hamiltonian  $H^{\text{D}}$ , Eq. (7) of the main text.

The operators  $\mathcal{T}$  and  $\mathcal{J}$  appearing in  $X$  and  $Y$  have no time dependence since they commute with  $H^{\text{D}}$  (and in fact also the full  $H$ ). The time dependence of the operators  $X$  and  $Y$  is determined by that of the bosonic operators

$$\begin{aligned}
a(t)_{H^{\text{D}}} &= a e^{-i\mathcal{W}t} \\
a^\dagger(t)_{H^{\text{D}}} &= a^\dagger e^{i\mathcal{W}t}.
\end{aligned} \tag{S9}$$

Evaluating the electronic part of the expectation value will yield  $\mathcal{W} \rightarrow \tilde{\omega}$  restoring a simple time dependence with the dressed cavity frequency  $\tilde{\omega}$ .

Reconsidering the expectation value Eq. (S8) we note that moving the fermionic operators through  $X$  and  $Y$  will only yield higher order corrections such that we can write

$$\langle c_k(t)c_k^\dagger \rangle = e^{\Phi(t)} (1 - n_k) \langle 0 | X_{\psi_{\text{GS}}}(t)_{H_b^{\text{D}}} Y_{\psi_{\text{GS}}}(t)_{H_b^{\text{D}}} X_{\psi_{\text{GS}}}^\dagger Y_{\psi_{\text{GS}}}^\dagger | 0 \rangle \tag{S10}$$

where  $n_k = \langle c_k^\dagger c_k \rangle$ . Here we have evaluated the time dependence of the fermionic annihilators that yields the time dependent phase factor  $e^{\Phi(t)}$ . We find, only keeping the leading order as before

$$c_k(t)_{H^{\text{D}}} = c_k e^{\mathcal{F}(t)} ; \quad \mathcal{F}(t) = -i\varepsilon_k t + i \frac{g^2 \varepsilon_k \omega_0}{L \mathcal{W}} \left( a^\dagger a + \frac{1}{2} \right) t - i \frac{g^2 \omega_0}{\mathcal{W}^2 L} v_k^2 t \tag{S11}$$

Evaluating the expectation of this yields

$$\langle e^{\mathcal{F}(t)} \rangle = e^{\Phi(t)} ; \quad \Phi(t) = -i\varepsilon_k t + i \frac{g^2 \varepsilon_k \omega_0}{2L \tilde{\omega}} t - i \Sigma_k t \tag{S12}$$

with

$$\Sigma_k = \frac{g^2 \omega_0}{\tilde{\omega}^2 L} v_k^2. \tag{S13}$$

In Eq. (S10) we have already executed the fermionic part of the expectation value performing

$$\begin{aligned}
\mathcal{T} &\rightarrow \langle \psi_{\text{GS}} |_f \mathcal{T} | \psi_{\text{GS}} \rangle_f = T_{\text{GS}} \\
\mathcal{J} &\rightarrow \langle \psi_{\text{GS}} |_f \mathcal{J} | \psi_{\text{GS}} \rangle_f = J_{\text{GS}}
\end{aligned} \tag{S14}$$

in the  $X^{(\dagger)}$  and  $Y^{(\dagger)}$  operator writing them as  $X_{\psi_{\text{GS}}}^{(\dagger)}$  and  $Y_{\psi_{\text{GS}}}^{(\dagger)}$ .

Since all operators act on the  $|0\rangle$  state, contributions come only from commutators of the operators in the exponentials. Therefore, all contributions from the  $Y$  operator are at least  $\exp\left(\mathcal{O}\left(\frac{1}{L^{\frac{3}{2}}}\right)\right)$ <sup>104,105</sup> and will thus be neglected. We are thus left with

$$\langle c_k(t)c_k^\dagger \rangle = e^{\Phi(t)}(1 - n_k) \langle 0| X_{\psi_{\text{GS}}}(t)_{H_b^D} X_{\psi_{\text{GS}}}^\dagger |0\rangle + \mathcal{O}\left(\frac{1}{L^{\frac{3}{2}}}\right). \quad (\text{S15})$$

The evaluation of the remaining expectation value is a standard textbook problem.<sup>106</sup>

Evaluating the other expectation value in the definition of the spectral function (Eq. (16) in the main part) yields the same result, just with a factor  $n_k$  instead of  $1 - n_k$  up front and the final expectation value in Eq. (S15) is complex conjugated as the order of the operators is reversed. This reflects the particle-hole symmetry of the half-filled system, which is inherited from the bare chain.

Performing the remaining FT we arrive at the final result reported in Eq. (18) in the main text.