Out-of-equilibrium actor-based system-dynamic modeling of the economics of climate change*

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Paper presented at

GSS Preparatory Workshop for the 3rd Open Global Systems Science Conference (2014)

29-30 October 2013, Beijing, China

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Abstract

The actor-based system-dynamic approach to macroeconomic modeling is illustrated for a simple model hierarchy consisting of a basic two-dimensional model with several alternative three-dimensional extensions. The hierarchy is based on an out-of-equilibrium approach: market clearing is not assumed, supply is not equal to demand, and there exists a stock of unsold goods. Depending on actor behaviour, the models exhibit stable exponential growth or instabilities leading to oscillations or economic collapse. In most cases, the simplicity and tractability of the models enables analytical solutions. The examples serve as illustration of more realistic models developed within the Multi Actor Dynamic Integrated Model System (MADIAMS) to assess the long-term impacts of climate mitigation policies.

1 Introduction

Integrated assessment models are the main tools for assessing the long-term options of climate mitigation policies. Until now, most of integrated assessment models of the coupled climate—socioeconomic system were deeply rooted in mainstream paradigms of economic theory (notably, the general equilibrium paradigm and neoclassical growth theory). However, the financial crisis, that undoubtedly became one of the major reasons for the stagnation of climate policy, the lack of agreement and even orientation among policy-makers with respect to long-term development goals, and the obvious failure of the general equilibrium paradigm in guiding economic policy, suggest the need for an alternative approach to integrated assessment.

The evolution of the socio-economic system is determined primarily by the strategies of key economic actors; very different evolution paths of model economies result from different hypotheses regarding actor behaviour. In addition to stable growth, unstable evolution paths are conceivable, and are indeed observed historically. Thus, economic models in general, and integrated assessment models in particular, should clearly state the hypothesized strategies of the actors. In our view, the most efficient way of computing the evolution of the coupled climate—socioeconomic system is to apply system dynamic modelling techniques, which already have a good track record in the analysis of global climate and environmental problems [1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 21, 23, 24, 25, 26, 27, 28, 29, 32].

In view of the large number of key economic actors (firms, households, — representing consumers and workers, — investors, governments etc.), the different production sectors that need to be taken into account (in particular with respect to green and carbon-based technology), and the different regions that must be included in assessing the impacts of possible international climate agreements, it is important to develop system-dynamic models in the form of a hierarchy, beginning with simple models that are successively made more complex as the simpler models are understood.

Our model hierarchy is based on a small number of key aggregated economic actors, pursuing different, often conflicting goals. In contrast to the multi-agent (or agent-based) modeling approach involving hundreds or even many thousand agents, our hypothesis is thus that realistic first order predictions are feasible without necessarily awaiting the emergence of unanticipated dynamics characterisic only of a large actor ensemble.

We have applied the actor-based system-dynamic approach previously in a number of publications [14, 15, 16, 17, 18, 22]. In the present paper we illustrate the basic concepts underlying these models of varying complexity in the form of a strongly simplified model hierarchy. We begin with the simplest possible economic growth model involving only two actors, a producer (a representative firm) and a consumer/worker (a representative household) (Sec. 2). These can produce already either stable exponential growth or instabilities, the latter in the form of oscillations or – depending on actor behaviour – economic collapse (Secs. 3-4). More complex models can then be built on these simple concepts by including further actors and economic processes, in particular, climate change, the financial sector, governments, etc. (Sec. 5). A summary of our approach in relation to present and future climate policy is given in the final section (Sec. 6).

2 The basic two-state-variable model

As simplest dynamic economic growth model economy we consider the following system consisting of two state variables governed by the actions of two aggregate actors (aggregate producer and aggregate consumer):

$$\dot{K} = I_K - \lambda_K K \quad (1)$$
$$\dot{G} = Y_G - C \quad (2)$$
$$Y = \nu K \quad (3)$$
$$Y_G = \rho_G Y \quad (4)$$
$$I_K = Y - Y_G \quad (5)$$
$$C = \rho_C Y. \quad (6)$$

Eq. (1) is a standard capital dynamics equation, where K is capital understood in a broad sense (including physical, human and social forms of capital), I_K is investment, and λ_K is a (constant) depreciation rate. Eq. (2) is an essential non-equilibrium feature of the model, since market clearing is not assumed, and a non-zero stock of unsold goods G can exist at any time. In the r.h.s. of Eq. (2), Y_G represents the production of consumer goods and C the consumption (both expressed in material units). Eq. (3) is a production function, linear in capital (as in the standard AK model of economic growth [3], in line with the broad treatment of the concept of capital, as mentioned above). Eq. (4) relates the production of consumer goods Y_G to total production Y through an important parameter of the model: the factor ρ_G, chosen by the producers. This determines then the decomposition, Eq. (5), of the total production Y between the production of consumer goods Y_G and investment goods I_K. Finally, Eq. (6) relates the consumption of consumer goods C to total production Y through another important parameter of the model: the factor ρ_C, which is essentially the choice of consumers.

In the basic model setup we assume (unrealistically, for illustrative and reference purposes only) that ρ_G and ρ_C are both constant (noting that, in general, $\rho_G \neq \rho_C$). The prognostic equations (1)–(2) can therefore be rewritten in the form

$$\dot{K} = \left[(1-\rho_G)\,\nu - \lambda_K\right] K, \tag{7}$$
$$\dot{G} = \nu\,(\rho_G - \rho_C)\,K. \tag{8}$$

In this model setup, Eq. (7) is a closed equation, with a growth rate of the economy given by

$$\gamma_K = (1-\rho_G)\,\nu - \lambda_K \tag{9}$$

($\gamma_K = \text{const}$), yielding as solution of the dynamic system

$$K = K_0 e^{\gamma_K t}, \tag{10}$$
$$G = G_0 + \frac{\nu\,(\rho_G - \rho_C)}{\gamma_K} K_0 \left(e^{\gamma_K t} - 1\right) \tag{11}$$

with initial conditions K_0, G_0. The economy either grows ($\gamma_K > 0$) or decays ($\gamma_K < 0$) exponentially. In the case of a growing economy ($\gamma_K > 0$), Eq. (11) implies that there is either an exponentially growing stock of unsold goods G, $\rho_G > \rho_C$, or, in the opposite case $\rho_G < \rho_C$ (over-consumption), after some finite time t_* the stock of unsold goods becomes zero: $G(t_*) = 0$, so that market clearing is reached. For $t > t_*$ the stock of unsold goods would become negative, implying that the model is no longer valid: either the aggregate producer has to increase ρ_G, or the aggregate consumer has to decrease ρ_C (Fig. 1).

We discuss various extensions of this simplest two-state-variable model to three state variables in the following sections (with a reference later in Sec. 5 to more complex models involving a larger number of state variables). This can yield a variety of economic evolution trajectories, depending on the strategies of the economic actors. In contrast to the standard efficient-market paradigm, these do not necessarily lead to a stable growth path. However, before considering various forms of instability, we discuss in the following section different versions of stabilizing actor behaviour.

3 Stable three-state-variable models

3.1 Supply-side control strategies

3.1.1 The "stocks" control strategy for ρ_G

We now consider ρ_G as a dynamic variable but still retain for the time the assumption $\rho_C = \text{const}$. In this model setup, the goal of the producer is to adjust production to the consumer's demand by adjusting ρ_G. The dynamic system takes the form

$$\begin{aligned} \dot{K} &= [(1-\rho_G)\,\nu - \lambda_K]\,K, & (12)\\ \dot{G} &= \nu\,(\rho_G - \rho_C)\,K, & (13)\\ \dot{\rho}_G &= \lambda_G \frac{\alpha_G K - G}{K} & (14) \end{aligned}$$

where Eq. (14) is essentially the "stocks" control for ρ_G from our previous work on MADIAMS development [17] (an alternative "flows" control strategy will be introduced in the next section).

We now reduce the 3D dynamic system (12)–(14) to a closed second-order nonlinear ODE for ρ_G.

First, Eq. (14) can be rewritten as

$$\dot{\rho}_G = \lambda_G \left(\alpha_G - \frac{G}{K}\right). \tag{15}$$

Differentiating Eq. (15) we obtain:

$$\ddot{\rho}_G = -\lambda_G \frac{d}{dt}\left(\frac{G}{K}\right), \tag{16}$$

At the same time,

$$\frac{d}{dt}\left(\frac{G}{K}\right) = \frac{\dot{G}}{K} - \frac{G}{K}\frac{\dot{K}}{K}. \tag{17}$$

The derivatives $\dot{G}$ and $\dot{K}/K$ in the r.h.s. of Eq. (17) can be expressed in terms of non-differentiated variables using Eq. (13) and (12), yielding:

$$\frac{d}{dt}\left(\frac{G}{K}\right) = \nu\,(\rho_G - \rho_C) - [(1-\rho_G)\nu - \lambda_K]\,\frac{G}{K}. \tag{18}$$

The ratio G/K in the r.h.s. of Eq. (18) can then be rewritten using Eq. (15) as

$$\frac{G}{K} = \alpha_G - \frac{\dot{\rho}_G}{\lambda_G}. \tag{19}$$

Substituting Eq. (19) into Eq. (18), and the resultant equation into Eq. (16), we obtain finally, after some rearrangement:

$$\ddot{\rho}_G + [\nu - \lambda_K - \nu\rho_G]\,\dot{\rho}_G + \lambda_G\nu\,(1+\alpha_G)\,\rho_G = \lambda_G\,(\nu\rho_C + (\nu - \lambda_K)\,\alpha_G)\,. \tag{20}$$

Eq. (20) has the unique stationary solution

$$\rho_G^* = \frac{\nu\rho_C + (\nu - \lambda_K)\,\alpha_G}{\nu\,(1+\alpha_G)}, \tag{21}$$

which is always less than unity. Indeed,

$$\rho_G^* < \frac{\nu\rho_C + \nu\alpha_G}{\nu(1+\alpha_G)} = \frac{\rho_C + \alpha_G}{1+\alpha_G} < 1 \tag{22}$$

since $\rho_C < 1$.

To consider now the deviation

$$r(t) = \rho_G(t) - \rho_G^* \tag{23}$$

from equilibrium (and noting that r is *not* necessarily small), we introduce two auxiliary constants:

$$\mu_0 = \frac{\nu(1-\rho_C) - \lambda_K}{1+\alpha_G} \tag{24}$$

$$\mu_1 = \lambda_G\nu\,(1+\alpha_G)\,. \tag{25}$$

The constant μ_1 is always positive, as is also μ_0 for normal regimes of the economy (no over-consumption). Eq. (20) can then be rewritten as

$$\ddot{r} + (\mu_0 - \nu r)\,\dot{r} + \mu_1 r = 0. \tag{26}$$

Eq. (26) represents a nonlinear damped oscillator. Introducing the "velocity" $v = \dot{r}$ it can be rewritten as the 2D nonlinear first-order system

$$\dot{r} = v, \tag{27}$$

$$\dot{v} = -\,(\mu_0 - \nu r)\,v - \mu_1 r, \tag{28}$$

and studied in the phase plane (r, v). The unique stationary point is $r = 0$, $v = 0$. The matrix of the linearized system has the form

$$\mathbf{A} = \begin{pmatrix} 0, & 1 \\ -\mu_1, & -\mu_0 \end{pmatrix}, \tag{29}$$

and the secular equation is

$$\det|\mathbf{A} - \lambda \mathbf{I}| = \lambda^2 + \mu_0 \lambda + \mu_1 = 0. \tag{30}$$

If, as mentioned above, $\mu_0 > 0$ and $\mu_1 > 0$, the real parts of both eigenvalues $\lambda_\pm$ are negative, and the equilibrium is stable (either we have a stable focus with damped oscillations of ρ_G converging to equilibrium (Fig. 2) or a stable node with exponential convergence of ρ_G to equilibrium (Fig. 3)).

After $\rho_G(t)$ is found by solving Eq. (20), it can be substituted into Eq. (12), yielding $K(t)$, and then $G(t)$, using Eq. (19).

3.1.2 The "flows" control strategy for ρ_G

We now supplement the basic 2D dynamic system (1)–(2) with the alternative "flows" control strategy for ρ_G [17],

$$\dot{\rho}_G = \lambda_G \frac{C - Y_G}{Y}, \tag{31}$$

while still keeping $\rho_C =$ const. In this case the goal of the producers is to balance the input and output flows of goods, rather than to maintain a constant ratio of the stock of goods to capital. This yields a 3D dynamic system of the form

$$\begin{aligned} \dot{K} &= [(1 - \rho_G)\,\nu - \lambda_K]\,K, && (32) \\ \dot{G} &= \nu\,(\rho_G - \rho_C)\,K, && (33) \\ \dot{\rho}_G &= \lambda_G\,(\rho_C - \rho_G)\,. && (34) \end{aligned}$$

Eq. (34) is a closed first-order linear ODE for ρ_G. The solution, for the initial value $\rho_G(t = 0) = \rho_G^0$, is given by

$$\rho_G(t) = \rho_C + \left(\rho_G^0 - \rho_C\right) e^{-\lambda_G t}, \tag{35}$$

and converges to ρ_C for large t. By substituting the solution (35) into Eqs. (32)–(33) one can obtains then $K(t)$ and $G(t)$.

Applying Eq. (35), the capital dynamics equation (32) can then be rearranged in the form

$$\frac{d}{dt}\ln K = \gamma_K^0 - \nu\left(\rho_G^0 - \rho_C\right)e^{-\lambda_G t} \tag{36}$$

where the asymptotic growth rate γ_K^0 is given by Eq. (9) with ρ_G replaced by ρ_C:

$$\gamma_K^0 = (1-\rho_C)\,\nu - \lambda_K. \tag{37}$$

Integration of Eq. (36) yields

$$\ln\frac{K}{K_0} = \gamma_K^0 t - \frac{\nu\left(\rho_G^0 - \rho_C\right)}{\lambda_G}\left(1 - e^{-\lambda_G t}\right). \tag{38}$$

The asymptotic growth rate is therefore equal to γ_K^0, but the amplitude of the growth (the factor before the exponent) differs in the long run from the basic 2D case (Sec. 2) for $\rho_G^0 \neq \rho_C$ (out-of-equilibrium initial conditions, cf. Fig. 4).

As example, consider the case $\rho_G^0 > \rho_C$ (initial overproduction); the asymptotic magnitude of the growth is then reduced, and it follows from Eq. (38) that

$$K(t) < K_0 e^{\gamma_K^0 t}. \tag{39}$$

Making use of the estimate (39), it can be shown that the stock of unsold goods $G(t)$ converges in this case to a constant value, provided that λ_G is sufficiently large. Indeed, it follows from Eqs. (33), (35), and (39) that in the case of initial overproduction

$$\dot{G} \equiv \nu\left(\rho_G^0 - \rho_C\right)e^{-\lambda_G t}K(t) < \nu\left(\rho_G^0 - \rho_C\right)K_0 e^{-(\lambda_G - \gamma_K^0)t}. \tag{40}$$

If $\lambda_G > \gamma_K^0$, by integrating the estimate (40) from 0 to ∞ we obtain an estimate from above on the asymptotic value of the increment of unsold goods:

$$\Delta G_\infty \equiv G(+\infty) - G(0) < \frac{\nu\left(\rho_G^0 - \rho_C\right)}{\lambda_G - \gamma_K^0}K_0. \tag{41}$$

3.2 Demand-side extensions

3.2.1 The consumption dynamics equation

In contrast to Sec. 3.1, we now keep again $\rho_G = \text{const}$, so consumption has to adjust to production. Instead of the diagnostic equation for C (Eq. (6)), we introduce a further time-adjusted prognostic equation for C:

$$\dot{C} = \lambda_C(qC^{\max} - C), \qquad q = \text{const}, \qquad 0 < q < 1, \tag{42}$$

where λ_C is a constant adjustment rate and $C^{\max}$ is the (time-dependent) maximum possible consumption corresponding to minimum possible level of investment $I^{\min}$ just balancing the depreciation of capital:

$$C^{\max} = \left(Y - I^{\min}\right) \frac{p_0}{p}. \tag{43}$$

In Eq. (43) $p_0 = \text{const}$ is the *production price;* initially, we keep the *consumption price* p constant as well. It follows from Eq. (1) that

$$I^{\min} = \lambda_K K. \tag{44}$$

The rationale behind Eqs. (42)–(44) is very close to the wage adjustment equation used in our previous work [15, 17, 22, 31]. It simulates the process of wage negotiation between entrepreneurs and the representatives of wage-earners. Wage-earners are assumed to strive to consume as much as $C^{\max}$ (Eq. (43)), which would lead the economy to its maximal stationary point, with no residual economic growth (but also not yet decay). However the negotiational power of entrepreneurs, parameterized in Eq. (42) by a constant factor q ($0 < q < 1$), reduces the maximal demands of wage-earners, enabling economic growth. The inevitable inertia of the wage negotiation process is modelled by an adjustment-rate parameter λ_C.

In our case, the full 3D dynamic system takes the form

$$\begin{aligned}
\dot{K} &= \left[(1 - \rho_G)\, \nu - \lambda_K\right] K, && (45)\\
\dot{G} &= \rho_G \nu K - C, && (46)\\
\dot{C} &= \lambda_C \left[q\,(\nu - \lambda_K)\, \frac{p_0}{p} K - C\right]. && (47)
\end{aligned}$$

For $\rho_G = \text{const}$, $p_0 = \text{const}$, $p = \text{const}$ yields a 3D linear system which can be readily solved analytically. We first integrate the closed Eq. (45), which yields exactly the same exponential growth of $K(t)$ as in Eq. (10). We then substitute $K(t)$ from Eq. (10) into Eq. (47), integrate the latter equation and obtain $C(t)$. Finally, we substitute $K(t)$ and $C(t)$ into Eq. (46), which is integrated to yield $G(t)$. The explicit form of the solution is:

$$K(t) = K_0 e^{\gamma_K t} \tag{48}$$

$$G(t) = G_0 + \frac{1}{\gamma_K}\left[\nu\rho_G - \frac{\lambda_C}{\lambda_C+\gamma_K} q\left(\nu-\lambda_K\right)\frac{p_0}{p}\right] K_0\left(e^{\gamma_K t}-1\right) - \\ - \frac{1}{\lambda_C}\left[C_0 - \frac{\lambda_C}{\lambda_C+\gamma_K} q\left(\nu-\lambda_K\right)\frac{p_0}{p}K_0\right]\left(1-e^{-\lambda_C t}\right), \quad (49)$$

$$C(t) = \frac{\lambda_C}{\lambda_C+\gamma_K} q\left(\nu-\lambda_K\right)\frac{p_0}{p}K_0 e^{\gamma_K t} + \\ + \left[C_0 - \frac{\lambda_C}{\lambda_C+\gamma_K} q\left(\nu-\lambda_K\right)\frac{p_0}{p}K_0\right] e^{-\lambda_C t} \quad (50)$$

(K_0, G_0, C_0 being the initial conditions).

For $t \gg 1/\lambda_C$, the second term in the r.h.s. of Eq. (50) is negligible, so that consumption grows exponentially for large times as a constant fraction of capital. A comparison with Eqs. (43), (44) shows that

$$C(t) \sim \frac{\lambda_C}{\lambda_C+\gamma_K} q C^{\max}(t) \quad \text{for} \quad t \to \infty, \quad (51)$$

The asymptotic solution contains a dynamic footprint caused by the wage adjustment process and manifested by a correction factor $\lambda_C / \left(\lambda_C + \gamma_K\right)$ which is less than but close to unity for typical values of model parameters.

The stock of unsold goods $G(t)$ given by Eq. (49) also grows exponentially in the long run at the same rate as the capital (provided that the difference in square brackets in the second term in the r.h.s. of Eq. (49) is positive, i.e. that the rate of consumption is less than the rate of goods production). However, there is also a constant shift dependent on the initial conditions (Fig. 5).

3.2.2 The "Walrasian" price adjustment law.

An alternative approach to modelling the consumption adjustment is to model the consumption price dynamics by introducing the (time-independent) production price p_0 and the (time-dependent) consumption price $p(t)$ directly in the (now time-dependent) factor ρ_C in Eq. (6):

$$\rho_C(t) = \frac{p_0}{p(t)}\rho_{C0} \quad (52)$$

where $\rho_{C0} = \text{const}$.

We use a slightly modified version of the text-book Walrasian price adjustment law

$$\dot{p} = \alpha \left(D(p) - S(p) \right), \tag{53}$$

where $D(p)$ is demand, $S(p)$ is supply, and $(D(p) - S(p))$ the excess demand, by normalizing the r.h.s.:

$$\dot{p} = \alpha \frac{D(p) - S(p)}{S(p)}. \tag{54}$$

In our notation,

$$\begin{aligned} D(p) &= C = \frac{p_0}{p(t)} \rho_{C0} Y, & (55) \\ S &= Y_G = \rho_G Y, & (56) \end{aligned}$$

so Eq. (54) takes the form

$$\dot{p} = \alpha \left(\frac{p_0}{p} \frac{\rho_{C0}}{\rho_G} - 1 \right). \tag{57}$$

Introducing the equilibrium consumption price

$$p_{\mathrm{eq}} = \frac{\rho_{C0}}{\rho_G} p_0, \tag{58}$$

the full 3D system then takes the form

$$\begin{aligned} \dot{K} &= \left[(1 - \rho_G) \nu - \lambda_K \right] K, & (59) \\ \dot{G} &= \nu \rho_G \left(1 - \frac{p_{\mathrm{eq}}}{p} \right) K, & (60) \\ \dot{p} &= \alpha \left(\frac{p_{\mathrm{eq}}}{p} - 1 \right). & (61) \end{aligned}$$

As before, Eq. (59) is a closed equation with an exponential solution (10). Note that Eq. (61) is then a closed first-order nonlinear ODE for $p(t)$ which can be solved analytically (by separating the variables). The solution has the (implicit) form

$$p(t) + p_{\mathrm{eq}} \ln \left| p(t) - p_{\mathrm{eq}} \right| = -\alpha t + p^0 + p_{\mathrm{eq}} \ln \left| p^0 - p_{\mathrm{eq}} \right| \tag{62}$$

where p^0 is the initial value of consumption price: $p^0 = p(t = 0)$. Substitution of $p(t)$ into Eq. (60), together with $K(t)$ from Eq. (10), then enables the calculation of $G(t)$.

Similar to the "flows" control strategy considered above in Sec. 3.1, the stock of unsold goods $G(t)$ converges to a finite value for sufficiently large α in Eq. (61). Indeed, it follows from Eq. (62) that the term in brackets in the r.h.s. of Eq. (60) decays in the long run as $\sim \exp\left(-(\alpha/p_{\mathrm{eq}})t\right)$, while $K(t)$ grows as $\sim \exp(\gamma_K t)$. Thus for sufficiently large α the r.h.s. of Eq. (60) decays exponentially, and $G(t)$ tends to some finite value $G(\infty)$ as $t \to \infty$ (Fig. 6).

4 Unstable three-state-variable models

The various actor responses to deviations from the basic exponential equilibrium response ($\rho_G = \rho_C = 0.6$, Fig. 1) introduced as reference in the previous section represent different expressions of the standard view that market forces are invariably stabilizing. However, alternative, equally plausible actor behaviours which lead to instabilities are also conceivable, and are in fact observed historically. We consider in the following two important examples. Further instability cases, with more detailed analyses, are presented in [14, 15, 17, 18].

4.1 Recessions

In the stabilizing strategies considered so far, a decrease in consumption relative to the equilibrium curve of Fig. 1 initiated a response of the consumer or the producer, or both (via the price mechanism), that brought goods production and consumption back again to an equilibrium growth curve. For example, in the Walrasian model (Sec. 3.2.2), a sudden step-function decrease in consumption through some external factor induces a decrease in the goods price, restoring again demand.

In practice, however, the response of producers to a decrease in demand can also be to lower supply rather than to reduce prices. This is achieved by laying off workers and idling productive capital, leading to a further decrease in demand. The result is a positive feedback loop producing a vicious cycle, culminating in a depression or (depending on further feedbacks) a business cycle.

To simulate this in our model, we need to allow for unemployment and idle production capital. This can be achieved by introducing an additional prognostic variable, the employment level ξ (with $0 < \xi \leq 1$), which is incorporated as a factor in the production equation (3) of our reference model of Sec. 2:

$$Y = \xi \nu K. \tag{63}$$

The remaining system equations of the reference model remain unchanged.

As additional prognostic equation for ξ we assume

$$\dot{\xi} = \lambda_C (C - C_{ref}) \tag{64}$$

where λ_C is a feedback constant characterizing the producer's response to changes in demand and C is again defined by Eq. (6), Y now being given by Eq. (63), with

$$C_{ref} = \rho_C \nu K \tag{65}$$

representing the consumption in the case of full employment (Eqs. (3), (6)). Expressing also C in terms of K using Eq. (63), Eq. (65) reduces to

$$\dot{\xi} = -\lambda_\xi K (1 - \xi) \tag{66}$$

with a net feedback constant

$$\lambda_\xi = \rho_C \nu \lambda_C. \tag{67}$$

The full 3D system then becomes

$$\dot{K} = \left[(1 - \rho_G) \nu \xi - \lambda_K \right] K, \tag{68}$$

$$\dot{G} = \nu (\rho_G - \rho_C) \xi K, \tag{69}$$

$$\dot{\xi} = -\lambda_\xi (1 - \xi) K. \tag{70}$$

For an initial state representing full employment and a consumption level $C = C_{ref}$, corresponding to the initial value $\xi_0 = 1$, we recover the growth paths of Sec. 2 with $\xi(t) = 1$. However, a small initial deviation from full employment and the consumption level $C = C_{ref}$, corresponding to $\xi_0 < 1$, results in an initially quadratic and subsequently monotonically increasing level of unemployment. Initially K and even $Y = \nu \xi K$ may increase, but ultimately they begin to decrease, and a recession evolves (see Fig. 7, where an instability is introduced at model year 20 by abruptly reducing ξ from 1.0 to $\xi_0 = 0.95$).

Eqs. (68) and (70) together form a closed first-order 2D system for the state variables K and ξ. The variables can be separated by dividing Eq. (68) by Eq. (70):

$$\frac{dK}{d\xi} = \frac{(1 - \rho_G) \nu \xi - \lambda_K}{\lambda_\xi (\xi - 1)}, \tag{71}$$

or, equivalently,

$$\frac{dK}{d\xi} = \frac{(1-\rho_G)\,\nu}{\lambda_\xi} + \frac{\gamma_K}{\lambda_\xi}\frac{1}{\xi - 1}, \tag{72}$$

where the notation (9) is used.

By integrating Eq. (72) over ξ, K can be expressed in terms of ξ:

$$K = K_0 + \frac{(1-\rho_G)\,\nu}{\lambda_\xi}(\xi - \xi_0) + \frac{\gamma_K}{\lambda_\xi}\ln\frac{1-\xi}{1-\xi_0}, \tag{73}$$

with initial values K_0 and ξ_0.

Eq. (73) can be rewritten in the form

$$K = K_0 + \frac{(1-\rho_G)\,\nu}{\lambda_\xi}\left[f(1-\xi) - f(1-\xi_0)\right] \tag{74}$$

in terms of the auxiliary function

$$f(z) = a\ln z - z + 1 \qquad (z \equiv 1 - \xi) \tag{75}$$

with a parameter

$$a = \frac{\gamma_0}{(1-\rho_G)\,\nu} \equiv 1 - \frac{\lambda_K}{(1-\rho_G)\,\nu} < 1. \tag{76}$$

An analysis of $f(z)$ in the interval $0 < z < 1$ $(1 > \xi > 0)$ shows that $f(z)$ starts with the value $-\infty$ at $z = 0$, then increases monotonically, changing sign from negative to positive, reaching a (positive) maximum value at $z = a$, and then monotonically decreasing to zero at $z = 0$ (Fig. 8).

For small initial perturbations of ξ $(\xi_0 \sim 1)$, and for realistic value of the model parameters, we have $1 - \xi_0 < a$. According to Eq. (70), ξ then monotonically decreases with time; this implies that for realistic values of model parameters, K initially increases, but then attains a maximum and begins to decrease. Ultimately the economy comes to a state for which $K > 0$ but $\xi = 0$ (and therefore $Y = 0$), unless other feedback mechanisms are taken into account.

In practice, the collapse will be arrested before $\xi = 0$ by further feedback processes not considered in our simple model. For example, wage reductions induced by decreases in the employment level can lead to business cycles ([17]) or, depending on parameter settings, a slow recovery after a period of stagnation ([14]).

4.2 Boom and bust events

The previous instability example concerned imbalances between the supply and demand of goods, formally independent of price signals. The consideration of prices and, more generally, the financial system, opens up a wide calatalogue of possible instabilities, as evidenced by the recent financial crisis and the many explanations of its origin offered in the literature. We consider here only one much discussed price-induced instability, namely boom-and-bust events in asset markets. This is readily amenable to the elementary model structure considered in our present overview of the lower-level realizations of the MADIAMS hierarchy. (A more detailed representation of the interaction between the production and financial sectors of the economy in relation to climate policy and the recent euro crisis is given in [14]).

The Walrasian stability of Section 3.2.2 was based on decreasing demand $D(=$ consumption $C)$ with increasing price p of a good (Eq. (55)). The boom-and-bust instabilities of asset markets result from the opposite response: an increase in the price of an asset generates an increase in demand of investors in anticipation of further price increases.

This can be readily implemented in our model by replacing Eq. (55) by the equation

$$D(p) = C = \frac{p(t) - p_0}{p_0} \rho_{C0} Y, \tag{77}$$

which yields in place of Eq. (57)

$$\dot{p} = \alpha \left\{ \frac{(p - p_0)}{p_0} \frac{\rho_{C0}}{\rho_G} - 1 \right\}. \tag{78}$$

or

$$\dot{p'} = \beta p', \tag{79}$$

where

$$p' = p - \bar{p}_{eq}, \tag{80}$$

$$\bar{p}_{eq} = p_0 \left(1 + \frac{\rho_g}{\rho_{C0}} \right), \tag{81}$$

$$\beta = \frac{\alpha \rho_{C0}}{\rho_G p_0}. \tag{82}$$

The deviation p' from the equilibrium price then grows exponentially:

$$p' = p'_0 e^{\beta t}, \tag{83}$$

where p'_0 is the initial value.

As in the recession example, the exponential growth of the deviation from the equilibrium is finally arrested through nonlinear feedbacks not included in the present simple model. An example of a nonlinear feedback triggered by a negative curvature in the price evolution curve is given in [17]. We note that in contrast to the recession example, in which the instability applies only to increasing unemployment – the opposite case of increasing employment is quickly terminated when full employment is reached – boom-and-bust events can have either sign, leading not only to booms followed by busts, but also to irrationally motivated busts followed by recovery.

5 Applications to economics of climate change

The actor-based system dynamic approach illustrated above by a simple model hierarchy, for some members of which it was even possible to obtain analytical solutions, was implemented in several more realistic models tailored to study the impacts of various global climate policies.

A Multi-Actor Dynamic Integrated Assessment Model MADIAM described in [31] was developed to study interactions between climate and the socioeconomic system by coupling a nonlinear impulse response model of the climate sub-system (NICCS) [19] to a multi-actor dynamic economic model (MADEM). The basic concept implemented in MADIAMS was that the principal driver of economic growth is the increase in human capital generated by endogeneous technological change. Impacts of government taxes on CO_2 emissions (assumed to be recycled into the economy in the form of various subsidies) were assessed, and it was found that substantial emission reduction can be achieved at an affordable cost of about 1 per cent of world GDP. This estimates are in broad agreement with other estimates available in publications on economics of climate change, including the Stern Review [30].

While designed along the lines of actor-based system dynamic approach, this earlier version of MADIAM however still applied the concept of market clearing for computing the relative prices and therefore retained some features of mainsteam general equilibrium models. This shortcoming was overcome in the later version of MADIAMS (a Multi-Actor Dynamic Integrated Assessment Model System) [17] consistently based on out-of-equilibrium approach, and therefore capable of modelling both stable and unstable macroeconomic dynamics. Many of the concepts explored in [17] by numeric simulations have been addressed in the present paper analytically using simplified models (notably the "stocks" and "flows" producer control strategies studied in Sec. 3.1).

A close connection between the stabilisation of the global financial system and effective climate mitigation policies was demonstrated for a family of actor-based system-dynamic models in [14]. In particular, examples of stabilisation policies were presented that can lead to green growth (stable economic growth supported by an accelerated decarbonisation of the economy).

6 Conclusions and outlook

The implementation of effective climate mitigation policies requires an adequate understanding of the interrelationship between climate change and the socio-economic system that one wishes to transform. This, in turn, depends on an effective communication between policy makers and the community of climate scientists and socio-economic experts striving to understand this interrelationship. Unfortunately, this communication has suffered in recent years through the global financial crisis and its aftermath, which was not foreseen by economists. Needed is a more realistic representation of the coupled climate-socio-economic-financial system that includes not only the traditional stabilizing forces of the market, but also the various forms of inherent instability of the system resulting from the behaviour of an ensemble of interacting economic actors pursuing divergent goals. Rather than maximizing an abstract global utility based on the general equilibrium paradigm of main-stream economic models, a new class of integrated assessment models needs to focus on the dynamics of the hypothesized actor strategies.

We have argued that the actor-based models required to capture the complex system-dynamic behaviour of the real coupled climate-socio-economic-financial system should to be developed in the form of a hierarchy, in which successive model complexity levels are introduced after clarification of the basic dynamics of previous hierarchy levels. The present paper has illustrated this approach by considering only the first two levels of the model hierarchy MADIAMS, progressing from two to three state variables. Our emphasis has been on clarifying the detailed mathematical basis of the models, rather on presenting simulation results.

For higher hierarchy levels, including details of the climate system and the multiple interactions between the production and financial sectors of the economy, a mathematical analysis will often no longer suffice and needs to be supplemented or replaced by computer simulations (see, for example, [5, 6, 14, 17, 22, 31], and the review article [11]). Computer simulations have the advantage of providing graphical stocks-and-flows representations of the interactions involved, which can be more readily understood by policy-makers and stakeholders not necessarily versed in the mathematics of differential equations.

Modern software tools greatly simplify the coding of system dynamic integrated assessment models, as well as the communication of the simulation results to non-experts. Thus, a wider application of actor-based system-dynamic integrated assessment models could provide an important contribution to the reinvigoration of the currently stalled attempts to transform today's carbon-based global economy into a sustainable low-fossil system.

Acknowledgements

The research leading to these results has received funding from the European Community's Seventh Framework Programme under Grant Agreement No. 308601 (COMPLEX). One of the authors (DVK) gratefully acknowledges also the financial support from the Russian Foundation for Basic Research (Projects No. 10-06-00238 and No. 12-06-00381).

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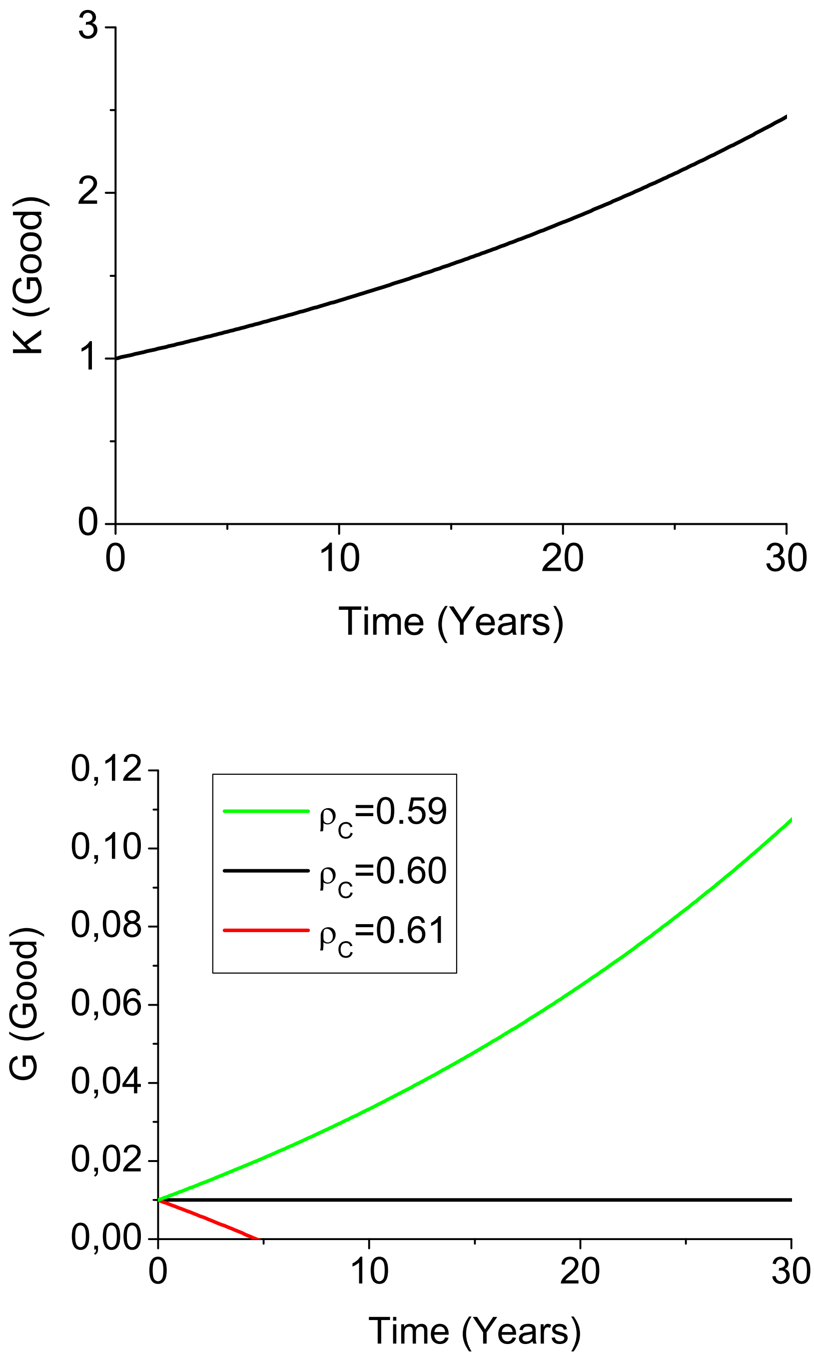

Figure 1: Basic 2D out-of-equilibrium model: upper panel — capital K; lower panel — stock of unsold goods G in case of overproduction ($\rho_G = 0.60$, $\rho_C = 0.59$), balanced growth ($\rho_G = 0.60$, $\rho_C = 0.60$) and overconsumption ($\rho_G = 0.60$, $\rho_C = 0.61$).

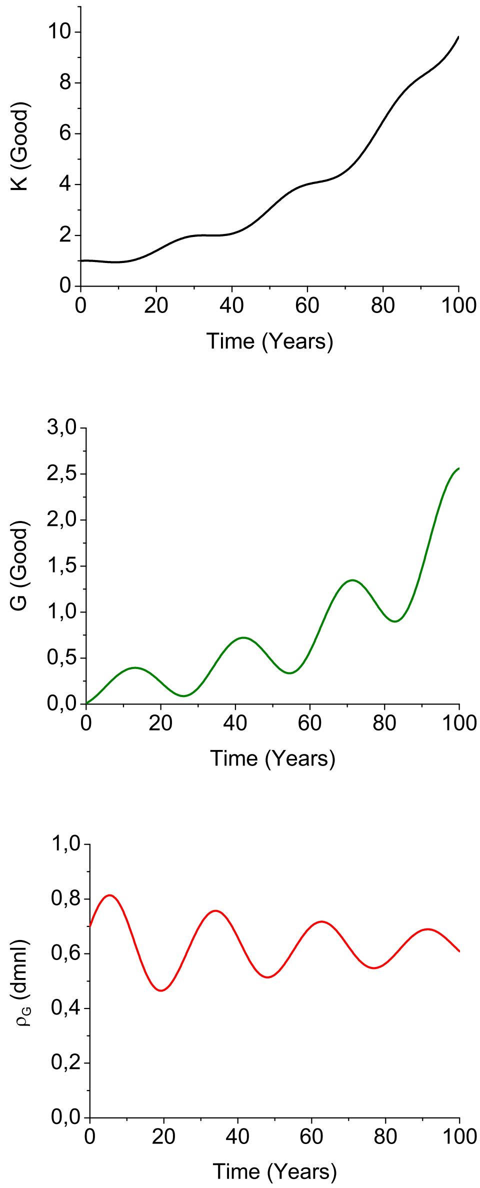

Figure 2: 3D model with the "stocks" control strategy ($\lambda_G = 0.2$): capital K, stock of unsold goods G and factor ρ_G. A pronounced oscillatory behaviour is manifested.

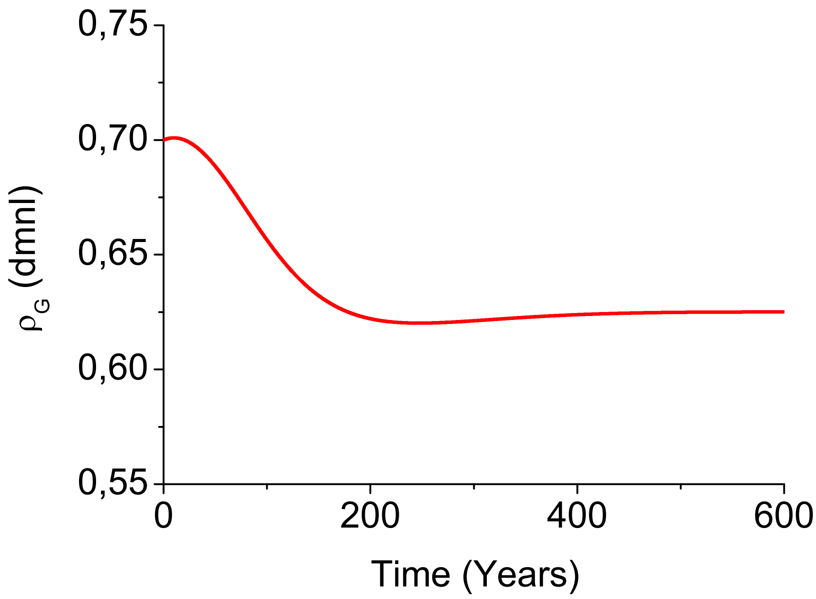

Figure 3: A non-oscillatory behaviour in case of the “stocks” control strategy for unrealistically small value of λ_G ($\lambda_G = 0.001$).

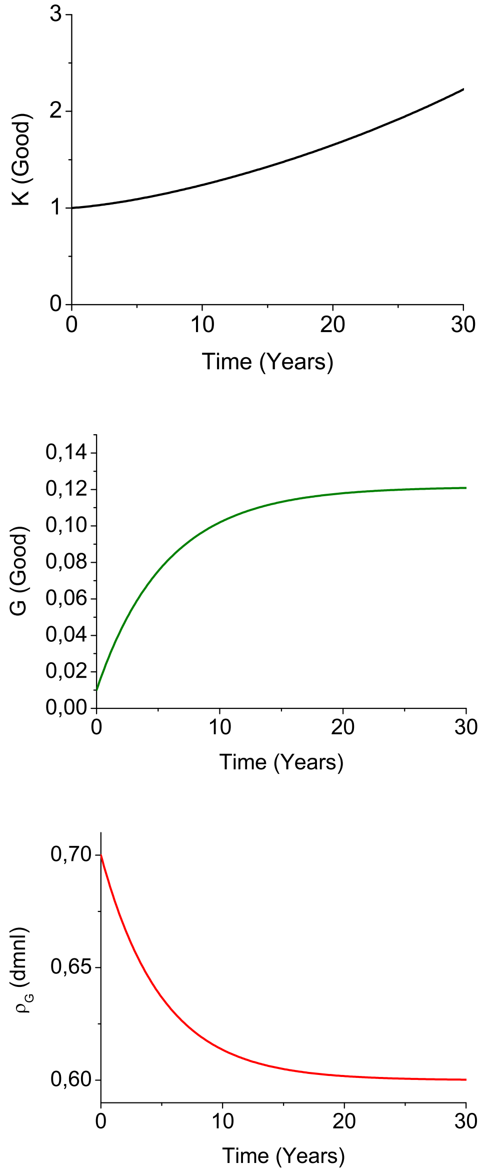

Figure 4: 3D model with the “flows” control strategy: capital K, stock of unsold goods G and factor ρ_G.

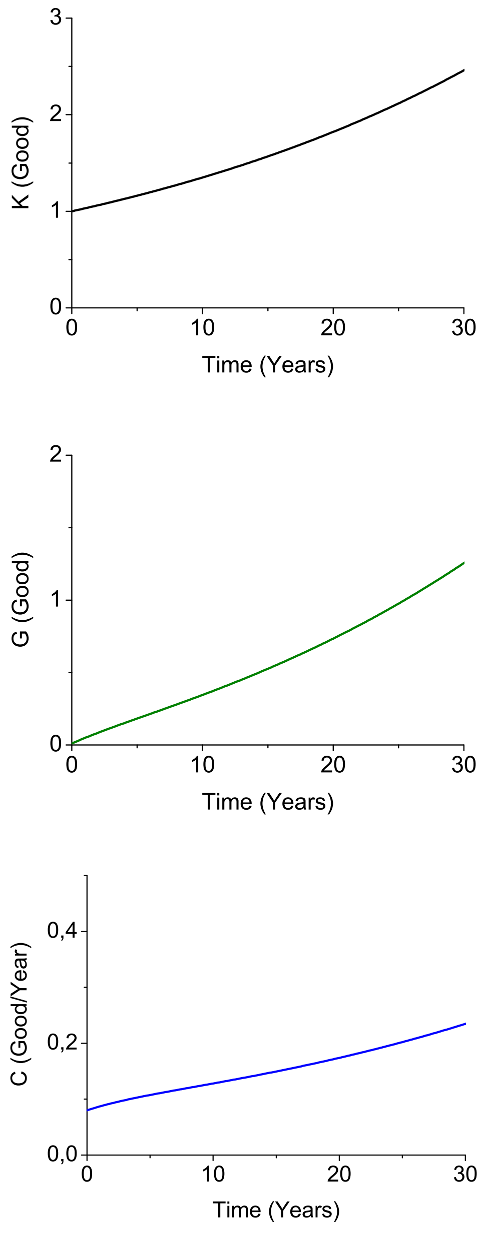

Figure 5: 3D model with the consumption dynamics equation: capital K, stock of unsold goods G and consumption C.

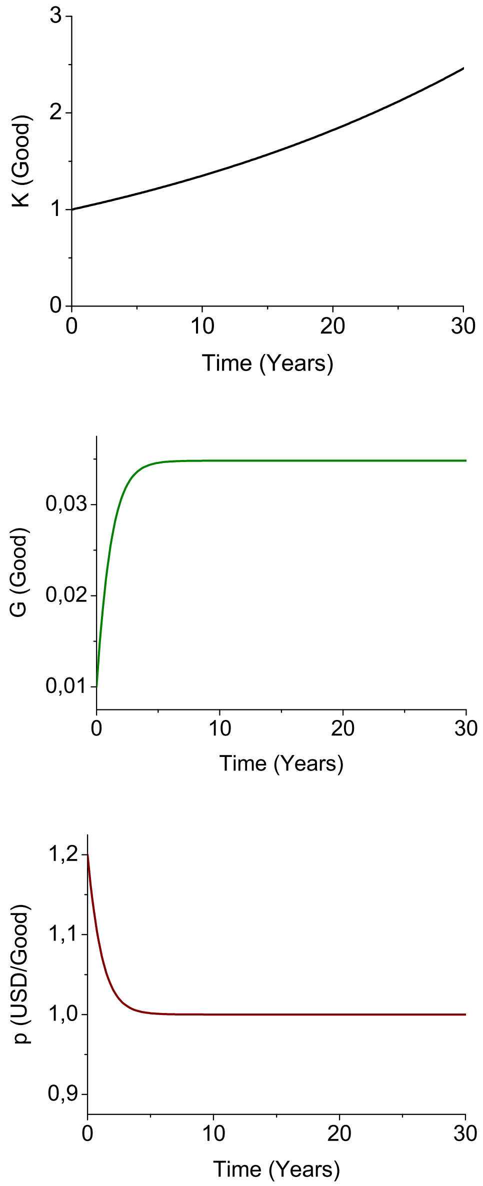

Figure 6: 3D model with the Walrasian price adjustment mechanism: capital K, stock of unsold goods G and price p.

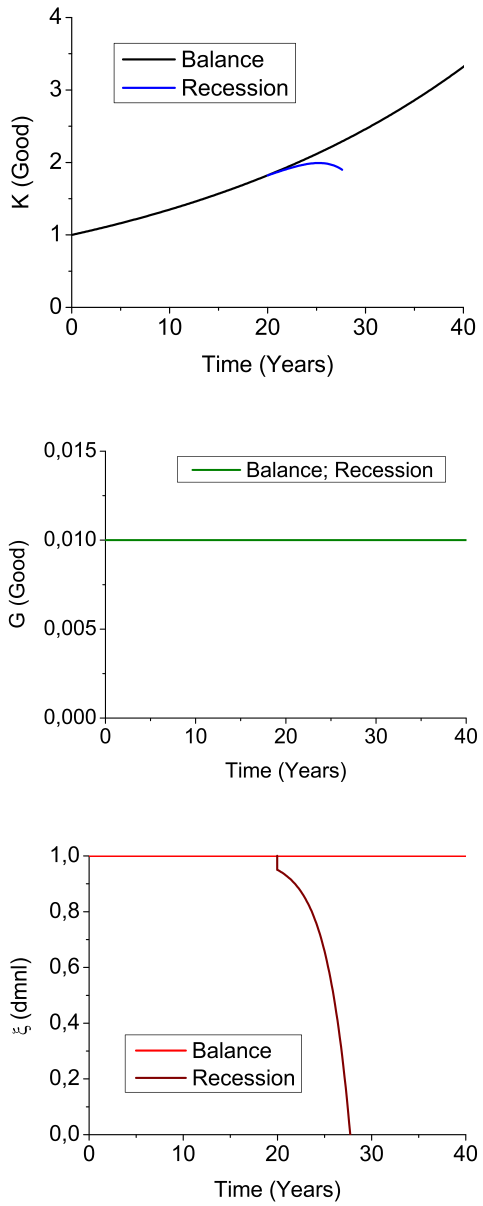

Figure 7: 3D model with an instability incurred at year 20 (recession): capital K, stock of unsold goods G and employment level ξ (superimposed also the balanced solution from Fig. 1).

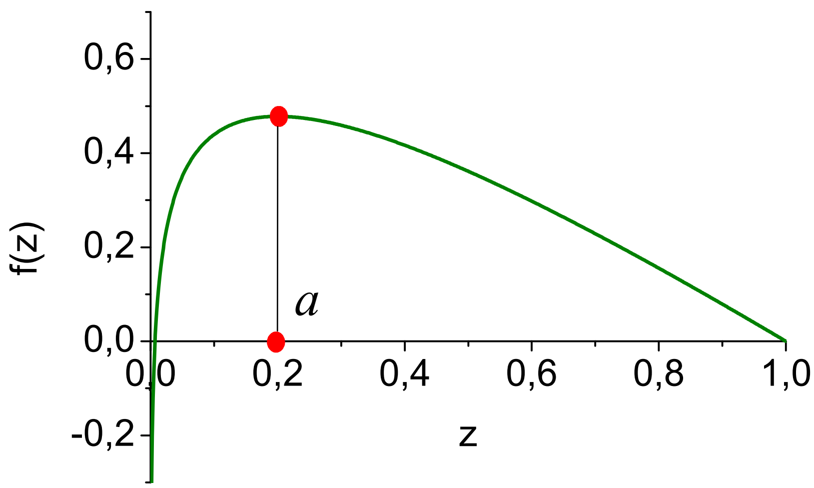

Figure 8: Auxiliary function $f(z)$ defined by Eq. (75) for $a = 0.2$.